AUA CS108, Statistics, Fall 2020

Lecture 43

Michael Poghosyan

07 Dec 2020

Contents

Linear Regression

Intro to Linear Regression

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- to Test a Hypothesis about θ, say, is it likely that $\theta = 3.1415926$ or not.

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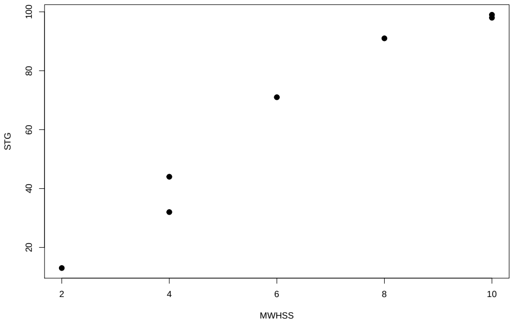

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Say, see Wiki page for BMI.

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The simplest Regression Model is the Linear Model: in the Linear Regression Problem, we assume

$$Y = \beta_0 + \beta^T \cdot \mathbf{X} + \varepsilon,$$

where ε is a r.v., for each value of $\mathbf{X}$, with $\mathbb{E}(\varepsilon) = 0$.

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so our Model, in the expanded way, is

$$Y = \beta_0 + \beta_1 \cdot X^1 + \beta_2 \cdot X^2 + ... + \beta_d \cdot X^d + \varepsilon.$$

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This is called the **Simple Linear Regression Model**.

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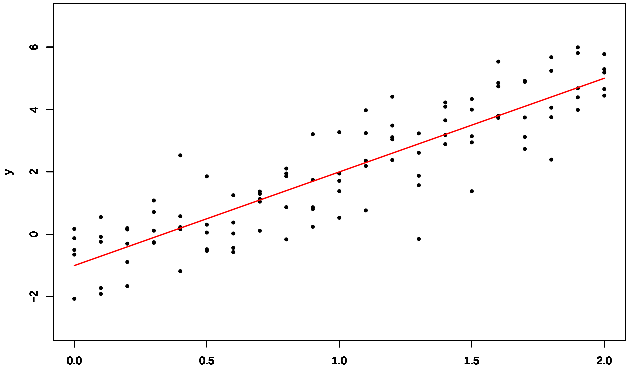

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where ε_k-s are Independent, and our aim is to find good Estimators for β_0 and β_1.

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Now, the idea of the Ordinary Least Squares Method for Estimating the Parameters β_0, β_1 is the following: Find

$$(\hat{\beta}_0, \hat{\beta}_1) = \underset{(\beta_0,\beta_1)\in\mathbb{R}^2}{argmin} \sum_{k=1}^{n} \Big(Y_k - \beta_0 - \beta_1 \cdot X_k \Big)^2 .$$

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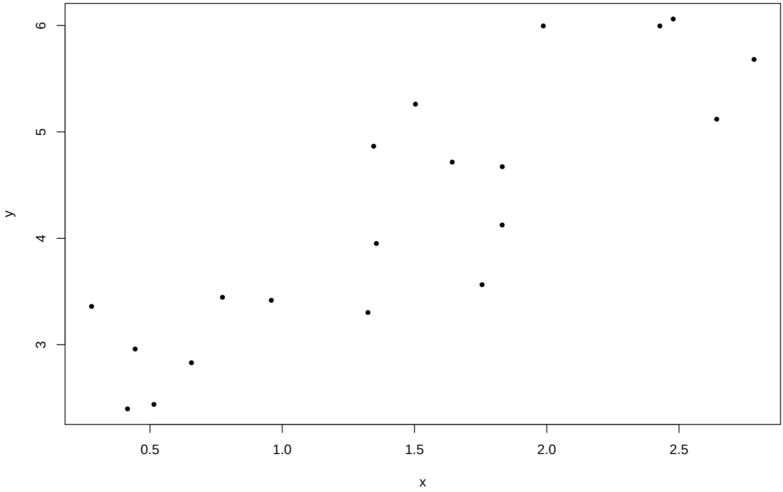

