AUA CS108, Statistics, Fall 2020

Lecture 14

Michael Poghosyan

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Example: Say, is the following Dataset

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##  [1]  0.033  0.136  0.887  0.764 -0.749  0.987  0.347  0
## [11] -0.405 -0.645  0.612  0.401  0.233 -0.920 -0.133  0
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from a Normal Distribution?

To answer this question, we again take some levels of quantiles, say, for some n,

$$\alpha = \frac{1}{n}, \frac{2}{n}, ..., \frac{n-1}{n}$$

and then draw the points (q_α^F, q_α^x), where q_α^F is the α-quantile of the Theoretical Distribution, and q_α^x is the α-quantile of x.

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To answer this question, we again take some levels of quantiles, say, for some n,

$$\alpha = \frac{1}{n}, \frac{2}{n}, ..., \frac{n-1}{n}$$

and then draw the points (q^F_α, q^x_α), where q^F_α is the α-quantile of the Theoretical Distribution, and q^x_α is the α-quantile of x.

Idea: If x is from the Distribution given by F, then we need to have $q^F_\alpha \approx q^x_\alpha$, so, graphically, the point will be close to the bisector.

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Another **R** command is `qqline` which adds a line passing (by default) through the first and third Quartiles,

$$(q^F_{0.25}, q^x_{0.25}) \qquad and \qquad (q^F_{0.75}, q^x_{0.75}).$$

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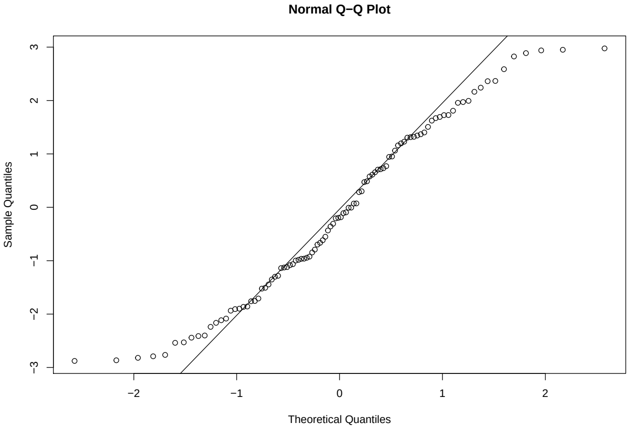
Some Experiments

Here are some experiments with `qqnorm`

```r
x <- runif(100,-3,3)
qqnorm(x)
qqline(x)
```



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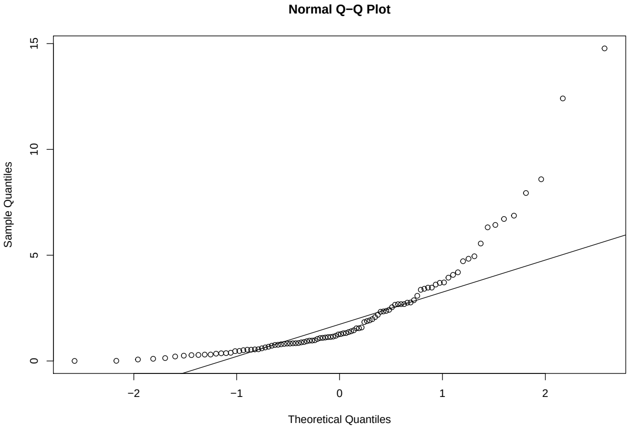
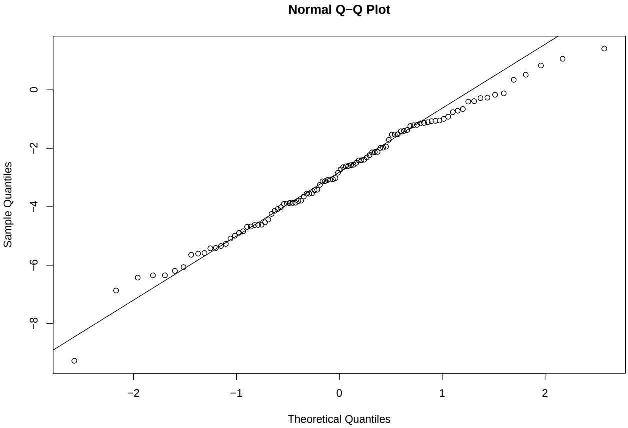
```
x <- rexp(100,0.4)
qqnorm(x)
qqline(x)
```



Some Experiments

Here are some experiments with `qqnorm`

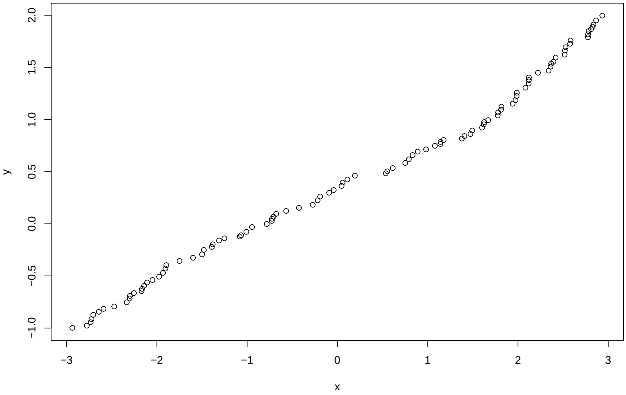
```
x <- rnorm(100, mean = -3, sd = 2)
qqnorm(x)
qqline(x)
```



Some Experiments

Now, assume we want to see if our Dataset x is from $Unif[-1,2]$:

```
x <- runif(100,-3,3)
y <- runif(1000,-1,2)
qqplot(x,y)
```



Important Note

It is important, that, using `qqnorm`, we can check if our Dataset comes from a Normal Distribution, *with some mean and variance*.

