

Optogenetic Maxwell Demon to Exploit Intrinsic Noise and Control Cell Differentiation Despite Time Delays and Extrinsic Variability

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Abstract

Synthetic biology seeks to create modular components that generate complex and controllable biological behaviors, often at microscopic scales. In single-cell applications, where important regulatory RNA or proteins are present in small numbers, intrinsic fluctuations cause biochemical ‘noise’ that frequently results in undesired effects, and this noise is usually treated as a nuisance that must be removed. However, recent theoretical analyses have shown that noise can sometimes be used to one’s advantage when seeking to manipulate nonlinear regulatory systems that would be difficult or impossible to control in a fully deterministic setting. Specifically, it was shown that a noise-exploiting optogenetic controller reminiscent of Maxwell’s Demon could systematically force multiple identical systems to reach different independently-assigned stable points, even for a few special cases where the model is inexact or if observations were incomplete. This work expands upon those analyses to explore in greater detail how control performance is affected by errors or approximations in the model or by temporal delays in the observations. We find that noise-exploiting controllers can remain highly effective despite coarse approximations to the model’s scale or incorrect estimations of key model parameters, and these controllers can even retain performance for significant time delays. Together, these findings suggest that noise-exploiting control should be possible in real experiments where models

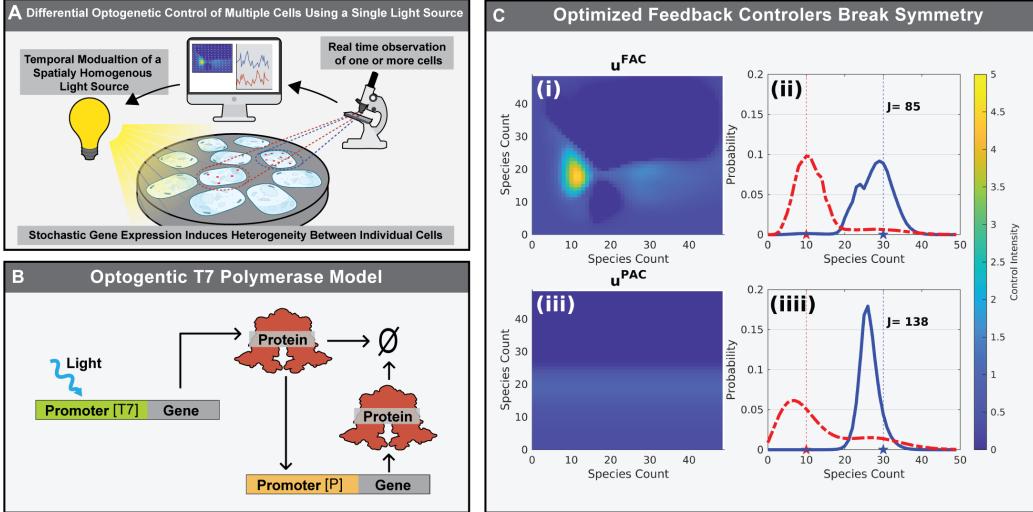


Figure 1: Optogenetic control of multiple cells using a single light source

(A) Schematic of the gene expression system. UV light activates T7 polymerase based transcription and protein expression from the first promoter-gene construct, and the protein actuates its own production via a second promoter-gene construct. (B) Schematic of the optogenetic control system. An observer is partially (PAC) or fully (FAC) aware of cell expression. The controller modulates a single spatially homogeneous light signal in attempt to drive each genetically identical cell to its own specified state. (C) The FAC (top) and PAC (bottom) control laws for two cells ($\mathbf{u}^{XAC}(x_1, x_2)$ shown on left) break the symmetry between Cell 1 and Cell 2 and result in distinct marginal distributions for the cells' responses (right). Figure adapted from [1].

are always approximate, where parameters are always uncertain, and where observations are corrupted by errors.

Keywords: Optogenetic control, Stochastic gene regulation, Synthetic biology

1 Introduction

Synthetic biology seeks to create modular [2] and orthogonal [3] components to sense and actuate [4] complex logical systems [5], and which are capable of

performing a variety of advanced biological behaviors [6]. New optogenetic tools have increased our ability to actuate embedded systems within cells reliably and with strong control performance [4, 7–9]. Great advances in these two fields have enabled computer programmable control of cellular protein production through the use of external optogenetic inputs and smart microscope techniques [10–12]. These types of digital-synthetic actuators allow for fine-tuned, computer-modulated control of cellular systems that were previously impossible [7, 13] and with fast response times in comparison to chemical diffusion.

Classical and modern control methods like PID control and model predictive control have been implemented in such systems [] which can control synthetic systems to different stable points. Theoretical works have shown that stochastic systems can be controlled to unstable points by oscillating around those points [14] but new control techniques which exploit the full probability distribution information of the system can be used in certain scenarios to control simple systems to unstable points by exploiting the natural noise of protein production to its advantage. Previous control effort focus on deterministic ODE formulations of the control problem, but only a few studies seek to analyze the full chemical master equation to exploit noise to *improve* the control of synthetic biology processes [1, 16].

