AUA CS108, Statistics, Fall 2020

Lecture 39

Michael Poghosyan

25 Nov 2020

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Asymptotic CIs

Asymptotic CI for the Mean of General Distribution

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Asnwer: The Random Interval (or, rather, the sequence of Intervals)

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is a $(1-\alpha)$-level Asymptotic CI for μ.

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Note: We have obtained the following $(1-\alpha)$-level Asymptotic CI for μ:

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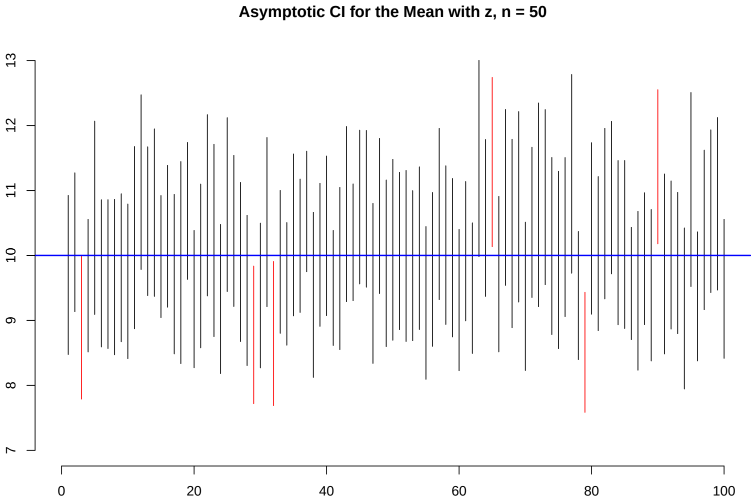
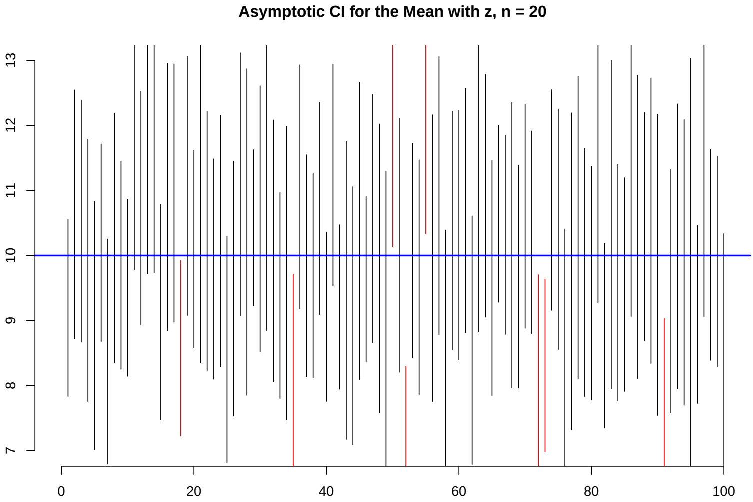
- although in the theory these intervals work for large n, but, in practice, the latter one works also for small n

Example



Asymptotic CI for the Mean with z, n = 50
13
12
11
10
9
8
7
0
20
40
60
80
100

Example



Asymptotic CI for the Mean with z, n = 20
7
8
9
10
11
12
13
0
20
40
60
80
100

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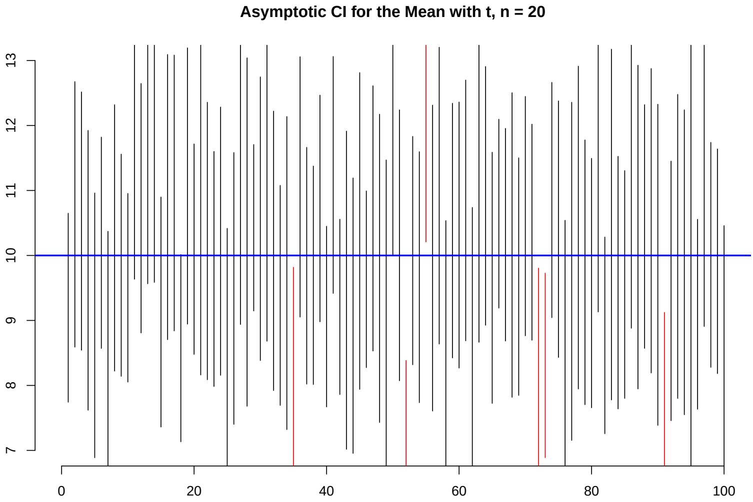
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We model our problem like this: we assume the skull sizes of Italians are coming from some Distribution with some Mean μ and Variance σ^2, σ^2 is unknown.

Example, Cont'd

If we believe that Etruscans are Italians, then we have a Sample from that Distrib:

$$X_1, X_2, ..., X_{84}.$$

where X_k is the skull size of k-th Etruscan person.

