AUA CS108, Statistics, Fall 2020

Lecture 30

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Example: Say, for the Mean μ of the Population,

$$\hat{\mu} = \frac{X_1 + X_2 + ... + X_n}{n+1}$$

is a Biased, but Asymptotically Unbiased Estimator. OTB, please!

Bias-Variance Decomposition

This is an important result:

Theorem(Bias-Variance Decomposition of the MSE): If $\hat{\theta}$ is an Estimator for θ, then

$$MSE(\hat{\theta}, \theta) = \left(Bias(\hat{\theta}, \theta)\right)^2 + Var_\theta(\hat{\theta}).$$

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Proof: OTB

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Nice Graphical Interpretation: Link, see also the next slide.

Bias-Variance Decomposition/Tradeoff

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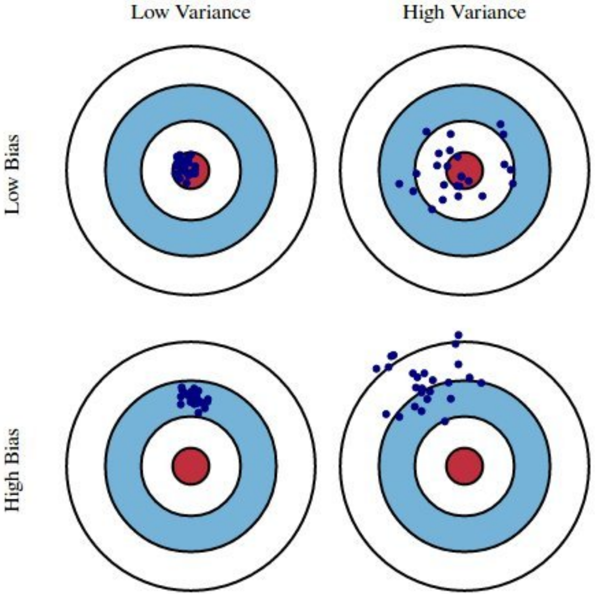

Standard Error and Estimated Standard Error

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And statisticians, when reporting the Estimate, usually report also the Estimated Standard Error, as a measure how precise is the result. If the Standard Error is small (and we are using a nice Estimator, say, it is Unbiased), then this is a sign that the result is close to real/actual one.

Example

Example: Assume we are facing an election with Parties A and B, and we want to estimate the percentage of voters for A in advance. So we do a poll, asking 10 persons to give their preferences. Let the result be:

$$A, B, B, B, A, B, B, A, B, B.$$

Problem: Estimate the percentage of voters for the Party A, and give the Estimated Standard Error.

Solution: OTB.

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Corollary: If $\hat{\theta}_1$ and $\hat{\theta}_2$ are both *Unbiased* Estimators for the unknown Parameter θ, then $\hat{\theta}_1$ is preferable to $\hat{\theta}_2$ if and only if

$$\mathit{Var}(\hat{\theta}_1) \le \mathit{Var}(\hat{\theta}_2), \qquad \text{for all } \theta,$$

with a strict inequality for at least one value of θ.

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