Szymanska et al. [16] examined a theoretical system inspired by the genetic toggle switch from Kobayashi et al. [?], where two gene products repress one another, but where degradation of one protein was under the control of UV radiation via the XXX pathway. Their analysis showed that control was possible despite certain parameter errors and time delays due to maturation of fluorescent proteins and limited observation of the regulatory proteins. To explore this concept in the context of more realistic models and parameters, May et al. [1] fit a simplified stochastic model to reproduce data measured in Baumschlager, et al. [] for the expression from the XXX promoter under the optogenetic control of a UV-activated T7 polymerase (see model in Figure 1A, top promoter). An extended model showed that addition of a positive auto-regulation module (Figure 1A, bottom promoter) could help to maintain an elevated expression phenotype in the presence of UV excitation. Using discrete stochastic models based on direct solutions to the chemical master equation, it was shown that the combination of biochemical noise, nonlinear auto-regulation, and a *single* optogenetic feedback could allow two genetically identical cells with arbitrary initial conditions to be controlled simultaneously to reach different specified phenotypes (Figure 1B). For ex-

ample, Figure 1C(left) reproduces the heat maps for two control laws, where the level of a single external UV excitation signal was modulated depending on the observed cellular activity, and Figure 1C(right) shows the corresponding marginal stationary distributions that could be achieved under these control laws. Effective control was possible not only for a Fully Aware Feedback Controller (FAC, Figure 1C(top)) that observed the protein counts of all observed cells, but control could be achieved nearly as well for a Partially Aware Feedback Controller (PAC, Figure 1C(bottom)) that observed the protein counts of only one cell. Moreover, it was again seen that these control laws remained effective even in the circumstance where the model used to design the control law was an inexact simplification of the process dynamics to which the controller was later applied.

In this paper, the analysis of this Single-Input-Multiple-Output (SIMO) multicellular control problem is extended to examine how the FAC and PAC controllers are affected by additional model uncertainties, including coarse grained approximations to the system dynamics, uncertainties or errors in system parameters, and delays in the biochemical sensors or optogenetic controllers through which the external feedback is implemented. In the next section, we briefly describe our methods to formulate the chemical master equation to analyze multiple individual cells in the same spatially homogeneous environment; we define the FAC and PAC controllers and their effects the dynamics of cells within that environment; and we explain how the control law is optimized to achieve desired performance objectives. Next, in the ‘Results’ section, we explore how granularity of the model formulation, inaccuracy of parametric assumptions, or introduction of time delays would affect the FAC and PAC control performance. Finally, in ‘Conclusions’ we summarize our findings and discuss the implications that these controllers, and their robustness to modeling errors, may have on future systems in synthetic biology.

2 Methods

In previous work [1], a six-species gene regulation model was postulated for the light-activated association of two T7 split domains (species 1 and 2) to form active T7 polymerases (species 3) which could then in turn associate with inactive T7 promoters (species 4) to create an active gene construct (species 5) that would produce its eventual protein product (species 6). A

second, far simpler single-species model was also postulated by assuming quasi-steady equilibrium for the first five species. Both models were independently parametrized by fitting to the same experimental data from Baumschlager, et al. [?], and then extended to include a secondary self-activated promoter-gene construct for which the expression rate was defined by a Hill function (see Eqn. 1 below). In [1], we showed that a control law identified using the simple model would work to achieve differential control for the more complex model, thus motivating our current study’s efforts to explore further how control performance would change upon additional simplifying assumptions or errors in model specification. In the following subsections, we discuss our methods to formulate this model, show how it is analyzed numerically, and discuss the specification of the control objective and the determination of an optimal controller.

2.1 Model

To examine how model approximations would affect the application of noise-enhanced control strategies, we start with the one species model from [1] (see Fig. 1A), which consists of two reactions: protein production and degradation. The nonlinear light-activated protein production rate is given by

$$\nu_1(x, t) = \kappa \frac{x^\eta}{x^\eta + \beta^\eta} + k_0 + u(UV(t)), \quad (1)$$

where x is the instantaneous protein level; κ is the maximum strength of the auto-regulation promoter; β is the concentration at which auto-regulation promoter reaches half its strength; η is the cooperativity in the auto-regulation promoter; k_0 is the promoter leakage rate; and $u(UV(t))$ is the T7 promoter strength. Through external feedback, the level of light excitation, and therefore the T7 promoter strength, $u(UV(t))$, can be modulated in response to the state of the system, thus removing explicit dependence on time. For example, in a system with the observation of two cells, one could define $u(UV(t)) = u^{(\text{FAC})}(x_1, x_2)$ as shown in Fig. 1C(top left) for the fully aware control law, and for a system with the observation of only one cell, one could define $u^{(\text{PAC})}(UV(t)) = u(x_1)$ as shown in Fig. 1C(bottom left) for the partially aware control law. Protein degradation is assumed to be a first order process with rate γ :

$$\nu_2(x) = \gamma x. \quad (2)$$

Table 1: Model Parameters

Parameter	Value	units
κ	0.406	Molecules Per Minute
β	20.0	Molecules
η	8.00	unit less
k_0	0.0001	Molecules per Minute
γ	0.0203	per Minute
u	dynamic	Molecules per Minute

Parameters describing the auto-regulation promoter, κ , η , β , and k_0 , and the degradation rate γ are presented in Table 1 [1].

