AUA CS108, Statistics, Fall 2020

Lecture 07

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Contents

Visualizing 2D Data

In case we have a 2D numerical Dataset

$$(x_1, y_1), (x_2, y_2), ..., (x_n, y_n),$$

we usually do the ScatterPlot - the plot of all points (x_i, y_i), $i = 1, ...n$.

Example

Example: Graph the ScatterPlot for the following data:

Person ID	Age	Weight
1	20	69
2	22	57
3	40	65
4	20	70

Example

Say, consider again the *cars* Dataset:

```r
head(cars, 3)
```

```
##   speed dist
## 1     4    2
## 2     4   10
## 3     7    4
```

```r
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## 'data.frame':    50 obs. of  2 variables:
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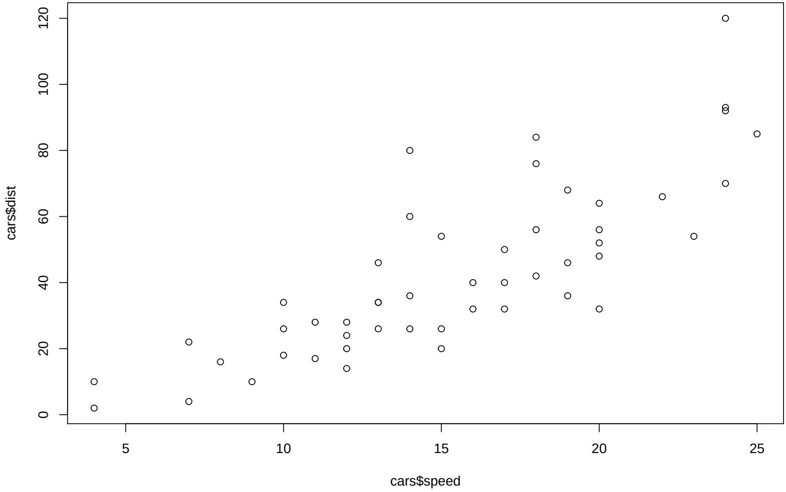
It has 2 Variables: *Speed* and *Distance*, and 50 Observations. Let us do the ScatterPlot of Observations:

ScatterPlot

```
plot(cars$speed, cars$dist)
```



Notes

- In this graph you can see that there is some relationship between the *Speed* and *Distance*, there is a *trend*: if the speed gets larger, the (stopping) distance is tending to increase.

Additions: Multidimensional Graphs

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- One can draw 3D in 2D, using the 3rd variable as the Color (not in all cases, of course)
- One can add the 4th Dimension by using the Size of Points
- And add the 5-th one by using the Shape of Points, ...

Examples

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Or, the following one: Gender Gap in Earnings per University

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- etc ...

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- Summaries (Statistics) about the Spread, Variability

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In particular,

$$x_{(1)} = \min\{x_1, x_2, ..., x_n\} \qquad \text{and} \qquad x_{(n)} = \max\{x_1, x_2, ..., x_n\}.$$

Example

Example: Let x be the Dataset

$$-2, 1, 3, 2, 2, 1, 1$$

Find the 4-th and 5-th Order Statistics.

Statistical Measures for the Central Tendency/Location

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Here we want to answer to the questions: what are the typical values of our Dataset, where is our Data located at?

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Note: Sometimes this property is a plus, not a drawback! Say, if we want to have an estimator which is sensitive to outliers.