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me <- t*s/sqrt(n)
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Asymptotic CI for the Bernoulli Model

By plugging the values for our case, we'll obtain the following Asymptotic CI of level $(1-\alpha)$ for p:

$$\left(\overline{X}_n - z_{1-\alpha/2} \cdot \sqrt{\frac{\overline{X}_n(1-\overline{X}_n)}{n}};\ \ \overline{X}_n + z_{1-\alpha/2} \cdot \sqrt{\frac{\overline{X}_n(1-\overline{X}_n)}{n}}\right).$$

Example, Asymptotic CI for Poisson Model

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Taking an $\alpha \in (0, 1)$, we want to construct a $(1 - \alpha)$-level Asymptotic CI for λ.

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We know that $\hat{\lambda}_n^{MLE} = \overline{X}_n$, and also, $\mathcal{I}(\lambda) = \dfrac{1}{\lambda}$. Then, using the formula above, we'll obtain the following Asymptotic CI of level $(1-\alpha)$ for λ:

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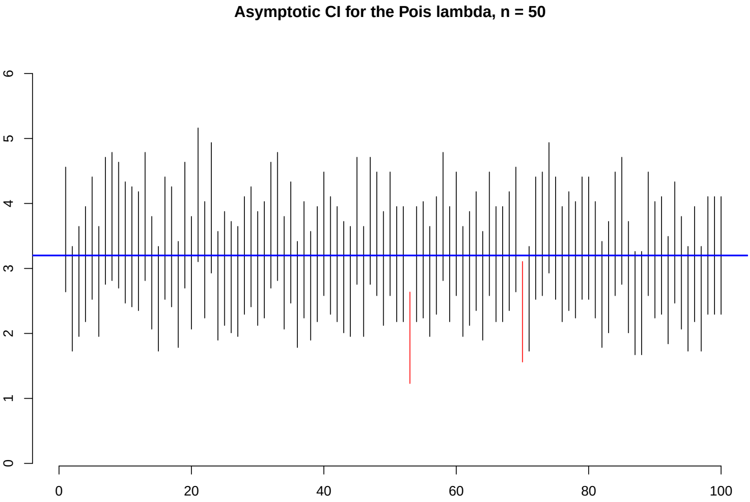
Example, in **R**



Asymptotic CI for the Pois lambda, n = 50
0
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Example, in **R**, Code

```r
lambda <- 3.2
conf.level <- 0.95; a = 1 - conf.level
sample.size <- 15; no.of.intervals <- 100
z <- qnorm(1-a/2)

plot.new()
plot.window(xlim=c(0,no.of.intervals),ylim=c(lambda-3,lambda+3))
axis(1); axis(2)
title("Asymptotic CI for the Pois lambda, n = 50")
for(i in 1:no.of.intervals){
  x <- rpois(sample.size, lambda = lambda)
  ME <- z*sqrt(mean(x)/sample.size) #Margin of Error
  lo <- mean(x) - ME;  up <- mean(x) + ME
  if(lo > lambda || up < lambda){
    segments(c(i), c(lo), c(i), c(up), col = "red")
  }
  else{
    segments(c(i), c(lo), c(i), c(up))
  }
}
abline(h = lambda, lwd = 2, col = "blue")
```

Hypothesis Testing

Hypothesis Testing, Intro

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As always, we assume we have a Dataset coming as a realization of a Random Sample from some unknown Parametric Distribution $\mathcal{F}_\theta$:

$$X_1, X_2, ..., X_n \sim \mathcal{F}_\theta.$$

In this case we want to Test a Hypothesis about θ: say, see whether $\theta = \theta_0$, a given number, or not.

Example

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The Idea The idea is the following: even if our coin was fair, the Probability of Heads $p = 0.5$, it is possible to have some deviation from the expected number of Heads, 50 (in 100 tosses).

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The Idea The idea is the following: even if our coin was fair, the Probability of Heads $p = 0.5$, it is possible to have some deviation from the expected number of Heads, 50 (in 100 tosses). The question is: how extreme, how unbelievable is that deviation. If the deviation from the expected is large, we will not believe that the Hypothesis is True. Otherwise, we will keep believing that it is True.

Hypothesis Testing: Problem Setting and Formalization

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Next, we have a Random Sample from $\mathcal{F}_\theta$:

$$X_1, X_2, ..., X_n \sim \mathcal{F}_\theta, \qquad \theta \in \Theta,$$

and using this Sample, we want to Test the Hypothesis, we want to see if we can **Reject $\mathcal{H}_0$ in favor of $\mathcal{H}_1$ or not**, i.e., we want to see if **we have enough evidence in our Data to Reject $\mathcal{H}_0$**.

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Example: In the above example about the coin fairness, if p is the Probability of a Head, then our Hypotheses are:

$$\mathcal{H}_0: \ p = \frac{1}{2} \qquad vs \qquad \mathcal{H}_1: \ p \neq \frac{1}{2}.$$

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So, when we will have our Data, we will see if we can **Reject** $\mathcal{H}_0$.

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So the conclusion of the Hypothesis Testing need to be either:

Reject $\mathcal{H}_0$ or **Fail to Reject** $\mathcal{H}_0$.

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