AUA CS108, Statistics, Fall 2020

Lecture 17

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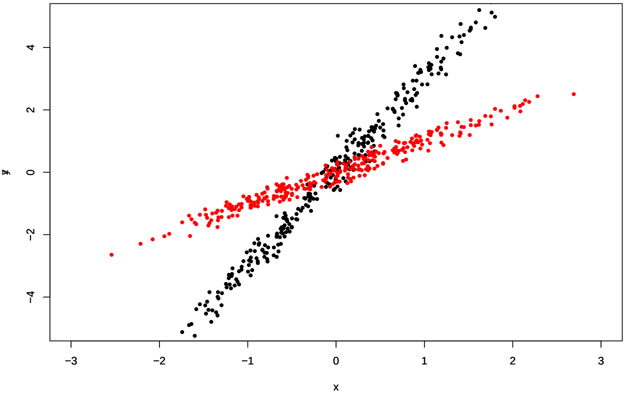
05 Oct 2020

Contents

- Sample Correlation Coefficient
- Quick reminder on R.V.s
- Important Discrete Distributions

Example

For which of the following pairs the Correlation is higher ((x, y) pairs are in black, and (x, z) pairs are in red)?



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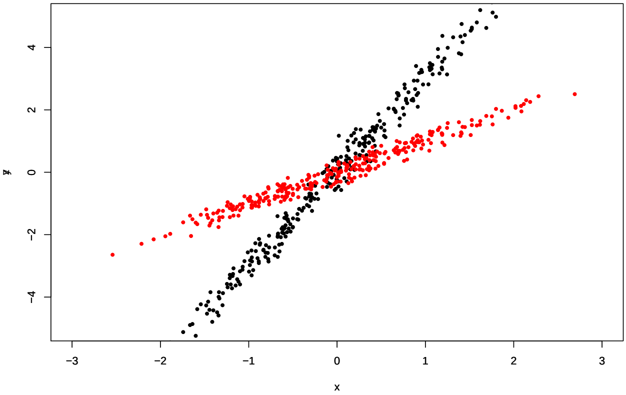

```
c(cor(x,y), cor(x,z))
```

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## [1] 0.9941472 0.9831613
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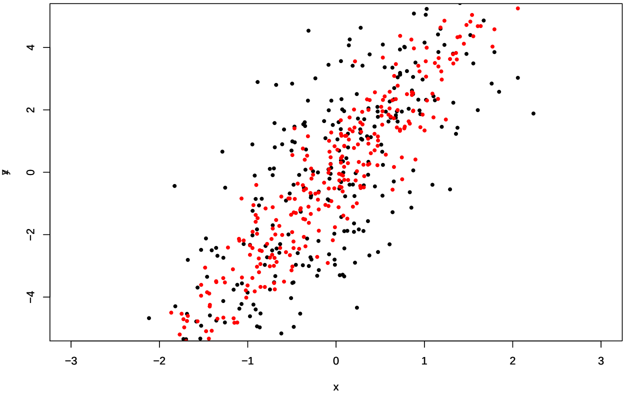

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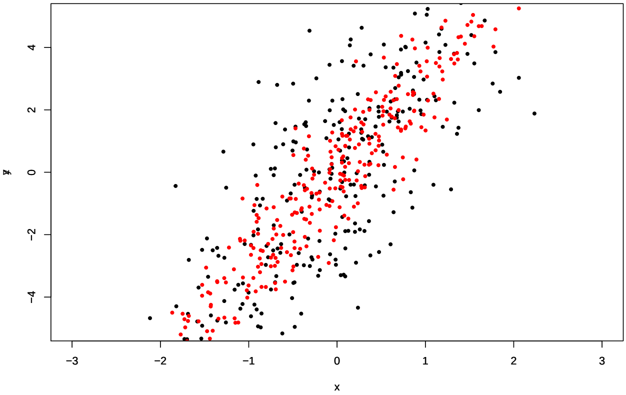

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## [1] 0.7955134 0.9440403
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Note: We will talk about this and about the relationship of slope with the Correlation Coefficient during the Linear Regression lectures.

Correlation is a Measure of Linear Relationship

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x <- runif(2000, -2,2)
y <- x^2 + 0.3*rnorm(2000)
plot(x,y, pch = 20)
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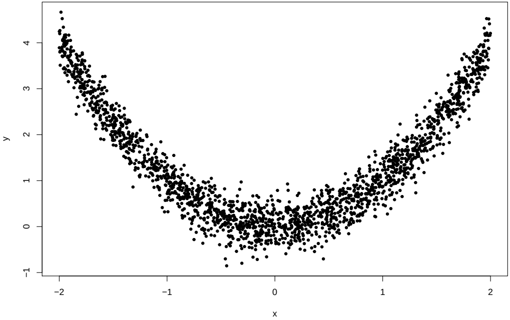

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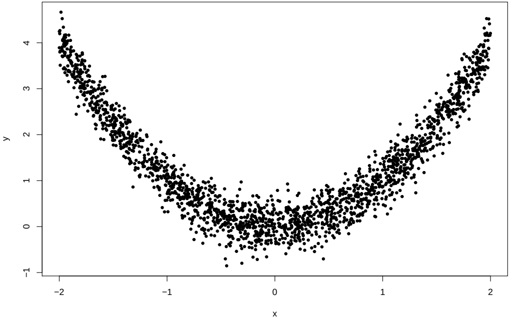

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See more at Wiki

Another Relationship between the Correlation and Covariance

Assume we have two datasets x and y of the same size. We standardize them, i.e., we consider

$$\frac{x-\bar{x}}{s_x}, \qquad \frac{y-\bar{y}}{s_y},$$

then the Correlation Coefficient is just the Covariance between these standardized daatasets:

$$cor(x,y) = cov\left(\frac{x-\bar{x}}{s_x}, \frac{y-\bar{y}}{s_y}\right).$$

Correlation is not Causation

- Some Examples: Spurious Correlations

Supplements, Other Measures of Correlation

- if working with several variables, we can calculate pairwise Correlations (Correlation Matrix) and plot the HeatMap

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- If working with multiple variables, one can calculate the Multiple correlation
- One can interpret the Correlation Coefficient as a Cosine of the angle between the r.v.s (or observations), see Wiki
- There are other measures of Association between variables, such as Rank Correlations, say, Kendal's τ

In **R**, the `cor` function has a parameter *method*, where you can change the Correlation Coefficient type.

Reminder on Random Variables and Distributions

Random Variables

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So $X = X(\omega)$, but usually we forget about ω, and use X.

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So for a Continuous r.v., another complete characteristic, besides the CDF, is its PDF.

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or, in a table form,

Values of X	x_1	x_2	...
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- The Variance

$$Var(X) = \mathbb{E}\Big((X - \mathbb{E}(X))^2\Big) = \mathbb{E}(X^2) - \Big[\mathbb{E}(X)\Big]^2.$$

Important Discrete Distributions

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