2.2 Stochastic analyses of the model

To describe the discrete behavior of the above model for a population system with N_c cells, we define the current state of the system as the tuple of the non-negative numbers of proteins in each cell: $\mathbf{X}_i = [x_1, x_2, \dots, x_{N_c}]_i \in \mathbb{Z}_{\geq 0}$, where the index i denotes the enumeration of the state within the countably infinite set of all possible states, $\mathbf{X}_i \in \mathcal{X} = \{\mathbf{X}_1, \mathbf{X}_2, \dots\}$. The stoichiometry vector for a reaction in any given cell is then defined as the change in state following that reaction event (e.g., $\mathbf{X}_i \rightarrow \mathbf{X}_i + \mathbf{s}_\mu$), where the $2N_c$ possible reactions are defined in pairs corresponding to production (odd indices) and degradation (even indices) as:

$$\begin{aligned}
 \mathbf{s}_1 &= \mathbf{e}_1, \quad \mathbf{s}_2 = -\mathbf{e}_1, \\
 \mathbf{s}_3 &= \mathbf{e}_2, \quad \mathbf{s}_4 = -\mathbf{e}_2, \\
 &\vdots \\
 \mathbf{s}_{2N_c-1} &= \mathbf{e}_{N_c}, \quad \mathbf{s}_{2N_c} = -\mathbf{e}_{N_c},
 \end{aligned} \tag{3}$$

where each $\mathbf{e}_i \in \mathbb{Z}^{N_c}$ is a Euclidean vector (i.e., unity for the i^{th} entry and otherwise zero). The corresponding propensity functions are:

$$\begin{aligned} w_1(\mathbf{X}) &= u(\mathbf{X}, t) + \kappa \frac{x_1^\eta}{x_1^\eta + \beta^\eta} + k_0, \quad w_2(\mathbf{X}) = \gamma x_1, \\ w_3(\mathbf{X}) &= u(\mathbf{X}, t) + \kappa \frac{x_2^\eta}{x_2^\eta + \beta^\eta} + k_0, \quad w_4(\mathbf{X}) = \gamma x_2, \\ &\vdots \\ w_{2N_c-1}(\mathbf{X}) &= u(\mathbf{X}, t) + \kappa \frac{x_N^\eta}{x_N^\eta + \beta^\eta} + k_0, \quad w_{2N_c}(\mathbf{X}) = \gamma x_N. \end{aligned} \quad (4)$$

With these stoichiometry and propensities, we can formulate and run the Gillespie stochastic simulation algorithm [17, 18] to generate representative trajectories of the stochastic process. However, quantifying performance and designing an optimal controller requires a more direct analysis of the chemical master equation (CME). For the same specifications of the stoichiometry and propensity functions, the CME is a linear ODE that describes how probability mass changes in time due to the specified reaction propensities and stoichiometries, which can be written:

$$\frac{d}{dt} P(\mathbf{X}_i) = \sum_{\mu=1}^{2N_c} (-w_\mu(\mathbf{X}_i)P(\mathbf{X}_i) + w_\mu(\mathbf{X}_i - \mathbf{s}_\mu)P(\mathbf{X}_i - \mathbf{s}_\mu)). \quad (5)$$

For convenience, the CME can also be formulated more compactly in matrix/vector form as:

$$\frac{d}{dt} \mathbf{P} = (\mathbf{A}_0 + \mathbf{B}\mathbf{u}^c)\mathbf{P}, \quad (6)$$

where $\mathbf{P} = [P(\mathbf{X}_1), P(\mathbf{X}_2), \dots]^T$ is the enumerated probability mass vector for all possible states of the system; \mathbf{A}_0 is the infinitesimal generator of the stochastic process due to the autoregulation promoter and degradation events; $\mathbf{u}^c = [u^c(\mathbf{X}_1), u^c(\mathbf{X}_2), \dots]^T$ is the collection of control inputs associated with each state; and $\mathbf{B}\mathbf{u}^c$ is the contribution that these control inputs make to the infinitesimal generator when included into the feedback process.

More specifically, the open-loop infinitesimal generator, \mathbf{A}_0 , is constructed according to:

$$[\mathbf{A}_0]_{ij} = \begin{cases} -\sum_{\mu=1}^{2N_c} w_\mu(\mathbf{X}_j), & \text{for } i = j, \\ w_\mu(\mathbf{X}_j), & \text{for } \mathbf{X}_i = \mathbf{X}_j + \mathbf{s}_\mu \\ 0, & \text{otherwise,} \end{cases} \quad (7)$$

and the feedback infinitesimal generator, $\mathbf{B}\mathbf{u}^{\mathcal{C}}$, of the controller is constructed according to

$$[\mathbf{B}\mathbf{u}^{\mathcal{C}}]_{ij} = \begin{cases} -N_c u^{\mathcal{C}}(\mathbf{X}_j), & \text{for } i = j \\ u^{\mathcal{C}}(\mathbf{X}_j), & \text{for } \mathbf{X}_i = \mathbf{X}_j + \mathbf{e}_{i_c} \text{ and } i_c = 1, \dots, N_c \\ 0, & \text{otherwise,} \end{cases} \quad (8)$$

where $u^{\mathcal{C}}(\mathbf{X}_j)$ is the specification of the \mathcal{C} controller in terms of the state, or its partial observations.

