CS 107, Probability, Spring 2020
Lecture 08

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Content

- Classical Probability Models: Finite Sample Spaces with Equally Likely (Equiprobable) Outcomes = Combinatorial Problems

Air Passenger Problem

A fully-booked plane is about to be boarded by its hundred passengers. The first passenger loses his boarding card and sits down in a random place. From then on, a passenger, whenever finding his seat taken, very politely sits somewhere at random. What is the probability that the hundredth passenger sits in his own seat?

The Answer is:

Last Lecture ReCap

Last time we were talking about Combinatorial Probability Problems.

Without Replacements Model, Solution 3:

Problem: We have a box containing 100 balls, from which 25 are white and the rest are black. We choose at random 12 balls. What is the probability of having exactly 4 white balls chosen?

Now, the correct solution: OTB

Think about the with-replacements model by yourself!

An Idea for Solving Probability Problems

- Assume we want to calculate the Probability of an Event A.
- Sometimes, when calculation of $\mathbb{P}(A)$ is not so easy, one can try calculating the probability of an Opposite Event, $\overline{A}$, and use
$$\mathbb{P}(A) = 1 - \mathbb{P}(\overline{A}).$$
- This trick helps in different situations

Example: Birthday Problem

Problem: We have 27 participants in our group of Probability class, including the instructor. What is the probability that at least two participants share the same birthday?
Assumptions: There are 365 days in a year, and the probability of being born on each day is the same.

Example: Birthday Problem, another version

Problem: We have 27 participants in our group of Probability class, including the instructor. What is the probability that at least one of our students will share the instructor's (MP's) birthday?

Example: Julius Caesar Problem

Do you remember our Caesar?

Figure: Gaius Julius Caesar

Example: Julius Caesar Problem

Do you remember our Caesar?

Problem: What are the chances that I have just inhaled a molecule which the great Caesar exhaled when saying his last words?

Assume we have N molecules of air in the world, and Caesar exhaled n of them, and they are uniformly spread over the world. And assume I have just inhaled m molecules.

Physics says that $N \sim 10^{44}$, $n, m \sim 2.2 \cdot 10^{22}$

The rest on the board!

Girls and Boys Problem

Problem: We randomly choose a family with three children. Find the probability that they have exactly one daughter.
Sol'n 1: We model the outcome to be the number of b/g child in the family:

$$\Omega = \left\{3g, \mathrm{2g1b}, \mathrm{1g2b}, 3b\right\},$$

The Favorable Event is:

$$A = \{\mathrm{1g2b}\}$$

So,

$$\mathbb{P}(A) = \frac{1}{4}.$$

Correct? No!

Girls and Boys Problem, 2nd Sol'n

Problem: We randomly choose a family with three children. Find the probability that they have exactly one daughter.
Sol'n 2: We model the outcome to be which child is b/g in the family:

$$\Omega = \{ggg, ggb, gbg, bgg, gbb, bgb, bbg, bbb\}.$$

The Favorable Event is:

$$A = \{gbb, bgb, bbg\}.$$

So,

$$\mathbb{P}(A) = \frac{3}{8}.$$

Correct? Aha!

Geometric Probabilities

Now let us talk about Experiments, where the Sample Space is some non-countable infinite subset of $\mathbb{R}^n$ for some $n \geq 1$, say, the Sample Space can be

- an interval in $\mathbb{R}$; e.g., No. 2 marshutka waiting time when arriving at the bus stop;
- a circle or a rectangle in $\mathbb{R}^2$; e.g., hitting a Darts board with the missile;
- a cube in $\mathbb{R}^3$; lifetimes for 3 different parts of the above marshutka;
- an n-dimensional cube in $\mathbb{R}^n$, arrival times of our Probability Students to our class today (? on Wed?)

We want to give a basic Probabilistic model for our Experiment.

Geometric Probabilities, Cont'd

So assume that our Experiment's Sample Space is $\Omega \subset \mathbb{R}^n$.
We need an assumption to define our Classical Geometric Probability Model.

Assumption

We assume that we have **uniform probability distribution**.

What we mean by that:

- **Incorrect interpretation:** the probability of choosing each point in Ω is the same. Well, in this Geometric Model (non-countably infinite Sample Space!) we will have that the probability of choosing any particular point is 0.
- **Correct Interpretation:** the probability of choosing any equal-measure (length/area/volume) subsets (Events) is the same.

LZ, part 2

Entracte

Assume we choose a point at random from $[0, 1]$, so $\Omega = [0, 1]$. We know that the probability to choose the point 0 is 0. So we will not choose 0 for 100%. Also, we will not choose the point 1 for 100%. Similarly, we will not choose the point 0.3241 for sure. The same for 0.6473241321.

Then which point we will choose if we will not choose any point for sure?

:D

Geometric Probabilities, Cont'd

Let's continue:

- Our Experiment's Sample Space is $\Omega \subset \mathbb{R}^n$;
- We assume that Ω is measurable[1], i.e., it has a finite length (if $\Omega \subset \mathbb{R}$) or it has a finite area (if $\Omega \subset \mathbb{R}^2$) or it has a finite volume (if $\Omega \subset \mathbb{R}^n$, $n \geq 3$).
- If $A \subset \Omega$ is an Event (if A has a finite measure), the we define

$$\mathbb{P}(A) = \frac{measure(A)}{measure(\Omega)}.$$

[1] In the Lebesgue's sense $(18+)$.