Trading Probability and Surplus

The point of this notebook is to compute the average number of trades and the expected surplus in the reverse directed search model 2 tier case. The question is what happens when the difference in quality between the 2 tiers increases. This is part of a bigger question about what happens when the accuracy of the mapinator classification improves. Does this create enough friction (competition for the top tier candates) to make things worse off.

In [1]:
```
using SymPy, Plots
x,z,y, v_1, v_2, n, m_1, m_2 = symbols("x,z,y, v_1, v_2, n, m_1, m_2")
```

Out[1]:
```
(x, z, y, v_1, v_2, n, m_1, m_2)
```

There are two functions

$$Q_1(x) = (1 - \overline{\pi}\frac{(F(x^*) - F(x))}{m_1} - \frac{1 - F(x^*)}{m_1})^{(n-1)}$$

when $x < x^*$ and

$$Q_1(x) = (1 - (\frac{1 - F(x)}{m_1}))^{n-1}$$

otherwise.

In [2]:
```
# find the cutoff
rhs = v_1*((1-(1-y)/(m_1))^(n-1))
z=solve(v_2-rhs,y)[1]
```

Out[2]:

$$m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}} - m_1 + 1$$

In [3]:
```
pi =(v_2/v_1)^(1/(n-1))*m_1/m_2/(1+(v_2/v_1)^(1/(n-1))*m_1/m_2)
```

Out[3]:

$$\frac{m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2\left(\frac{m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2}+1\right)}$$

In [4]:
```
Q_1 = (1-(max(z-x,0)*pi)/m_1 - ((1-max(z,x))/m_1))^(n-1)
```

Out[4]:

$$\left(1 - \frac{\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}\max\left(0, m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}} - m_1 - x + 1\right)}{m_2\left(\frac{m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2}+1\right)} - \frac{1 - \max\left(x, m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}} - m_1 + 1\right)}{m_1}\right)^{n-1}$$

In [5]:
```
pbar = sympy.Piecewise((pi, Le(x,z)), (1, Gt(x,z)))
```

Out[5]:

$$\begin{cases} \frac{m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2\left(\frac{m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2}+1\right)} & \text{for } x \leq m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}} - m_1 + 1 \\ 1 & \text{otherwise} \end{cases}$$

In [6]:

```
Q_2 = (1-max(z-x,0)*(1-pi)/m_2)^(n-1)
```

Out[6]:

$$\left(1 - \frac{\left(-\frac{m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2\left(\frac{m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2}+1\right)}+1\right)\max\left(0, m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}-m_1-x+1\right)}{m_2}\right)^{n-1}$$

In [7]:

```
Prob1 = pbar*Q_1
Prob2 = (1-pbar)*Q_2
```

Out[7]:

$$\left(1 - \frac{\left(-\frac{m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2\left(\frac{m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2}+1\right)}+1\right)\max\left(0, m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}-m_1-x+1\right)}{m_2}\right)^{n-1} \cdot \left(1 - \begin{cases} \frac{m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2\left(\frac{m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2}+1\right)} & \text{for } x \leq m_1\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}} - m_1 + 1 \\ 1 & \text{otherwise} \end{cases}\right)$$

In [8]:

```
Prob=pbar*Q_1+(1-pbar)*Q_2
surplus=pbar*v_1*Q_1+(1-pbar)*v_2*Q_2
```

Out[8]:

$$
v_1 \left(1 - \frac{\left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}} \max\left(0, m_1 \left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}} - m_1 - x + 1\right)}{m_2 \left(\frac{m_1 \left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2} + 1\right)} - \frac{1 - \max\left(x, m_1 \left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}} - m_1 + 1\right)}{m_1} \right)^{n-1} \left(\begin{cases} \frac{m_1 \left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2 \left(\frac{m_1 \left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2} + 1\right)} & \text{for } x \leq m_1 \left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}} - m_1 + 1 \\ 1 & \text{otherwise} \end{cases} \right)
$$

$$
+ v_2 \left(1 - \frac{\left(- \frac{m_1 \left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2 \left(\frac{m_1 \left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2} + 1\right)} + 1 \right) \max\left(0, m_1 \left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}} - m_1 - x + 1\right)}{m_2} \right)^{n-1}
$$

$$
\cdot \left(1 - \begin{cases} \frac{m_1 \left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2 \left(\frac{m_1 \left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}}}{m_2} + 1\right)} & \text{for } x \leq m_1 \left(\frac{v_2}{v_1}\right)^{\frac{1}{n-1}} - m_1 + 1 \\ 1 & \text{otherwise} \end{cases} \right)
$$

