CS 107, Probability, Spring 2019
Lecture 13

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Content

- Repeated, Independent Trials: Binomial Distribution

I am lying.

LZ

I am lying.

Question: Am I lying?

Example, from the previous lecture:

Problem: (Network Reliability Problem) Assume we have some computer network joining two nodes through some intermediate nodes. The probability that each intermediate node is working is p. What is the probability that the connection between the initial and terminal nodes is working, given that

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Solution: OTB

Repeated Independent Trials Model

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And if the Trials are independent (i.e., the knowledge of the result of some Trial is not changing the Probabilities of results in the other Trial), then we say that we have Repeated Independent Trials model.

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Here we consider two problems: The first one is the Binomial Model (of repeated trials):

- We have a Simple Experiment, called a Trial;
- We have an Event in this Experiment, say, A;
- We repeat our Simple Experiment n times (n=Number of Trials);
- We are interested in: how many times A will appear during that n Trials?

Examples:

Some Examples:

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- We are rolling a die 4 times. And we are interested in how many 6s we will have in that 4 rolls.

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- Again, from the above box, 20 times we draw a ball at random, fix its color, and return that ball into the box.

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- Can you give some real-world type problems?

Repeated Indep Trials: Binomial Probabilities

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Now Assume:

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- We assume the probability of having A in one Trial is p, so $\mathbb{P}(A) = p$;
- We repeat our Trial n times;
- We are interested in the probability that we will have exactly k successes in these n Trials, i.e., k times we will have A, and $n-k$ times we will have $\overline{A}$.

Binomial Probabilities

For any $k = 0, 1, ..., n$,

$$\mathbb{P}(\text{Exactly } k \text{ successes in } n \text{ trials}) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}.$$

or, if we will denote the number of Successes in that n Trials by X,

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Intuition: OTB

Repeated Indep Trials: Binomial Probabilities

We can represent this in the form of a Table:

X	0	1	2
$\mathbb{P}(X=k)$	$\binom{n}{0}\cdot p^0\cdot(1-p)^n$	$\binom{n}{1}\cdot p^1\cdot(1-p)^{n-1}$	$\binom{n}{2}\cdot p^2\cdot(1-$

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- Simple Experiment = Trial = Picking a Card at random;
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- $\mathbb{P}(\text{Event}) = \frac{1}{4}$

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- Number of Trials (Repetitions) $= 20$

So

$$\mathbb{P}(\text{Exactly 12 Hearts}) = \binom{20}{12} \cdot \left(\frac{1}{4}\right)^{12} \cdot \left(1 - \frac{1}{4}\right)^{20-12}.$$

Example: Cont'd

Problem: In the problem above, what is the probability that we will have Hearts more in than 17 cases? In not more than 17 cases?

Solution: OTB