AUA CS 108, Statistics, Fall 2019

Lecture 44 (The Last One)

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Contents

Last Lecture ReCap

- We will skip this, sorry!

Intro to Linear Regression

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And also we will consider the simplest case, when we seek the Regression Function among the Linear Functions.

Intro to Linear Regression

The simplest Regression Model is the Linear Model: we will assume that

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$$Y = \beta_0 + \beta^T \cdot \mathbf{X} + \varepsilon,$$

where ε is a r.v., for each value of $\mathbf{X}$, with $\mathbb{E}(\varepsilon) = 0$.

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so our Model, in the expanded way, is

$$Y = \beta_0 + \beta_1 \cdot X^1 + \beta_2 \cdot X^2 + ... + \beta_d \cdot X^d + \varepsilon.$$

Simple Linear Regression

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This is called the **Simple Linear Regression Model**.

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Assume we have the actual Relationship between X and Y:

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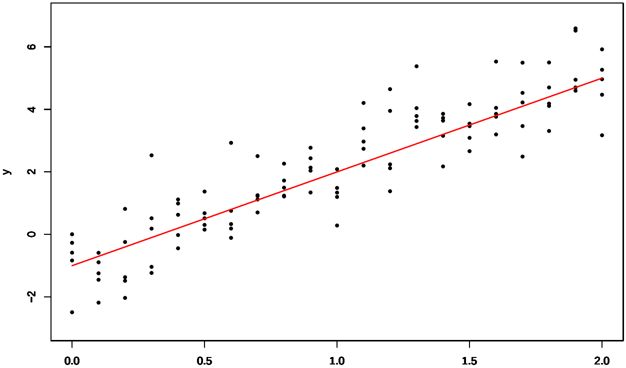

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$$(x_1, y_1), ..., (x_n, y_n),$$

and we want to Estimate β_0 and β_1.

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$$Y_k = \beta_0 + \beta_1 \cdot X_k + \varepsilon_k, \qquad \mathbb{E}(\varepsilon_k) = 0$$

where ε_k-s are Independent, and our aim is to find good Estimators for β_0 and β_1.

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Now, the idea of the Ordinary Least Squares Method for Estimating the Parameters β_0, β_1 is the following: Find

$$(\hat{\beta}_0, \hat{\beta}_1) = \underset{(\beta_0, \beta_1) \in \mathbb{R}^2}{argmin} \sum_{k=1}^{n} \Big(Y_k - \beta_0 - \beta_1 \cdot X_k \Big)^2 .$$

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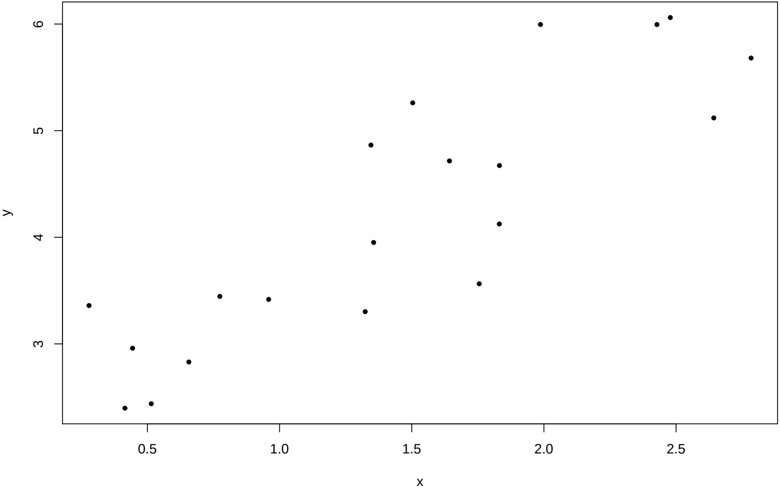

