AUA CS108, Statistics, Fall 2020

Lecture 10

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- BoxPlot
- Sample Quantiles
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Example (Quartiles and IQR)

Example: Find the Quartiles and IQR of

$$x: \ 1, 1, 2, 3, 1, 1, 3, 4, 5, 2$$

Quartiles and IQR

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Note: The interval $[Q_1, Q_3]$ contains almost the half of the Datapoints. So the IQR shows the Spread of the middle half of our Dataset, it is a measure of the Spread/Variability.

Quartiles in **R**

In **R**, one can use the commands `quantile(x, 0.25)` and `quantile(x, 0.75)` to find Q_1 and Q_3.

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x <- 1:10
quantile(x,0.25)
```

```
##  25%
## 3.25
```

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```

```
##  25%
## 3.25
```

If you will not give a parameter to `quantile`, **R** will calculate 0% (minimum datapoint), 25%, 50%, 75% and 100% (maximum datapoint) quartiles:

```
x <- 1:10
quantile(x)
```

```
##    0%   25%   50%   75%  100%
##  1.00  3.25  5.50  7.75 10.00
```

Quartiles in **R**

Also, you can use the following commands:

```
x <- 1:10
fivenum(x)
```

```
## [1]  1.0  3.0  5.5  8.0 10.0
```

```
summary(x)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    1.00    3.25    5.50    5.50    7.75   10.00
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```

To calculate the IQR in **R**, we can use the `IQR` command:

```
x <- 1:10
IQR(x)
```

```
## [1] 4.5
```

Note

Note: Please note that **R** is not using our definition of the Quartiles, so sometimes we will get different results when calculating by a hand or by **R**.

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- The Quartiles $Q_1, Q_2 = Median, Q_3$
- the Lower and Upper Fences
  $W_1 = \min\{x_i : x_i \geq Q_1 - 1.5 \cdot IQR\}$ and
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$$\left[Q_1 - \frac{3}{2}IQR,\ Q_3 + \frac{3}{2}IQR\right];$$

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- the set of all Outliers

$$O = \left\{x_i : x_i \notin \left[Q_1 - \frac{3}{2}IQR, Q_3 + \frac{3}{2}IQR\right]\right\}$$

BoxPlot, Example

Then we draw the points W_1, Q_1, Q_2, Q_3, W_2 on the real line and add all outliers, and make a box over $[Q_1, Q_3]$.

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Example: Draw the Boxplot of

$$x: \ 0, -2, 2, 1, 5, 6, 4, 1, 2, 1, 12$$

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Example: Draw the Boxplot of

