CS 107, Probability, Spring 2019
Lecture 05

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AUA

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Content

- Classical Probability Models: Countably Infinite Sample Spaces
- Classical Probability Models: Finite Sample Spaces with Equally Likely Outcomes

Surprize

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Quiz Time!

Let us do a scientific experiment:

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Figure: Gaius Julius Caesar

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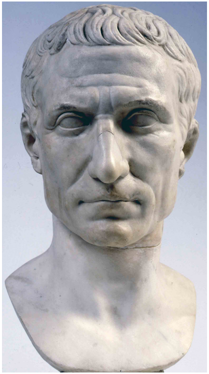

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You too, Brutus.

Vnimanie, the question: what are the chances that you just inhaled a molecule which the great Caesar exhaled when saying his last words?

The answer is: the probability is larger than 99%! $\ddot\smile$

Classical Probability Models: Countably Infinite Sample Spaces

We continue considering Discrete Models. We assume that we have a countably infinite Sample Space:

$$\Omega = \{\omega_1, \omega_2, ..., \omega_n, \omega_{n+1}...\},$$

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$$p_k \geq 0, \quad k \in \mathbb{N} \qquad \text{and} \qquad \sum_{k=1}^{\infty} p_k = 1,$$

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and define

$$\mathbb{P}(\{\omega_k\}) = p_k, \qquad k \in \mathbb{N}.$$

Classical Probability Models: Countably Infinite Sample Spaces

In the table form, this looks like

Outcome	ω_1	ω_2	ω_3	$\dots$	ω_n	$\dots$
Probability	p_1	p_2	p_3	$\dots$	p_n	$\dots$

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Now, we define for any nonempty event $A \in \mathcal{F}$ (i.e., for any nonempty subset $A \subset \Omega$),

$$\mathbb{P}(A) = \sum_{\omega_k \in A} p_k,$$

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Now we have the complete Model.

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Example:

Equally Likely Outcomes

Now assume we have a Discrete Model with finitely many outcomes:

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One of the important Probability Models is the Equally Likely Outcomes (or the uniform discrete distribution) case:

$$\mathbb{P}(\{\omega_1\}) = \mathbb{P}(\{\omega_2\}) = \dots = \mathbb{P}(\{\omega_n\}) =$$

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Equally Likely Outcomes, cont'd

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$$= \frac{1}{n} \cdot (\text{number of elements in } A) =$$

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Or, in other words,

$$\mathbb{P}(A) = \frac{\text{number of elements favorable for the event } A}{\text{total number of possible outcomes}} = \frac{\#A}{\#\Omega}.$$

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- What is the Equally Probable Outcomes (or the Discrete Uniform) Model?
- Give an example of an Equally Probable Outcomes model.
- Is it possible to define the Equally Probable Outcomes Model for the Countably Infinite Sample Space case?
- How to calculate the probability of an event in the Equally Probable Outcomes Case?

Examples

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- Why this kind of problems are interesting/important in Probability Theory? Can you give a real -life example?

Example: In our Parliament we have 132 seats. 88 Seats are for "My Step" Alliance, 26 seats are for "Prosperous Armenia" party and 18 are for the "Bright Armenia". What is the probability that a randomly chosen parliamentarian will be from the "Bright Armenia"?