

# AUA CS 108, Statistics, Fall 2019

## Lecture 36

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# Contents

- ▶ Hypothesis Testing

# Last Lecture ReCap

- ▶ Give the Hypo Testing Framework.

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- ▶ Give the Hypo Testing Framework.
- ▶ Give the usual types of Hypotheses

## Moral, and Choosing Null Hypotheses

**Moral:** In Hypothesis testing, if we have enough evidence from Data against  $\mathcal{H}_0$ , we Reject it, otherwise, we say that we do not have enough evidence to Reject  $\mathcal{H}_0$ , so we Fail to Reject it, and keep believing in it.

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One is using this general idea to choose the Null and Alternative Hypotheses: **we will keep believing in Null, if the Data will not show strong evidence against.**

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This is an example of **A/B Testing**.

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# Type I and II errors (aka Confusion Matrix)

Assume we are Testing the Hypothesis

$$\mathcal{H}_0 \quad \text{vs} \quad \mathcal{H}_1.$$

Then the following cases can happen:

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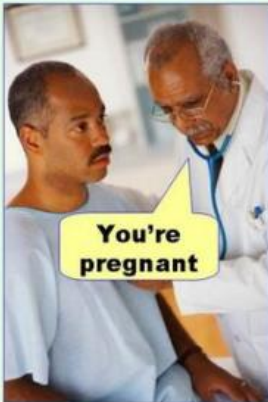
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Do Not Reject $\mathcal{H}_0$	Correct Decision (True Positive)	<b>Type II Error (False Negative)</b>

Can you guess the Null Hypo?

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**Type I error**  
(false positive)



**Type II error**  
(false negative)