For a given controller, the equilibrium distribution of the system (\mathbf{P}^*) can be found by solving Eq. 6 and is given by:

$$\mathbf{P}^* = \text{null}(\mathbf{A} + \mathbf{B}\mathbf{u}^{\mathcal{C}}). \quad (9)$$

2.3 Quantification and optimization of control performance

For the single-input-multiple-output control of the multiple cell system, any input signal that is applied to one cell in the population will be felt by all cells in the population. As a result, it is not possible to independently control any individual cell without introducing changes to the behavior of the others. Therefore, an effective controller must be able to strike a balance among the desired behaviors of all cells in the system.

To quantify performance success, we define the performance error score, J , as the expected steady state squared Euclidean distance of the process from the specified target state, \mathbf{T} :

$$J = E\{(\mathbf{X} - \mathbf{T})^2\}. \quad (10)$$

This score is easily calculated by applying a linear operation on steady state probability distribution \mathbf{P}^* from Eq. 9) as follows:

$$\begin{aligned} J &= \lim_{t \rightarrow \infty} \mathbb{E}\{|\mathbf{X}(t) - \mathbf{T}|_2^2\}, \\ &= \sum_{i_1, i_2, \dots} P^*(x_1 = i_1, x_2 = i_2, \dots) [(i - \mathbf{T}_1)^2 + (j - \mathbf{T}_2)^2 + \dots], \\ &= \sum_{i, j, \dots} P_{ij\dots}^*(t) C_{ij\dots} = \mathbf{C}\mathbf{P}^*, \end{aligned} \quad (11)$$

where \mathbf{C} is simply a vector that contains the squared Euclidian distance of each state from the specified target \mathbf{T} , i.e., $C_i = |\mathbf{X}_i - \mathbf{T}|_2^2$. As a result of this calculation, J is a non-negative scalar that is zero if and only if \mathbf{P}^* is a delta distribution located exactly at the target vector \mathbf{T} .

In this work, we explore two different controller designs that allow for the controller to use two different types of information. A fully aware controller (\mathbf{u}^{FAC}) uses direct observation of the protein count of two cells simultaneously while making its control input decision, while the partially aware controller \mathbf{u}^{PAC}) uses only the direct observation of a single cell:

$$\mathbf{u}^{FAC} = u(x_1, x_2) \quad (12)$$

$$\mathbf{u}^{PAC} = u(x_1) \quad (13)$$

where x_1 and x_2 are discrete integers greater than or equal to zero that represent the number of proteins in cell one and cell two. In [1], the controllers \mathbf{u}^{FAC} and \mathbf{u}^{PAC} controllers were optimized to minimize J , and both \mathbf{u}^{FAC} and \mathbf{u}^{PAC} were saved as lookup tables after being optimized. In what follows, we will use these controllers exactly and without modification to explore how performance changes when the controllers are applied to systems that are approximations of the model under they were originally designed.

2.4 Scaling for system granularity

One of the major computational obstacles in using the CME or FSP formulations to analyze discrete stochastic chemical kinetics is that the dimension of the FSP scales linearly with each chemical species, and therefore exponentially with the number of independent species. To circumvent this issue, several previous studies have projected the FSP onto lower dimensional spaces based using various techniques such as time scale separations [?], Krylov subspaces [?], coarse meshes [?, ?], or principle orthogonal decompositions [?]. On the other hand, an experimental obstacle to using CME analyses is that gene expression quantification based on fluorescence intensities (e.g., using fluorescent protein reporters) usually does not provide molecular resolution, but rather measures only relative changes in expression over time.

To address how variations in model or observation scales might impact the effectiveness of noise-based control, we use the controller optimized at one resolution and ask how effective it would be when applied to systems with the same dynamics but with different resolution. To accomplish this

rescaling, let M denote some metric for the size of the original system (e.g., the average number of proteins at steady state and maximal UV) and let M' denote the same metric for the system at a different resolution. Toward this goal, we define a granularity parameter ($\alpha = M'/M$) that linearly scales the species populations to increase ($\alpha > 1$) or decrease ($\alpha < 1$) the maximum populations size, while maintaining the dynamics and general behavior of the model. To apply the granularity parameter, we assume that each propensity function, w_μ , from Eqns. 4 is rescaled to a different level of discreteness by substituting

$$w'(\mathbf{X}) = w(\mathbf{X}/\alpha). \quad (14)$$

For example, the production and degradation of protein in cell one would become:

$$w'_1(\mathbf{X}) = u(\mathbf{X}/\alpha, t) + \kappa \frac{(x_1/\alpha)^\eta}{(x_1/\alpha)^\eta + \beta^\eta} + k_0, \quad w'_2(\mathbf{X}) = \gamma x_1/\alpha. \quad (15)$$

After this rescaling, if $\alpha < 1$ the system dynamics will now occur over a smaller range of protein counts, and if $\alpha > 1$ the system dynamics occurs over a larger range of protein counts.