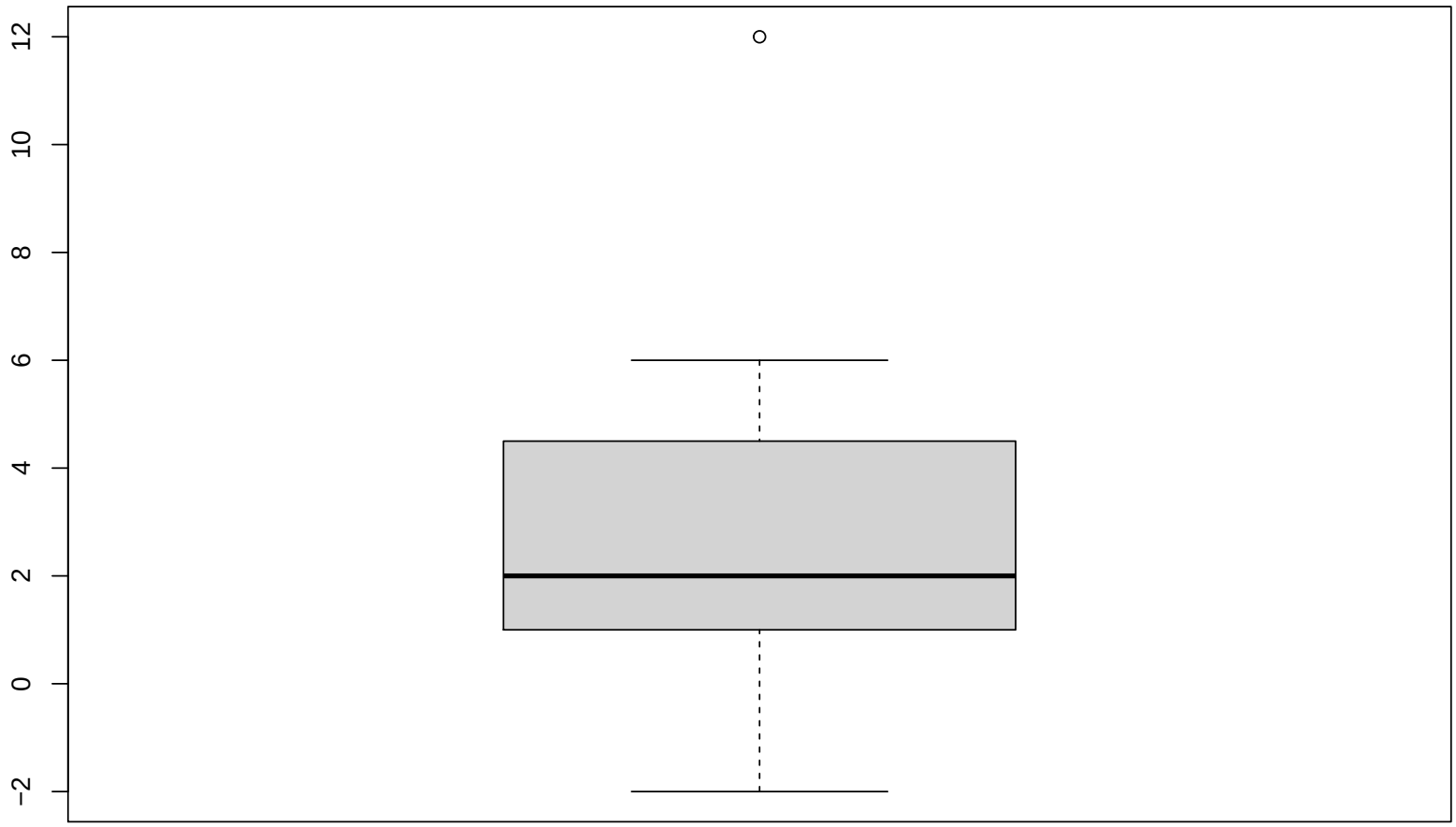
$$x: \ 0, -2, 2, 1, 5, 6, 4, 1, 2, 1, 12$$

Solution: OTB

BoxPlot, Example

Now, using **R**:

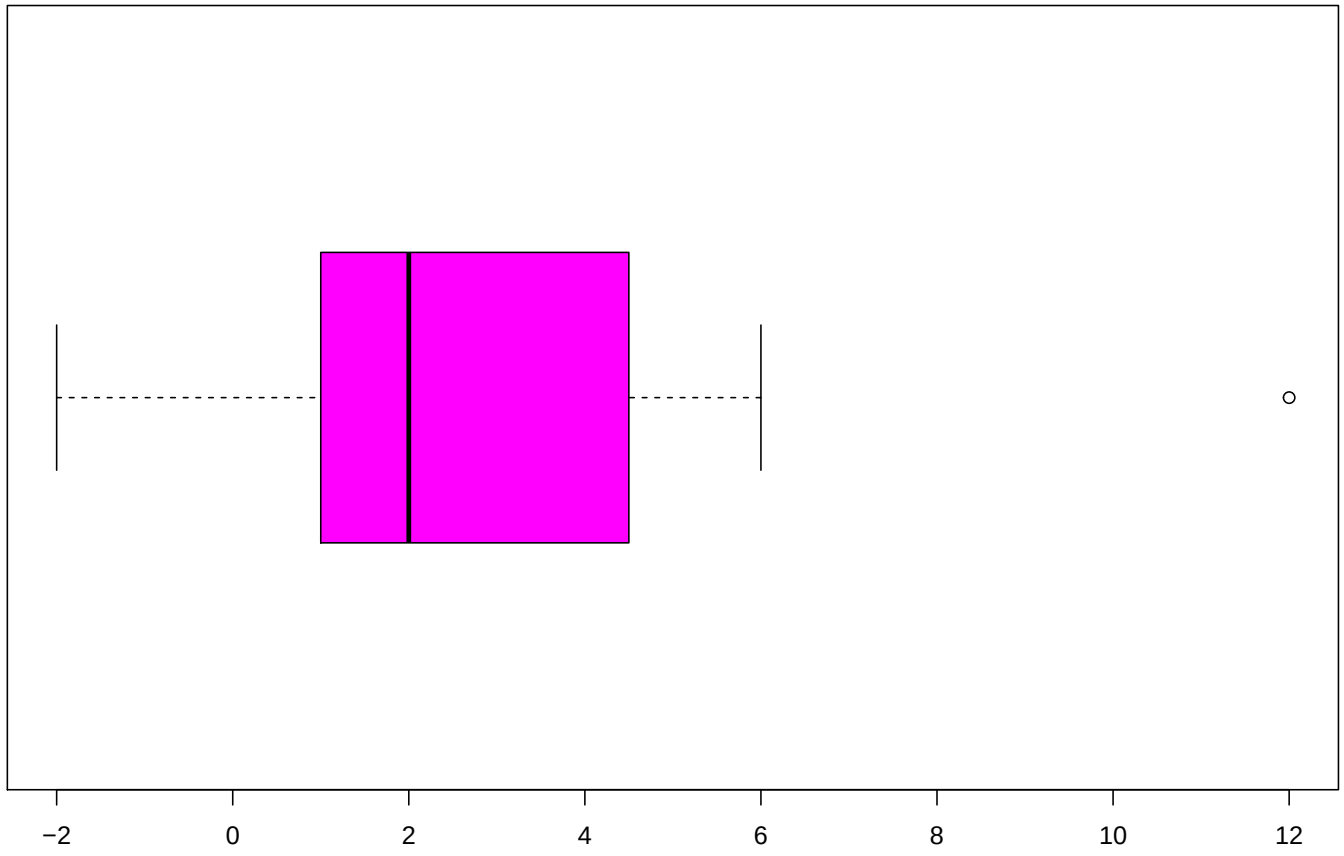
```
x <- c(0, -2, 2, 1, 5, 6, 4, 1, 2, 1, 12)
boxplot(x)
```



