AUA CS 108, Statistics, Fall 2019

Lecture 37

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Contents

Last Lecture ReCap

- How to choose the Null Hypothesis?

From the last lecture: Type I and II errors

Assume we are Testing the Hypothesis

$$\mathcal{H}_0 \qquad vs \qquad \mathcal{H}_1.$$

Then the following cases can happen:

Test Decision \ Reality	$\mathcal{H}_0$ is True	$\mathcal{H}_0$ is False (i.e., $\mathcal{H}_1$ is True)
Reject $\mathcal{H}_0$	**Type I Error (False Positive)**	Correct Decision (True Negative)
Do Not Reject $\mathcal{H}_0$	Correct Decision (True Positive)	**Type II Error (False Negative)**

Significance and Power of Test

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  $(1 - \beta)$ is called the **Power** of the Test.

It is easy to see that

$$\textit{Power} = 1 - \beta = \mathbb{P}(\text{Reject } \mathcal{H}_0 \mid \mathcal{H}_0 \text{ is False}).$$

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Probabilities of Correct/InCorrect Decisions:

Test Decision \ Reality	$\mathcal{H}_0$ is True	$\mathcal{H}_0$ is False (i.e., $\mathcal{H}_1$ is True)
Reject $\mathcal{H}_0$	$\alpha =$**Significance**	$1-\beta =$**Power**
Do Not Reject $\mathcal{H}_0$	$1-\alpha$	β

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Common values of α are $0.05, 0.01$ and 0.1 (corresponding to $95\%, 99\%$ and 90% Confidence Level!).

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- What is means that the Significance level of our Test, α, is small ?
- What it means that the Power of our Test, $1 - \beta$, is high ?

Hypo Testing: Constructing a Test

To test a Hypothesis, one is following the following steps ⌣̈:

Step 1: Choosing a Model: We assume our Data comes from a Parametric Model $\mathcal{F}_\theta$, $\theta \in \Theta$

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Step 2: We State the Hypotheses: we take Θ_0 and Θ_1 such that $\Theta = \Theta_0 \cup \Theta_1$ and $\Theta_0 \cap \Theta_1 = \varnothing$, and state the Hypothesis:

$$\mathcal{H}_0: \ \theta \in \Theta_0 \qquad \textit{vs} \qquad \mathcal{H}_1: \ \theta \in \Theta_1$$

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$$X_1, X_2, ..., X_n \sim \mathcal{F}_\theta.$$

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Test Statistics: So, we will base our Test-Statsitics on $\overline{X}$: as the name is suggesting, we take

$$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}.$$

Let us check[2] that it satisfies the requirements:

- If we will have the Observation, we can calculate it, for sure;
- If the Null Hypo is True, then $\mu = \mu_0$. Recall what is the connection between our Statistics, Z and μ: the Distribution of X_k in Z is $\mathcal{N}(\mu, \sigma^2)$. So, under $\mathcal{H}_0$, $X_k \sim \mathcal{N}(\mu_0, \sigma^2)$, so

$$\overline{X} \sim \mathcal{N}\left(\mu_0, \frac{\sigma^2}{n}\right) \qquad \text{and} \qquad Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim \mathcal{N}(0,1).$$

Great!

[2] Note the difference from the Pivot Z: when constructing a CI for μ, we were taking Z with μ.

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Rejection Region: Now we choose the **RR**. The idea is:

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Example

Example: I have generated in **R** a Sample of Size 50 from $\mathcal{N}(3, 2^2)$ and made some rounding:

```
set.seed(20112019)
s.size <-50; sigma <- 2
obs <-  rnorm(s.size, mean = 3, sd = sigma)
obs <- round(obs, digits = 2); obs
```

```
##  [1]  1.68  5.48  0.98  3.08  4.79  5.03  1.64  2.35  0.
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I will assume I do not know μ (which is 3, of course), and will just assume my Observation is coming from $\mathcal{N}(\mu, 2^2)$, with some μ. And I will test, at the 5% Significance Level, the following Hypothesis:

$$\mathcal{H}_0 : \; \mu = 4 \qquad vs \qquad \mathcal{H}_1 : \; \mu \neq 4.$$

Example, Cont'd

First, I calculate Z-statistic:

```r
mu0 <- 4
Z <- (mean(obs) - mu0)/(sigma/sqrt(s.size)); Z
```

```
## [1] -3.63665
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Example, Cont'd

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Now, I am calculating the quantile $z_{1-\alpha/2}$:

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a <- 0.05
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```
abs(Z) > z
```

```
## [1] TRUE
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So the decision is:

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So the decision is: **Reject** $\mathcal{H}_0$.

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## [1] TRUE
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So the decision is: **Reject** $\mathcal{H}_0$. In this case we say that the result was **Statistically Significant**.

Example

Example: Now, with the same Observations from the last example, let us test, at the 5% level, the following Hypothesis:

$$\mathcal{H}_0: \ \mu = 3.3 \qquad \textit{vs} \qquad \mathcal{H}_0: \ \mu \neq 3.3.$$

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$$\mathcal{H}_0 : \ \mu = 3.3 \qquad vs \qquad \mathcal{H}_0 : \ \mu \neq 3.3.$$

```
mu0 <- 3.3; a <- 0.05
Z <- (mean(obs) - mu0)/(sigma/sqrt(s.size))
cat("Z-statistics = ", Z)
```

```
## Z-statistics =  -1.161776
```

```
z <- qnorm(1-a/2)
cat("critical value = ", z)
```

```
## critical value =  1.959964
```

```
if (abs(Z) > z) cat("Reject") else cat("Do Not Reject")
```

```
## Do Not Reject
```