We note that in transforming the propensity functions, the inputs to the controller have also be scaled by $1/\alpha$ before computing the appropriate functions of that state, e.g., $u^C = u^C(\mathbf{X}/\alpha)$. Because identification of the original control formulation, $\mathbf{u}^{FAC}(x_1, x_2)$ and $\mathbf{u}^{PAC}(x_1)$ only considered integer values for (x_1, x_2) , fractional number inputs for after rescaling $(x_1/\alpha, x_2/\alpha)$ are handled by 2D cubic interpolation of the nearest values. Finally, to provide a consistency relative scoring, the definition of the performance score is also adjusted according to sale magnitudes, i.e.:

$$\begin{aligned} J' &= \sum_{i=0}^{M'} \sum_{j=0}^{M'} P^*(x_1 = i, x_2 = j)((i/\alpha - \mathbf{T}_1)^2 + (j/\alpha - \mathbf{T}_2)^2), \\ &= \mathbf{C}' \mathbf{P}^* \end{aligned} \quad (16)$$

2.5 Observation and actuation time delays

No real optogenetic control system can provide instantaneous feedback – there will always be delays in biochemical reactions needed for observations

(e.g., transcription and translation dynamics as well as folding and maturation of fluorescent proteins), delays in data analysis and decision making (e.g., image processing and calculation of the corresponding control signals), and delays in the actuation dynamics (e.g., activation of optogenetically excitable molecules). To explore the impacts of delays between the time a cellular fluctuation occurs and the time at which the corresponding control action can take effect, we developed a simple time-delay stochastic simulation algorithm. In this analysis, the state history is recorded following every stochastic event, allowing for reconstruction of the piecewise constant history of the species' populations, e.g., $x_1(s)$ and $x_2(s)$ for all $s \in [0, t]$. Using this information, the time-delayed control signal at time t can be specified as:

$$u_\tau(t) = \begin{cases} 0, & \text{for } t \leq \tau, \\ u^C(x_1(t - \tau), x_2(t - \tau)), & \text{for } t > \tau, \end{cases} \quad (17)$$

where τ is the time delay between observation and actuation, and u^C is the previously optimized control law (e.g., \mathbf{u}^{FAC} or \mathbf{u}^{PAC}). We note that the time delay stochastic process was only simulated using the SSA because to our knowledge an appropriate direct FSP/CME integration procedure has not yet been developed.

3 Results

To assess the potential robustness of noise-enhanced control of the proposed synthetic auto-repression toggle switch, we adopt the model and parameters from [1] as well as the fully aware and partially aware controllers, $u^{\text{FAC}}(x_1, x_2)$ and $u^{\text{FAC}}(x_1)$, respectively as described in the methods section. In the following subsections, we explore how well these controllers would perform in the more realistic setting where the approximate model used to define the controller no longer matches exactly to the dynamics of the process that the controller is being used to manipulate.

We first explore how the model defined for a given assumed system size (i.e., in which protein numbers have a range of M) when applied to a system with larger or smaller systems sizes (i.e., where protein numbers now have a range of $M' = \alpha M$). This modification could arise from two situations related to numerical convenience or inexact resolution in the measurement of concentrations. First, from the perspective of numerical convenience when optimizing a controller, it can be advantageous to reduce the assumed range

of protein levels in order to reduce the dimension of the CME and FSP analyses, thereby achieving a more tractable optimization problem. For example, the current model (see Methods) takes XXX seconds to solve the FSP analysis when $\alpha = 1$ but the same analysis takes XXX seconds to solve when $\alpha = 10$. Since numerical optimization of the controller can take many millions of calculation for different control laws, learning a controller using $\alpha = 1$ that works for $\alpha = 10$ would yield substantial savings in computational effort. The second situation under which sensitivity to system scaling could become important is when measurements are relative and do not provide single-molecule resolution. For example, in measurements like those on [?] which are based on fluorescence protein intensities, it is not clear how many proteins correspond to what level of fluorescence intensity, and the scalar α is an unknown quantity that must be estimated (e.g., through calibration with single-molecule measurements or dilution experiment [?]).

It is well established that the relative size of stochastic fluctuations (i.e., noise) in a chemical process decreases with the inverse square root of the process scale [?]. As the system size increases (with all species being scaled by the same factor), the dynamics approach those of a deterministic process with probability one (with zero-measure exceptions only for initial conditions lying on manifolds that separate attractors for different steady state behaviors) [?]. For the purpose of noise-induced control, this introduces a tradeoff where noise is critical to break symmetry and enable differential control, yet noise is also deleterious to maintaining desired states once they have been achieved. Considering these competing objectives, it is not immediately clear how the reduction of noise through the addition of system granularity will affect control performance.

