AUA CS 108, Statistics, Fall 2019

Lecture 03

Michael Poghosyan
YSU, AUA
michael@ysu.am, mpoghosyan@aua.am

30 Aug 2019

Descriptive Statistics

Contents

- Different Types of Variables
- Measurement Levels
- Frequency Tables and Plots

- **Mane's PSS:** Thursdays, 3:30 - 5PM, room TBD
- **Mane's OH:** Thursdays, 5:10 - 7:10PM, room TBD

Last Lecture ReCap

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- What is a **Statistics** (not the Subject!)?
- What is a **Representative Sample**?

Few Sampling Methods

Let us recall the Definition of the **Representative Sample** (of size k):

Definition: We say that our Sample is *Representative* (obtained by a Simple Random Sampling), if it is obtained in the process where all Samples of size k have the same probability of being chosen.

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Simple Random Sampling is not so easy to perform, so people are using different simpler Sampling Strategies (although they are not always giving exactly Representative Samples):

Few Sampling Methods

- *Systematic (Interval) Sampling*, we fit the population into a list, enumerate it; choose n; choose at random a starting element from the first n elements in the list; and from that element on, every n-th member of the population is selected;

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- *Stratified Sampling*, where the total population is divided into subgroups (strata), that share similar characteristics, and then random sample (of corresponding size) is choosen from each strata.
- *Cluster Sampling*, where the total population is divided into subgroups (clusters), then some clusters are randomly chosen. Then we include all elements of chosen clusters into our Sample.

Classification of Data wrt its Dimension

Data can be

- **Univariate** (1D) - here the observations are on a single Variable
- **Bivariate** (2D) - here the observations are on two Variables
- **Multivariate** (n-D, $n \geq 2$) - when the observations are on more than a one Variable (usually, more than two)

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Examples:

- *No. of Children*, *No. of Customers* ,. . . are Discrete
- *Height*, *Weight*, *Age*, . . . are Continuous

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Let me give by an example: when talking about the number of children in the family, we can have the following data: 0, 2, 1, 2, 4, 6, and we can calculate, say, the average number of children in families, here 2.5.

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But even if we are enumerating the Sex or the Color, the average Sex or the average Color is not meaningful, we cannot deal with the assigned numbers as above!

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Yeah, there is an **order** in the second Variables, *Stat Final Letter Grade* and *Year of University Study*.

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Strongly Disagree | Disagree | Neither | Agree | Strongly Agree

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Descriptive Statistics for a Univariate Data

Let me recall that we use the Descriptive Statistics at the beginning of our Statistical Study to examine, explore the dataset.

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This part of Statistics is sometimes called **Exploratory Data Analysis**, EDA.

And we start by describing some of the *Graphical Summaries*.

Here, for the beginning, we will assume that we have a univariate (mostly numerical) data (dataset), $x_1, x_2, ..., x_n$. In this case we will say that we are given a (univariate, 1D) dataset x.

Frequency Tables

Here we assume that we have a univariate *discrete* numerical or categorical data $x_1, x_2, ..., x_n$.

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Definition: The **frequency** of a value t in observations $x_1, x_2, ..., x_n$ is the number of times t occurs in observations:

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Definition: The **relative frequency** (or percentage) of a value t in observations $x_1, x_2, ..., x_n$ is the ratio of frequency of t divided by the total number of observations, n:

$$\text{Relative Frequency of } t = \frac{\text{Frequency of } t}{\text{Total Number of Observations}} =$$

$$= \frac{\text{Frequency of } t}{n}.$$

Frequency Tables, Example

Example: Given the following Dataset:

$$1, 2, 4, 7, 2, 3, 2, 1, 2, 1, 4, 1, -1$$

obtain the Frequency and Relative Frequency Tables.

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Example: Let's construct the Frequency Table of the above Dataset using **R**:

```
x <- c(1, 2, 4, 7, 2, 3, 2, 1, 2, 1, 4, 1, -1)
table(x)
```

```
## x
## -1  1  2  3  4  7
##  1  4  4  1  2  1
```

