

# CS 107 Section A - Probability

Spring 2020, AUA

## Homework No. 03

Due time/date: 09:35AM, 17 February, 2020

**Note:** Supplementary Problems will not be graded, but you are very advised to solve them and to discuss later with TA or Instructor.

**Problem 1.** What is the probability that a randomly chosen 8-bit binary number will

- represent an even decimal number;
- represent a decimal number divisible by 5;
- will be less than  $(10001)_2$ ;
- (supplementary) will contain the substring 1011 ?

**Problem 2.** We will consider here Armenian car license plates. For simplicity, you can assume that license plates have the form  $nnLLnnn$ , and plates like 00AA000 are possible, that is, all numbers and all letters are allowable.

- What is the probability that a randomly chosen Armenian car license plate will contain the letters  $MP$  (in this order) ?
- What is the probability that among randomly chosen 3 Armenian car license plates at least one will contain the letters  $MP$  (in this order) ?

**Problem 3.** We want to build a team from AUA to participate in a TV game. The team needs to have 5 members. We have the following candidates for participation: 10 CS students, 6 BUS students and 3 EC students.

- We choose 5 members randomly. What is the probability that we will have 3 CS students and 1 BUS and 1 EC students in that team?
- We choose 5 members randomly. What is the probability that all departments will be represented in that team?

**Problem 4.** a. We toss a biased coin 3 times, and our biased coin has twice higher probability of getting Tails than Heads. I am interested in the event that I will have at least 2 Heads in 3 tosses. I solve this problem in the following way: the Sample Space is the set of all possible outcomes:

$$\Omega = \text{Sample Space} = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\},$$

so  $\#\Omega = 8$ , and the event in which I am interested is

$$A = \text{At Least 2 Heads} = \{HHH, HHT, HTH, THH\},$$

with  $\#A = 4$ . So I calculate

$$\mathbb{P}(A) = \frac{\#A}{\#\Omega} = \frac{4}{8} = \frac{1}{2}.$$

Is this solution correct? Explain your reasoning. If it is not correct, give the correct solution.

- b. Again, I am tossing a coin 3 times, in this case my coin is unbiased (fair). And again I am interested in the event that I will have at least 2 Heads in 3 tosses. I solve the problem in this way: What can be the outcomes of my experiment? I can get either 0 Heads, or 1 Heads, or 2 Heads or 3 Heads in 3 tosses, so

$$\Omega = \text{Sample Space} = \{0\text{Heads}, 1\text{Heads}, 2\text{Heads}, 3\text{Heads}\},$$

and  $\#\Omega = 4$ , and the event I am interested in is

$$A = \text{At Least 2 Heads} = \{2\text{Heads}, 3\text{Heads}\},$$

with  $\#A = 2$ . So the probability of  $A$  is

$$\mathbb{P}(A) = \frac{\#A}{\#\Omega} = \frac{2}{4} = \frac{1}{2}.$$

Is this solution correct? Explain your reasoning. If it is not correct, give the correct solution with this Sample Space.

**Problem 5.** We have 15 computers in our server room, and at some instant 15 jobs are arriving to our server room. We have a server that assigns the jobs to computers randomly (each choice is equiprobable, and it is possible that the server will assign all jobs to just one of the computers). What is the probability that at least one of the computers will have more than one jobs assigned to it?

**Problem 6.** Our class consists of 27 students and the instructor, MP. The number of female students is 10.

- What is the probability that a randomly chosen person in our class will be a male?
- We chose 2 persons randomly in our group. What is the probability that they will be of the same sex?
- We choose 5 persons randomly. What is the probability that all of them will be females?

**Problem 7.** I have a Baunty Chocolate stick, which is 15cm long, and I want to share it with you. I am breaking the chocolate stick at a random (with uniform probabilities) place along the length, and give the right-hand piece to you. What is the probability that you will get more than twice longer piece than me? ☹

**Problem 8.** We are playing the following game: me and you are choosing each randomly (with uniform probabilities) a real number from  $[0, 2]$ . I win, if the sum of our numbers is smaller than 1 or if their product is larger than 3. Otherwise, you win. Is this game fair? (The game is fair, if the chances to win are the same for me and you). Explain your reasoning. If not, who has more chances to win?

**Problem 9.** (Supplementary) A stick of length  $a$  is broken at random at 2 places (mathematically, this can be stated as: we choose uniformly 2 points in  $[0, a]$ ). What is the Probability that we can form a triangle from the obtained 3 parts of our stick?

**Problem 10.** (Supplementary) Assume Tatiki Lotto issued 10000 lotto tickets - each ticket contains a rectangle, where, under the cover, either "Muraba" or "Skhtor" is written. Playing is easy - you erase the cover of the rectangle, and if "Muraba" happens, you win. Otherwise, you loose. Tatiki Lotto announced that there are 250 winning tickets. I am buying 20 tickets. What is the probability that I will have at least one winning ticket?