CS 107, Probability, Spring 2020
Lecture 06

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Content

- Classical Probability Models: Finite Sample Spaces with Equally Likely (Equiprobable) Outcomes = Combinatorial Problems

Surprize

Quiz Time!

Birthday Problem

We have 27 participants in our group of Probability class, including the instructor. What is the probability that at least two participants share the same birthday, i.e., they were born on the same day and month (but maybe not in the same year)?

Answer: the Probability is larger than 62.6%.

Last Lecture ReCap

Last time we were talking about some Classical Probability Models:

- Finite or Countably Infinite Sample Space case: assume $\Omega = \{\omega_1, \omega_2, ...\}$ is finite or Countably Infinite. To define a Probability Model on this Sample Space, it is enough to give the following Table:

Outcome	ω_1	ω_2	...
$\mathbb{P}(\{\omega\})$	p_1	p_2	...

  where
  $$p_k \geq 0, \quad p_1 + p_2 + ... = 1.$$

Last Lecture ReCap

- One important particular case is the Equally Likely Outcomes (Equiprobable) Model: here Ω is finite, $\Omega = \{\omega_1, \omega_2, ..., \omega_n\}$, and

Outcome	ω_1	ω_2	ω_3	...	ω_n
Probability	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	...	$\frac{1}{n}$

In this case, for any Event A,

$$\mathbb{P}(A) = \frac{\text{number of elements favorable for the event } A}{\text{total number of possible outcomes}} = \frac{\#A}{\#\Omega}.$$

Examples

Example: Assume we have a box containing 12 red balls, 21 white balls and 14 green balls. We pick a ball at random from the box.

- What is the probability that we will choose a green ball?
- Why this kind of problems are interesting/important in Probability Theory? Can you give an example without balls?

Example: In our Parliament we have 132 seats. 88 Seats are for "My Step" Alliance, 26 seats are for "Prosperous Armenia" party and 18 are for the "Bright Armenia". What is the probability that a randomly chosen parliamentarian will be from the "Bright Armenia"?

Example:

Before the Example - 52 Playing Cards Deck consists of:

- Suits:
  - **Hearts** $= \heartsuit$,
  - **Diamonds** $= \diamondsuit$,
  - **Clubs** $= \clubsuit$,
  - **Spades** $= \spadesuit$.
- Cards: **Ace**, **King**, **Queen**, **Jack**, **2, 3, 4, 5, 6, 7, 8, 9, 10**.

Interesting Question: How many shuffling (arrangement of cards) of this deck exist? Do you think that all possible shuffling appeared at least once during the whole history of our world?

Example: Cont'd

- **Problem:** We shuffle our deck and pick at random 2 cards. What is the probability that none of these cards will be hearts?
- Natural Assumption: the probability of choosing any card is the same = Equiprobable Case!
- **Case 1:** We take cards **without replacement**:

$$\# \text{ of all Possible Outcomes: } \boxed{}\boxed{}$$

$$\# \text{ of all Favorable Outcomes: } \boxed{}\boxed{}$$

$$\text{Probability} = \frac{\#\text{of favorable outcomes}}{\#\text{of all possible outcomes}} =$$

Example: Cont'd 2

- **Case 1:** We take cards **without replacement**, almost the same approach in other way: clearly, our outcomes are pairs of cards. Each pair is equally probable to be chosen (why?). Then

$$\mathbb{P}(\textit{no} \heartsuit \textit{ in } 2 \textit{ cards}) = \frac{\#\text{pairs without a } \heartsuit}{\#\text{all pairs}} = \frac{\binom{39}{2}}{\binom{52}{2}}$$

- What is the difference between these two approaches?
- For the first case, we consider all **ordered** pairs:

$$\{(A\heartsuit, A\diamondsuit), (A\diamondsuit, A\heartsuit), (A\heartsuit, K\clubsuit), (K\clubsuit, A\heartsuit), \ldots\}$$

- For the second case, we consider all **unordered** pairs:

$$\{(A\heartsuit, A\diamondsuit), (A\heartsuit, K\clubsuit), \ldots\}$$

- **Case 2:** We take cards **with replacement**