CS 107, Probability, Spring 2019
Lecture 14

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AUA

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Content

- Repeated, Independent Trials: Binomial Distribution, Cont'd
- Repeated, Independent Trials: Multinomial Distribution

Quiz:

LZ

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a. We are rolling 2 fair dice. What is the probability that the difference between the shown numbers will be 2, if we know that one of the shown numbers was less than 3?

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b. I am choosing 2 persons from our group of 36 participants. We know that 10 of them are female. What is the probability that one of the chosen persons will be a female and and the other one - a male?

Binomial Probabilities formula, again:

Binomial Probabilities

For any $k = 0, 1, ..., n$,

$$\mathbb{P}(\text{Exactly } k \text{ successes in } n \text{ trials}) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}.$$

or, if we will denote the number of Successes in that n Trials by X,

$$\mathbb{P}(X = k) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k}.$$

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Proof: ONB

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- He/she will get 25 points?

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- He/she will get 25 points?
- He/she will get more than 25 points?

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Problem: Some entrance exam test consists of 30 multiple choice problems. Every correct choice gives 1 points. Each problem has suggested 5 choices for the answer, from which only one is correct. If the person will choose randomly the answers, what is the probability that

- He/she will get 25 points?
- He/she will get more than 25 points?
- He/she will get less than 29 points?

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- Clearly, we need to have (we are doing n trials)

$$k_1 + k_2 + ... + k_m = n.$$

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- We are interested in having exactly k Successes in n Trials, that is, we are interested in having exactly $k_1 = k$ times A_1 (Success) and exactly $k_2 = n - k_1$ times A_2 (Failure)

Repeated Indep Trials: Multinomial Probabilities

Multinomial Probabilities

The probability that exactly k_1 times we will have the event A_1, exactly k_2 times we will have the event A_2, ... , exactly k_m times we will have the event A_m in the above described n trials, is

$$P_n(k_1, k_2, ..., k_m) = \binom{n}{k_1, k_2, ..., k_m} \cdot p_1^{k_1} \cdot p_2^{k_2} \cdot ... \cdot p_m^{k_m},$$

where $p_k = \mathbb{P}(A_k)$, $k = 1, ..., m$.

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Note that in the binomial case, i.e., when $m = 2$, we get the Binomial Probabilities formula.