CS 107, Probability, Spring 2019
Lecture 21

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AUA

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Content

- Discrete r.v., PMF
- Continuous r.v., PDF

Zeno's Paradoxes

- To reach the wall in front of me, I need to pass the half of the way first.

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- To reach the wall in front of me, I need to pass the half of the way first. Then I need to pass the half of the remaining half.

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LZ

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- Achilles and the tortoise paradox, read the Wiki Page at `https://en.wikipedia.org/wiki/Zeno's_paradoxes`

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- Graph the PMF of X;
- Graph the CDF of X.

The Graph of CDF of a Discrete r.v

In general, if the r.v. X is Discrete, given through its PMF:

Values of X	x_1	x_2	x_3	$\dots$
$\mathbb{P}(X = x)$	p_1	p_2	p_3	$\dots$

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Graph on the board!

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Give the graph!

Continuous R.V.

Intro: Continuous Random Variables

There are many r.v.s that are not Discrete. For example, the lifetime of a light bulb, the height of a (randomly chosen) person, the exact age (not rounded) of a person etc, the waiting time for the quiz grades

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In fact, there is a famous Lebesgue Decomposition Theorem:

Intro: Continuous Random Variables

Lebesgue Decomp Theorem, 18+

Any r.v. X can be represented in the form:

$$X = X_{discr} + X_{cont} + X_{sing},$$

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We know already what Discrete r.v.s are, the next slides introduce Continuous r.v., and we will not talk about the Singular ones, because they are Singular $\ddot\smile$

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Continuous r.v.

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$$F(x) = \int_{-\infty}^{x} f(t)dt, \qquad \forall x \in \mathbb{R}.$$

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Remark: By a Theorem of $RA \cup Calc2$, if X is a continuous r.v., then $F(x)$ will be continuous at every x.

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Characterization of PDF

What are the characteristic properties of a PDF? The following Theorem give necessary and sufficient conditions in a function for being a PDF for some r.v.

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Inversely, if f is an integrable function satisfying the above two conditions, then there is a Probability Space and a r.v. on that Space such that f is the PDF of that r.v.

Geometric Interpretation of PDF

Recall that

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And from $RA \cup Calc2$ you know that the integral on the RHS is the *Area under the curve* $y = f(x)$ *over the interval* $(-\infty, x]$. Hence,

$$\mathbb{P}(X \leq x) = F(x) = Area(\text{region under } y = f(x) \text{ on } (-\infty, x]).$$

Geometric Interpretation of PDF

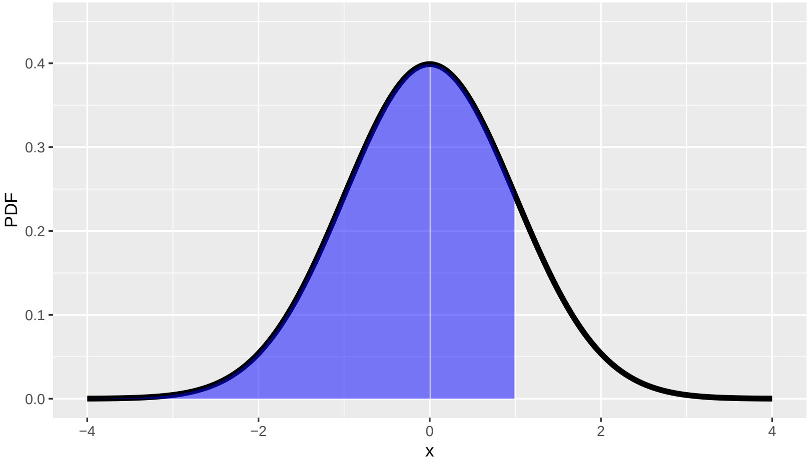
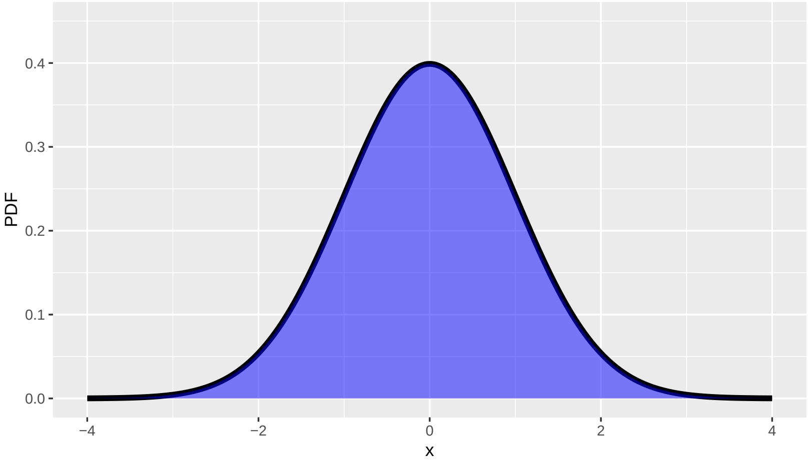


Figure: The Shaded Region area is $\mathbb{P}(X \leq 1) = F(1)$

Geometric Interpretation of PDF

Fact: The area under the PDF graph is 1.



Using PDF to calculate Probabilities

Recall again that, if f and F are the PDF and CDF of r.v. X, then for any $x \in \mathbb{R}$,

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$$=$$

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$$\mathbb{P}(a < X \le b) = F(b) - F(a) = \int_{-\infty}^{b} f(x)dx - \int_{-\infty}^{a} f(x)dx =$$

$$= \int_{a}^{b} f(x)dx.$$

So we have obtained the following relation:

$$\mathbb{P}(a < X \le b) = \int_{a}^{b} f(x)dx.$$

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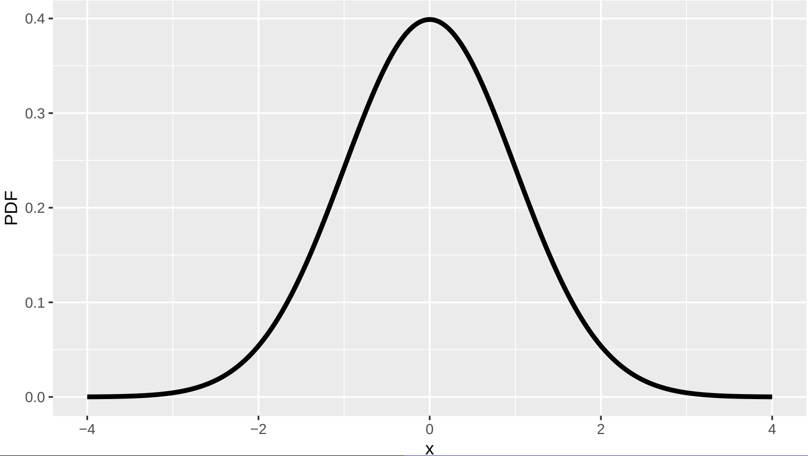
$$= \mathbb{P}(a < X < b) = \int_a^b f(x)dx$$

Example:

Example: Below is the graph of the PDF $f(x)$ for some r.v. X.

- Which one is larger: $\mathbb{P}(X = 0)$ or $\mathbb{P}(X = 2)$?



Example:

- Show geometrically and calculate approximately $\mathbb{P}(X = 0)$, $\mathbb{P}(X < -1)$, $\mathbb{P}(1 \leq X \leq 2)$