AUA CS 108, Statistics, Fall 2019

Lecture 38

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Last Lecture ReCap

- How many Errors we can do when Testing a Hypo?

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- What is the Significance of the Test?
- What is the Power of the Test?
- Describe the Z-Test.

Z-Test, relation to the Normal μ CI

Generally, it can be proven that there is a duality/association between Hypothesis Testing and a CI construction. Let me give the relation for our case, for Normal $\mathcal{N}(\mu, \sigma^2)$ Model, when σ is known.

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$$\left(\overline{X} - z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \; ; \; \overline{X} + z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

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$$\mathcal{H}_0 : \; \mu = \mu_0 \qquad vs \qquad \mathcal{H}_1 : \; \mu \neq \mu_0$$

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- Reject Null if $|Z| > z_{1-\alpha/2}$, otherwise, Do Not Reject Null.

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- Reject Null if $|Z| > z_{1-\alpha/2}$, otherwise, Do Not Reject Null.

This is equilvalent to: Do Not Reject, if

$$-z_{1-\alpha/2} \leq \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \leq z_{1-\alpha/2},$$

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i.e., if

$$\mu_0 \in \left[\overline{X} - z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \; ; \; \overline{X} + z_{1-\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right].$$

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Hence, the relation: we Reject $\mathcal{H}_0$, if μ_0 is not in the CI, and otherwise, we Fail to Reject[1].

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t-test Example

Example: Again, I have generated in **R** a Sample of Size 20 from $\mathcal{N}(3.12, 2^2)$ and made some rounding:

```
set.seed(20112019)
s.size <-20; sigma <- 2
obs <-  rnorm(s.size, mean = 3.12, sd = sigma)
obs <- round(obs, digits = 2); obs
```

```
##  [1]  1.80  5.60  1.10  3.20  4.91  5.15  1.76  2.47  0.
## [13]  3.98  4.79  1.98  4.50  3.52  4.13 -0.08  3.87
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Now, let us forget about the fact that the actual value of μ is 3.12 and that $\sigma = 2$, and do some Testing, just assuming that our Observation is coming from a Normal Distribution.

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Now, let us forget about the fact that the actual value of μ is 3.12 and that $\sigma = 2$, and do some Testing, just assuming that our Observation is coming from a Normal Distribution. Say, let us Test, at the 5% Significance Level, the following Hypothesis:

$$\mathcal{H}_0 : \ \mu = 4 \qquad vs \qquad \mathcal{H}_1 : \ \mu \neq 4.$$

Example, Cont'd

First, we calculate *t*-statistic:

```
mu0 <- 4
t <- (mean(obs) - mu0)/(sd(obs)/sqrt(s.size)); t
```

```
## [1] -1.795358
```

Example, Cont'd

First, we calculate t-statistic:

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Now, we calculate the critical value, the quantile $t_{n-1,1-\alpha/2}$:

```
a <- 0.05
c <- qt(1-a/2, df = s.size-1); c
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## [1] 2.093024
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Finally, we check if t is in RR, i.e., if $|t| > t_{n-1,1-\alpha/2}$:

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abs(t) > c
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## [1] FALSE
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So the decision is:

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So the decision is: **Fail to Reject** $\mathcal{H}_0$ at 5% level.

Example, Cont'd

Now, the same, but with an **R** built-in function `t.test`:

```r
t.test(obs, mu = mu0, conf.level = 0.95)
```

```
##
##  One Sample t-test
##
## data:  obs
## t = -1.7954, df = 19, p-value = 0.08852
## alternative hypothesis: true mean is not equal to 4
## 95 percent confidence interval:
##  2.524009 4.112991
## sample estimates:
## mean of x
##    3.3185
```

Example, Cont'd

Now, for the same Data, let us Test, at the 10% Significance Level, the following Hypothesis:

$$\mathcal{H}_0 : \ \mu = 3 \qquad \textit{vs} \qquad \mathcal{H}_1 : \ \mu > 3.$$

Example, Cont'd

Now, for the same Data, let us Test, at the 10% Significance Level, the following Hypothesis:

$$\mathcal{H}_0: \ \mu = 3 \qquad \textit{vs} \qquad \mathcal{H}_1: \ \mu > 3.$$

```
t.test(obs, mu=3,alternative="greater", conf.level=0.9)
```

```
##
##  One Sample t-test
##
## data:  obs
## t = 0.83906, df = 19, p-value = 0.2059
## alternative hypothesis: true mean is greater than 3
## 90 percent confidence interval:
##  2.814508      Inf
## sample estimates:
## mean of x
##    3.3185
```