CS 107, Probability, Spring 2019
Lecture 12

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AUA

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Content

- Independence of Events

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Question: Who will win? Wrong question, of course $\ddot\smile$. Now, the correct one: which order you'll choose to maximize the winning probability?

The Bayes Rule: Medical Diagnosis Example

Problem: Medical Test gives a correct answer in 95% of cases: if the person is ill, it is saying that he/she is ill with 95% probability, and if the person is healthy, it is saying that he/she is healthy with 95% probability (i.e., in 95% of cases).

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Solution: OTB

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And you know that events do not happen in an isolated way, do not come alone, there are a lot of other event affecting somehow or not affecting to the event under our interest. So we want to formalize the notion of Independence and Dependence in the Probability (Statistical) sense.

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Remark: It is easy to see that the condition above is equivalent to[1]

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and is equivalent to

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The last two are intuitive: say,

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Remark: We say two Events A and B are **Dependent**, if they are not Independent.

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- We suppose (from the intuition of the problem) that Events are Independent, and use Independence to calculate Probabilities.

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Clearly, $A \cap B = \{HH\}$, so

$$\mathbb{P}(A \cap B) = \frac{1}{4}, \qquad \mathbb{P}(A) = \frac{2}{4} = \frac{1}{2} = \mathbb{P}(B),$$

so

$$\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B).$$

So our events are independent!!!

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So, because of the Independence of this Events,

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BTW, can you describe the Sample Space here?

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Problem: Assume we are choosing a real number from $[0, 4]$. Let A be the Event that our number is less than 3, and the Event $B = [1.5, 3.5]$. Are these Events Independent?

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If the Events A and B are disjoint, then they are Dependent, unless $\mathbb{P}(A) = 0$ or $\mathbb{P}(B) = 0$.

Interpretation: Knowing that, say, B happened, we will know for sure that A cannot happen, so knowing B changes the probability of having A!

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- If A is independent of B and A is independent of C, and also $B \cap C = \varnothing$, then A is independent of $B \cup C$.

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Pairwise Independence

We will say that the events $A_1, A_2, ..., A_n$ are **Pairwise Independent**, if every pair A_i and A_j are Independent, for any $i \neq j$, i.e., if

$$\mathbb{P}(A_i \cap A_j) = \mathbb{P}(A_i) \cdot \mathbb{P}(A_j), \qquad i \neq j.$$

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Mutual Independence

We say that $A_1, ..., A_n$ are **Mutually Independent** or just **Independent**, if for any subgroup of events $A_{i_1}, A_{i_2}, ..., A_{i_k}$,

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Give an Example! I mean you, not MP! $\ddot\smile$

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Problem: (Network Reliability Problem) Assume we have some computer network joining two nodes through some intermediate nodes. The probability that each intermediate node is working is p. What is the probability that the connection between the initial and terminal nodes is working, given that

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- Give the definition of the Mutual Independence of several Events.

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Solution: OTB **Solution:** Again, do it using Conditional Probabilities