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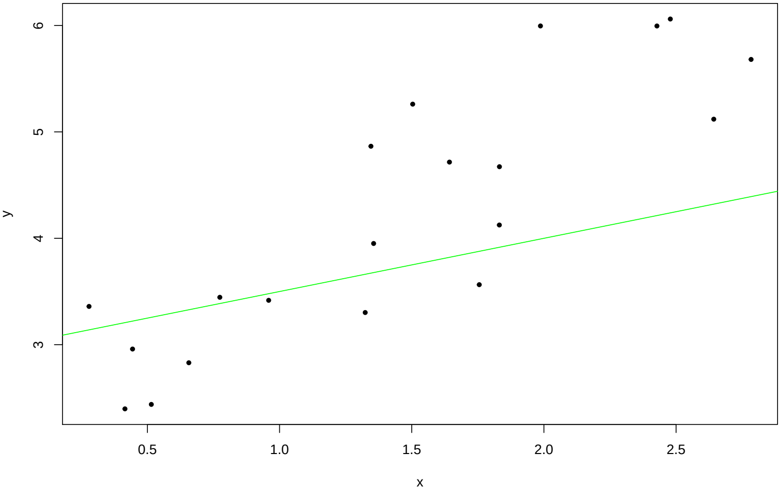

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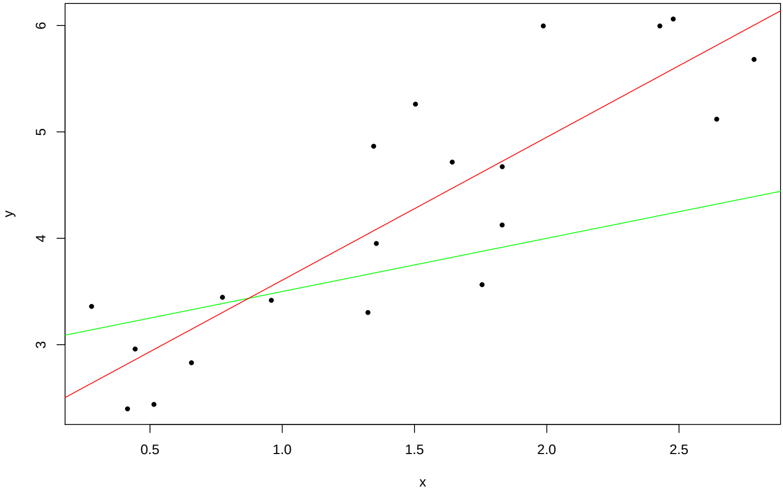

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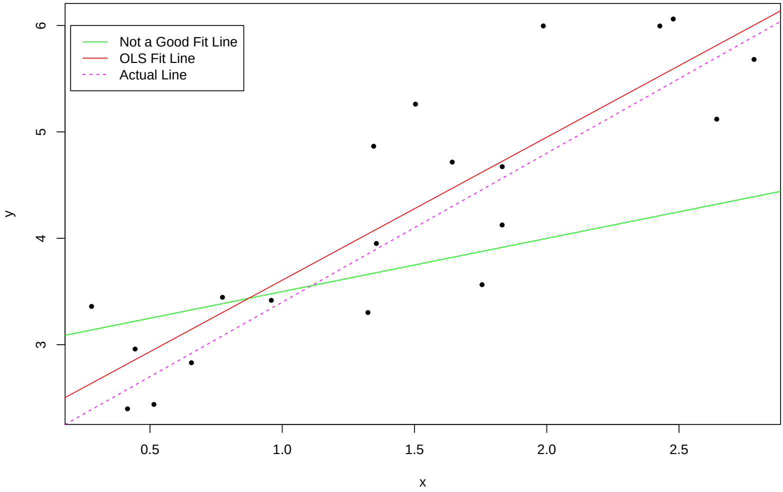

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Calculation of the best fit line is easy: we just define

$$\varphi(\beta_0, \beta_1) = \sum_{k=1}^{n} \Big(Y_k - \beta_0 - \beta_1 \cdot X_k \Big)^2, \qquad (\beta_0, \beta_1) \in \mathbb{R}^2,$$

and using our Calc 3 knowledge, find the Minimum Point of φ by solving the System

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Note: So the $cor(X, Y)$ is not the Slope of the Regression Line, but

