CS 107 Section B - Probability

Spring 2019, AUA

Homework No. 07

Due time/date: 10:35AM, 22 March, 2019

Note: Supplementary Problems will not be graded, but you are very advised to solve them and to discuss later with TA or Instructor.

Note: Please provide your answers in the form of a decimal number, by calculating and simplifying fractions, with the accuracy of 2 digits after the period.

Problem 1. We are rolling two dice. Let X be the r.v. showing the maximum of the two numbers shown on the top faces. Construct the PMF of X.

Problem 2. Assume X is a discrete random variable given by its PMF:

Values of X	-3	2	7
$\mathbb{P}(X = x)$	0.2	0.5	0.3

Find the CDF $F(x)$ of X (i.e., write $F(x)$ in the analytic form) and give the graph of F.

Problem 3. Assume X is a random variable given by its CDF $F(x)$ (see Fig. 1). Find the PMF of X.

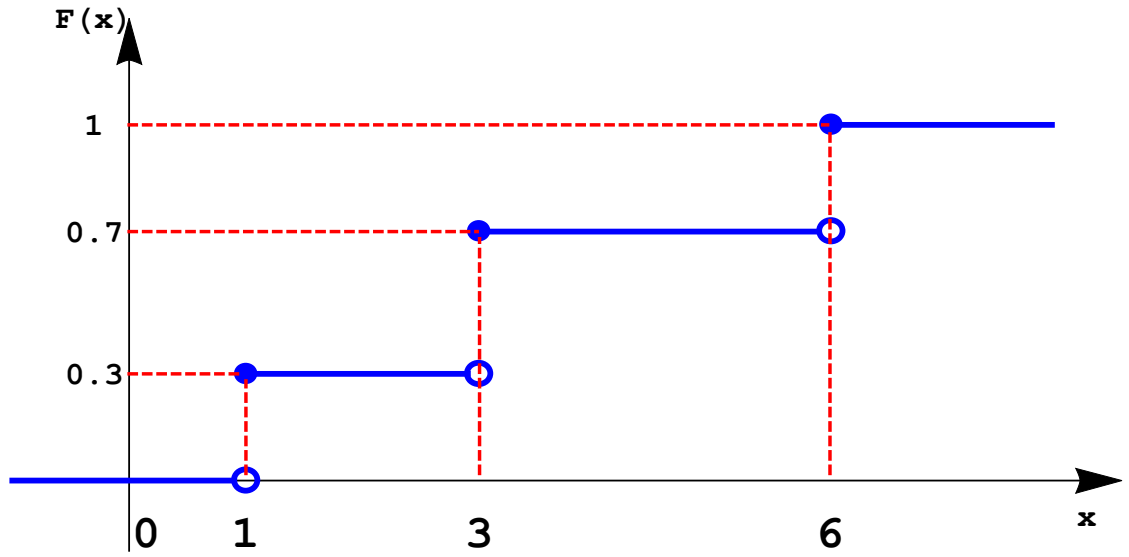

Figure 1: The CDF of X

Problem 4. Fig. 2 shows some function $f(x)$.

a. Is f a PDF for some r.v. X? Explain. If it is, proceed to next tasks, otherwise go to the next problem.

b. Calculate the probability $\mathbb{P}(X = 0.3 \cup X = 7)$.

c. Calculate is the probability $\mathbb{P}(1 \leq X \leq 7)$?

d. What is the range of X (see the note above)?

e. Calculate the probability $\mathbb{P}(X \leq 5)$.

f. Which is more probable: $X \in [-1, 1]$ or $X \in [5, 7]$?

g. (Supplementary) Construct the CDF of X.

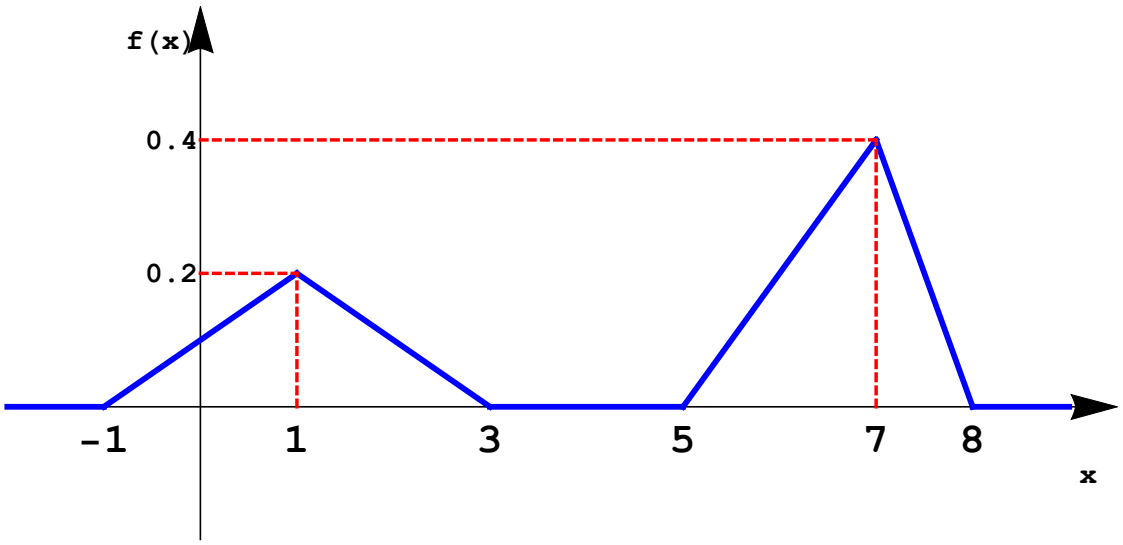

Figure 2: The graph of $f(x)$

Problem 5. Assume the PDF of the r.v. X is given by:

$$f(x) = \begin{cases} c \cdot (x^2 + 1), & \text{if} \quad x \in [0, 1] \\ 0, & \text{otherwise.} \end{cases}$$

a. Find the constant c;

b. Calculate the probability $\mathbb{P}(X \leq 0.2)$;

c. Find the CDF of X.

Problem 6. Let X be a r.v. with the PDF

$$f(x) = \frac{K}{1 + x^2}, \qquad x \in \mathbb{R}.$$

a. Find K;

b. Calculate $\mathbb{P}(X = 2.3)$;

c. Calculate $\mathbb{P}(X \in [0, 1])$;

d. Calculate $\mathbb{P}(X < 1 | X > 0)$.

Problem 7. I have a Bounty Chocolate stick, which is 15cm long, and I will share it with you. I am breaking the chocolate stick at a random place (with uniform probabilities) along the length, and give the right-hand piece to you. My r.v. X is the calories I will get eating my piece, and it is calculated by $X(\omega) = 20 \cdot \omega^2$, where ω is the length of my share.

a. Construct the CDF of X.

Note: Use the definition of the CDF:

$$F(x) = \mathbb{P}(\omega : X(\omega) \leq x), \qquad x \in \mathbb{R}.$$

b. Find the PDF of X.

Problem 8. (from [R][1]) Solve the Problem 4.10, page 163.

Problem 9. (from [R]) Solve the Problem 4.19, page 164.

Problem 10. (from [R]) Solve the Problem 5.2, 5.3 page 212.

[1] Ross's Textbook