CS 107 Section A - Probability

Spring 2020, AUA

Homework No. 08

Due time/date: 7 April, 2020

Note: Supplementary Problems will not be graded, but you are very advised to solve them and to discuss later with TA or Instructor.

Important Discrete Random Variables

A Discrete Uniform Distribution

Problem 1. Assume $X \sim DiscreteUnif(\{-10, 2, \pi, 10\})$.

a. Write the PMF of X;

b. Construct the CDF of X.

Problem 2. I am choosing a weekday at random, with equal probabilities. Let WD be the number of the chosen weekday (I will take $Monday = 1$, $Tuesday = 2$, ...). Write WD as a DiscrUnif Distributed r.v. (i.e., write in the form $WD \sim DiscreteUnif(...)$).

B Bernoulli Distribution

Problem 3. Assume $X \sim Bernoulli(0.9)$.

a. Construct the PMF of X;

b. Construct the CDF of X;

c. What is the distribution of the r.v. $Y = X^2$?

d. What is the distribution of the r.v. $Z = 1 - X$?

e. Will $T = -X$ be a Bernoulli r.v.? If yes, what is the parameter p?

Problem 4. We have 11 female students in our group of 28 students. Let G be the gender of a randomly chosen student in our group, with the convention that $G = 1$, if the chosen person is a female, and $G = 0$ otherwise. Write G as a Bernoulli Distributed r.v. .

C Binomial Distribution

Problem 5. Assume $X \sim Binom(3, 0.2)$.

a. Construct the PMF of X;

b. Find $\mathbb{P}(X > 1)$;

c. Find the distribution of the r.v. $Y = 3 - X$;

d. (Supplementary) Will the r.v. $Z = 2 \cdot X$ be a Binomial r.v?

e. (Supplementary) Give an example of an experiment, and a r.v. X in that experiment with the above Distribution

Problem 6. 28% of population in Armenia are smokers[1]. I am choosing 100 persons in Armenia (a sample), randomly, independently (so I can choose the same person several times). Let X be the number of non-smokers in that sample.

a. Write X as a Binomial Distributed r.v. ;

b. Write the PMF of X in the form $\mathbb{P}(X = k) = ...$, for any $k = ...$;

c. What is the probability that we will have more than 3 smokers in that sample?

d. (Supplementary) Find the most probable value of X.

D Geometric Distribution

Problem 7. Assume $X \sim Geom\left(\frac{1}{4}\right)$.

a. Construct the PMF of X;

b. Write the PMF of X in the form $\mathbb{P}(X = k) = ...$, for any $k = ...$;

c. Find $\mathbb{P}(X \text{ is divisible by } 3)$;

d. For any possible value of m, calculate the probability

$$\mathbb{P}(X = m | X \text{ is divisible by } 3).$$

e. Assume also $Y \sim Geom(0.5)$. Which one is larger, $\mathbb{P}(Y > 100)$ or $\mathbb{P}(X > 100)$? Can you find the largest one without any calculations, just by the idea of the Geometric r.v.?

f. (Supplementary) By using the idea of the Geometric r.v., can you find the average value of X? (Later we will define and calculate the mean value of the r.v.).

g. (Supplementary) Give an example of an experiment, and a r.v. X in that experiment with the above Distribution

Problem 8. Assume I am playing the Minesweeper game[2], with the board of size 10×10, with 20 hidden mines. But, as a Probability lecturer, I am hitting on the game board randomly at some square. Let B be the number of hits I will do until "blowing up" (hitting on a mine).

a. Write B as a Geometric Distributed r.v. (i.e., write it in the form $B \sim Geom(...)$);

b. Calculate $\mathbb{P}(B \leq 3)$.

[1] 2019 Dec data.

[2] https://en.wikipedia.org/wiki/Minesweeper_(video_game)

E Poisson Distribution

Problem 9. Assume $X \sim Pois(1.2)$.

a. Construct the PMF of X;
b. Write the PMF of X in the form $\mathbb{P}(X = k) = ...$, for any $k = ...$;
c. Find $\mathbb{P}(6 \leq X < 8.7)$;
d. What is the average value of X?
e. Is $\mathbb{P}(X > 90)$ big? Explain!
f. (Supplementary) Will the r.v. $Y = 2 \cdot X$ be again a Poisson r.v.?

