CS 107, Probability, Spring 2019
Lecture 07

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AUA

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Content

- Classical Probability Models: Finite Sample Spaces with Equally Likely Outcomes = Combinatorial Problems, Cont'd

Strange Mailman Problem

Mailman needs to deliver 100 letters to given addresses. But he is putting letters at random to the mailboxes. What is the probability that at least one person will receive the correct letter?

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Answer: the Probability is about 63%.

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- **Case 2:** We take balls with a replacement:

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Our Experiment is taking 12 balls at random. Let us fix, as an Outcome of our Experiment, the number of white and black balls in the chosen set of 12 balls. Say, if we will get 3 whites and 9 blacks, our Outcome will be $(3, 9)$, or if we will have 5 whites and 7 blacks, then our Outcome will be $(5, 7)$.

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Now, the correct solution:

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- This trick helps in different situations

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Assumptions: There are 365 days in a year, and the probability of being born on each day is the same.