CS 107, Probability, Spring 2020
Lecture 02

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Content

- Experiment, Outcomes and the Sample Space
- Events, Operations with Events

LZ

Assume our mobile phone Weather App says that there is a 50% chance of snow for this Saturday, and also 50% chance of snow this Sunday.
Is it true that it will snow for sure (i.e., with probability 1) this weekend?

Last Lecture Recap

- We run over the Syllabus: MP OH is on Wed, 11:30 - 13:30
- PSS, Tuesday, 11:00-12:30, Gayane Tonoyan's OH 12:30 -14:30
- We use Probability to model Mathematically the uncertainty, to talk about/study phenomena with unpredictable (random) outcomes
- Probabilists do Experiments with coins, dice, cards, darts and things like these, and various situations can be modeled by these type of Experiments

Modeling by Coins/Dice/Balls...

In general,

- Coin tossing type Experiments model Binary Outcome ($0/1$, *Success*/*Failure*, *Yes*/*No*, *Loves*/*Doesn't* ...) phenomena;
- Die rolling type Experiments model multiple-Outcome phenomena, where all Outcomes are Equiprobable;
- Ball picking type Experiments model multiple-Outcome phenomena, where Outcomes can have different Probabilities;
- Cards picking type Experiments model multiple-Outcome, multiple class phenomena - we can differentiate cards by their color, nomination, suit. Say, we can talk about choosing a person at random: we can differentiate persons by their gender (model by a color), age groups (model by suits), birthplace (model by, say, nominations).

Modeling by Coins/Dice/Balls...

- Darts throwing type Experiments model infinite-Outcome Equiprobable (*Uniform*) geometric phenomena.

Q: Can you give different examples of the above type phenomena?

Experiment, Outcomes and the Sample Space

- A **random (or probabilistic) Experiment** is a situation, where we are uncertain about the result.
- An **Outcome** is a possible result of an Experiment.

So if our "Experiment" has just one Outcome, then it is not a random Experiment, it is not an Experiment in our sense.

- The set of all Outcomes of an Experiment is called the **Sample Space** of that Experiment:

$$\Omega = \text{the Sample Space of the Experiment} =$$

$$= \text{the set of all outcomes of our Experiment}$$

Examples

- Our Experiment: we are tossing a (fair) coin.
- **Heads** is one of the outcomes.
- The Sample Space in this Example is:

$$\Omega = \text{Sample Space} = \{\text{Heads, Tails}\} = \{H, T\}$$

Examples

- Experiment: we are rolling a (fair) die.
- One of the outcomes is  .
- The Sample Space in this Example is:

Examples

- Experiment: we are watching (interested) if the produced detail is defective.
- One of the outcomes is that it is Not Defective.
- The Sample Space in this Example is:

Examples

- Experiment: we are interested in the remaining lifetime (in years) of a person (for insurance reasons, say).
- One of the outcomes is 30.1.
- The Sample Space in this Example is:

Examples

- Experiment: we are watching for the Google Stock closing Price for the next Friday.
- One of the outcomes is $1501.31\$$.
- The Sample Space in this Example is:

Examples

- Experiment: we are rolling 2 dice.
- Important: we need to specify (depends on the problem we are solving) if the order is important for us or not.

Case 1: Order is important. **Example:** 2 persons are rolling dice - wins the person obtaining the largest number.

- Some outcomes are $(1,1), (2,3), (3,2)$
- Say, first person rolls a black die, the second one - white:

- The Sample Space is:

Examples

- Experiment: we are rolling 2 dice.
- Important: we need to specify (depends on the problem we are solving) if the order is important for us or not.

Case 2: Order is not important. **Example:** 2 persons are playing Nardi at Besedka.

- Some outcomes are
  $(1,1), (2,3), \textit{panj} - \textit{u} - \textit{du}, \textit{dord} - \textit{djhar}$
- In this case rolling $(2,3)$ or $(3,2)$ are the same
- The Sample Space is:

Events

Usually in the Probability Theory (and in real-life situations), we are interested not in particular outcomes and their probabilities, but also in some groups of outcomes and their probabilities.
An **Event** in some Experiment is some collection of Outcomes of that Experiment. Mathematically, every subset of the Sample Space is called an **Event**[1].

[1] Some Restrictions may apply!

Events Examples

- Experiment: Rolling a die
- Sample Space $= \Omega = \{1, 2, 3, 4, 5, 6\}$
- Some Events:
  - The Result is Odd $= \{1, 3, 5\}$
  - The Result is larger than 2 $= \{3, 4, 5, 6\}$
  - Any Result $= \Omega$
  - No Result $= \varnothing$

Events Examples

- Experiment: Waiting Time (in minutes) for the Metro train
- An example of an outcome: 3.24. Another one: π
- Sample Space $= \Omega = [0, 20]$
- It is not interesting to have the probability of one outcome: say, what is the probability that the waiting time will be 3.24312456231? **Exactly**, I mean. The answer is 0.
- So in this case we are interested in events' probabilities rather than in particular outcome probability.
- Some Events:
  - The WT is larger than $3 = (3, 20]$
  - The WT is between 2 and 5, included $= [2, 5]$
  - The WT is anything $= \Omega$
  - No Result $= \varnothing$

Operations with Events

Assume we have an Experiment with a Sample Space Ω. And assume A and B are two Events in this experiment, i.e.,

$$A \subset \Omega \qquad \textit{and} \qquad B \subset \Omega.$$

Then we can obtain new Events from A and B:

- $A^c = \Omega \setminus A$ - the complement of A, the negation of A;
- $A \cap B$ - the intersection of A and B, the Event "A and B";
- $A \cup B$ - the union of A and B, the Event "A or B";
- $A \setminus B$ - the Event "A but not B" ...

And having Events $A_1, A_2, ..., A_k, ...$ in our Experiment, i.e., $A_k \subset \Omega$, we can form

$$\bigcup_k A_k, \qquad \bigcap_k A_k, ...$$

Some Questions to Answer

- What is an Experiment?
- What is an Outcome?
- What is the Sample Space?
- What is an Event?
- Is an Outcome an Event?
- If the Sample Space of an Experiment has cardinality n, $\#\Omega = |\Omega| = card(\Omega) = n$ (i.e., the number of elements in Ω is n), how many different Events our Experiment have?