CS 107 Section B - Probability

Spring 2019, AUA

Homework No. 04

Due time/date: 10:35AM, 15 February, 2019

Note: Supplementary Problems will not be graded, but you are very advised to solve them and to discuss later with TA or Instructor.

Problem 1. I have a Snickers Chocolate stick, which is 15cm long, and I want to share it with you. I am breaking the chocolate stick at a random (with uniform probabilities) place along the length, and give the right-hand piece to you. What is the probability that you will get more than twice longer piece than me ? $\ddot\frown$

Problem 2. We are playing the following game: me and you are choosing each randomly (with uniform probabilities) a real number from $[0, 2]$. I win, if the sum of our numbers is smaller than 1 or if their product is larger than 3. Otherwise, you win. Is this game fair? (The game is fair, if the chances to win are the same for me and you). Explain your reasoning. If not, who has more chances to win?

Problem 3. Consider the following events:

A= the student is highly motivated;

B= the student graduates the University with a high GPA;

C= the student will be hired within the next 3 months of his/her graduation.

Write the following statement in mathematical form, using conditional probabilities: *"If the student is highly motivated and graduates the University with a high GPA, then the chance that he/she will get hired within the next 3 months of his/her graduation is* 95%".

Problem 4. Two fair dice are rolled. What is the conditional probability that at least one shows a number less than 5 if we know that the sum of shown numbers is larger than 9?

Problem 5. Assume we pick a point (x, y) at random (the choice is uniform) inside the square $[-1, 1] \times [-1, 1]$. We know that the chosen point's coordinates satisfy the inequality $y \leq 1 - |x|$. What will be then the probability that the chosen point is inside the unit disk with the center at the origin?

Problem 6. Assume we have 2 parallel servers A and B working in a network (parallel means that if one of the servers will be down, the next one will do the job for the other one, and the servers have two states: working or down). The probability that B is down is 0.3. If B is working, then the probability that A is working is 0.95, and if B is down, then the probability that A is working is 0.6. What is the probability that A is down?

Problem 7. There are 3 coins in a box. One is two-headed coin, another is a fair coin and the third is a biased coin that comes up heads 75 percent of the time. When one of the 3 coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin?

Problem 8. I am tossing a fair coin. If the result is a head, then I am rolling a fair die, but if the result is a tail, then I am rolling two fair dice. What is the probability that I will have 6 in a total on die/dice (if only one die is rolled, the total is just the number shown on that die, but if 2 dice are rolled, the total is the sum of shown numbers on that dice)?

Problem 9. A server is transmitting either 0 or 1. The probability that 0 is transmitted is 0.7, and the probability that 1 is transmitted is 0.3. The channel is noisy, so if 0 is sent, then there is a probability that 1 will be received, and that probability is 0.1. Also, if 1 is sent, then the probability that 0 is received is 0.2. If 0 is received, what is the probability that 0 is sent?

Problem 10. Which of the following pairs of events are independent? And which of that pairs are mutually exclusive (disjoint)? Explain.

a. We are rolling a die. The first event is that an even number will be shows, the second event is that the number shown number will be 5;
b. We are rolling a die. The first event is that an even number will be shows, the second event is that the number shown number will be divisible by 3;
c. We are rolling 2 dice. The first event is that the maximum of two numbers will be exactly 4, and the second event is that one of the dice will show 3;
d. We are rolling 2 dice. The first event is that the maximum of two numbers will be exactly 4, and the second event is that one of the dice will show 5;
e. We are rolling 2 dice. The first event is that the maximum of two numbers will be exactly 4, and the second event is that one of the dice will show 4.

Problem 11. (Supplementary) Assume we are spinning a casino roulette composed of numbers from 0 to 36, and assume that all outcomes are equiprobable. We make a bet on two events: that the ball will show an odd number, or that the ball will show a number from 1 to 18 (including 1 and 18).

a. What is the probability that we will win?
b. What is the probability that we will win, if the ball shows a number divisible by 3?

Problem 12. (Supplementary) Our Probability test will be composed by 2 instructors and 2 TA's. Out of 10 problems, 8 will be composed by instructors (4 problems each), and 2 will be composed by TA's (1 problem each). The probability that student will solve the problem is

- 90%, if the problem is composed by first TA;
- 85%, if the problem is composed by second TA;
- 80%, if the problem is composed by first instructor;
- 60%, if the problem is composed by second instructor.

a. A problem is chosen by random. What is the probability that student will solve that problem?

b. Student solved the problem. What is the probability that the problem was composed by the second instructor?

c. (supplementary) What is the probability that student will pass the test, if he/she needs to solve at least 9 problems?

Problem 13. (Supplementary) The AUA juniors' closed group at Facebook consists of 70 students of CS faculty, 210 students of BUS faculty, 30 EC students. For CS students, the chance to solve some particular math problem (say, to calculate the antiderivative of $e^x \cos(2x)$) is 90%. For BUS students, the chances are only 30% to solve that problem, and chances are 10% for the EC students. What is the probability that a randomly chosen student from that closed group will solve that math problem?

Problem 14. (Supplementary) This example concerns one of the important notions of Actuarial Mathematics. Let T denote the lifetime of a fixed individual measured in years from the birth. Say, $T_{H.Tumanyan} = 54.1$, stating that the lifetime of Hovhannes Tumanyan was 54.1 years. Clearly, T is not known in advance for any living individual. Actuaries need to calculate probabilities for T to calculate correctly insurance premiums (how much a person will pay to enter into a life insurance contract).

Now, we fix some person and assume T is his/her lifetime from the birth. So $\mathbb{P}(T = 57)$ is the probability that the person under consideration will live exactly 57 years.

Explain, as in the previous example, what the following probabilities mean (here x, t are some fixed positive numbers):

a. $\mathbb{P}(T \geq x)$;

b. $\mathbb{P}(T \geq x + 5 | T \geq x)$;

c. $\mathbb{P}(T < x + t | T \geq x)$;

Now denote $f(x) = \mathbb{P}(T \geq x)$ and $g(x, t) = \mathbb{P}(T < x + t | T \geq x)$.

d. Is f a monotonic function? Explain.

e. Is g a monotonic function with respect to variable $t > 0$ for fixed x? Explain.

f. Assume $\mathbb{P}(T \geq 50) = 0.89$ and $\mathbb{P}(50 \leq T < 51) = 0.07$. Calculate $\mathbb{P}(T < 51 | T \geq 50)$.

g. Assume that $\mathbb{P}(T < 51 | T \geq 50) = 0.09$. We consider a random cohort of $10,000$ individuals of age 50. How many of them will be expected to live more than 51 years?

h. Is it possible to express g in terms of the function f?

i. Is it possible to calculate $f(0)$ or $\mathbb{P}(T = 57)$?

j. Suggest a method to calculate $f(x)$ for any x (you can use any real data you want, just give the method one can use to estimate the probabilities).