AUA CS108, Statistics, Fall 2020

Lecture 03

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- Different Types of Variables
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Random Sampling

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Definition: We say that our Sample is *Representative* (obtained by a Simple Random Sampling), if it is obtained in the process where all Samples of size k have the same probability of being chosen.

Example

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Which one gives a Representative (Simple Random) Sample?

Few Sampling Methods

Simple Random Sampling is not always easy to perform, so people are using different simpler Sampling Strategies (although they are not always giving exactly Representative Samples):

Few Sampling Methods

- *Systematic (Interval) Sampling*, we fit the population into a list, enumerate it; choose n; choose at random a starting element from the first n elements in the list; and from that element on, every n-th member of the population is selected;

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- *Stratified Sampling*, where the total population is divided into subgroups (strata), that share similar characteristics, and then random sample (of corresponding size) is chosen from each strata.
- *Cluster Sampling*, where the total population is divided into subgroups (clusters), then some clusters are randomly chosen. Then we include all elements of chosen clusters into our Sample.

Classification of Data wrt its Dimension

Data can be

- **Univariate** (1D) - here the observations are on a single Variable
- **Bivariate** (2D) - here the observations are on two Variables
- **Multivariate** (n-D, $n \geq 2$) - when the observations are on more than a one Variable (usually, more than two)

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Examples: *Gender* is a Categorical Variable, taking values *Male, Female*; Or *Color* and *Model* (of a car) are again Categorical

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Examples:

- *No. of Children*, *No. of Customers* ,. . . are Discrete
- *Height*, *Weight*, *Age*, . . . are Continuous

Remark

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Let me give by an example: when talking about the number of children in the family, we can have the following data: 0, 2, 1, 2, 4, 6, and we can calculate, say, the average number of children in families, here 2.5.

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But even if we are enumerating the Gender or the Color, the average Gender or the average Color is not meaningful, we cannot deal with the assigned numbers as above!

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Maybe one of the well-known Ordinal Scale Measurements is the **Likert Scale**: This is our famous

Strongly Disagree | Disagree | Neither | Agree | Strongly Agree

Descriptive Statistics

Why we need it?

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> *This dataset contains a subset of the fuel economy data that the EPA makes available on http://fueleconomy.gov. It contains only models which had a new release every year between 1999 and 2008.*

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Consider, for example, the dataset `mpg` from the `ggplot2` package. From the official description of the data,

> *This dataset contains a subset of the fuel economy data that the EPA makes available on http://fueleconomy.gov. It contains only models which had a new release every year between 1999 and 2008.*

Lets look at the first 3 rows of our dataset:

```
head(ggplot2::mpg, 3)
```

```
## # A tibble: 3 x 11
##   manufacturer model displ  year   cyl trans      drv      cty
##   <chr>        <chr> <dbl> <int> <int> <chr>      <chr> <int>
## 1 audi         a4      1.8  1999     4 auto(l5)   f        18
## 2 audi         a4      1.8  1999     4 manual(m5) f        21
## 3 audi         a4      2    2008     4 manual(m6) f        20
```

Descriptive Statistics

Descriptive Statistics

The variable `cty` is the *city miles per gallon*, and the variable `cyl` is the *number of cylinders*. Let's separate that Variables:

```
cty <- ggplot2::mpg$cty
cyl <- ggplot2::mpg$cyl
```

Descriptive Statistics

Let's see the results:

```
cyl
```

```
##   [1] 4 4 4 4 6 6 6 4 4 4 4 6 6 6 6 6 8 8 8 8 8 8 8 8 8 8
##  [38] 4 6 6 6 6 6 6 6 6 6 6 6 6 6 6 8 8 8 8 8 6 8 8 8 8 8 8 8
##  [75] 8 8 8 6 6 6 6 8 8 6 6 8 8 8 8 8 6 6 6 6 8 8 8 8 8 4 4 4
## [112] 4 6 6 6 4 4 4 4 6 6 6 6 6 6 8 8 8 8 8 8 8 8 8 8 8 6 6
## [149] 6 6 6 6 6 8 6 6 6 6 8 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 6
## [186] 6 4 4 4 4 6 6 6 4 4 4 4 4 8 8 4 4 4 6 6 6 6 4 4 4 4 6 4
## [223] 4 4 4 5 5 4 4 4 4 6 6 6
```

