CS 107 Section B - Probability

Spring 2019, AUA

Homework No. 11

Due time/date: 10:35AM, 29 April, 2019

Note: Supplementary Problems will not be graded, but you are very advised to solve them and to discuss later with TA or Instructor.

Note: Please provide your answers in the form of a decimal number, by calculating and simplifying fractions, with the accuracy of 2 digits after the period.

Problem 1. a. Assume that X and Y are Discrete r.v.'s, and assume X and Y are Independent: $X \perp\!\!\!\perp Y$. Find the Joint PMF of X and Y, if

Y \ X	-2	1	2	PMF of Y
-10				$\frac{1}{4}$
10				$\frac{3}{4}$
PMF of X	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	

b. Assume that the Joint PMF of Discrete r.v. X and Y is given by

Y \ X	2	3
-3	0.1	0.3
1	0.2	0.4

Are X and Y Independent? Prove your statement.

c. Assume X and Y are Discrete r.v. with the following PMFs:

X	0	2
$\mathbb{P}(X = x)$	0.3	0.7

and

Y	0	2
$\mathbb{P}(Y = y)$	0.3	0.7

Are X and Y Dependent? Explain.

Problem 2. Assume $X \sim Binom(4, 0.2)$ and $Y \sim Pois(1)$ and X and Y are Independent: $X \perp\!\!\!\perp Y$. Calculate

$$\mathbb{P}(X + Y \leq 2).$$

Problem 3. Assume $(X, Y) \sim Unif(D)$, where D is the square $D = \{(x, y) : |x| + |y| \leq 1\}$. Are X and Y Independent? Prove your statement.

Problem 4. Assume X and Y are Independent. Prove that $2X+1$ and Y^3 are Independent too.

Problem 5. Assume $X \sim Unif[-1,2]$ and $Y \sim Exp(3)$, and X and Y are Independent.

a. Find the Joint PDF of X and Y;
b. Calculate $\mathbb{P}(X \in [1,2], Y \in [0,1])$.

Problem 6.

a. Assume $X \sim \mathcal{N}(1,2^2)$ and $Y \sim \mathcal{N}(2,4^2)$. Are X and Y Independent? Explain.
b. Assume again that $X \sim \mathcal{N}(1,2^2)$ and $Y \sim \mathcal{N}(2,4^2)$, and now assume that X and Y are Independent. Calculate the Joint PDF of X and Y.
c. Assume $(U,V) \sim \mathcal{N}(\mu,\Sigma)$, where

$$\mu = \begin{bmatrix} -2 \\ 3 \end{bmatrix}, \qquad \Sigma = \begin{bmatrix} 1 & 0 \\ 0 & 6 \end{bmatrix}.$$

Prove that U and V are Independent.

Problem 7. Assume X, Y and Z are IID, i.e., Independent and Identically Distributed, i.e., they all have the same CDF $F(x)$. Calculate the CDF of

$$U = \max\{X,Y,Z\} \qquad \text{and} \qquad V = \min\{X,Y,Z\}.$$

Generalize for n IID random variables.

Note: This result is important in Statistics.

Problem 8. (from [R]) Solve the Problem 6.14, page 271.

Problem 9. (from [R]) Solve the Problem 6.17, page 272.

Problem 10. (from [R]) Solve the Problem 6.22, page 272.

Problem 11. (from [R]) Solve the Problem 6.31, page 273.

Problem 12. (from [R]) Solve the Problem 6.32, page 273.

Supplementary Problems

Problem 13. (Supplementary) Assume $X \sim Pois(\lambda_1)$, $Y \sim Pois(\lambda_2)$ and $X \perp\!\!\!\perp Y$. Prove that $X+Y \sim Pois(\lambda_1+\lambda_2)$. Give some intuition behind this property.

Problem 14. (Supplementary) Assume $X, Y \sim Unif[0,1]$ and $X \perp\!\!\!\perp Y$. Using convolutions, find the PDF of $X+Y$.

Problem 15. (Supplementary)

a. Assume X is a discrete r.v. Is X independent of X? Prove your statement.
b. Assume X is any r.v.. Can X be independent of X?

Hint: Assume the CDF of X is $F(x)$, and $F(x,y)$ is the Joint CDF of X with X, i.e., the CDF of the random vector (X,X). Express $F(x,y)$ in terms of $F(x)$.

Problem 16. (Supplementary, from [R]) Solve the Problem 6.54, page 274.

Problem 17. (Supplementary, from [R]) Solve the Problem 6.56, page 274.