In [9]:

```
# calculate the weighted average of values of the two tiers
function surplus_test(a)
    vbar = a[3]*(a[1]/(a[1]+a[2]))+a[4]*(a[2]/(a[1]+a[2]))
    return vbar
end
## calculate trading probabilities Q_1 and Q_2 with no ai then ai
function p_lot(x,b)
    vbar = surplus_test(b)
    #use average quality for all grads, then break the tiers down using
    plot(Prob1(m_1 => b[1],m_2=>b[2],v_1=>vbar,v_2=>vbar,n=>a[5]),0,1)
    plot!(Prob1(m_1 => b[1],m_2=>b[2],v_1=>b[3],v_2=>b[4],n=>b[5]),0,1)
    plot!(Prob2(m_1 => b[1],m_2=>b[2],v_1=>vbar,v_2=> vbar,n =>b[5]),0,1)
    plot!(Prob2(m_1 => b[1],m_2=>b[2],v_1=>b[3],v_2=>b[4],n=>b[5]),0,1)
end
function p_e(x,a)
    # for testing
    plot(Prob1(m_1 => a[1],m_2=>a[2],v_1=>a[3],v_2=>a[4],n=>a[5]),.9,1)
end
# calculate Q_1 with different parameters for comparison
function Q_lot(x,a,b)
    plot(Q_1(m_1 => a[1],m_2=>a[2],v_1=>a[3],v_2=>a[4],n=>a[5]),0,1)
    plot!(Q_1(m_1 => b[1],m_2=>b[2],v_1=>b[3],v_2=>b[4],n=>b[5]),0,1)
end
# calculate overall trading probability first without then with ai
```

```julia
# values in b are averaged to get the without part
function q_lot(x,b)
    vbar = surplus_test(b)
    plot(Prob(m_1 => b[1],m_2=>b[2],v_1=>vbar,v_2=>vbar,n=>b[5]),0,1)
    plot!(Prob(m_1 => b[1],m_2=>b[2],v_1=>b[3],v_2=>b[4],n=>b[5]),0,1)
end
# calculate overall surplus with parameters
function s_lot(x,b)
    vbar = surplus_test(b)
    plot(surplus(m_1=>b[1],m_2=>b[2],v_1=>vbar,v_2=>vbar,n=>b[5]),0,1)
    plot!(surplus(m_1 => b[1],m_2=>b[2],v_1=>b[3],v_2=>b[4],n=>b[5]),0,1)
end
function i_surplus_with(x,a)
    return surplus(m_1 => a[1],m_2=>a[2],v_1=>a[3],v_2=>a[4],n=>a[5])
end
function i_surplus_without(x,a)
    vbar = surplus_test(a)
    return surplus(m_1 => a[1],m_2=>a[2],v_1=>vbar,v_2=>vbar,n=>a[5])
end
function i_Prob_with(x,a)
    return Prob(m_1 => a[1],m_2=>a[2],v_1=>a[3],v_2=>a[4],n=>a[5])
end
function i_Prob_without(x,a)
    vbar = surplus_test(a)
    return Prob(m_1 => a[1],m_2=>a[2],v_1=>vbar,v_2=>vbar,n=>a[5])
end
```

Out[9]:
```
i_Prob_without (generic function with 1 method)
```

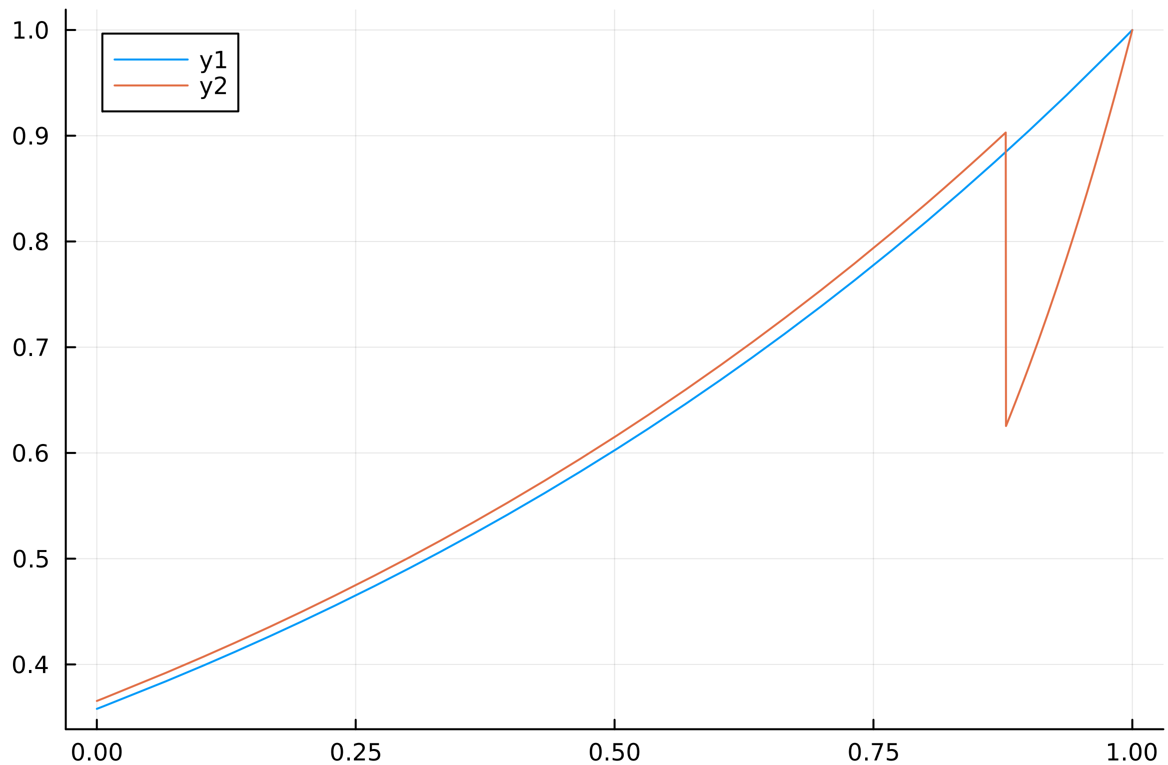
In [10]:
```julia
#m_1 => 7,m_2=>7,v_1=>.65,v_2=>.65,n=>14
a = [3,8,.6,.5,14];
b = [5,14,.8,.5,20];
surplus_test(b)
```