Descriptive Statistics

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cyl
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```
##   [1] 4 4 4 4 6 6 6 4 4 4 4 6 6 6 6 6 8 8 8 8 8 8 8 8 8 8
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## [186] 6 4 4 4 4 6 6 6 4 4 4 4 4 8 8 4 4 4 6 6 6 6 4 4 4 4 6 4
## [223] 4 4 4 5 5 4 4 4 4 6 6 6
```

Can you describe this data? What can be said about the No. of Cylinders of these cars?

Descriptive Statistics

Let's see the results for `cty`:

```
cty
```

```
##   [1] 18 21 20 21 16 18 18 18 16 20 19 15 17 17 15 15 17 16 1
##  [26] 16 15 15 14 11 11 14 19 22 18 18 17 18 17 16 16 17 17 1
##  [51] 13 14 14 14  9 11 11 13 13  9 13 11 13 11 12  9 13 13 1
##  [76] 11 12 14 15 14 13 13 13 14 14 13 13 13 11 13 18 18 17 1
## [101] 24 25 23 24 26 25 24 21 18 18 21 21 18 18 19 19 19 20 2
## [126] 14  9 14 13 11 11 12 12 11 11 11 12 14 13 13 13 21 19 2
## [151] 14 15 14 12 18 16 17 18 16 18 18 20 19 20 18 21 19 19 1
## [176] 15 15 16 14 21 21 21 21 18 18 19 21 21 21 22 18 18 18 2
## [201] 15 16 17 15 15 15 16 21 19 21 22 17 33 21 19 22 21 21 2
## [226] 20 20 21 18 19 21 16 18 17
```

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##   [1] 18 21 20 21 16 18 18 18 16 20 19 15 17 17 15 15 17 16 1
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## [101] 24 25 23 24 26 25 24 21 18 18 21 21 18 18 19 19 19 20 2
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## [176] 15 15 16 14 21 21 21 21 18 18 19 21 21 21 22 18 18 18 2
## [201] 15 16 17 15 15 15 16 21 19 21 22 17 33 21 19 22 21 21 2
## [226] 20 20 21 18 19 21 16 18 17
```

Again, can you describe this data? What can be said about the City Miles per Gallon values of these cars?

Descriptive Statistics

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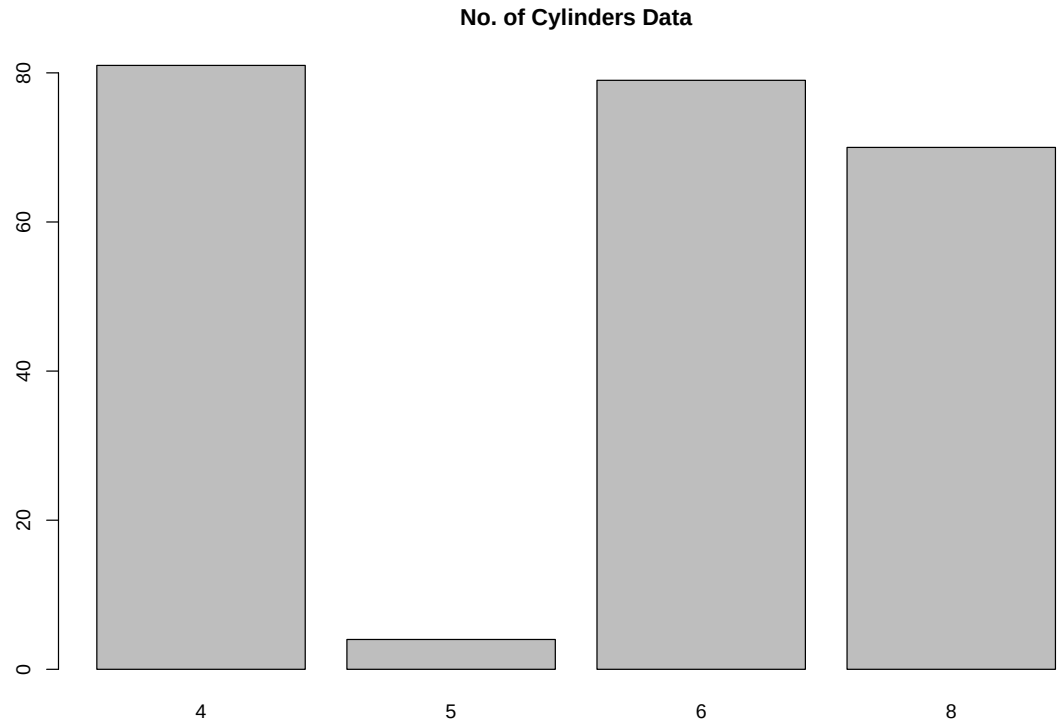
For example, let us draw the BarPlot for the frequencies of the `cyl` variable:

```
barplot(table(cyl), main = "No. of Cylinders Data")
```