To shed light on this tradeoff, control performance scores for the \mathbf{u}^{FAC} and \mathbf{u}^{PAC} controllers were calculated at different levels of granularity (α) between 0.1 and 2.0. Figure 2 (Ai - Fi) shows the joint (left plots) and marginal (right plots) distributions of the two cells relative to the specified target position (circles in joint distributions and vertical dashed lines in marginal distributions) for the FAC controller and for $\alpha=0.2$ (A,B), 1 (C,D) and 2 (E,F). As the granularity changes from $\alpha=0.2$, to $\alpha=2$, the distributions become focused at a point near the target, and the performance score improves from $J = 333$ to 85 to 27, respectively. The PAC controller performance also improves considerably with alpha as shown in Fig. 2 (G-L).

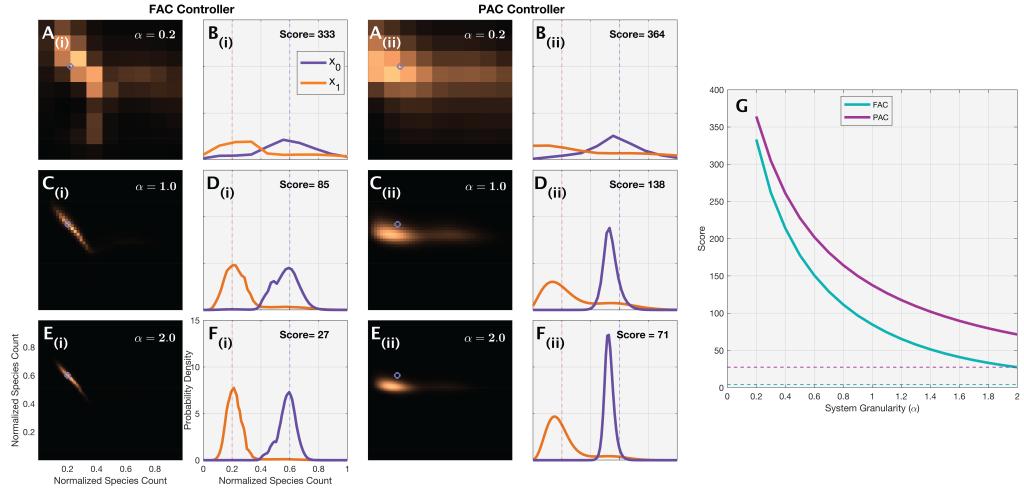


Figure 2: Effects of Scale Changes. (A-F) Joint (A,C,D,) and marginal (B,D,F) probability distributions for the FAC control method at $\alpha = 0.2$ (A,B), $\alpha = 1$ (C,D) and $\alpha = 2.0$ (E,F). Target point shown as blue circle. (G-L) Same as (A-F), but for PAC controller. (M) FAC (cyan) and PAC (magenta) controller performance as function of system volume. Horizontal dashed lines correspond to SSA analyses at $\alpha = 100$. In all analyses, the system model and PAC and FAC controllers are unchanged from [1] other than their adaptation to the new system volume size (see Methods).

3.1 Effects of changes to system granularity

Figure 2M shows the trend of the performance score versus alpha for both the FAC (solid cyan line) and the PAC (solid magenta line) controller. This improvement in performance appears to approach a small value as the granularity goes to infinity, but since the size of the FSP increases with the square of the system size, systems much larger than $\alpha=2$ (where $\mathbf{A}_0 \in \mathbb{R}^{10^4 \times 10^4}$) become more difficult to calculate. To bypass this limit in the FSP, sixteen SSA simulations were used to sample the CME of a system with a much larger volume of $\alpha=100$. Each SSA was ran for 5×10^7 minutes and only the last 4×10^7 minutes were sampled to estimate the stationary distribution and the calculate the performance score. The performance score estimates of this high granularity SSA using the FAC and PAC were 4.13 and 27.5 respectively, which are plotted as dashed lines in Fig. 2G. Although it is unclear if further