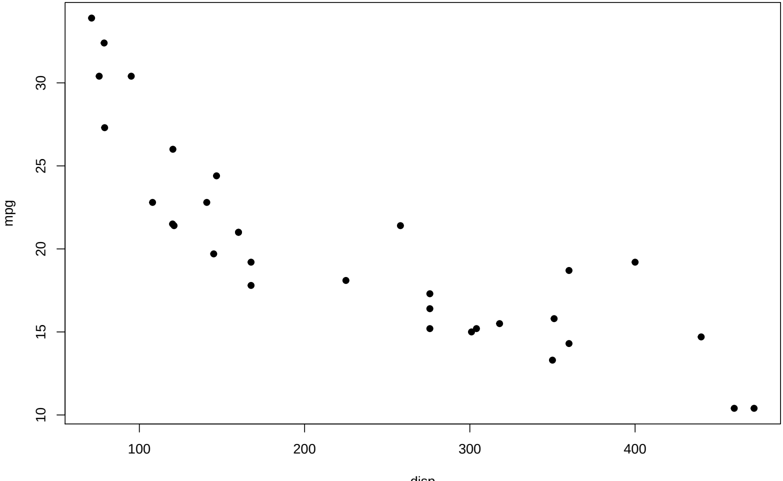
$$cor(X, Y) \cdot \frac{sd(Y)}{sd(X)}$$

is. Recall our Descriptive Statistics part!

Example

Example: We will use the `mtcars` Dataset from **R**:

```
plot(mpg ~ disp, data = mtcars, pch = 19)
```



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```
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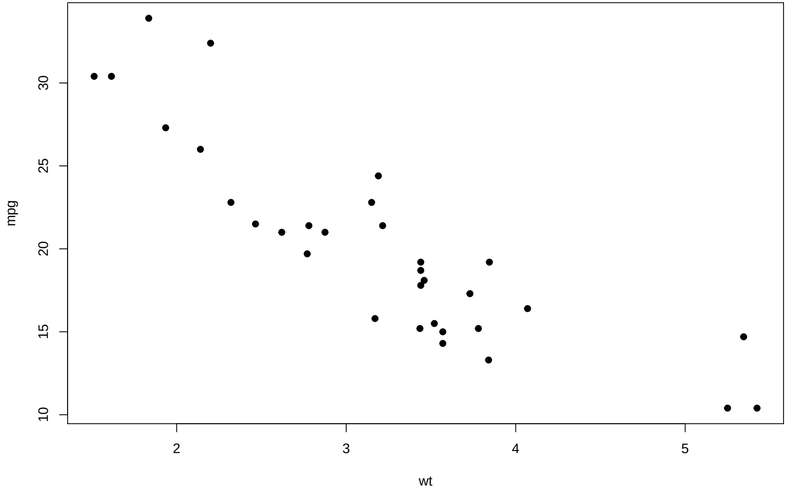

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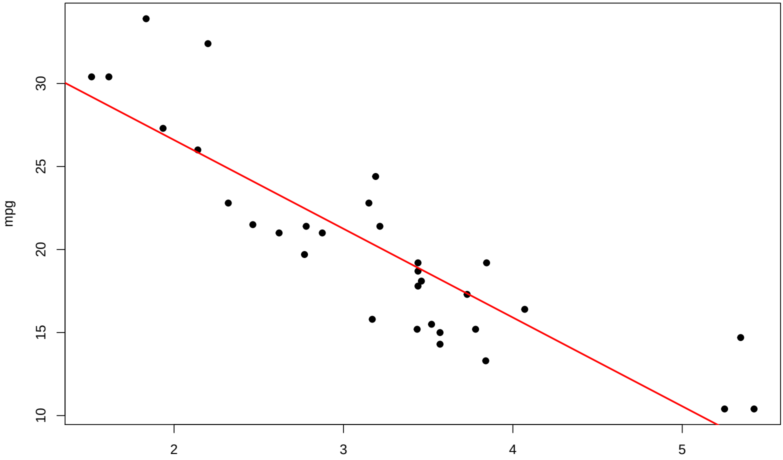
```
plot(mpg ~ wt, data = mtcars, pch = 19)
model <- lm(mpg ~ wt, data = mtcars)
abline(model, col = "red", lwd = 2)
```



Example, Cont'd

```r
model <- lm(mpg ~ wt, data = mtcars)
summary(model)
```

```
##
## Call:
## lm(formula = mpg ~ wt, data = mtcars)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.5432 -2.3647 -0.1252  1.4096  6.8727
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  37.2851     1.8776  19.858  < 2e-16 ***
## wt           -5.3445     0.5591  -9.559 1.29e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
##
## Residual standard error: 3.046 on 30 degrees of freedom
## Multiple R-squared:  0.7528, Adjusted R-squared:  0.7446
## F-statistic: 91.38 on 1 and 30 DF,  p-value: 1.294e-10
```

Example, Cont'd

Now, we predict the value of `mpg` for a new values of a `wt` Variable:

```
pred <- predict(model, data.frame(wt=4.7))
pred
```

