CS 107, Probability, Spring 2019
Lecture 39

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AUA

29 April 2019

Content

- The Expected Value of a R.V.
- The Variance of a R.V.
- Markov and Chebyshev Inequalities

Example:

Question: Is it true that

$$\mathbb{E}(X^2) = \Big(\mathbb{E}(X)\Big)^2 ?$$

Expectation and Independence

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Recall the Jensen's Inequality for convex functions: if $g : [a, b] \to \mathbb{R}$ is a convex function, $x_1, \dots, x_n \in [a, b]$ and $\alpha_1, \dots, \alpha_n \in [0, 1]$ with $\alpha_1 + \alpha_2 + \dots + \alpha_n = 1$, then

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Example: In Economics/Finance, one is defining Risk Aversion using this Inequality.

Partial Numerical Characteristics of R.V.s: Variance and Standard Deviation of a R.V

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Say, if we know that the AMZN stock Expected price at the end of the next week will be $\$2K$, then this is an important information. And we imagine that the price at the end of the next week will be around $\$2K$. Is this giving us a nice picture what the actual price will be?

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Consider 2 cases:

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- The AMZN stock price at the end of the next week can be anything from the interval $[1950, 2050]$, uniformly. In this case too, the Expected price will be 2000. But here the information is much more concrete about the price.

So not only the average is important, but also the spread around that average matters!

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Another example: if X is the wage of a (randomly chosen working) person in Armenia, then $\mathbb{E}(X) = 172,056$AMD (by Feb 2019, see `https://www.armstat.am/en/`).
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So the spread around the Expected Value is an important characteristic!

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which is a r.v.. Then we calculate the Square of that: $(X - \mathbb{E}(X))^2$, which is again a r.v.. Finally, we calculate the Expected Squared Deviations, $\mathbb{E}\Big((X - \mathbb{E}(X))^2\Big)$, which we call the Variance of X or the Dispersion of X.

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So people use the next important characteristic of the Spread, for reporting, the Standard Deviation:

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The **Standard Deviation** of a r.v. X is the square root of the Variance:

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Note: Another important numerical characteristic of the spread around the mean is the Mean Absolute Deviation (MAD) or the Mean Absolute Error:

$$MAD(X) = \mathbb{E}(|X - \mathbb{E}(X)|).$$

Calculation Formula for the Variance

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Important Consequence:

$$\mathbb{E}(X^2) = Var(X) + \Big(\mathbb{E}(X)\Big)^2.$$

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- The Deviation from the Mean is NOT Changed: $Var(X + \alpha) = Var(X)$.

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- The property $Var(X+Y) = Var(X) + Var(Y)$ is NOT TRUE for any r.v.s. It is true, in particular, when $X \perp\!\!\!\perp Y$. Later we will see that this property holds only for *uncorrelated* r.v.s.

Example

Example: Assume $X, Y \sim Bernoulli(0.5)$ and $X \perp\!\!\!\perp Y$. Calculate

$$Var(2X - 3Y + 5).$$