Descriptive Statistics

Now, let us give some numerical summaries for `cty`: calculate the average Miles per Gallon for a City, and its max and min.

```
cat("mean = ", mean(cty))
```

```
## mean =  16.85897
```

```
cat("Max = ", max(cty))
```

```
## Max =  35
```

```
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```
## Min =  9
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```
## Min =  9
```

And we can use the `summary` command to get some numerical info:

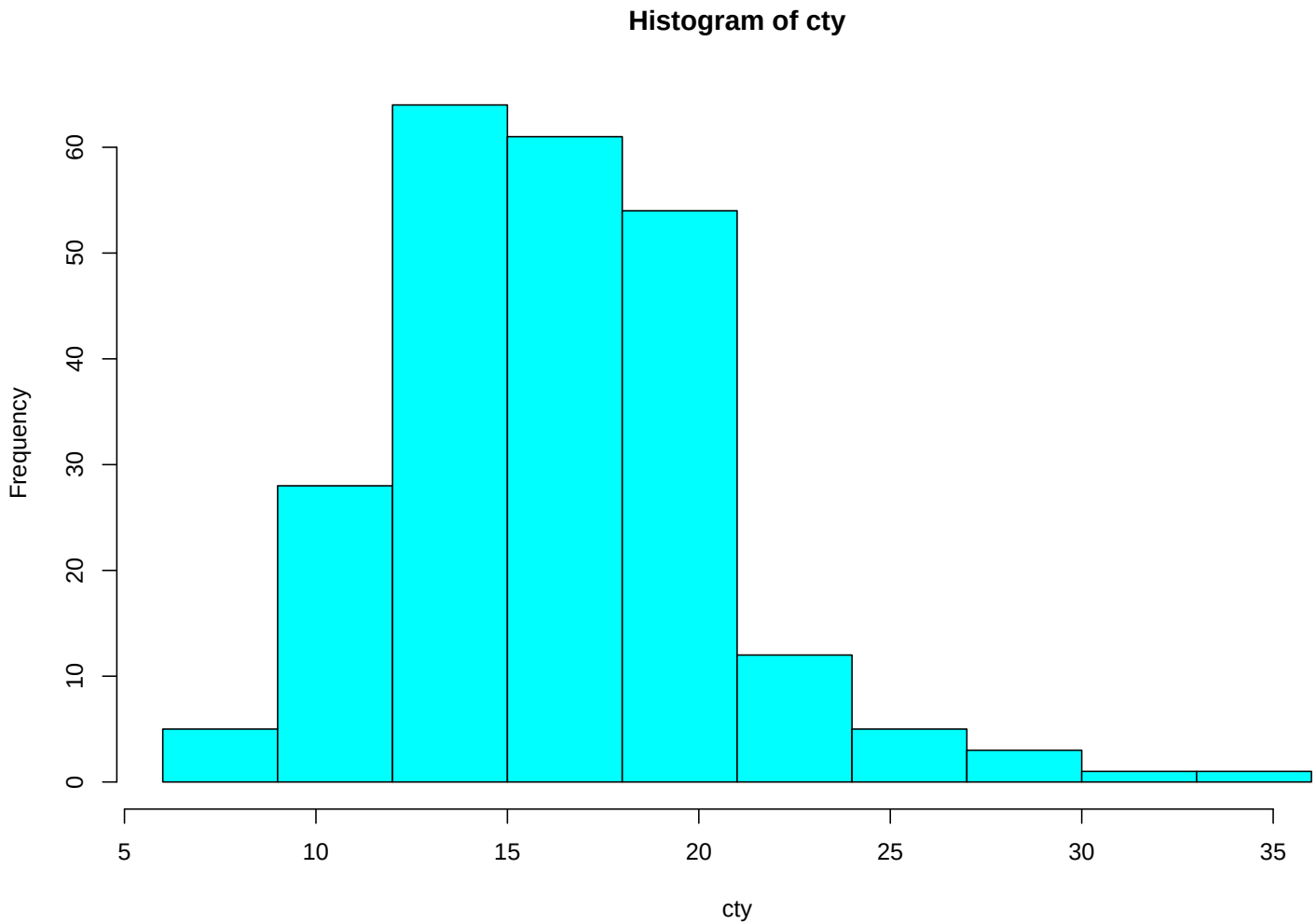
```
summary(cty)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    9.00   14.00   17.00   16.86   19.00   35.00
```

