AUA CS 108, Statistics, Fall 2019

Lecture 14

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Last Lecture ReCap

- Why we use the Sample Covariance?

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- Why we use the Sample Covariance?
- Why we use the Sample Correlation Coefficient?

Important Discrete Distributions

Bernoulli Distribution

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- Example:

```
rgeom(10,prob = 0.3)
```

```
##  [1] 1 2 6 0 2 4 4 3 1 4
```

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- Example:

```
rpois(10, lambda = 2)
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Important Continuous Distributions

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runif(10, min = 2, max = 5)
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- Models: Different things. Important is because of the CLT

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- Parameters: μ (mean) and σ^2 (variance);
- Notation: $X \sim \mathcal{N}(\mu, \sigma^2)$;
- Support: $\mathbb{R}$
- PDF:
$$f(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$
- Mean and Variance: $\mathbb{E}(X) = \mu$, $Var(X) = \sigma^2$.
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- Example:

```
rnorm(10, mean = 2, sd = 3)
```

```
##  [1] -0.4924022  4.0012314  6.4835360  2.3935659  4.966
##  [7]  2.7634180  4.5808330  2.4298559 -1.0900586
```

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Additional Properties:

- If $X \sim \mathcal{N}(\mu, \sigma^2)$, then $Z = \dfrac{X - \mu}{\sigma} \sim \mathcal{N}(0, 1)$ and

$$\mathbb{P}(a < X < b) = \mathbb{P}\left(\frac{a-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right) =$$

$$= \Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right).$$

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- If $X \sim \mathcal{N}(\mu, \sigma^2)$, then

$$\mathbb{P}(-\sigma < X - \mu < \sigma) \approx 0.6827,$$

$$\mathbb{P}(-2\sigma < X - \mu < 2\sigma) \approx 0.9545$$

and

$$\mathbb{P}(-3\sigma < X - \mu < 3\sigma) \approx 0.9973.$$

Additions

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