CS 107, Probability, Spring 2019
Lecture 45

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AUA

the Last One

Content

- Intro to Markov Chains

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Some Problems Concerning Markov Chains

Recall that last time we were considering one of the problems concerning MCs: the Probabilities of Paths

$$\mathbb{P}(X_0 = i_0, X_1 = i_1, ..., X_n = i_n),$$

which can give us the idea, say, about the best possible scenario for our System for the next n time instants.

Some Problems Concerning Markov Chains

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$$P^{(n)} = \left[p_{ij}^{(n)} \right].$$

Note: Please note that because our MC is Time-Homogeneous, then, for any $m \in \mathbb{N}$,

$$\mathbb{P}(X_{n+m} = j | X_m = i) = \mathbb{P}(X_n = j | X_0 = i) = p_{ij}^{(n)}.$$

n-Step Transition Probabilities

Using the Total Probability Formula, one can prove the following theorem:

Chapman - Kolmogorov Equation

For any $n \in \mathbb{N}$,

$$P^{(n)} = P^n, \quad \text{and, consequently,} \quad P^{(n+m)} = P^{(n)} \cdot P^{(m)}.$$

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In the terms of the matrix elements, the last equality can be written in the form

$$p_{ij}^{(n+m)} = \sum_{k=1}^{N} p_{ik}^{(n)} \cdot p_{kj}^{(m)}.$$

Interpretation: Give ⌣̈

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Describe this System's n-step Transition Probability Matrix.

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$$P = \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix}.$$

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		Next Gen		
Current Gen	State	1	2	3
	1	0.65	0.28	0.07
	2	0.15	0.67	0.18
	3	0.12	0.36	0.52

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Now, the Transition Matrix (TPM) is

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(i.e., the proportion of class 1 families is 0.25, class 2 families is 0.6), find the distribution of the income classes after 2 generations.

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To be continued ...

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This means, that in the long run, the Probability to be at the State 0 is approximately $\frac{q}{p+q}$, and the Probability to be at the State 1 is approximately $\frac{p}{p+q}$, irrespective of the initial State!

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Now, it turns out, that, under some conditions on the TPM P, the sequence π_n will converge to some vector π, $\pi_n \to \pi$, which is called the **Stationary (or the Equilibrium) Distribution of MC**.

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Important Note 1: It is important that the Stationary Distribution π is *independent* of the initial Distribution π_0. So for any Initial Distribution of States Probabilities, after some time the Distribution will stabilize to the same Distribution.

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and P is the $N \times N$ TPM, and we have a SLE to solve. But it turns out that the rank of this system is less than N, so there are infinitely many solutions to this SLE. But we need to remember that π needs to be a Probability Distribution, so the sum of its elements needs to be 1. Adding this to our SLE gives a unique solution.

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Example: In the Income Classes Example, find the Stationary Distribution of States and give an interpretation.
Solution: OTB

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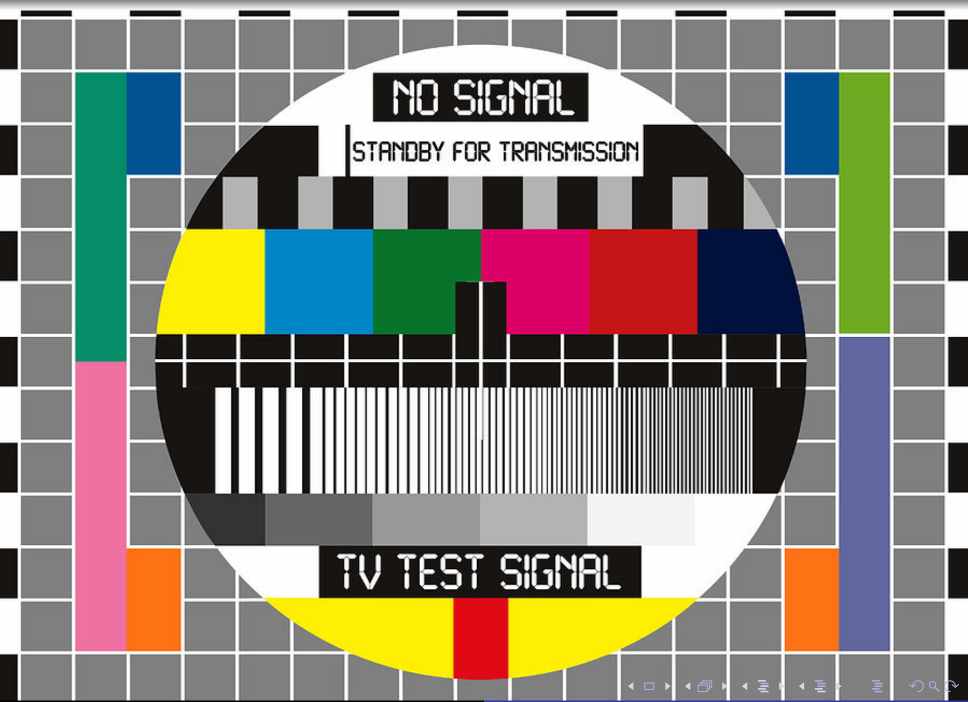
Idea: If our MC is Irreducible, then the initial State Probabilities are thoroughly mixed.

PRTS! URAA!


NO SIGNAL
STANDBY FOR TRANSMISSION
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