Descriptive Statistics

To get some visual information about the Variable `cty`, its distribution, we can draw the Histogram:

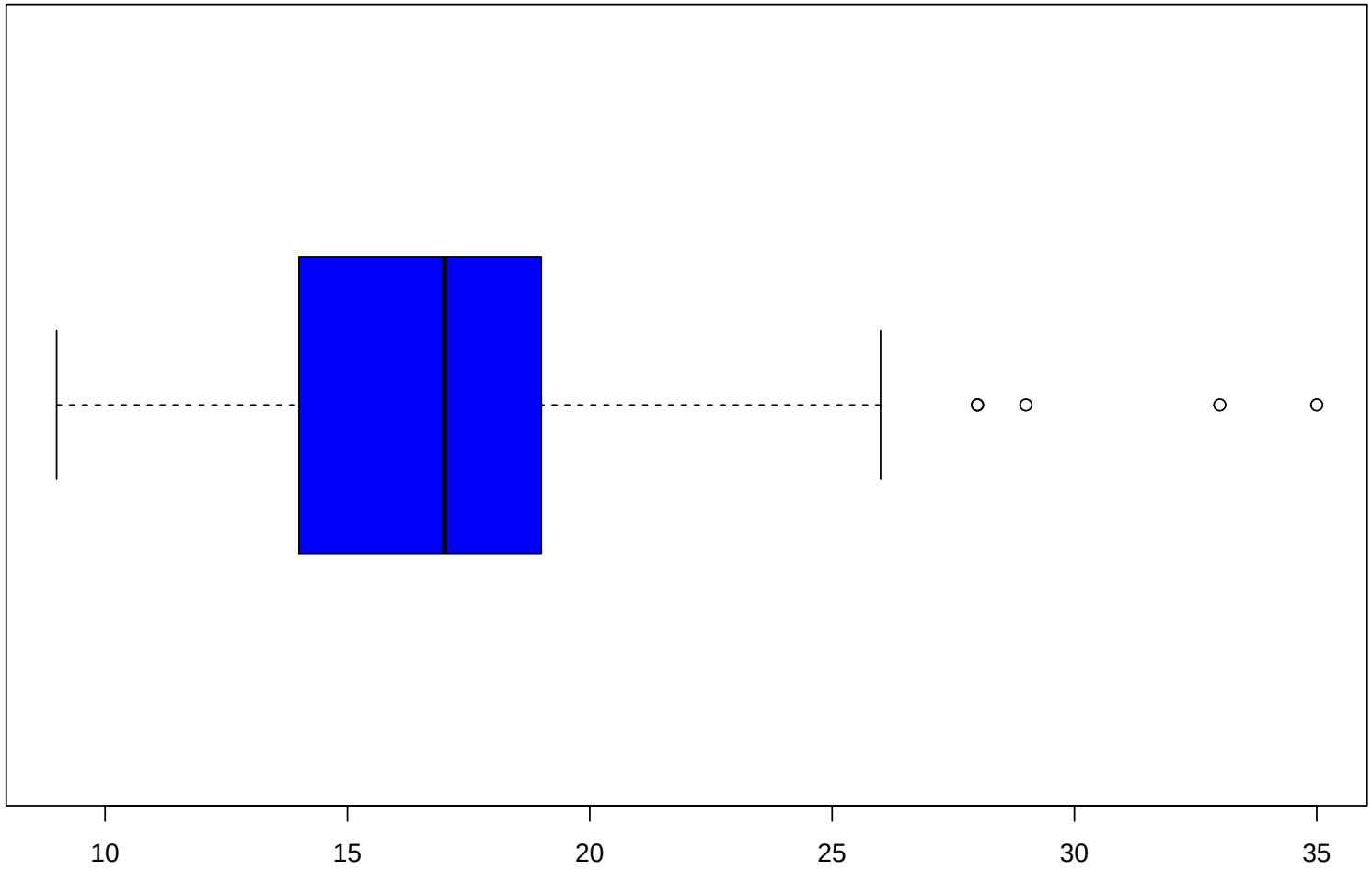
```
hist(cty, breaks=seq(6,36, 3),col="cyan")
```



