# CS 107, Probability, Spring 2019 Lecture 14

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#### Content

- Repeated, Independent Trials: Binomial Distribution, Cont'd
- Repeated, Independent Trials: Multinomial Distribution

## LZ



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#### Quiz:

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- b. I am choosing 2 persons from our group of 36 participants. We know that 10 of them are female. What is the probability that one of the chosen persons will be a female and and the other one a male?

## Binomial Probabilities formula, again:

#### **Binomial Probabilities**

For any k = 0, 1, ..., n,

$$\mathbb{P}(\mathsf{Exactly}\ k\ \mathsf{successes}\ \mathsf{in}\ n\ \mathsf{trials}) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}.$$

or, if we will denote the number of Successes in that n Trials by X,

$$\mathbb{P}(X=k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}.$$

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Proof: ONB



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• He/she will get 25 points?

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- He/she will get 25 points?
- He/she will get more than 25 points?

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- He/she will get 25 points?
- He/she will get more than 25 points?
- He/she will get less than 29 points?

Now we consider an Experiment consisting of repetition of a Simple Experiment, which can result in more than 2 events. More precisely,

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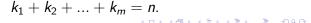
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- We repeat our Trial *n* times, independently;
- We are interested in the probability that we will have exactly  $k_1$  outcomes from  $A_1$ , exactly  $k_2$  outcomes from  $A_2,...$ , and exactly  $k_m$  outcomes from  $A_m$ .



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- Clearly, we need to have (we are doing n trials)



Note that the Binomial Case is the particular case of the Multinomial: in that case

• Each trial can result either in A (Success) or in  $\overline{A}$  (Failure);

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- We are interested in having exactly k Successes in n Trials, that is, we are interested in having exactly  $k_1 = k$  times  $A_1$  (Success) and exactly  $k_2 = n k_1$  times  $A_2$  (Failure)

## Repeated Indep Trials: Multinomial Probabilities

#### Multinomial Probabilities

The probability that exactly  $k_1$  times we will have the event  $A_1$ , exactly  $k_2$  times we will have the event  $A_2$ , ..., exactly  $k_m$  times we will have the event  $A_m$  in the above described n trials, is

$$P_n(k_1, k_2, ..., k_m) = \binom{n}{k_1, k_2, ..., k_m} \cdot p_1^{k_1} \cdot p_2^{k_2} \cdot ... \cdot p_m^{k_m},$$

where  $p_k = \mathbb{P}(A_k)$ , k = 1, ..., m.

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$$\binom{n}{k_1, k_2, \dots, k_m} = \frac{n!}{k_1! \cdot k_2! \cdot \dots \cdot k_m!}$$



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Note that in the binomial case, i.e., when m=2, we get the Binomial Probabilities formula.

