CS 107 Section B - Probability

Spring 2019, AUA

Homework No. 05

Due time/date: 10:35AM, 01 March, 2019

Note: Supplementary Problems will not be graded, but you are very advised to solve them and to discuss later with TA or Instructor.

Note: Please provide your answers in the form of a decimal number, by calculating and simplifying fractions, with the accuracy of 2 digits after the period.

Problem 1. Assume we are rolling a die with sides

$$\{\boxdot, \boxdot, \boxdot, \boxdot, \boxdot, \boxdot\},$$

and the probability of rolling each side is the same. We repeat to roll this die 10 times. What is the probability that

a. We will have $\boxdot$ shown exactly 7 times;

b. We will not have any $\boxdot$ shown in 10 rolls;

c. We will have $\boxdot$ shown at least 8 times;

d. We will have $\boxdot$ shown 6 times, $\boxdot$ shown 3 times and $\boxdot$ shown 1 times.

e. We will not have $\boxdot$ and $\boxdot$ shown in these 10 rolls.

Problem 2. We have 37 Probability course participants, including the students, TA and the instructor. Assuming that all week days are equiprobable for the birth of a person, find the probability that

a. Exactly 12 participants have their birthdays on Sunday this year;

b. Less than 5 participants have their birthday on Monday this year;

c. 10 participants have their birthday on Monday this year, 5 on Tuesday, 6 on Wednesday, 8 on Thursday, 4 on Friday and the rest on Saturday.

d. 7 participants have their birthday on Saturday, 5 on Sunday, and all others have their birthdays on other (i.e., working) days.

Problem 3. When preparing for a birthday party, the host person invited 12 friends. She knows that for each friend there is a 5% chance that he/she will not be able to join the party.

a. What is the probability that exactly 10 persons will attend the party?

b. The host wants to buy a birthday cake. She wants to be sure by 80% that everybody will have his/her piece of cake. For how many persons at least she needs to order the cake?

Remark: You need to interpret what it means that "She wants to be sure by 80% that everybody will have his/her piece of cake".

Problem 4. Assume we want to test the recent elections results. The election was between 3 candidates, A, B and C, and the results of the election was:

Candidate A - 45% of votes;

Candidate B - 30% of votes;

Candidate C - 25% of votes.

To test the results, we pick by random 11 times a phone number (it is possible to choose the same phone number several times), call by that number and ask to report anonymously for whom the vote was given. What is the probability to have that 4 persons called gave their votes for the candidate A, 5 persons voted for B and 2 persons voted for C?

Problem 5. We have a box with 10 white, 5 red and 7 green balls. We draw a ball at random, fix the color and then return to the box. We repeat this experiment 10 times.

a. What is the probability that we will have exactly 6 white balls in 10 experiments?

b. What is the probability that the number of the white balls in 10 experiments will be between 5 and 7 (including 5 and 7)?

c. What is the probability that we will have exactly 3 white, 6 red and 1 green balls drawn?

d. What is the probability that we will have at least 6 white and at least 3 red balls drawn?

Problem 6. Assume we have a Server, which is sending an information packet through 3 Routers given in the Figure below. The probability that Router 1 will go out of order is 0.2, that Router 2 will go out of order is 0.1, and for the Router 3 that probability is 0.12. What is the probability that the End User will get the information sent by the Server, if we know that the events that the Routers will go out of order are mutually independent?