Out[10]:
```
0.5789473684210527
```

In [11]:
```julia
# probability Q_1 first a has a low v_1 b has high v_1
#Q_lot(x,a,b)
```

In [12]:
```julia
## overall trading probabilities without ai first then with ai
# in a every grad has expected quality in b tiers are distinguished
q_lot(x,b)
```

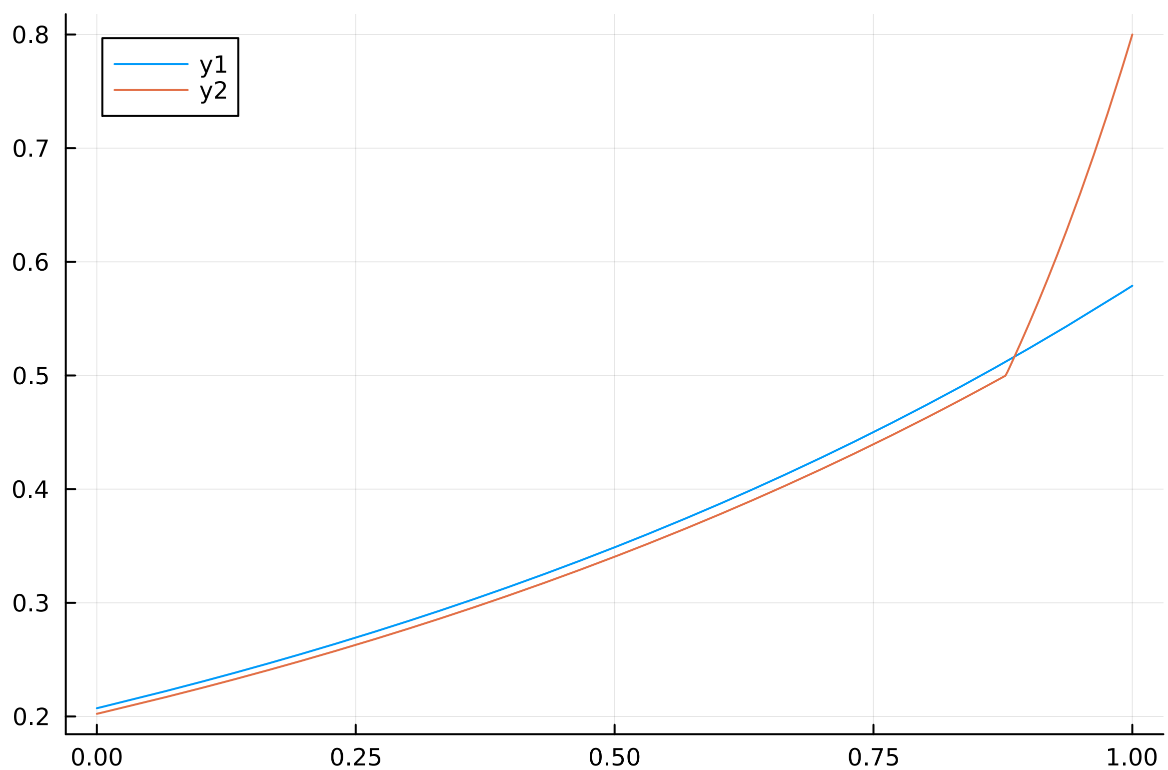
Out[12]:



In [13]:

```
# expected surplus without then with
s_lot(x,b)
```

Out[13]:



In [14]:

```
(integrate(i_surplus_without(x,b),0,1)-integrate(i_surplus_with(x,b),0,1))/integrate(i_s
```

Out[14]: -0.012043443052472

In [15]:

```
integrate(i_Prob_without(x,b),0,1)-integrate(i_Prob_with(x,b),0,1)
```

Out[15]: 0.006720508796781