BoxPlot, Example

Another view:

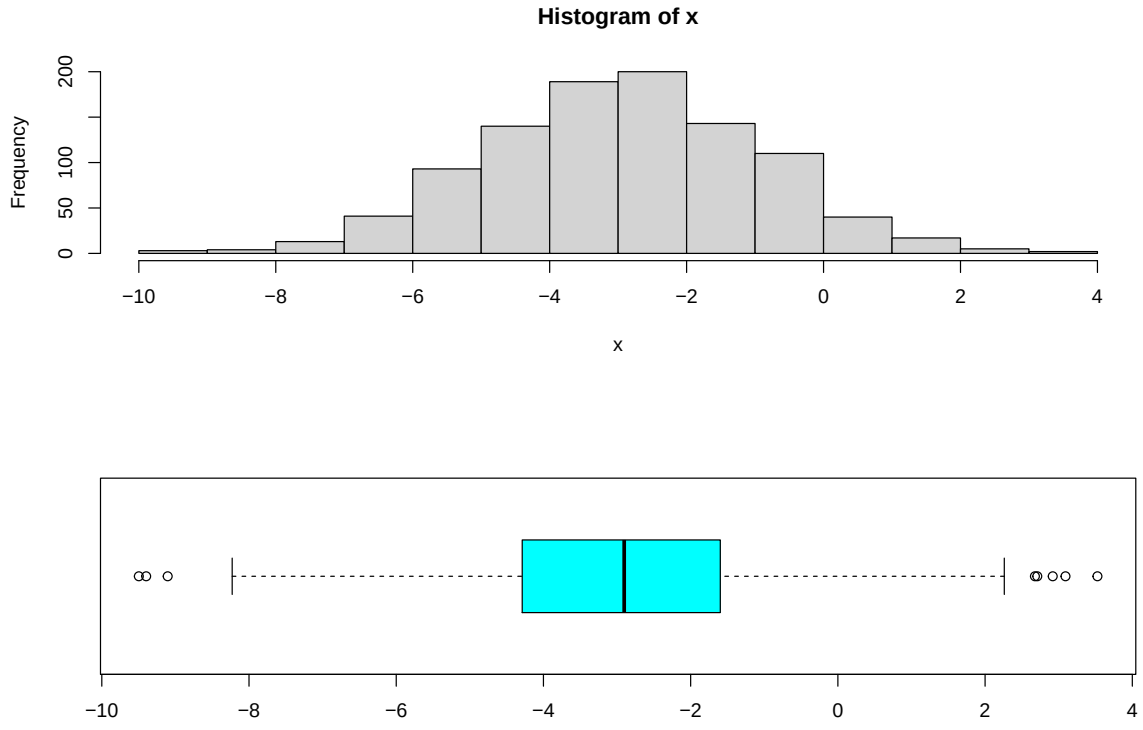
```
x <- c(0, -2, 2, 1, 5, 6, 4, 1, 2, 1, 12)
boxplot(x, horizontal = T, col = "magenta")
```