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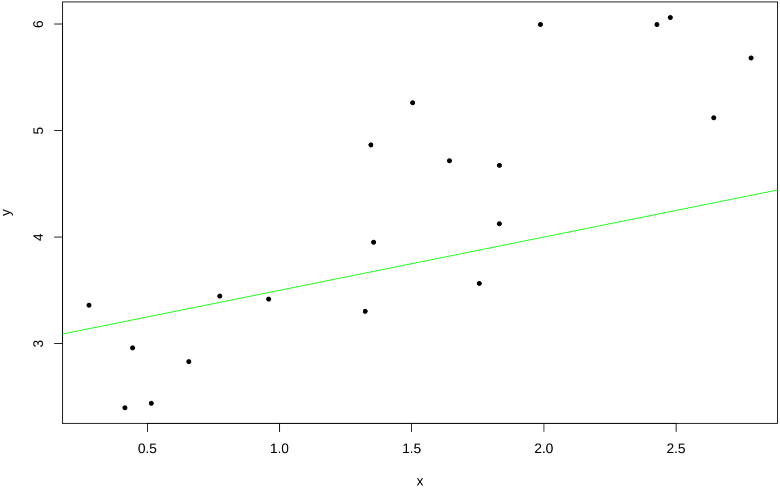

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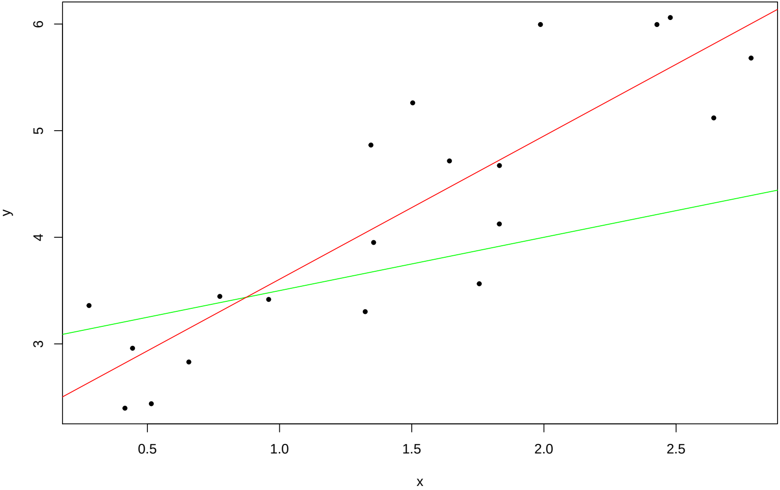

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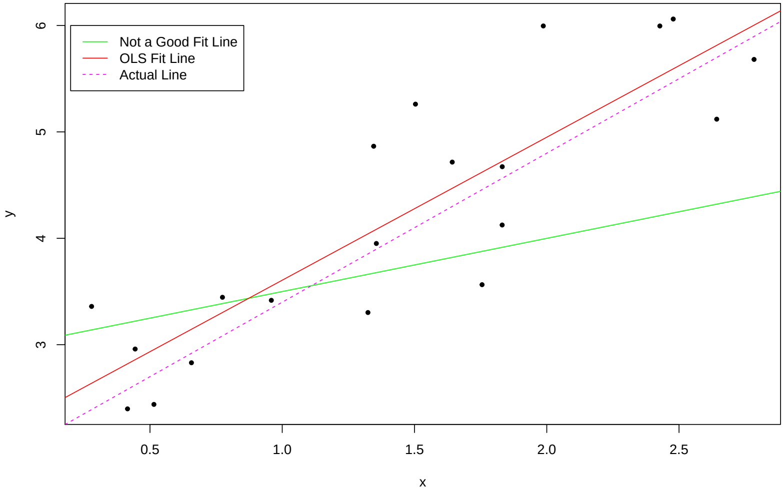

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Calculation of the best fit line is easy: we just define

$$\varphi(\beta_0, \beta_1) = \sum_{k=1}^{n} \Big(Y_k - \beta_0 - \beta_1 \cdot X_k \Big)^2, \qquad (\beta_0, \beta_1) \in \mathbb{R}^2,$$

and using our Calc 3 knowledge, find the Minimum Point of φ by solving the System

$$\begin{cases} \dfrac{\partial \varphi}{\partial \beta_0} = 0 \\ \dfrac{\partial \varphi}{\partial \beta_1} = 0 \end{cases}$$

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The Solution is:

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and

$$\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \cdot \overline{X}.$$

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The obtained Line

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Note: So the $cor(X, Y)$ is not the Slope of the Regression Line, but

