AUA CS 108, Statistics, Fall 2019

Lecture 43 (-1)

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Contents

Last Lecture ReCap

- Maybe we can skip this this time?

Intro to Linear Regression

Recall that our Mission (for these last few lectures $\ddot\smile$) was to give a Model for the Relationship between two and more Variables.

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[1] And, in fact, we can recover the Joint Distribution, having the Distribution of X and the Conditional Distribution $Y|X = x$.

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Example: Say, $(X, Y) \sim \mathit{Unif}([0,1] \times [0,2])$. What is $Y|X = 1$, or, in general, $Y|X = x$?

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Example: Now, assume $(X, Y) \sim \mathit{Unif}(D)$, where D is the triangle with vertices at $(-1, 0)$, $(0, 1)$ and $(1, 0)$. What is $Y|X = x$?

---

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Example

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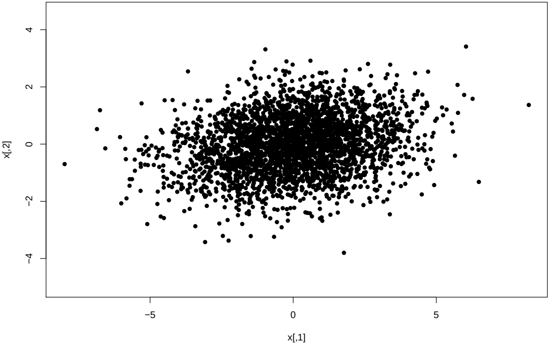
$$\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad \Sigma = \begin{bmatrix} 4 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$



What is $Y|X = x$?

Intro to Linear Regression

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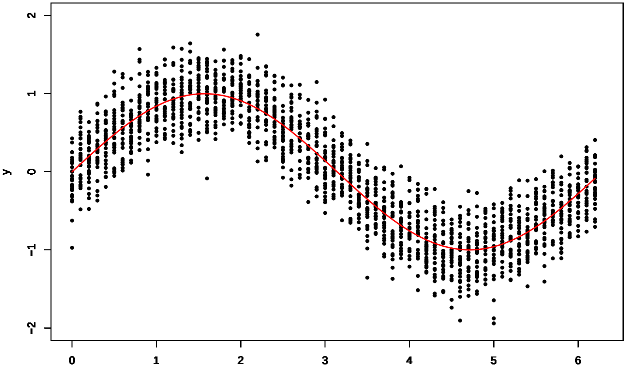

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So our r.v. $Y|\mathbf{X} = \mathbf{x}$ is a r.v. around $RegFun(\mathbf{x})$, or simply,

$$(Y|\mathbf{X} = \mathbf{x}) \quad = \quad RegFun(\mathbf{x}) + \varepsilon$$

where ε is a r.v. (just take $\varepsilon = (Y|\mathbf{X} = \mathbf{x}) - RegFun(\mathbf{x})$),

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Here, importantly, $\mathbb{E}(\varepsilon) = 0$, for any $\mathbf{x}$.

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Say, see Wiki page for BMI.