BoxPlot, Example

Here are some Datasets' Histograms along with the BoxPlots:

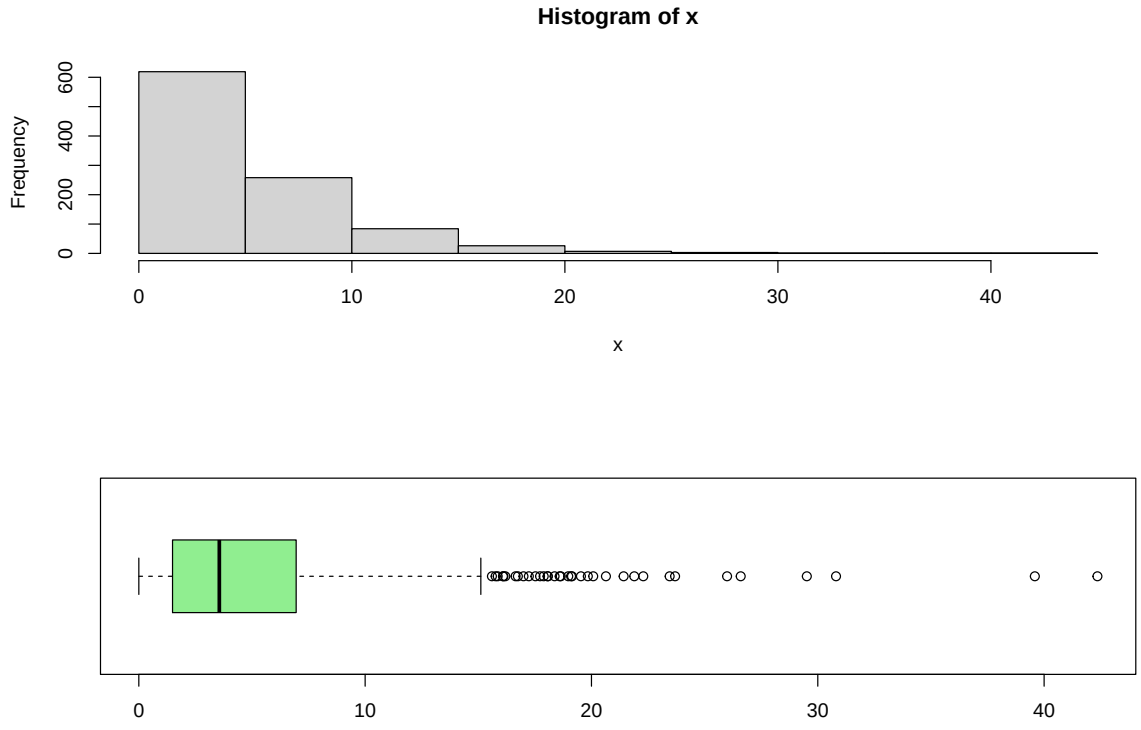
```
x <- rnorm(1000, mean = -3, sd = 2)
par(mfrow=c(2,1)); hist(x)
boxplot(x, horizontal = T, col = "cyan")
```



BoxPlot, Example

Here are some Datasets' Histograms along with the BoxPlots:

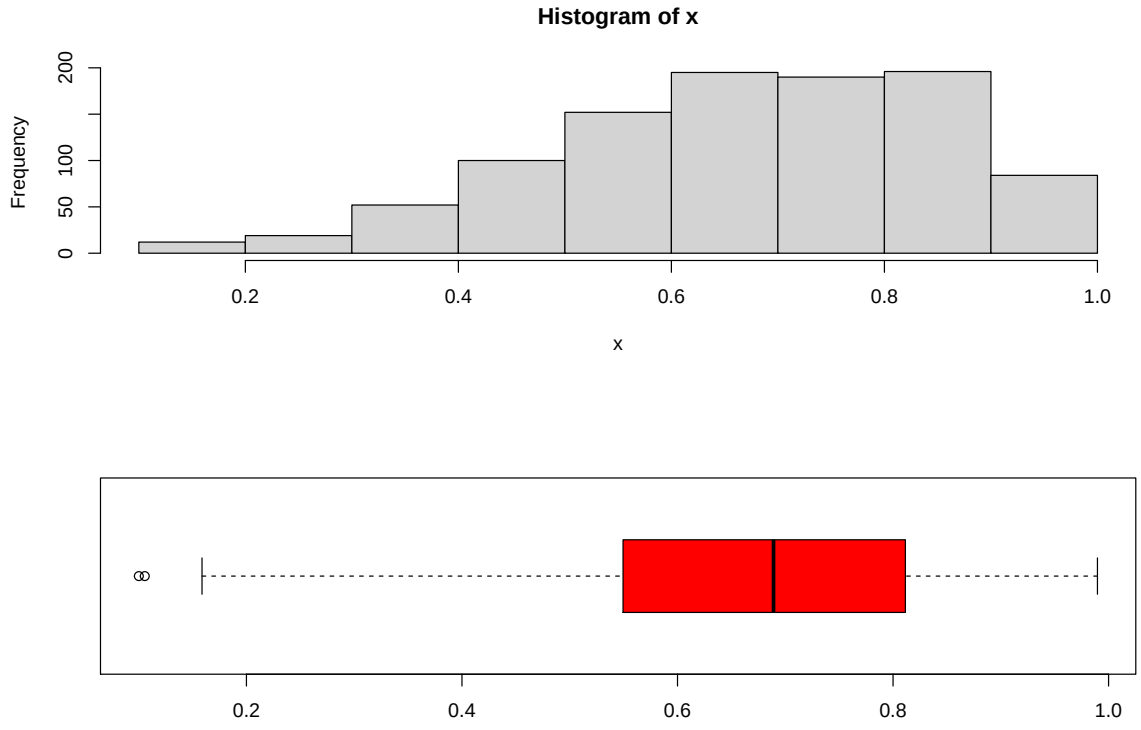
```
x <- rexp(1000, rate = 0.2)
par(mfrow=c(2,1)); hist(x)
boxplot(x, horizontal = T, col = "lightgreen")
```



BoxPlot, Example

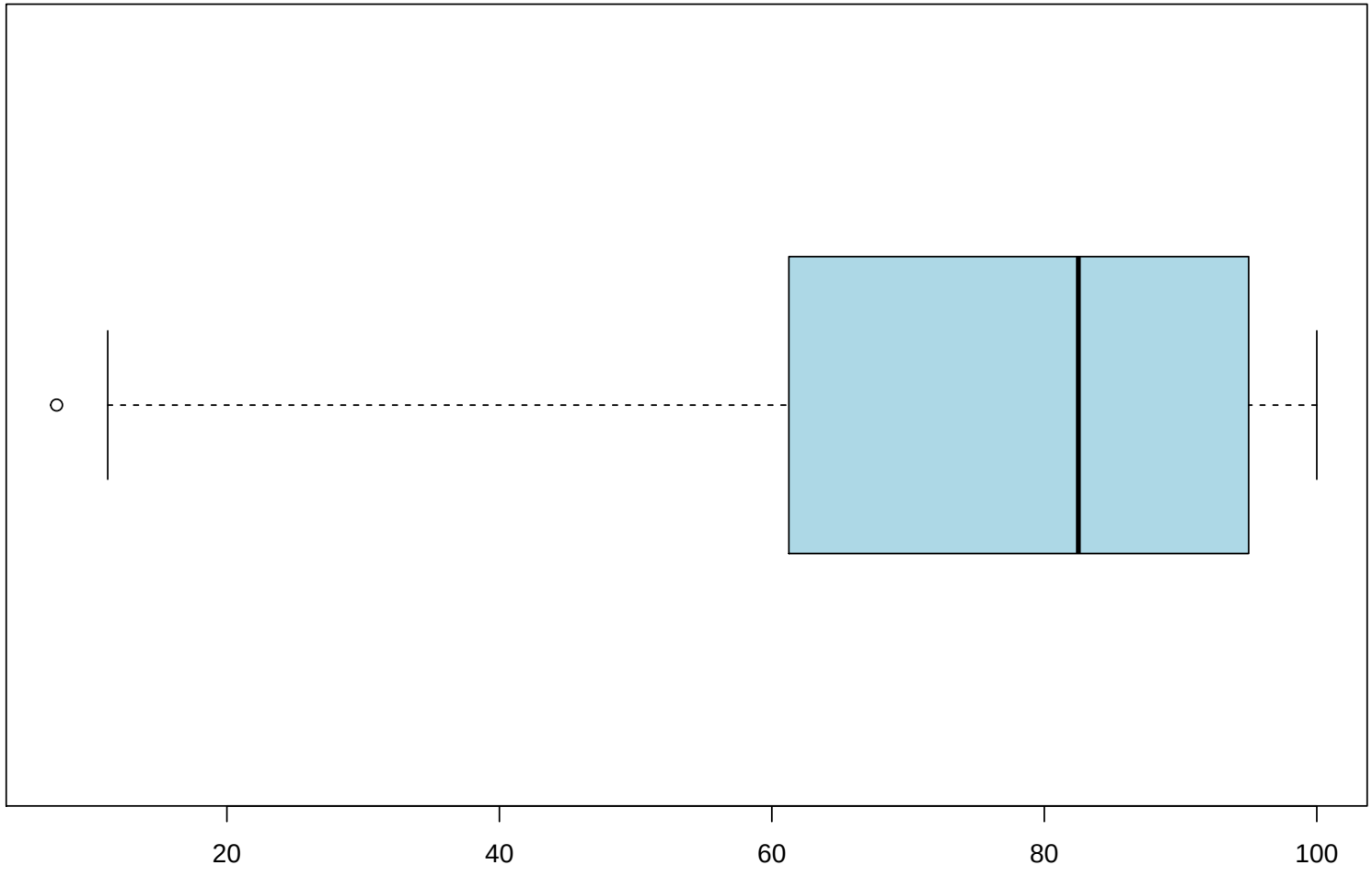
Here are some Datasets' Histograms along with the BoxPlots:

```
x <- rbeta(1000, shape1 = 4, shape2 = 2)
par(mfrow=c(2,1)); hist(x)
boxplot(x, horizontal = T, col = "red")
```