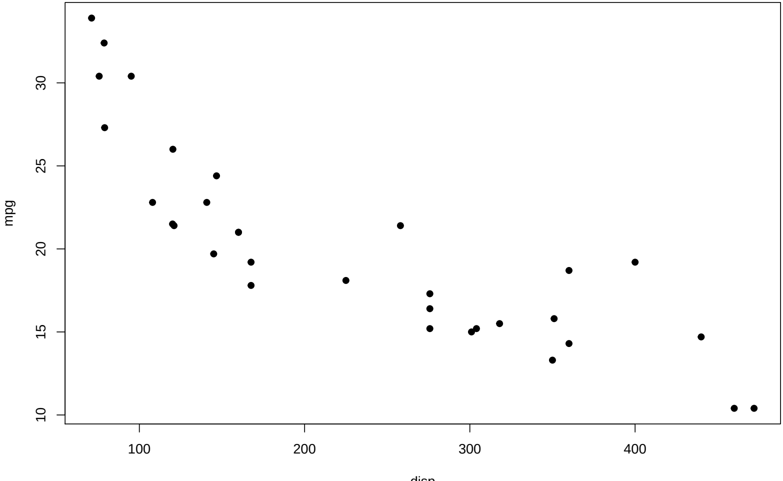
$$cor(X, Y) \cdot \frac{sd(Y)}{sd(X)}$$

is. Recall our Descriptive Statistics part!

Example

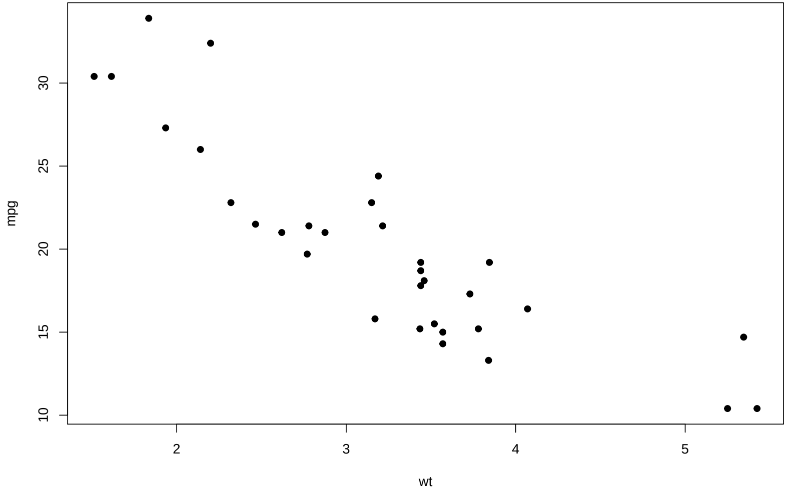
Example: We will use the `mtcars` Dataset from **R**:

```r
plot(mpg ~ disp, data = mtcars, pch = 19)
```



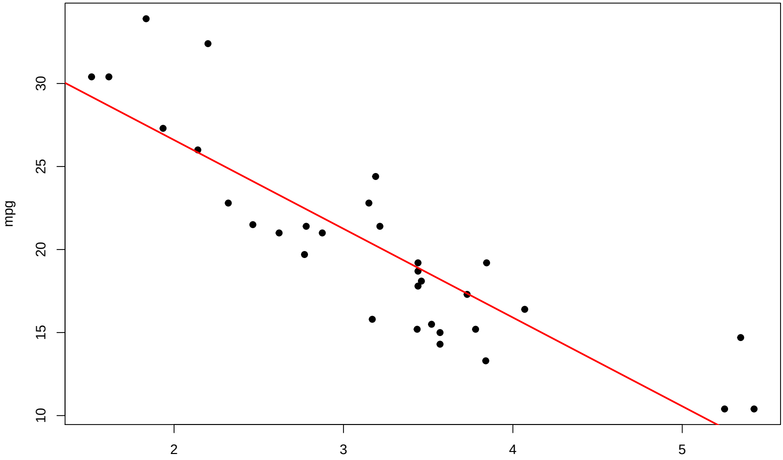
Example, Cont'd

```
plot(mpg ~ wt, data = mtcars, pch = 19)
```



Example, Cont'd

```
plot(mpg ~ wt, data = mtcars, pch = 19)
model <- lm(mpg ~ wt, data = mtcars)
abline(model, col = "red", lwd = 2)
```



Example, Cont'd

```r
model <- lm(mpg ~ wt, data = mtcars)
summary(model)
```

```
##
## Call:
## lm(formula = mpg ~ wt, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.5432 -2.3647 -0.1252  1.4096  6.8727
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  37.2851     1.8776  19.858  < 2e-16 ***
## wt           -5.3445     0.5591  -9.559 1.29e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
##
## Residual standard error: 3.046 on 30 degrees of freedom
## Multiple R-squared:  0.7528, Adjusted R-squared:  0.7446
## F-statistic: 91.38 on 1 and 30 DF,  p-value: 1.294e-10
```

