AUA CS108, Statistics, Fall 2020

Lecture 29

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02 nov 2020

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Example

Biased Estimator Case

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and we want to estimate λ.

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and we want to estimate λ. We consider the following Estimator:

$$\hat{\lambda} = \frac{X_1 + X_2 + ... + X_{10}}{10}.$$

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Biased Estimator Case

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$$X_1, X_2, ..., X_{10} \sim Exp(\lambda)$$

and we want to estimate λ. We consider the following Estimator:

$$\hat{\lambda} = \frac{X_1 + X_2 + ... + X_{10}}{10}.$$

Easy to see that $\hat{\lambda}$ is an Biased Estimator for λ (OTB!).

Example, cont'd

Now, the code:

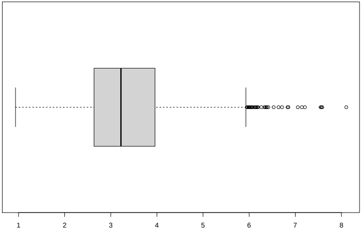
- observing once: generating a Sample just once and calculating one Estimate:

```
lambda <- 0.3
x <- rexp(10, rate = lambda)
lambda.hat <- mean(x)
lambda.hat
```

```
## [1] 4.088202
```

- observing many times: ganarating Samples many times, calculating Estimates, and then averaging:

```
lambda <- 0.3; n <- 10; m <- 2000
x <- rexp(n*m, rate = lambda)
x <- as.data.frame(matrix(x, ncol = m))
lambda.hats <- sapply(x, mean)
boxplot(lambda.hats, horizontal = T);
abline(v = lambda, col="red", lwd = 2, lty = 2)
```

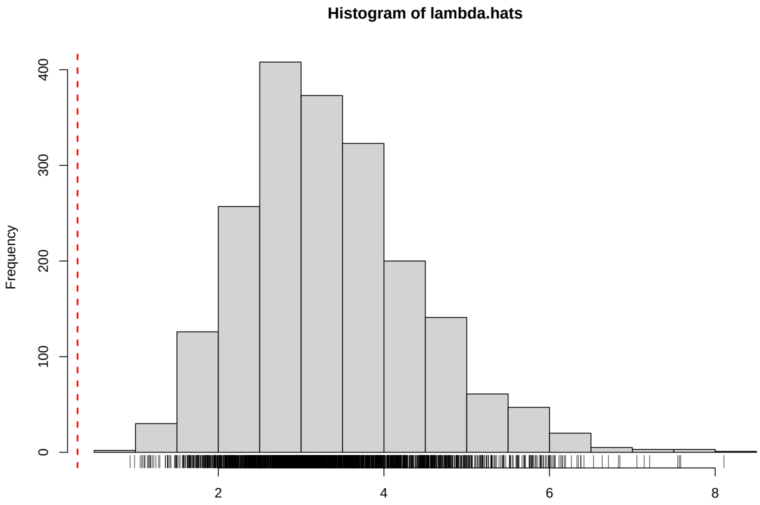



```
mean(lambda.hats)
```

```
## [1] 3.354296
```

With a Histogram:

```
hist(lambda.hats)
rug(lambda.hats)
abline(v = lambda, col="red", lwd = 2, lty = 2)
```



Example

Example: Assume we have a Random Sample for a some Distribution with the Mean μ and Variance σ^2:

$$X_1, X_2, ..., X_n \sim \mathcal{F}_{\mu,\sigma^2},$$

and we want to estimate the Parameters μ and σ^2.

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and we want to estimate the Parameters μ and σ^2.

We consider the following Estimators:

$$\hat{\mu} = \overline{X}_n = \frac{X_1 + X_2 + ... + X_n}{n}$$

and

$$\widehat{\sigma^2} = \frac{\sum_{k=1}^{n}(X_k - \overline{X}_n)^2}{n} \qquad \text{and} \qquad \widehat{\sigma^2} = S^2 = \frac{\sum_{k=1}^{n}(X_k - \overline{X}_n)^2}{n-1}$$

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Let us see (OTB) which ones are Biased and which ones are not.