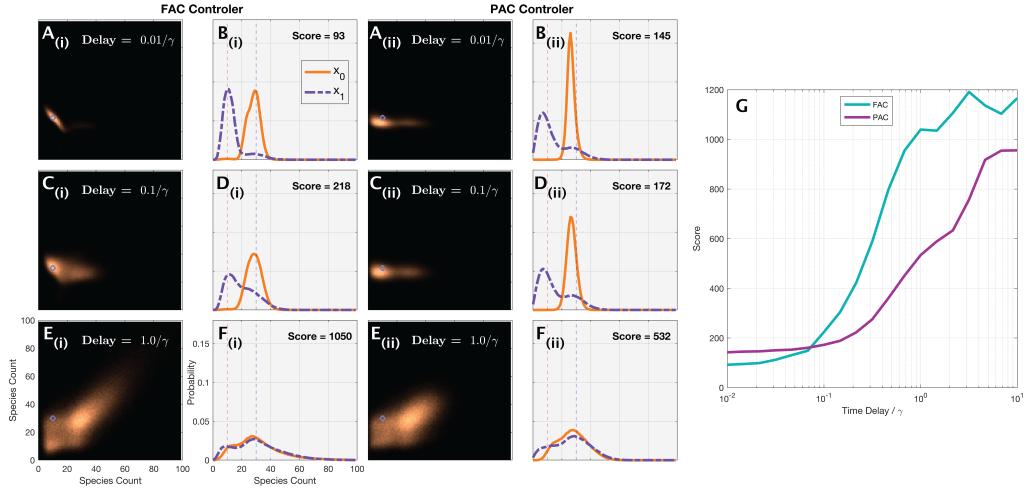


Figure 3: Effects of time delays. (Ai-Fi) Joint (Ai,Ci,Di) and marginal (Bi,Di,Fi) probability distributions for the FAC control method at three different time delays of $\tau/\gamma=0.01$ (top), 0.1 (middle) and 1 (bottom) relative to the target point (blue circles). (Aii-Fii) Same as (Ai-Fi), but for PAC controller. (G) FAC PAC performance as function of time delay show that PAC outperforms the FAC when time delay is greater than $0.07/\gamma$. In all analyses, the system model and PAC and FAC controllers are unchanged from [1].

performance improvements could be obtained with further increases to the system scale, for all cases considered so far, we found that both controllers monotonically improved with increased α and that \mathbf{u}^{FAC} always outperforms the \mathbf{u}^{PAC} .

We note however, that the improved steady state performance for systems with larger scales, comes at a cost that they also take longer to reach a steady state distribution. This tradeoff is illustrated in Fig. ?? which shows the performance score $J(t)$ as the system adapts to a change from the original target point $\mathbf{T} = [XXX, XXX]$ to its mirror point $\mathbf{T}_m = [XXX, XXX]$. From the figure, it is clear that the smaller systems XXX.

3.2 Effects of Time Delays on Controller Performance

Time delay was hypothesized to adversely affect control performance of the \mathbf{u}^{FAC} and \mathbf{u}^{PAC} controllers. To test the effect of time delay on control performance, groups of time delayed SSA were simulated while changing controller and the level of time delay between each group. Each SSA was sampled for 1000 times over 10000 minutes after reaching a steady state by simulating the system for 10000 minutes.

Figure 3 shows the joint and marginal of each controller and time-delay pair and the resulting score, with panels $A_i - F_i$ showing results for the FAC controller and panels $A_{ii}-F_{ii}$ showing results for the PAC controller, and panel G showing the score of both controllers versus the time delay. From the figures, it is clear that performance is rapidly degraded as the delay approaches and then exceeds the characteristic time scale of the process. At low time delays (below $\tau = 0.07/\gamma$), the FAC outperforms the PAC ($J = 87$ versus 146 at $\tau = 0.01/\gamma$) but at moderate time delays (above $\tau = 0.07/\gamma$) the PAC outperforms the FAC ($J = 170$ versus 219 at $\tau = 0.1/\gamma$).

This data taken together show that time delays much larger than $0.07/\gamma$ is detrimental to the FAC controller; delays beyond $0.2/\gamma$ are detrimental for the PAC controller; and the best choice in controller is dependent the level of time delay in the system. For extreme levels of time delay, both systems lose their asymmetry, and their scores become much worse ($J = 1041$ and 931).

3.3 Effects of Parameter Errors or Extrinsic Uncertainties

It is rare that any parameters are exactly known for biological processes, and real systems often exhibit slightly different parameters due to extrinsic variations. This parameter variation between cells is factor that could influence control performance. To test this, parameter sensitivity analysis was performed on the cell one while holding the parameters of cell two constant figure 4(Row I). For each perturbation analysis, both the \mathbf{u}^{FAC} and \mathbf{u}^{PAC} controller was used over a range of s between 0.5 and 2. The data for such perturbations are shown in figure 4. The analysis was then repeated, this time holding the parameters of Cell 1 constant and perturbing only the parameters of Cell 2 (shown in figure 4(Row II)). Finally, both cell one and cell two had their parameters jointly perturbed by the same amount (figure 4(Row III)).