Example, Cont'd

Now, we predict the value of mpg for a new values of a wt Variable:

```r
pred <- predict(model, data.frame(wt=4.7))
pred
```

```
##        1
## 12.16611
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Example, Cont'd

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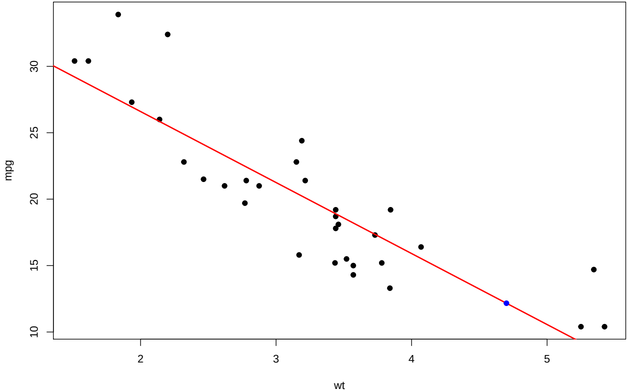

Example

```r
x <- rnorm(100, mean = -1, sd = 1)
y <- runif(100, 2, 10)
z <- 2.7 - 1.7*x + 13.5*y + rnorm(100)
head(x)
```

```
## [1]  0.5271618 -2.0473435  0.4363159 -0.1708086 -0.7298618 -1
```

```r
head(y)
```

```
## [1] 3.977900 9.480366 2.867116 6.922705 7.797675 7.675757
```

```r
head(z)
```

```
## [1]  55.39792 133.41440  42.64337  95.71013 109.41316 108.627
```

Example

```r
mod1 <- lm(z ~ x); summary(mod1)
```

```
##
## Call:
## lm(formula = z ~ x)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -58.755 -27.454   2.906  28.465  50.215
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   84.135      4.202  20.022   <2e-16 ***
## x             -3.130      2.917  -1.073    0.286
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
##
## Residual standard error: 31.23 on 98 degrees of freedom
## Multiple R-squared:  0.01161,    Adjusted R-squared:  0.00152
## F-statistic: 1.152 on 1 and 98 DF,  p-value: 0.2859
```

Example, Cont'd