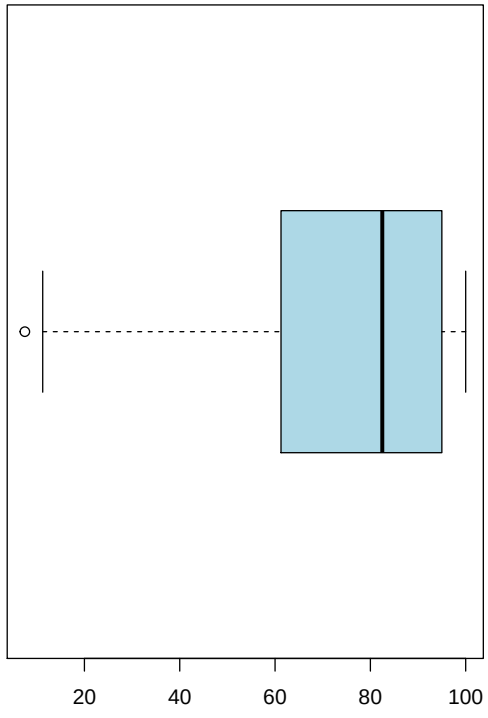
BoxPlot, Example

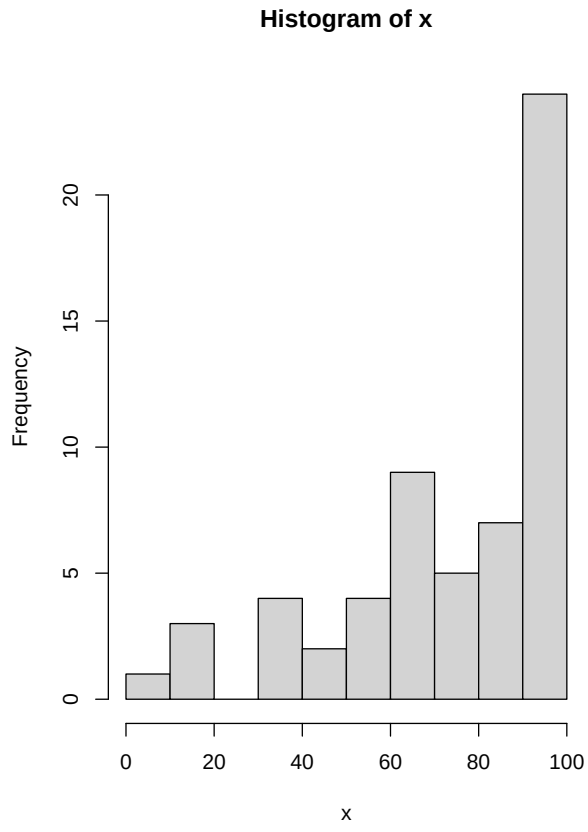
Here is the BoxPlot of our Quiz 01 grades: can you describe the result?



BoxPlot, Example

And here is the BoxPlot of our Quiz 01 grades along with the Histogram:






```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    7.50   61.25   82.50   74.63   95.00  100.00
```