Descriptive Statistics

Now, we can draw the BoxPlot of the `cty` data:

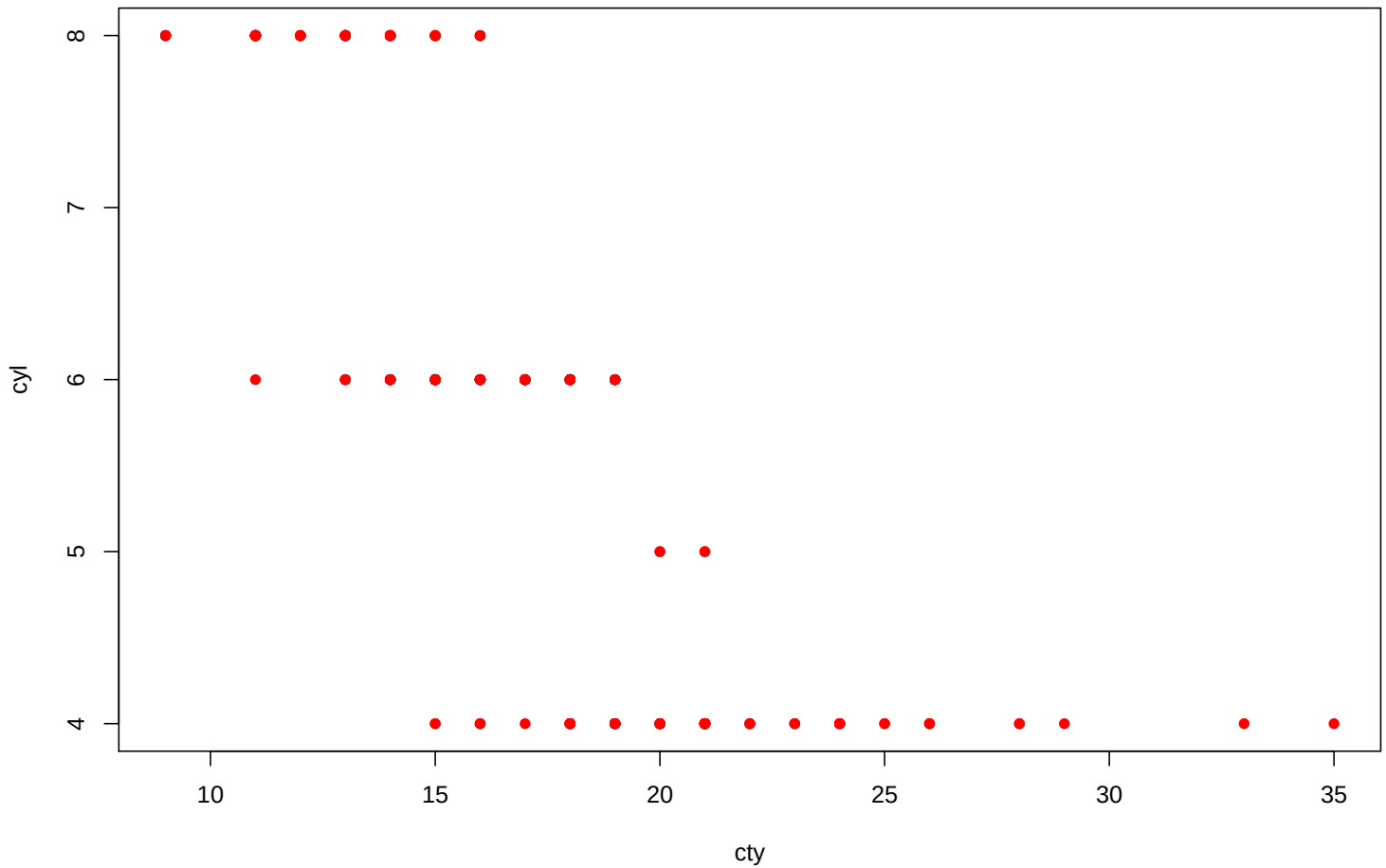
```
boxplot(cty, horizontal = T, col = "blue")
```



Descriptive Statistics

Now, instead of just getting information about `cyl` and `cty` separately, let us give visually the relationship between them:

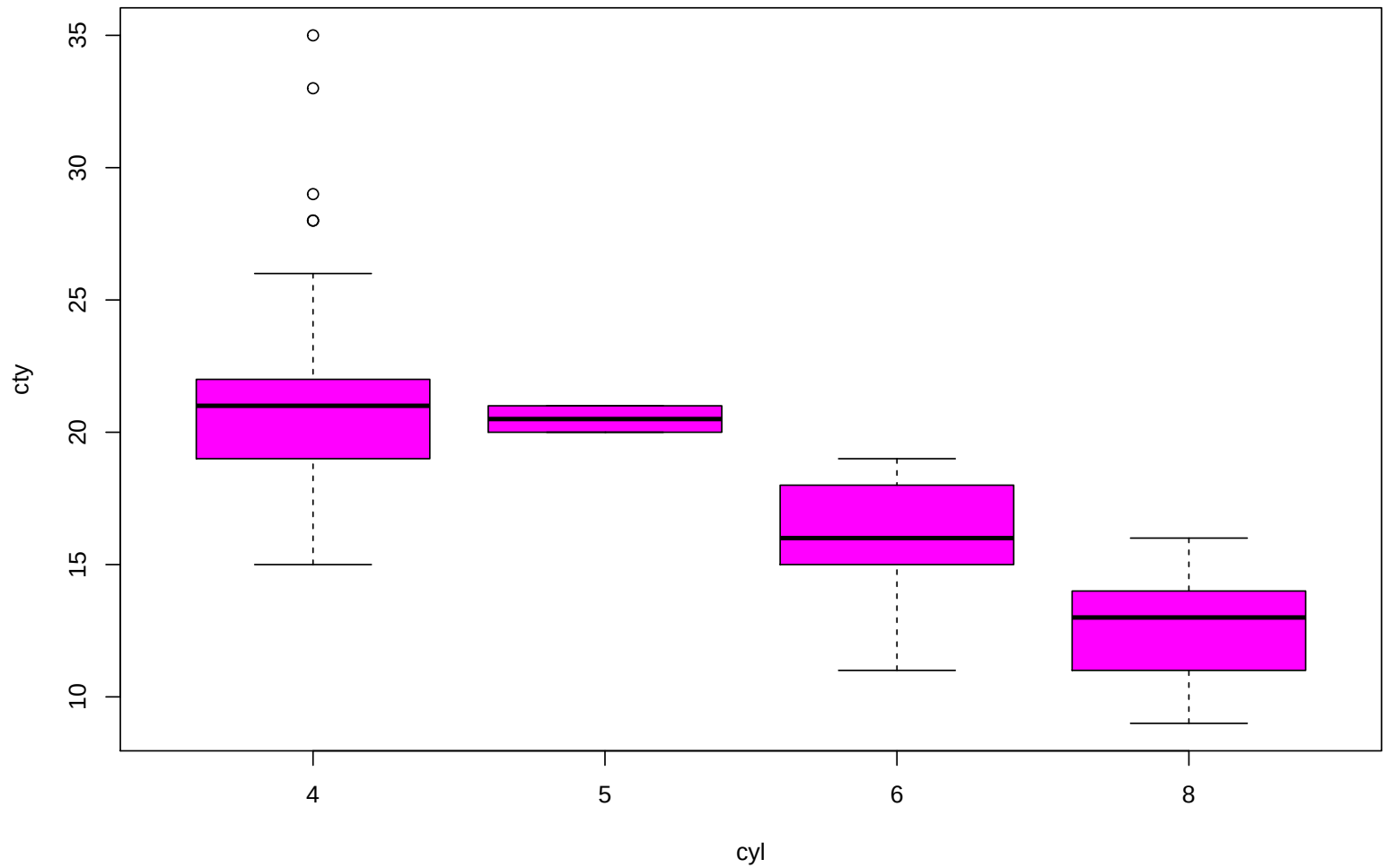
```
plot(cty, cyl, pch=16, col = "red")
```



Descriptive Statistics

. . . or draw a BoxPlot of `cty` for each type of the cylinder:

```
boxplot(cty~cyl, col="magenta")
```



Descriptive Statistics

Moral: our brain cannot get an insight from the list of numbers, but Descriptive Statistics can help ⌣̈

Descriptive Statistics

How to do it?

Descriptive Statistics for a Univariate Data

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Descriptive Statistics for a Univariate Data

So we use the Descriptive Statistics at the beginning of our Statistical Study to examine, explore the dataset.

This part of Statistics is sometimes called **Exploratory Data Analysis**, EDA.

And we start by describing some of the *Graphical Summaries*.

Here, for the beginning, we will assume that we have a univariate (mostly numerical) data (dataset), $x_1, x_2, ..., x_n$. In this case we will say that we are given a (univariate, 1D) dataset x.

Frequency Tables

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$$\text{Frequency of } t = \text{number of occurrences of } t \text{ in data.}$$

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Definition: The **relative frequency** (or percentage) of a value t in observations $x_1, x_2, ..., x_n$ is the ratio of frequency of t divided by the total number of observations, n:

$$\text{Relative Frequency of } t = \frac{\text{Frequency of } t}{\text{Total Number of Observations}} =$$

$$= \frac{\text{Frequency of } t}{n}.$$

Frequency Tables, Example

Example: Given the following Dataset:

$$1, 2, 4, 7, 2, 3, 2, 1, 2, 1, 4, 1, -1$$

obtain the Frequency and Relative Frequency Tables.

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Example: Let's construct the Frequency Table of the above Dataset using **R**:

```
x <- c(1, 2, 4, 7, 2, 3, 2, 1, 2, 1, 4, 1, -1)
table(x)
```

```
## x
## -1  1  2  3  4  7
##  1  4  4  1  2  1
```