```r
mod2 <- lm(z ~ x + y); summary(mod2)
```

```
##
## Call:
## lm(formula = z ~ x + y)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.25835 -0.69422 -0.04329  0.72539  2.12851
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.92180    0.29018   10.07   <2e-16 ***
## x           -1.50675    0.09194  -16.39   <2e-16 ***
## y           13.46759    0.04283  314.41   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
##
## Residual standard error: 0.9827 on 97 degrees of freedom
## Multiple R-squared:  0.999,  Adjusted R-squared:  0.999
## F-statistic: 5.001e+04 on 2 and 97 DF,  p-value: < 2.2e-16
```

Properties of the Estimators: $\hat{\beta}_0$ and $\hat{\beta}_1$

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Fact 1: Estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ are UnBiased:

$$\mathbb{E}(\hat{\beta}_0) = \beta_0, \qquad Var(\hat{\beta}_0) = \frac{\sigma^2}{n} \cdot \frac{\sum_{k=1}^n X_k^2}{\sum_{k=1}^n (X_k - \overline{X})^2}$$

$$\mathbb{E}(\hat{\beta}_1) = \beta_1, \qquad Var(\hat{\beta}_1) = \frac{\sigma^2}{\sum_{k=1}^n (X_k - \overline{X})^2}$$

Properties of the Estimators: $\widehat{\sigma^2}$

Fact 2: Assume σ^2 is unknown.

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$$s^2 = \frac{1}{n-2} \cdot \sum_{k=1}^{n} (\hat{\varepsilon}_k)^2$$

is an UnBiased Estimator for σ^2, and

$$\widehat{\sigma^2} = \frac{1}{n} \cdot \sum_{k=1}^{n} (\hat{\varepsilon}_k)^2$$

is the MLE for σ^2.

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is the MLE for σ^2. Here

$$\hat{\varepsilon}_k = Y_k - \hat{\beta}_0 - \hat{\beta}_1 \cdot X_k$$

is the k-th **residual**.

Goodness-of-Fit Tests

Intro to GoF Tests

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First we consider Pearson's χ^2-Test: a famous GoF Test for the Multinomial Distribution.

Goodness-of-Fit Tests: Pearson's χ^2 Test

Model: Here we assume that the result of an Experiment can be one of the $A_1, ..., A_m$ (different classes), with Probabilities

$$p_1 = \mathbb{P}(A_1), ..., p_m = \mathbb{P}(A_m).$$

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Data: We have the results of a repetition of the previous Experiment: The results are: the number of A_1 shown is X_1, the number of A_2 shown is X_2, . . . , the number of A_m shown is X_m;

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Significance Level: $\alpha \in (0, 1)$;

Goodness-of-Fit Tests: Pearson's χ^2 Test

Test Statistics:

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Test Statistics: $\chi^2 = \sum_{k=1}^{m} \frac{(X_k - n \cdot p_k)^2}{n \cdot p_k}$

Goodness-of-Fit Tests: Pearson's χ^2 Test

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Rejection Region: $\chi^2 > \chi^2_{m-1,1-\alpha}$

Example

Example: I am claiming that, for my Stat courses, the percentage of A-grade students is 15%, of B-grade students is 25%, of C-grades are 20%, for D I have 15%, and all others are Failing the course.

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$$\#A = 27,\ \#B = 22,\ \#C = 10,\ \#D = 10,\ \#F = 12.$$

Is this Data supporting my claim?

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Solution: We have $n = 27 + 22 + 10 + 10 + 12 = 81$. Next, we make the Table:

	A	B	C	D	E
Obs. Frq., O_k	27	22	10	10	12
Exp. Frq., E_k	$81 \cdot 0.15$	$81 \cdot 0.25$	$81 \cdot 0.2$	$81 \cdot 0.15$	$81 \cdot 0.25$

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Now, we can calculate the TS:

$$\chi^2 = \sum_{k=1}^{5} \frac{(O_k - E_k)^2}{E_k} = \frac{(27 - 81 \cdot 0.15)^2}{81 \cdot 0.15} + ... + \frac{(12 - 81 \cdot 0.15)^2}{81 \cdot 0.15}$$

Example, Cont'd

The rest is in **R**:

```
obsd <-  c(27, 22, 10, 10, 12)
expd <- 81* c(0.15, 0.25, 0.2, 0.15, 0.25)
xi2 <- sum((obsd-expd)^2/expd)
xi2
```

```
## [1] 24.41564
```

```
q <- qchisq(1-0.05, df = length(obsd)-1)
q
```

```
## [1] 9.487729
```

```
xi2 > q
```

```
## [1] TRUE
```

Example, Cont'd

```r
obsd <-  c(27, 22, 10, 10, 12)
p <-  c(0.15, 0.25, 0.2, 0.15, 0.25)
chisq.test(obsd, p = p)
```

```
##
##  Chi-squared test for given probabilities
##
## data:  obsd
## X-squared = 24.416, df = 4, p-value = 6.592e-05
```

Kolmogorov-Smirnov Test

```r
x <- rnorm(50, mean = 3, sd = 1)
ks.test(x, y = "pnorm", mean = 0, sd = 1)
```

```
##
##  One-sample Kolmogorov-Smirnov test
##
## data:  x
## D = 0.87385, p-value = 8.882e-16
## alternative hypothesis: two-sided
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##  One-sample Kolmogorov-Smirnov test
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## data:  x
## D = 0.87385, p-value = 8.882e-16
## alternative hypothesis: two-sided
```

```r
x <- rexp(50, rate = 3.1)
ks.test(x, y = "pnorm", mean = 0, sd = 1)
```

```
##
##  One-sample Kolmogorov-Smirnov test
##
## data:  x
## D = 0.50157, p-value = 3.672e-12
## alternative hypothesis: two-sided
```

Example

```r
x <- runif(40)
y <- rexp(30)
ks.test(x,y)
```

```
##
##  Two-sample Kolmogorov-Smirnov test
##
## data:  x and y
## D = 0.54167, p-value = 4.101e-05
## alternative hypothesis: two-sided
```

Shapiro-Wilk test

```r
x <- rnorm(25, mean = -2, sd = 10)
shapiro.test(x)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  x
## W = 0.94969, p-value = 0.2467
```

Fitting a Distribution Family, **R**

```r
library(MASS)
x <- rexp(45, rate = 3.4254)
fitdistr(x, densfun = "exponential")
```

```
##      rate
##   4.8935081
##  (0.7294811)
```

I FSYO!