CS 107, Probability, Spring 2019
Lecture 10

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AUA

8 February 2019

Content

- Conditional Probabilities

Air Passenger Problem

A fully-booked plane is about to be boarded by its hundred passengers. The first passenger loses his boarding card and sits down in a random place. From then on, a passenger, whenever finding his seat taken, very politely sits somewhere at random. What is the probability that the hundredth passenger sits in his own seat?

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The Correct Answer is: Raymond Queneau, French poet

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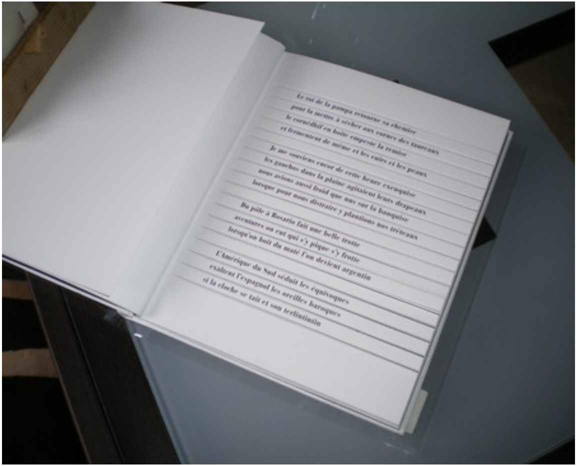

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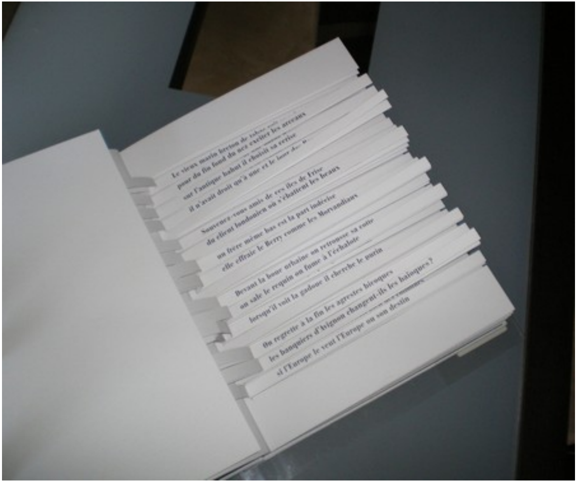

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The conditional probability of A given B (or the probability of A under the condition of B) is defined to be

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}.$$

And we define also $\mathbb{P}(A|B) = 0$, if $\mathbb{P}(B) = 0$.

Example

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e. If $A_1, ..., A_n, ...$ are some **mutually disjoint** events, then
$$\mathbb{P}(\cup_{n=1}^{\infty} A_n|B) = \sum_{n=1}^{\infty} \mathbb{P}(A_n|B);$$

Properties of Conditional Probabilities, Cont'd

f. If A_1, A_2, B are some events and $\mathbb{P}(B) \neq 0$, then

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h. (multiplication or chain rule) If $A_1, ..., A_n$ are some events, then

$$\mathbb{P}(A_1 \cap A_2 \cap ... \cap A_n) = \mathbb{P}(A_1) \cdot \mathbb{P}(A_2|A_1) \cdot \mathbb{P}(A_3|A_1 \cap A_2) \cdot ...$$

$$\cdot \mathbb{P}(A_n|A_1 \cap A_2 \cap ... \cap A_{n-1}).$$

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King's Sister Problem: In the middle ages there was a story about a King. The parents of the King had 2 children. What is the probability that the other child is the sister of the King?

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This property is called the Multiplication Rule.

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