AUA CS 108, Statistics, Fall 2019

Lecture 21

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11 Oct 2019

Contents

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- the CDF of $X + \frac{1}{n^2}$ is not $F_X(x) + \frac{1}{n^2}$.

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- State the Problem of the Point Estimation.
- Define the MSE.
- How we can define that $\hat{\theta}$ is close to θ?
- Define the Bias of an Estimator.
- What is the definition of the Unbiased/Biased Estimator?

Example

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Easy to se that $\hat{\lambda}$ is an Unbiased Estimator for λ (OTB!).

Example, cont'd

Now, the code

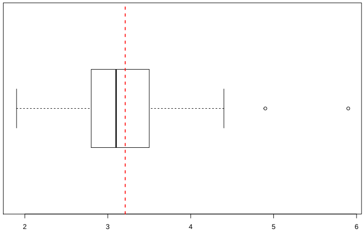
- observing once: ganarating a Sample just once and calculating one Estimate:

```r
lambda <- 3.21
x <- rpois(10, lambda = lambda)
lambda.hat <- mean(x)
lambda.hat
```

```
## [1] 3.4
```

- observing many times: ganarating Samples many times, calculating Estimates, and then averaging:

```
lambda <- 3.21; n <- 10; m <- 200
x <- rpois(n*m, lambda = lambda)
x <- as.data.frame(matrix(x, ncol = m))
lambda.hats <- sapply(x, mean)
boxplot(lambda.hats, horizontal = T);
abline(v = lambda, col="red", lwd = 2, lty = 2)
```

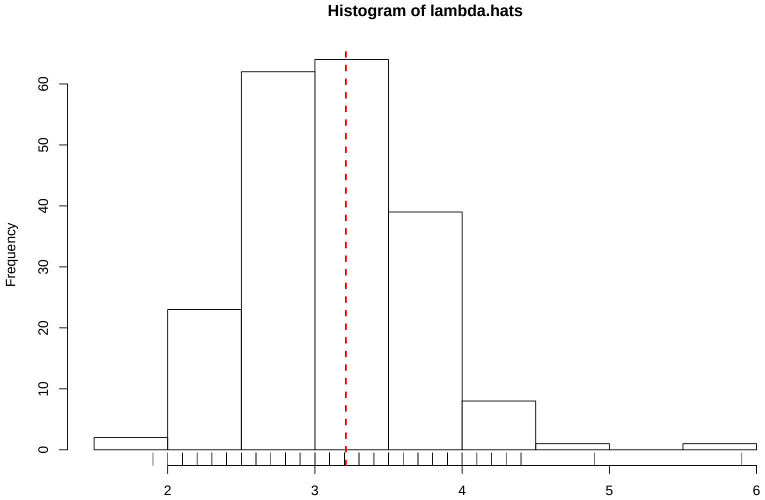



```
mean(lambda.hats)
```

```
## [1] 3.172
```

With a Histogram:

```r
hist(lambda.hats)
rug(lambda.hats)
abline(v = lambda, col="red", lwd = 2, lty = 2)
```



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Biased Estimator Case

Say, let us consider the Exponential Model:

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Easy to se that $\hat{\lambda}$ is an Biased Estimator for λ (OTB!).

Example, cont'd

Now, the code:

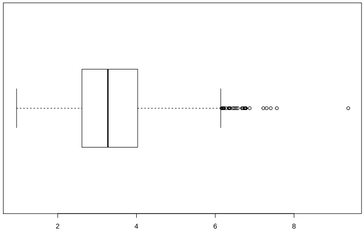
- observing once: ganarating a Sample just once and calculating one Estimate:

```
lambda <- 0.3
x <- rexp(10, rate = lambda)
lambda.hat <- mean(x)
lambda.hat
```

```
## [1] 2.3389
```

- observing many times: ganarating Samples many times, calculating Estimates, and then averaging:

```
lambda <- 0.3; n <- 10; m <- 2000
x <- rexp(n*m, rate = lambda)
x <- as.data.frame(matrix(x, ncol = m))
lambda.hats <- sapply(x, mean)
boxplot(lambda.hats, horizontal = T);
abline(v = lambda, col="red", lwd = 2, lty = 2)
```




```
mean(lambda.hats)
```

```
## [1] 3.380309
```

With a Histogram:

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hist(lambda.hats)
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abline(v = lambda, col="red", lwd = 2, lty = 2)
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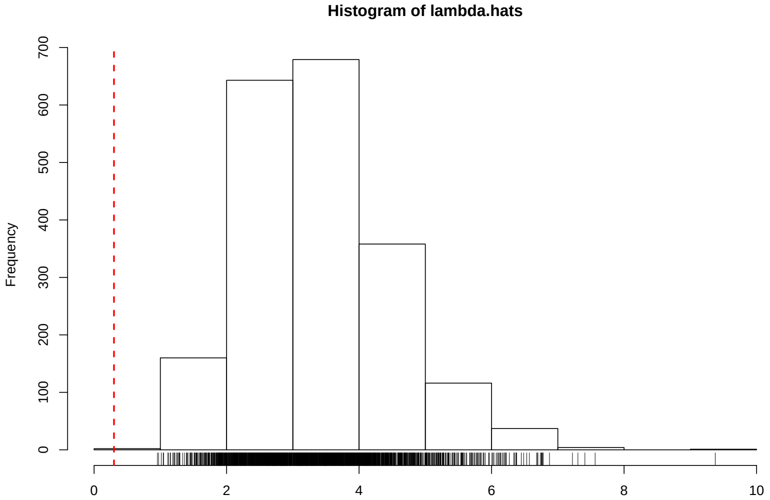

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$$Bias(\hat{\theta}_n, \theta) \to 0, \qquad \text{for any } \theta \in \Theta.$$

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Idea: If the Sample size is very large, then the behaviour of our Asymptotic Unbiased Estimator is close to an Unbiased one, $Bias(\hat{\theta}_n, \theta) \approx 0$

Example: Say, for the Mean μ of the Population,

$$\hat{\mu} = \frac{X_1 + X_2 + ... + X_n}{n+1}$$

is a Biased, but Asymptotically Unbiased Estimator. OTB, please!

Bias-Variance Decomposition

This is an important result:

Theorem(Bias-Variance Decomposition of the MSE): If $\hat{\theta}$ is an Estimator for θ, then

$$MSE(\hat{\theta}, \theta) = \left(Bias(\hat{\theta}, \theta)\right)^2 + Var_\theta(\hat{\theta}).$$

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Proof: OTB