AUA CS108, Statistics, Fall 2020

Lecture 13

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Contents

Theoretical Quantiles, again

Now, if q_α is the α-quantile of some Distribution, and X is a r.v. from that Distribution, then

$$\mathbb{P}(X \leq q_\alpha) \geq \alpha \qquad \text{and} \qquad \mathbb{P}(X \geq q_\alpha) \geq 1 - \alpha.$$

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Note: If $\alpha = 0.5$, we call $q_\alpha = q_{0.5}$ to be the **Median of the Distribution**. So if we consider a Continuous r.v. and draw the PDF of that r.v., then the Median is the (leftmost) point dividing the area under the PDF curve into 50%-50% portions.

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- two given Datasets (possibly, of different sizes) are from the same Distribution;
- a given Dataset comes from a given Distribution;
- given two theoretical Distributions, check if one of them is a shifted-scaled version of the other one, or check if one has *fatter tails* than the other one

Q-Q Plots, Data vs Data

Now, assume we have two Datasets, not necessarily of the same size:

$$x: \; x_1, x_2, ..., x_n \qquad \text{and} \qquad y: \; y_1, y_2, ..., y_m$$

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and then draw the points (q_α^x, q_α^y).

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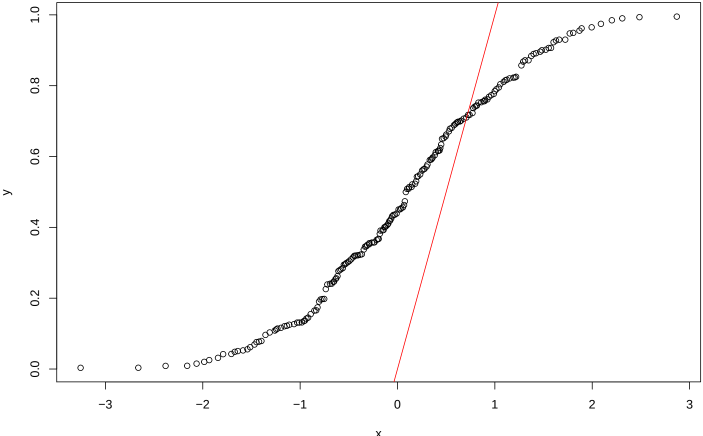
Idea: If x and y are coming from the same Distribution, then the Quantiles of x and y need to be approximately the same, $q_\alpha^x \approx q_\alpha^y$, so geometrically, the points (q_α^x, q_α^y) need to be close to the bisector line.

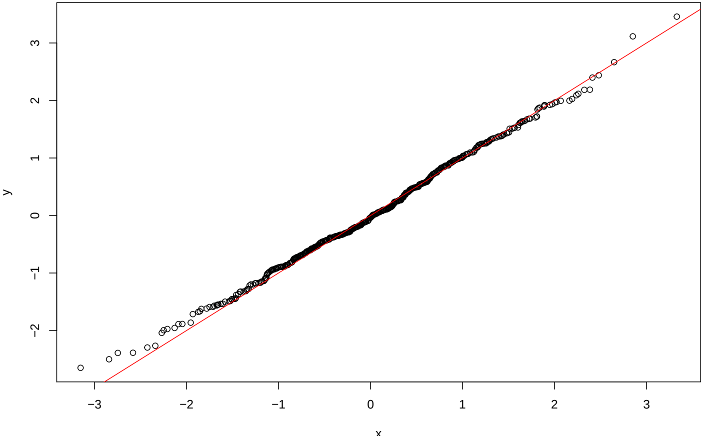
Example, Q-Q Plots, Data vs Data

```r
x <- rnorm(1000)
y <- runif(200)
qqplot(x,y)
abline(0,1, col="red")
```



Example, Q-Q Plots, Data vs Data

```
x <- rnorm(1000)
y <- rnorm(500)
qqplot(x,y)
abline(0,1, col="red")
```



Example, Q-Q Plot by Hands, Data vs Data

Example: Assume

$$x: \ -1, 2, 1, 2, 3, 2, 1 \qquad y: \ 0, 3, 4, 1, 1, 1, 1, 2$$

Draw the Q-Q Plot for x and y.