CS 107, Probability, Spring 2019
Lecture 30

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AUA

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Content

- Identically Distributed R.V.s
- Joint Distribution of two R.V.s

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Important about Kakutani: `https://www.uml.edu/docs/tangents_math_news_tcm18-59534.pdf`

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- **Reason 2:** X and Y can be defined on the same experiment, but the values can be different. Say, we are rolling a fair die. ω is the number shown on the top face. We take $X(\omega) = 0$, if ω is even, and $X(\omega) = 1$ otherwise.

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Identically Distributed r.v.s

Now, we give a definition:

Identically Distributed r.v.s

We will say that X and Y are ID (Identically Distributed), if they have the same CDFs, i.e., if

$$F_X(x) = F_Y(x), \qquad \forall x \in \mathbb{R},$$

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$$f_X(x) = f_Y(x) \qquad \textit{for almost all } x \in \mathbb{R}$$

Examples of ID r.v.

Example: When we are writing $X, Y \sim \mathcal{N}(0,1)$, this means that X and Y have the same distribution, Standard Normal, so they are ID.

Examples of ID r.v.

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Very important question is how two or more (random) quantities, i.e., r.v. in our terms, are related to each other.

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During several lectures, we will talk about the Joint Distribution of several r.v.s, and give methods to describe their Joint Distribution.

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The **Joint CDF of random variables** X **and** Y or the **CDF of a random vector** (X, Y) is the function

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Here, when writing $\mathbb{P}(X \leq x, Y \leq y)$ we mean the probability that both $X \leq x$ and $Y \leq y$, i.e.,

$$\mathbb{P}(X \leq x, Y \leq y) = \mathbb{P}(X \leq x \text{ and } Y \leq y).$$

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In fact, if F satisfies all above properties, then it is a Joint CDF of some random vector (X, Y).

Calculation of Probabilities through the Joint CDF

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Note: This is the sum from the above properties! That's why it is non-negative!

Expressing 1D Distributions through the Joint one

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