Frequency Tables, Example

Now, consider the *iris* dataset in **R**:

```
head(iris)
```

```
##   Sepal.Length Sepal.Width Petal.Length Petal.Width Species
## 1          5.1         3.5          1.4         0.2  setosa
## 2          4.9         3.0          1.4         0.2  setosa
## 3          4.7         3.2          1.3         0.2  setosa
## 4          4.6         3.1          1.5         0.2  setosa
## 5          5.0         3.6          1.4         0.2  setosa
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Frequency Tables, Example, Cont'd

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iris$Species
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Frequency Tables, Example, Cont'd

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And to calculate the Frequency of each of the Species, we use

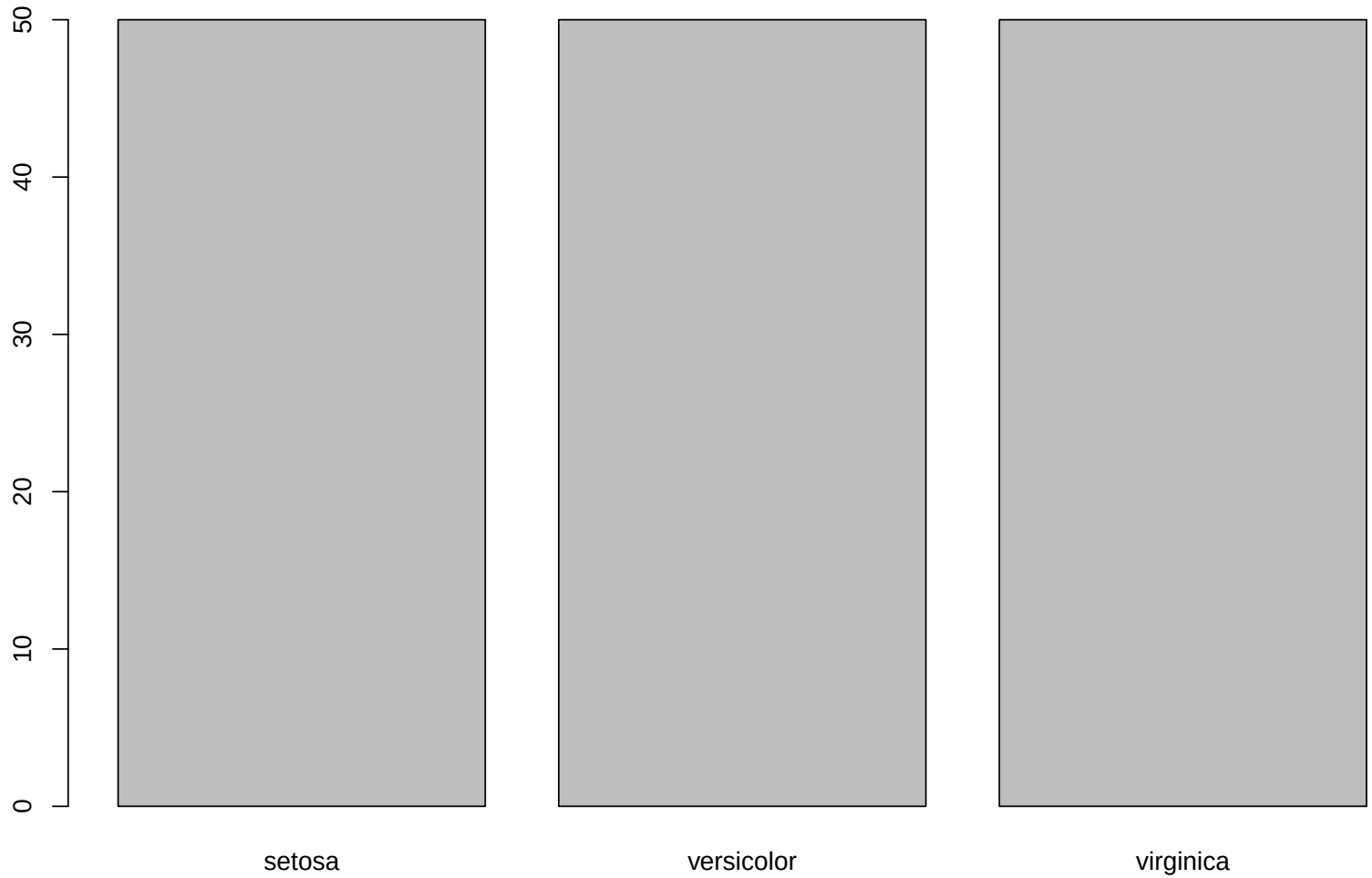
```
table(iris$Species)
```

```
##
##     setosa versicolor  virginica
##         50         50         50
```

LineGraph and BarPlot

Now, let us visualize our Frequency Table:

```
barplot(table(iris$Species))
```



LineGraph and Barplot

Another standard Dataset, *mtcars*, again about cars ◡̈:

```
head(mtcars, 3)
```

```
##                mpg cyl disp  hp drat    wt  qsec vs am gear c
## Mazda RX4     21.0   6  160 110 3.90 2.620 16.46  0  1    4
## Mazda RX4 Wag 21.0   6  160 110 3.90 2.875 17.02  0  1    4
## Datsun 710    22.8   4  108  93 3.85 2.320 18.61  1  1    4
```

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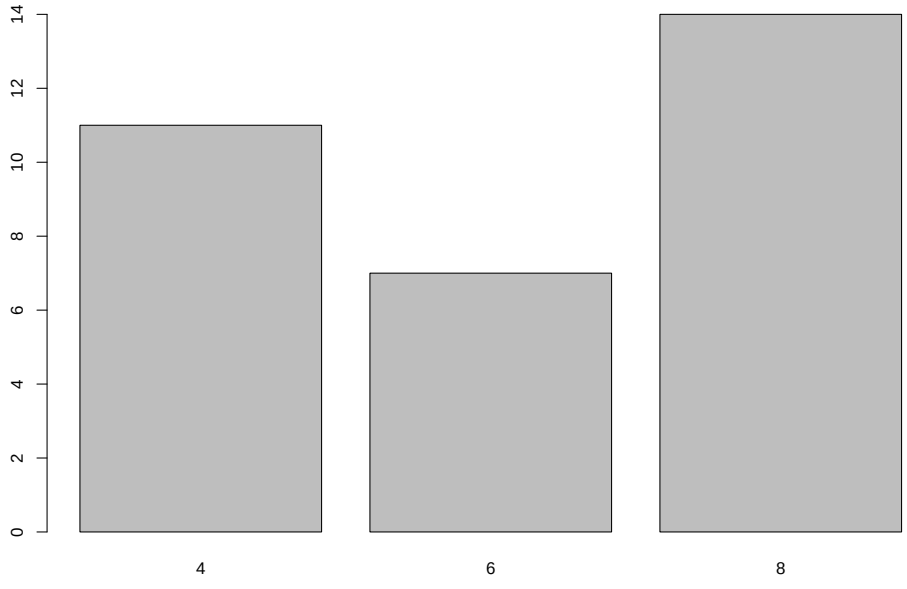
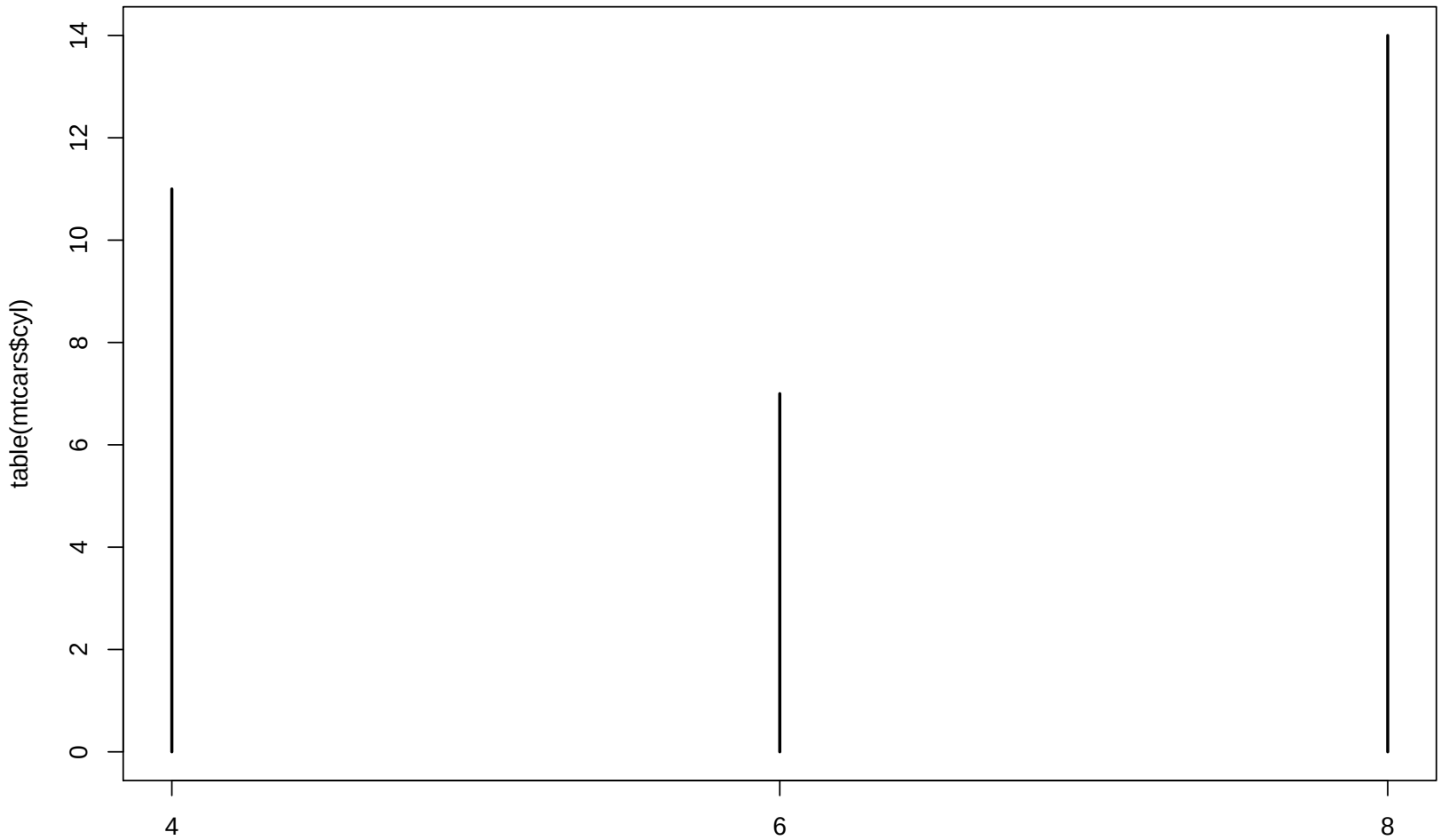
```
barplot(table(mtcars$cyl))
```



LineGraph and Barplot

Now, with the Line Graph:

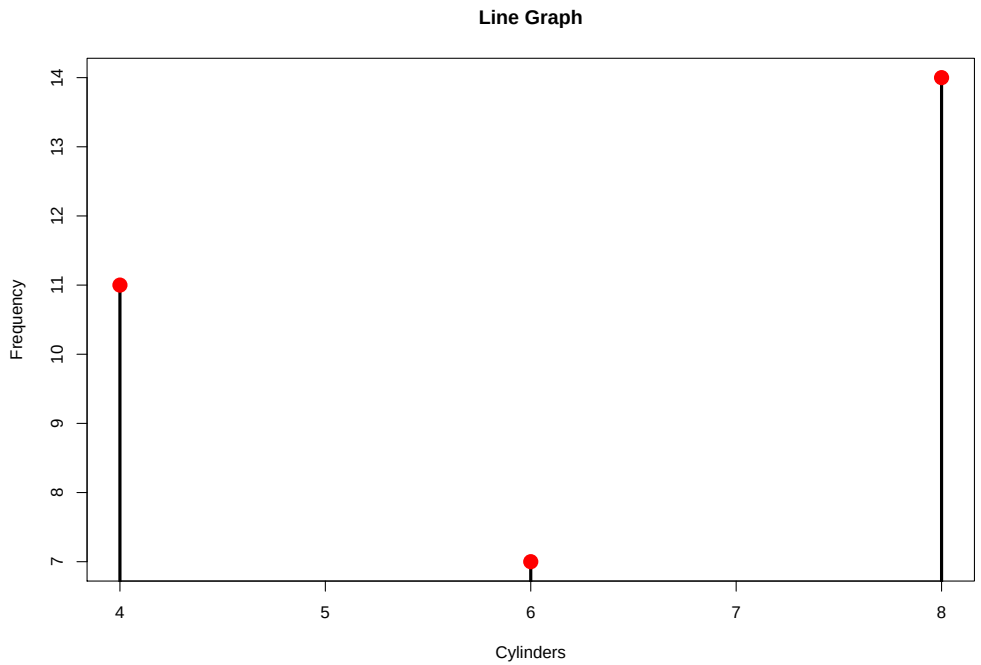
```
plot(table(mtcars$cyl))
```



LineGraph and Barplot

More sophisticated (titiz) version:

```
x <- mtcars$cyl; y <- as.data.frame(table(x))
a <- as.numeric(as.character(y$x)); b <- y$Freq
plot(a,b,type="h", lwd=3,  xlab = "Cylinders",
     ylab = "Frequency", main = "Line Graph")
points(a,b, pch=16, cex=2, col="red")
```



The Frequency Polygon

Again, same cars, but now the *carb* Variable Frequencies:

```
plot(table(mtcars$carb), type = "l")
```