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But, for the Normal Distribution, we can use the fact that all Normal Distributions can be obtained from the Standard Normal, by scaling and shifting.

[2] Can you state rigorously and prove this?

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But, for the Normal Distribution, we can use the fact that all Normal Distributions can be obtained from the Standard Normal, by scaling and shifting. This means that the Quantiles of any Normal Distribution can be obtained by a linear transform from the Standard Normal Quantiles[2].

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So if, say, x is a sample from $\mathcal{N}(2, 3^2)$, then

- when doing a Q-Q Plot of x vs $\mathcal{N}(2, 3^2)$, the Quantiles will be

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So if, say, x is a sample from $\mathcal{N}(2, 3^2)$, then

- when doing a Q-Q Plot of x vs $\mathcal{N}(2, 3^2)$, the Quantiles will be on the bisector;
- when doing a Q-Q Plot of x vs $\mathcal{N}(0, 1)$, the Quantiles will be on some line (can you find the line equation?);

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And if `qqnorm` shows that the quantiles are close to the bisector, that means that the Dataset is possibly from the Standard Normal Distribution.

Note: The theoretical justification of the above is in the following: if z_α is the quantile of order α of $\mathcal{N}(0, 1)$, and if q_α is the same order quantile of $\mathcal{N}(\mu, \sigma^2)$, then there is a linear relationship between q_α and z_α.

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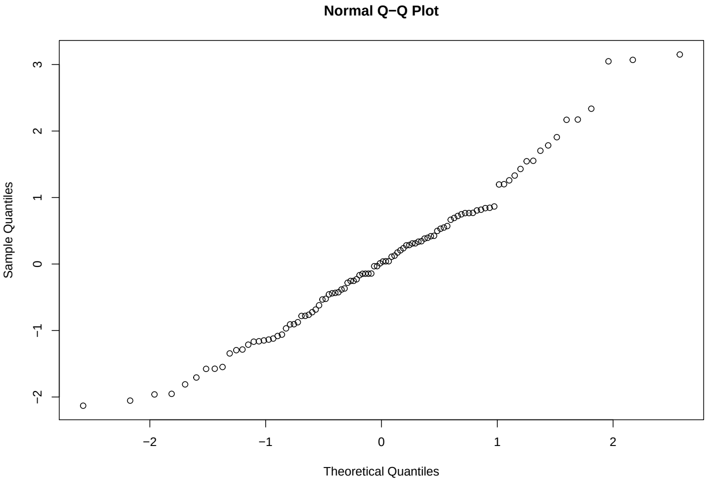
Exercise: Find that relationship in terms of μ and σ.

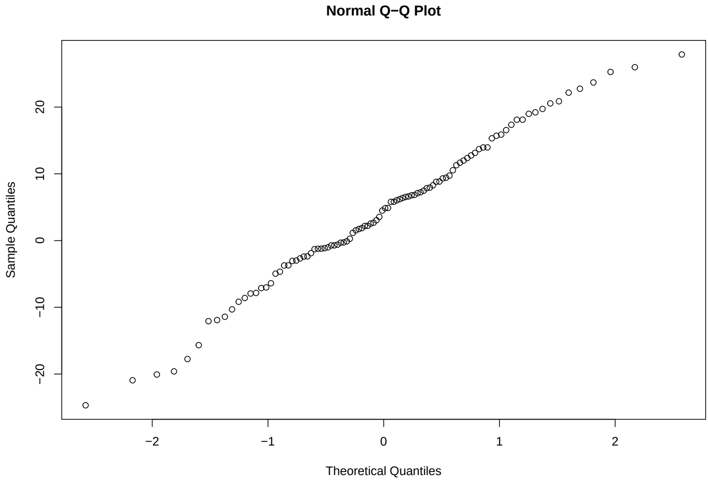
Some Experiments

```
x <- rnorm(100, mean=0, sd=1)
qqnorm(x)
```



Some Experiments

```r
x <- rnorm(100, mean=2, sd=12)
qqnorm(x)
```



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The above important note works also for the Uniform Distribution. This is again because all Uniform Distributions are the scaled-translated versions of the Standard Uniform $Unif[0, 1]$.

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So if you will compare your Dataset with $Unif[0,1]$, and Q-Q Plot will show that the Quantiles are close to a line, that means that probably your Dataset is from a Uniform Distribution, with some parameters.

Exercise: Find a relationship between the quantiles of $Unif[a,b]$ and $Unif[0,1]$.

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and then draw the points (q_α^F, q_α^G), where q_α^F is the α-quantile of the Theoretical Distribution with the CDF F, and q_α^G is the α-quantile of the Theoretical Distribution with the CDF G.