AUA CS 108, Statistics, Fall 2019

Lecture 18

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Last Lecture ReCap

- What is CLT about?

CLT, Again

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This means that, in particular, for large n and any real numbers $a < b$,

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Here $\Phi(x)$ is the CDF of Standard Normal Distribution, $\mathcal{N}(0,1)$.

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And this is **for any X_n (IID), with any distribution**. This will be our tool to construct Confidence Intervals and design Hypotheses Tests.

Inferential Statistics

Parametric Inference: Point Estimation

Parametric Statistics: General Problem

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then we think about this as a Sample, and we realize that if we would make another Sampling, we'd get other numbers. Even with the same cars!

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Example: If we consider the weights (in Kg) of 10 persons:

$$69.5, 77.1, ..., 109,$$

then we make the following model: let X_1 be the weight of the first person (say, the first person we will meet when performing the experiment), X_2 be the weight of the second person,. . . , X_{10} be the weight of the 10-th person.

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Our Dataset of weights is just one of the possible realizations of $X_1, ..., X_{10}$.

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rnorm(6, mean = 155, sd = sqrt(30))
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So, again, having a Dataset $x_1, ..., x_n$, statisticians work with a r.v.s $X_1, X_2, ..., X_n$ to work not only with a particular Sample, but with **all possible samples** from the Distribution (Process) behind the phenomenon.

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Now, if we have a Random Sample $X_1, ..., X_n$, then, because they are IID, we will have that all X_k-s are coming from the same Distribution:

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We will consider one of the main Problems of the Parametric Statistics: **Using the observations from our Random Sample, estimate the value of the Parameter θ.**

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And our problem here will be to estimate our unknown λ, using the realizations $x_1, x_2, ..., x_n$.