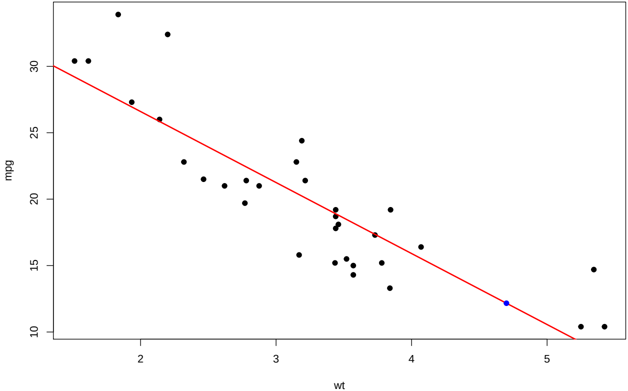
```
##        1
## 12.16611
```

Example, Cont'd

Now, we predict the value of `mpg` for a new values of a `wt` Variable:

```
pred <- predict(model, data.frame(wt=4.7))
pred
```

```
##        1
## 12.16611
```



Example

```r
x <- rnorm(100, mean = -1, sd = 1)
y <- runif(100, 2, 10)
z <- 2.7 - 1.7*x + 13.5*y + rnorm(100)
head(x)
```

```
## [1]  0.5271618 -2.0473435  0.4363159 -0.1708086 -0.7298618 -1
```

```r
head(y)
```

```
## [1] 3.977900 9.480366 2.867116 6.922705 7.797675 7.675757
```

```r
head(z)
```

```
## [1]  55.39792 133.41440  42.64337  95.71013 109.41316 108.627
```

Example

```
mod1 <- lm(z ~ x); summary(mod1)
```

```
##
## Call:
## lm(formula = z ~ x)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -58.755 -27.454   2.906  28.465  50.215
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   84.135      4.202  20.022   <2e-16 ***
## x             -3.130      2.917  -1.073    0.286
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
##
## Residual standard error: 31.23 on 98 degrees of freedom
## Multiple R-squared:  0.01161,    Adjusted R-squared:  0.00152
## F-statistic: 1.152 on 1 and 98 DF,  p-value: 0.2859
```

Example, Cont'd

```r
mod2 <- lm(z ~ x + y); summary(mod2)
```

```
##
## Call:
## lm(formula = z ~ x + y)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.25835 -0.69422 -0.04329  0.72539  2.12851
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.92180    0.29018   10.07   <2e-16 ***
## x           -1.50675    0.09194  -16.39   <2e-16 ***
## y           13.46759    0.04283  314.41   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
##
## Residual standard error: 0.9827 on 97 degrees of freedom
## Multiple R-squared:  0.999,  Adjusted R-squared:  0.999
## F-statistic: 5.001e+04 on 2 and 97 DF,  p-value: < 2.2e-16
```

Properties of the Estimators: $\hat{\beta}_0$ and $\hat{\beta}_1$

Here we assume that $\varepsilon \sim \mathcal{N}(0, \sigma^2)$.

Properties of the Estimators: $\hat{\beta}_0$ and $\hat{\beta}_1$

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Fact 1: Estimators $\hat{\beta}_0$ and $\hat{\beta}_1$ are UnBiased:

$$\mathbb{E}(\hat{\beta}_0) = \beta_0, \qquad Var(\hat{\beta}_0) = \frac{\sigma^2}{n} \cdot \frac{\sum_{k=1}^n X_k^2}{\sum_{k=1}^n (X_k - \overline{X})^2}$$

$$\mathbb{E}(\hat{\beta}_1) = \beta_1, \qquad Var(\hat{\beta}_1) = \frac{\sigma^2}{\sum_{k=1}^n (X_k - \overline{X})^2}$$

Properties of the Estimators: $\widehat{\sigma^2}$

Fact 2: Assume σ^2 is unknown.

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$$s^2 = \frac{1}{n-2} \cdot \sum_{k=1}^{n} (\hat{\varepsilon}_k)^2$$

is an UnBiased Estimator for σ^2, and

$$\widehat{\sigma^2} = \frac{1}{n} \cdot \sum_{k=1}^{n} (\hat{\varepsilon}_k)^2$$

is the MLE for σ^2.

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is the MLE for σ^2. Here

$$\hat{\varepsilon}_k = Y_k - \hat{\beta}_0 - \hat{\beta}_1 \cdot X_k$$

is the k-th **residual**.

Fsyo!

Thank You!