AUA CS108, Statistics, Fall 2020

Lecture 32

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Contents

Methods to find (good) Estimators

The Problem

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Problem: The Problem is to find/construct a good Estimator for θ, using our Random Sample.

The Method of Moments

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Note: Note that, in general, the Theoretical Moments of $\mathcal{F}_\theta$ are functions of θ.

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Note: The Empirical Moment is independent of the Parameter θ, it is just a Statistics, it is a function of $X_1, X_2, ..., X_n$, it is a r.v.

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from the following Model:

X	0	1	2
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