CS 107, Probability, Spring 2020
Lecture 09

Michael Poghosyan
mpoghosyan@aua.am

AUA

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Content

- Classical Probability Models: Geometric Probabilities
- Conditional Probabilities

The Power of Combinatorics, or How to write more than a Billion of Poems during a life. Do you know this guy?

The Answer is: Well, of course NO $\ddot\smile$

The Correct Answer is: Raymond Queneau, French poet

One of his books is called **Cent mille milliards de poèmes**, i.e., **A Hundred Thousand Billion Poems**:

LZ

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You will find only 10 sonets there, all 14 lines long. But...

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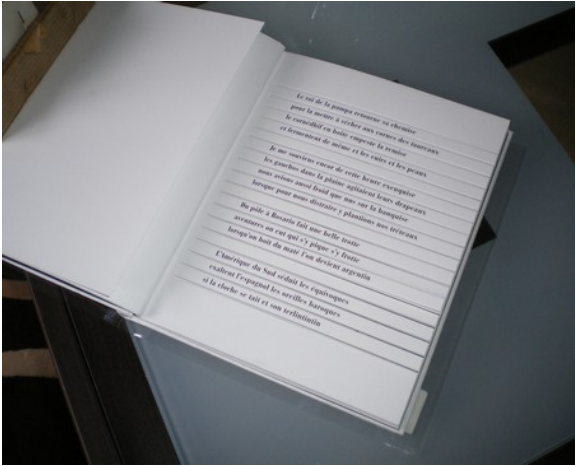

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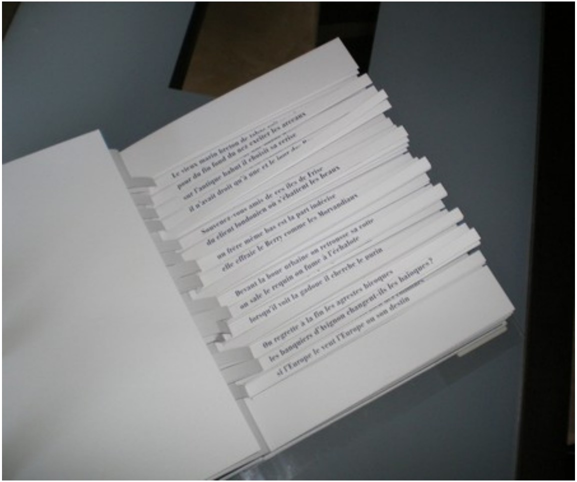

Last Lecture ReCap

Last time we were talking about Combinatorial Probability Problems, and also about Geometric Probabilities:

- We assume our Experiment's Sample Space is $\Omega \subset \mathbb{R}^n$;
- For any Event $A \subset \Omega$, we define
  - if $n = 1$,
  $$\mathbb{P}(A) = \frac{Length(A)}{Length(\Omega)}$$
  - if $n = 2$,
  $$\mathbb{P}(A) = \frac{Area(A)}{Area(\Omega)}$$
  - if $n \geq 3$,
  $$\mathbb{P}(A) = \frac{Volume(A)}{Volume(\Omega)}$$

Example:

Problem: Assume my waiting time for the train can be anything from $[0,12]$ (in min), $\Omega = [0,12]$.
We (silently) assume that the probabilities are uniformly distributed.

- What is the probability that I will wait more than 8 min?
  - Here our Event is $A = (8,12]$.
  - Since Ω is 1D ($\Omega \subset \mathbb{R}$), then *measure* $=$ *length*.
  - So

$$\mathbb{P}(A) = \frac{\mathit{length}(A)}{\mathit{length}(\Omega)} = \frac{\mathit{length}\big((8,12]\big)}{\mathit{length}\big([0,12]\big)} = \frac{12-8}{12-0} = \frac{4}{12} = \frac{1}{3}.$$

- What is the probability that I will wait exactly 7.3 min?
  - Here our Event is $A = \{7.3\}$. So $\mathbb{P}(A) = \frac{\mathit{length}(A)}{\mathit{length}(\Omega)} = \frac{0}{12} = 0.$

Example: Romeo and Juliet Problem

Problem: Two persons, R and J, arrive at random to the Siraharneri Aygi between [0, 2] hours from 1PM and stay there for 20 minutes ($= 1/3$ hours). What is the probability that they will meet?

Implicit assumptions: The arrival time has a uniform distribution.

Solution: MP, solve it on the board!

More Realistic Version: Now, R will wait for 30 min, but J only for 10 min. Can you calculate the probability? HW

Example: Calculating π by Monte-Carlo

Let us calculate π by a computer and Probability. Later we will talk about this method again, we will justify the reasoning.

Problem: Assume we are throwing a darts missile into the quadratic darts board $\Omega = [-1,1] \times [-1,1]$. What is the probability that we will hit a point inside the unit disk?

Example: 4D Case

Example: Assume (not a correct assumption!) that students can be late from a Prob lecture at most for 5 min from the beginning of the lecture, uniformly. 4 students are arriving independently to our course. What is the probability that none of them will be late more than 3 min?

Solution: OTB