AUA CS 108, Statistics, Fall 2019

Lecture 09

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- Define the Sample Variance and the Standard Deviation
- Give the Definition of the MAD

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- Define the Sample Variance and the Standard Deviation
- Give the Definition of the MAD
- What are the Quartiles?

BoxPlot

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- the set of all Outliers
  $$O = \left\{x_i : x_i \notin \left[Q_1 - \frac{3}{2}IQR, Q_3 + \frac{3}{2}IQR\right]\right\}$$

BoxPlot, Example

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$$x: \; 0, -2, 2, 1, 5, 6, 4, 1, 2, 1, 12$$

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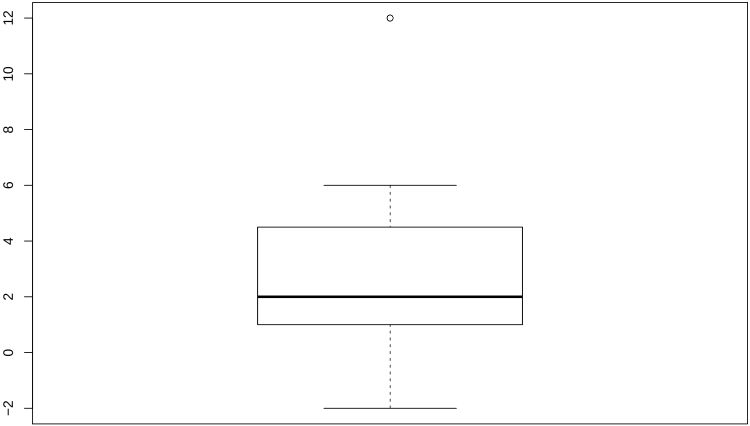
Solution: OTB;

BoxPlot, Example

Now, using **R**:

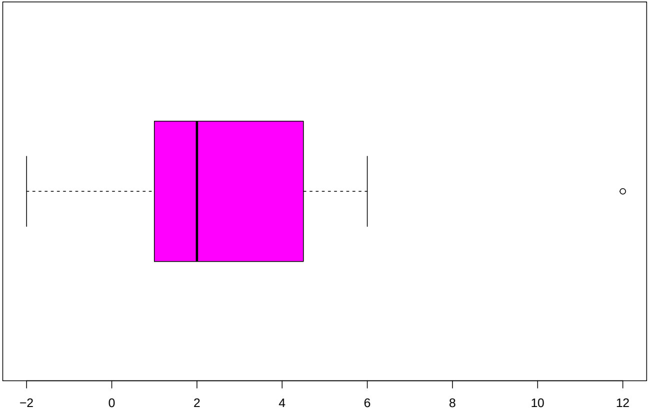
```r
x <- c(0, -2, 2, 1, 5, 6, 4, 1, 2, 1, 12)
boxplot(x)
```



BoxPlot, Example

Another view:

```
x <- c(0, -2, 2, 1, 5, 6, 4, 1, 2, 1, 12)
boxplot(x, horizontal = T, col = "magenta")
```



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Take as W_1 and W_2 the smallest and largest **Datapoints**, respectively, in

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Take as W_1 and W_2 the smallest and largest **Datapoints**, respectively, in

$$\left[Q_1 - \frac{3}{2} IQR, \; Q_3 + \frac{3}{2} IQR \right].$$

- One doesn't keep the scale. Say, one draws the Median exactly at the middle of the Quartiles, despite the Dataset is not symmetric at all.

Additions/Variations:

Some Variations:

- Variable Width BoxPlot

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See, for Example, this page.

Boxplot, Why we use it

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Boxplot, Why we use it

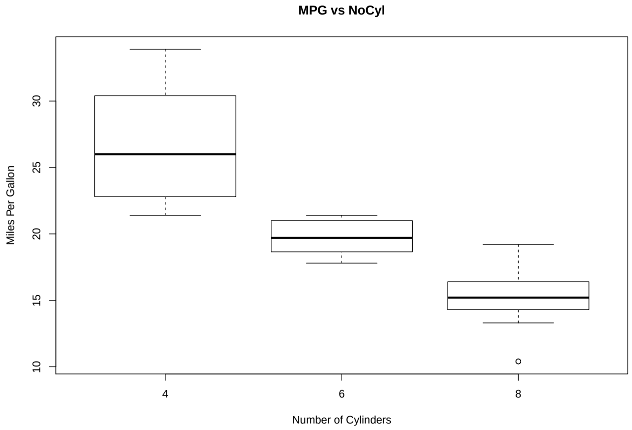
We use BoxPlots to:

- Visualize the distribution of the Dataset
- To compare two or more Datasets

Example

Here we use the `mtcars` Dataset:

```
boxplot( mpg~cyl, data=mtcars, main="MPG vs NoCyl",
   xlab="Number of Cylinders", ylab="Miles Per Gallon")
```

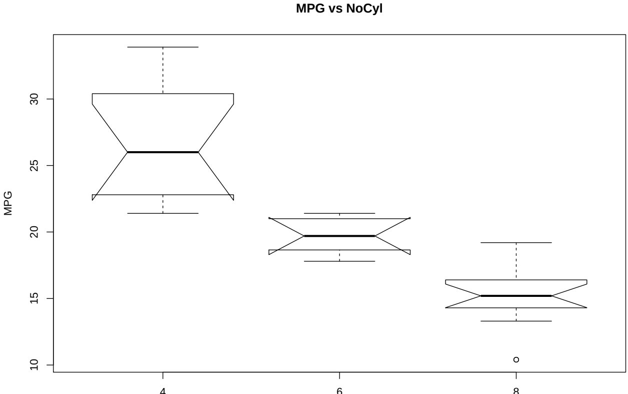


Example

Again,

```
boxplot( mpg~cyl, data=mtcars, notch = T,
         main="MPG vs NoCyl", xlab="Number of Cylinders", y
```

```
## Warning in bxp(list(stats = structure(c(21.4, 22.8, 26,
## notches went outside hinges ('box'): maybe set notch=FAI
```



Note

Recall that an **Outlier** in the BoxPlot sense is a Datapoint x_k with

$$x_k \notin \left[Q_1 - \frac{3}{2} IQR,\ Q_3 + \frac{3}{2} IQR \right].$$

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Another way to define an **Outlier:** Datapoint x_k is an Outlier, if

$$|x_k - \bar{x}| \geq 3 \cdot sd(x).$$

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- 75% of Datapoints are to the left of the Upper Quartile Q_3, and 25% are to the right, so Q_3 divides the (sorted) Dataset in the (approximate) proportion 75%-25%

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Now, let $\alpha \in (0, 1)$.

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Now, let $\alpha \in (0, 1)$. We want to find a real number q_α dividing our (sorted) Dataset into the proportion $100\alpha\% - 100(1 - \alpha)\%$, i.e., q_α is a point such that the α-portion of the Datapoints are to the left to q_α, and others are to the right.

Sample Quantiles

Let $x: \; x_1, x_2, ..., x_n$ be our 1D numerical Dataset. Assume also that $\alpha \in (0, 1)$.

Definition: The Quantile of order α (or $100\alpha\%$ order, the α-Quantile) of x is defined by

$$q_\alpha = q_\alpha^x = x_{([\alpha \cdot n])}.$$

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Note: There are different definitions of the α-quantile in the literature and in software implementations. Say, **R** has 9 methods to calculate quantiles.