CS 107, Probability, Spring 2020
Lecture 12

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Content

- Bayes Formula
- Independence of Events

LZ

You are offered to play a chess game consisting of 3 rounds (each round consists of 1 game). You will get the prize if you will win in at least 2 rounds. You have two opponents, and one of them is the local city chess champion (C), and the other one is the local yard (thagh) chess champion (T), and you need to play with them alternately: either in the $C - T - C$ order or $T - C - T$ order.
Question: Who will win? Wrong question, of course $\ddot{\smile}$. Now, the correct one: which order you'll choose to maximize the winning probability?

Last Lecture ReCap

Last time

- we were solving Conditional Probability Problems
- we were drawing Probability Trees
- we were talking about the TPF in the case of one Hypothesis and its negation:

$$\mathbb{P}(A) = \mathbb{P}(B) \cdot \mathbb{P}(A|B) + \mathbb{P}(\overline{B}) \cdot \mathbb{P}(A|\overline{B})$$

- we were talking about the TPF in the general case:

$$\mathbb{P}(A) = \mathbb{P}(B_1) \cdot \mathbb{P}(A|B_1) + \mathbb{P}(B_2) \cdot \mathbb{P}(A|B_2) + \ldots + \mathbb{P}(B_n) \cdot \mathbb{P}(A|B_n).$$

- we started to talk about the Bayes formula:

$$\mathbb{P}(B|A) = \frac{\mathbb{P}(A|B) \cdot \mathbb{P}(B)}{\mathbb{P}(A)}.$$

Bayes Formula, General Form

Now, let us give the general form of the Bayes Formula, and the interpretation of that Formula as an Inverse Probability Calculation Formula.

Assume we have an event A. We have also some Hypotheses B_1, B_2,..., B_n.

- The Direct Problem: Calculate the Probability of A, given the Hypotheses. Solution: TPF

$$\mathbb{P}(A) = \mathbb{P}(B_1)\cdot\mathbb{P}(A|B_1)+\mathbb{P}(B_2)\cdot\mathbb{P}(A|B_2)+...+\mathbb{P}(B_n)\cdot\mathbb{P}(A|B_n)$$

- The Inverse Problem: Calculate the Probability that B_k happened, given that we have observed A. Solution: Bayes Formula:

$$\mathbb{P}(B_k|A) = \frac{\mathbb{P}(A|B_k)\cdot\mathbb{P}(B_k)}{\mathbb{P}(A)} = \frac{\mathbb{P}(A|B_k)\cdot\mathbb{P}(B_k)}{\sum_{i=1}^{n}\mathbb{P}(A|B_i)\cdot\mathbb{P}(B_i)}.$$

Example

Let us recall our Gym-Lovers City Problem: We know that in some city, 15% of persons are below the age 15, 25% are between 15 and 30, and the rest of population in that city are above 30. The proportion of regular gym visitors in that city for the younger-age population is 20%, for the mid-age-population - 40%, and for the older-age-population - 55%. What is the probability that a person in that city is a regular gym visitor? We know the solution. Now, the Inverse Probability question: Assume that the randomly chosen person is a gym visitor. What is the probability that he/she is of younger-age-group?

Solution: OTB

From one of my Midterms

Problem: I saw a dream. I was in a magic forest, standing at the crossroads. There was a street sign at the crossroads, stating:

> Welcome to the Magic Forest. If you will go through the Left Road, then you will meet your Princess with probability 60% or you will meet our Dragon with probability 40%. If you will go through the Right Road, then the chances are equal that you will meet your Princess or our Dragon. Please find under this sign 3 sealed boxes. One of this boxes is empty, and other two contain golden rings. Choose one of the boxes and open it. If that box will be empty, go to the Left, otherwise go to the Right. Hope you will enjoy the Magical Tour with us!

I was very excited, and I wanted to calculate my chances before choosing a box. Please help me to do that:

From one of my Midterms, Cont'd

a. What is the probability that I will meet the Dragon?
b. I woke up in the morning from the nightmare that someone is eating me. I met the Dragon! Find the probability that I was going through the Right Road.

The Bayes Rule: Medical Diagnosis Example

Problem: Medical Test gives a correct answer in 95% of cases: if the person is ill, it is saying that he/she is ill with 95% probability[1], and if the person is healthy, it is saying that he/she is healthy with 95% probability[2] (i.e., in 95% of cases).
Now assume I am taking that test, and it says that I am ill. What is the probability that I am really ill?
Solution: OTB

[1] This is called the *Sensitivity* of the test
[2] This is called the *Specificity* of the test

Nice video about the Bayes Theorem

https://www.youtube.com/watch?v=HZGCoVF3YvM

Independence of Events

Independence (and, of course, dependence) of events is one of the central notions in Probability theory. Probability theory is the mathematical tool to model the uncertainty, say, in the future. Like in problems of predicting stock future returns, predicting the chance of being ill, predicting the chance that some server will go out of order in the huge web of servers, predicting the chance of having a large number of insurance claims for a day, predicting the chance that we will have a quiz today etc.
And you know that events do not happen in an isolated way, do not come alone, there are a lot of other event affecting somehow or not affecting to the event under our interest. So we want to formalize the notion of Independence and Dependence in the Probability (Statistical) sense.