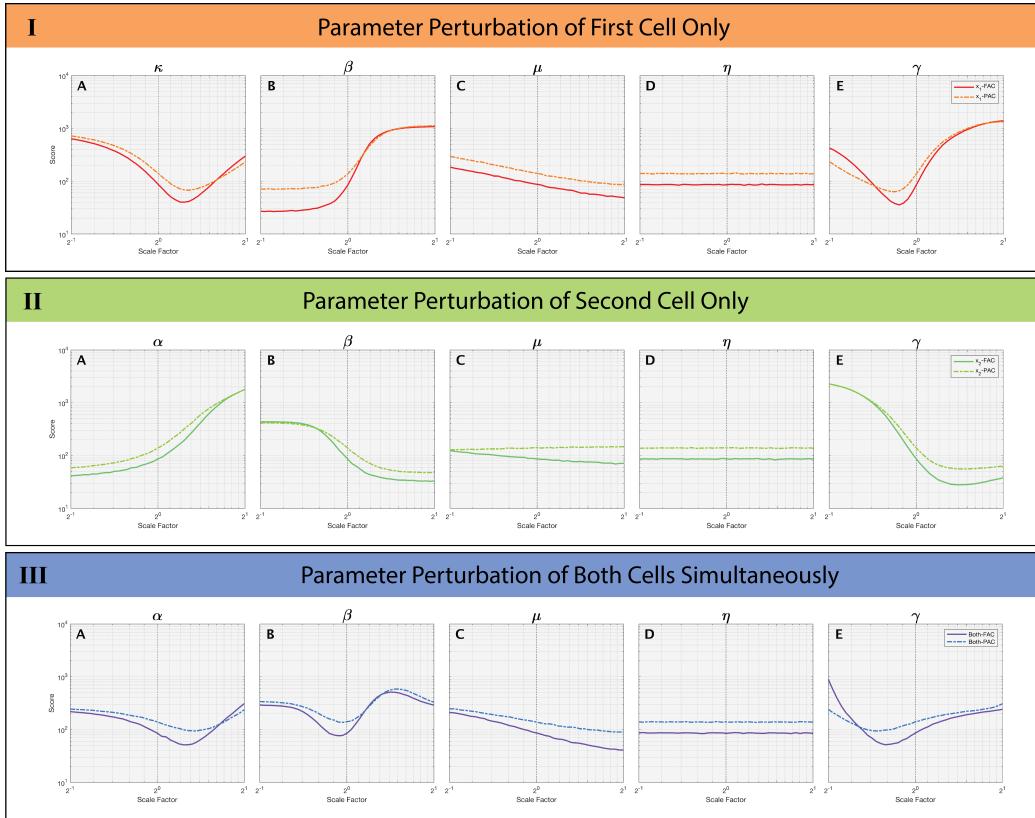


Figure 4: Effects of parametric variations to Cell 1 (top row), Cell 2 (middle row) or to both cells simultaneously (bottom row). Each column show the effect of changing a single model parameter (e.g., κ in the leftmost row). Trends for the FAC (solid lines) and PAC (dashed lines), but the optimal controller can change under parameter variations. In all analyses, the system model and PAC and FAC controllers are unchanged from [1].

Parameter changes were found to have a broad range of effects, including increasing performance, decreasing performance, or having little effect. Figure 4(Bi) shows that increasing β in Cell 1 worsens performance while increasing β in Cell 2 improves performance. In some cases, these effects were not monotonic; for example increasing κ in Cell 1 is highly advantageous up to a limit after which the control performance degrades rapidly. In other cases (such as for k_0), the effect of parameter perturbations on performance is insignificant even for relatively large ($s=2$) perturbations. When parameters of both Cell 1 and Cell 2 are jointly changed (Fig. 4 bottom row) some unique trends were observed. It was found that the \mathbf{u}^{FAC} often outperformed the \mathbf{u}^{PAC} , but for large s in a few parameters the inverse was found (figure ?? (Row III, E and At)).

When examining the effects of changing parameters when using the \mathbf{u}^{FAC} (solid lines) or the \mathbf{u}^{PAC} (dashed lines), general trends typically remained the same in that changes to a parameter which caused a decrease in the control performance of the FAC also tended to decrease the performance of the PAC. Some parameters (e.g., κ, γ) reached minima that performed better than the original, which suggests that the physical system itself has room for optimization beyond the original design that could lead to better control performance, *even without changing the control law*. One explanation for why changes to η had no effect on control performance is that the original parameter was so small that it may require a larger scaling on a different magnitude before changes are observed. For situations where the \mathbf{u}^{PAC} outperformed the \mathbf{u}^{FAC} at large s , this may be an indication that re-optimizing the controller for this parameter combination could yield better control results, since the controller is not re-optimized at each parameter combination.

4 Conclusion

Key results from [1] showed that noise was a key requirement for the development of a noise-exploiting controller, and that deterministic systems could not be controlled to two different stable points if they both started at the same initial condition. As a stochastic system becomes increasingly more granular, it also becomes less noisy and more similar to its ODE solution which is known to be impossible to control to two different fates, but here, we have shown that the removal of noise through system granularity led to better control performance. Therefore, stochastic controllers may perform

work best on models which are nearly deterministic, but not quite. Unfortunately, such systems were found to take longer time to achieve steady state.

The ability to analyze the model at one system granularity apply it elsewhere might help analyze models for which the computational effort may be too large. Since the computation time of the FSP solution to the CME grows with the square of the number of states, this can cause an explosion in computational requirements for large systems. One alternative is to learn a controller at a computationally feasible number of states and apply them to large systems which cannot be solved for using the FSP.

Parameter perturbation analysis showed that there is room to improve control performance by adjusting system parameters, and that joint optimization of the controller with the system parameters may lead to better control performance. It also shows that populations of cells where each cell might have its own parameter combination could still be reasonably controlled for small changes in parameter scale, but large changes could be detrimental.

Time delay analysis showed that increasing time delay decreased control performance. It was also found that a controller with less control information outperformed a controller with more information at higher levels of time delay. We believe this is happening because a controller with more information can afford to be more aggressive to implement its control, and time delay can cause this aggression to backfire. A controller was optimized which required no information in May et al which had a score of 402. This controller should be immune to time delay since no feedback is required for this controller. Here we saw both controllers did perform worse than 402 at very large time delay.

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