CS 107, Probability, Spring 2019
Lecture 22

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AUA

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Content

- Examples of Important Discrete R.V.s

LZ

$$0 = 1$$

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Proof:

$$\int \frac{1}{x} dx =$$

$$0 = 1$$

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$$\int \frac{1}{x} dx = (IBP)$$

LZ

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Proof:

$$\int \frac{1}{x} dx = (IBP) = x \cdot \frac{1}{x} - \int x \cdot (\frac{1}{x})' dx =$$

$$1 + \int \frac{1}{x} dx$$

And we cancel $\int \frac{1}{x} dx$ from the LHS and RHS.

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That is, we can calculate Probabilities about the possible values of a r.v. through the CDF or PMF/PDF.

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$\displaystyle\sum_k p_k = 1$	$\displaystyle\int_{-\infty}^{+\infty} f(x)dx = 1$
$F(x) = \displaystyle\sum_{x_k \le x} p_k$	$F(x) = \displaystyle\int_{-\infty}^{x} f(t)dt$
$\mathbb{P}(a \le X \le b) = \displaystyle\sum_{a \le x_k \le b} p_k$	$\mathbb{P}(a \le X \le b) = \displaystyle\int_a^b f(x)dx$

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Example: Let X be the number of emails I will get today. Then X is a r.v. Is it Discrete or Continuous? Then $\mathbb{P}(X)$ denotes "the Probability of number of emails I will get today". Nonsense, of course $\ddot\smile$ But we can talk about $\mathbb{P}(X = 0)$ or $\mathbb{P}(X > 10)$, and they have clear meanings.

Important Discrete R.V.

Discrete Uniform Distribution

Discrete Uniform Distribution

We will say that the r.v. X has a Discrete Uniform Distribution with (over) the values $x_1, x_2, ..., x_n$ ($x_i \neq x_j$, $i \neq j$), and we will write $X \sim DiscreteUnif(x_1, ..., x_n)$, if

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that is, $\mathbb{P}(X = x_k) = \frac{1}{n}$, for any $k = 1, 2, ..., n$.

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- We are rolling a fair die, X is the top face number.
- Can you give some more?

Discrete Uniform Distribution: R Examples

R Code

```
#Discrete Uniform on data
data <- c(-1,2,4)
sample(data, size = 10, replace = T)

#Generating a sample of size 100
s <- sample(data, size = 100, replace = T)
#Calculating the number of -1 in the sample s
length(s[s == -1])
```