AUA CS108, Statistics, Fall 2020

Lecture 40

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Contents

Moral, and Choosing Null Hypotheses

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One is using this general idea to choose the Null and Alternative Hypotheses: **we will keep believing in Null, if the Data will not show strong evidence against**.

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This is an example of **A/B Testing**.

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$$\mathcal{H}_0 \qquad vs \qquad \mathcal{H}_1.$$

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Do Not Reject $\mathcal{H}_0$	Correct Decision (True Positive)	**Type II Error (False Negative)**

Can you guess the Null Hypo?

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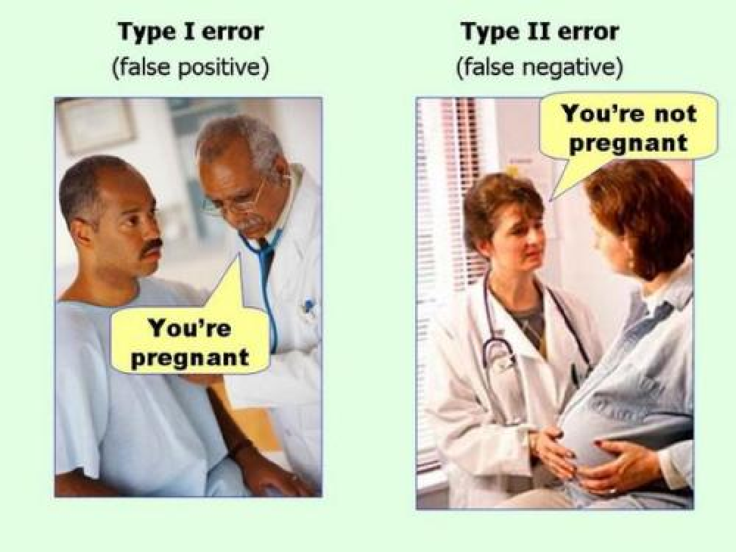

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It is easy to see that

$$\textit{Power} = 1 - \beta = \mathbb{P}(\text{Reject } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is False}).$$

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Probabilities of Correct/InCorrect Decisions:

Test Decision \ Reality	$\mathcal{H}_0$ is True	$\mathcal{H}_0$ is False (i.e., $\mathcal{H}_1$ is True)
Reject $\mathcal{H}_0$	$\alpha =$**Significance**	$1-\beta =$**Power**
Do Not Reject $\mathcal{H}_0$	$1-\alpha$	β

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Common values of α are $0.05, 0.01$ and 0.1 (corresponding to $95\%, 99\%$ and 90% Confidence Level!).

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- What is means that the Significance level of our Test, α, is small ?
- What it means that the Power of our Test, $1 - \beta$, is high ?

Hypo Testing: Constructing a Test

To test a Hypothesis, one is following the following steps $\ddot\smile$:

Step 1: Choosing a Model: We assume our Data comes from a Parametric Model $\mathcal{F}_\theta$, $\theta \in \Theta$

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Step 4: Assume we know how many Observations we will have[1], say, n. Then we take a Random Sample

$$X_1, X_2, ..., X_n \sim \mathcal{F}_\theta.$$

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