Problem 10. According to the Wikipedia list of countries by the share of millionaires[3], only 0.1% of the total population in Armenia are millionaires. If I will choose at random a sample of 5000 persons in Armenia (I can choose the same person several times),

a. What is the exact Probability of having more than 3 millionaires in that sample?
b. Calculate the above probability by using the Poisson approximation.

Problem 11. Assume that we have a statistics that in 10 days, in average 6 Samsung Galaxy S20+ (Red) phones are bought at some phone store. Let X be the number of customers that will buy the mentioned phone in a day.

a. Write a probabilistic model for X;
b. Calculate the probability that in a day, 2 customers will buy phones of that model;
c. Assume Y is the number of phones of that model bought in a week. Write a probabilistic model for Y;
d. Calculate the probability that in a week, more than 10 phones of that model will be bought;
e. (Supplementary) Give a suggestion to the manager of that store about at least how many phones of that model they need to have for a week to satisfy the following: the probability that there will be not enough phones in a week for customers is less than 0.03. Make a computer simulation to see what is the proportion of weeks when there was a shortage of phones.

Problem 12. In a day, I am doing several calls. The probability that I will dial zero wrong numbers in a day, is 0.98. What is the probability that I will dial 2 wrong numbers in a day?

F Supplementary Problems

Problem 13. (Supplementary) My Financial Mathematics exam program at YSU consists of 50 questions, and I am preparing for the exam 50 small paper pieces, writing a question on one side of the paper, one question for each piece of paper. A student knows the answers for 20 questions from that program. During the oral exam, he is taking

[3] https://en.wikipedia.org/wiki/List_of_countries_by_the_number_of_millionaires

at random a paper piece with a question, and is answering, if he knows the answer. After answering or not answering to the question, this student is returning the exam question paper back into the basket of questions, and then taking another question at random to answer. I assume that answering the same question is OK for me, if the student is taking the same question twice, and I am allowing to take as many questions as the student wants[4]. The student passes the exam, if he is answering correctly to one question. Let X be the number of trials (of taking a question) the student will do to pass the exam.

a. What is the distribution of X ?
b. What is the probability that the student will pass my exam doing no more than 7 trials?
c. What is the probability that the student will not pass my exam at all?
d. What about the parts **a.** and **b.**, if I am asking to answer 2 (or, say, 3) questions correctly to pass the exam?
e. What about the parts **a.** and **b.**, if I am not allowing to return back the taken question?

Problem 14. (Supplementary) I am rolling 3 dice 350 times[5]. In each roll, I am interested to have at least 2 sixes - this is my success in one trial. For the total experiment of 350 rolls, I am interested in the number of successes I will have. Let X be the number of successes in 350 trials.

a. What is the distribution of X?
b. I want to approximate X using the Poisson distribution. What is the parameter (rate) λ I will use for the Poisson distribution to approximate the distribution of X?
c. What is the exact probability that the number of successes will be in between 30 and 33?
d. What is the Poisson approximated probability that the number of successes will be in between 30 and 33? Calculate the numbers using some math software and compare with the above probability.

Problem 15. (Supplementary) We consider the number of car accidents during 10AM till 6PM in the Arshakunyats avenue. The average number of accidents is, say, 0.6 (for example, in average, we "have" 6 accidents in 10 days during that time interval). Let X will be the number of car accidents during 10AM till 6PM tomorrow in the Arshakunyats ave. .

a. Model the probability distribution of X;
b. What is the probability that the number of car accidents for tomorrow during the above time interval on the Arshakunyats ave. will be more than 2?
c. What is the probability that we will not have any car accidents tomorrow during the above time interval on the Arshakunyats ave.?

[4] Well, this is from a kind of scientific fiction series, of course ⌣̈

[5] Tiresome job, of course. There is nothing I would not do for a good Problem ⌣̈

Problem 16. (Supplementary) Assume one of our local taxi services is receiving in average 0.5 phone calls during 1 min interval. Let X be the number of calls that taxi service is receiving in a **10 min** interval.

a. Model the probability distribution of X;

b. What is the probability that our taxi service will not receive any calls during the next 10-min interval?

c. What is the probability that our taxi service will receive more than 20 calls during the next 10-min interval?