AUA CS 108, Statistics, Fall 2019

Lecture 29

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Contents

- The Maximum Likelihood Method (MLE)

Last Lecture ReCap

- I was asking: *Any Questions*? What was the answer?

The Maximum Likelihood Method

Examples, MLE

Example: Find the MLE Estimator for (μ, σ^2) in the $\mathcal{N}(\mu, \sigma^2)$ Model.

Solution: OTB

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$$0, 1, 1, 2, 1, 0, 0, 1, 1$$

from the following Model:

X	0	1	2
$\mathbb{P}(X = x)$	$\frac{\theta}{10}$	$\frac{\theta}{5}$	$1 - \frac{3\theta}{10}$,

where $\theta \in [0, \frac{10}{3}]$.

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where $\theta \in [0, \frac{10}{3}]$. Find the MLE Estimator and MLE Estimate for θ.

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Examples, MLE

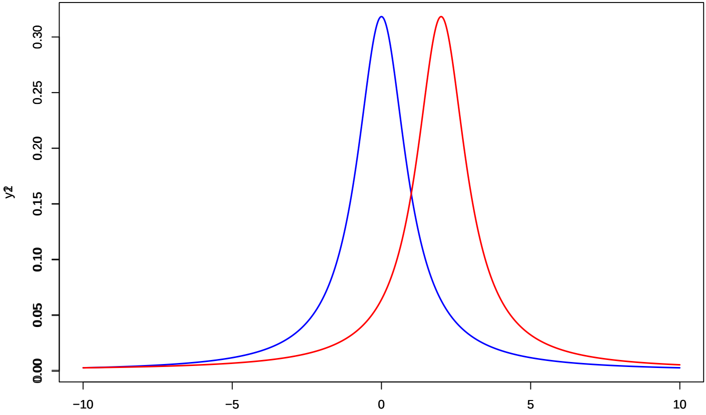
Example: Find the MLE Estimator for θ in the one-Parametric Cauchy Distribution *Cauchy*(θ) Model. Here, the PDF of $X \sim \mathit{Cauchy}(\theta)$ is given by

$$f(x|\theta) = \frac{1}{\pi(1 + (x - \theta)^2)}, \qquad x \in \mathbb{R},$$

and $\theta \in \mathbb{R}$ is called the *location parameter*.

PDF of Cauchy(0) and Cauchy(2)

```
x <- seq(from = -10, to = 10, by = 0.01)
y1 <- dcauchy(x); y2 <- dcauchy(x, location = 2);
plot(x, y1, type = "l", lwd = 2, xlim = c(-10,10), col = "blue")
par(new = T)
plot(x, y2, type = "l", lwd = 2, xlim = c(-10,10), col = "red")
```



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Note: Do you know what is the Distribution of the ratio $\dfrac{X}{Y}$, when X and Y are Independent Standard Normal Rvs? Cauchy(0) ! $\ddot\smile$