AUA CS108, Statistics, Fall 2020

Lecture 18

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Contents

Important Discrete Distributions

Bernoulli Distribution

- Parameter: $p \in [0, 1]$ (usually, $p \in (0, 1)$)

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- Mean and Variance: $\mathbb{E}(X) = p$, $Var(X) = p(1-p)$.
- Models: Models binary output, "success-failure" type Experiments, a lot of examples.

Bernoulli Distribution

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Bernoulli Distribution

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- Example:

```r
rbinom(10, size = 1, prob = 0.3)
```

```
##  [1] 0 0 0 0 0 0 0 1 0 0
```

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- Support: $\{0, 1, 2, ..., n\}$
- PMF:

Values of X	0	1	...	k	...	n
$\mathbb{P}(X = x)$	$\binom{n}{0} p^0 (1-p)^{n-0}$	$\binom{n}{1} p^1 (1-p)^{n-1}$	...	$\binom{n}{k} p^k (1-p)^{n-k}$	...	$\binom{n}{n} p^n (1-p)^0$

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- Mean and Variance: $\mathbb{E}(X) = n \cdot p$, $Var(X) = n \cdot p(1 - p)$.

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- Mean and Variance: $\mathbb{E}(X) = n \cdot p$, $Var(X) = n \cdot p(1-p)$.
- Models: Models the independent repetition of the *Bernoulli*(p) Experiment.

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- Additional: If $X_1, X_2, ..., X_n \sim Bernoulli(p)$ are independent, then $X_1 + X_2 + ... + X_n \sim Binom(n, p)$.

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- Additional: If $X_1, X_2, ..., X_n \sim Bernoulli(p)$ are independent, then $X_1 + X_2 + ... + X_n \sim Binom(n, p)$.
- Example:

```
rbinom(10, size = 5, prob = 0.3)
```

```
##  [1] 2 2 3 0 4 3 2 1 0 2
```

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- PMF:

Values of X	1	2	3	...
$\mathbb{P}(X = x)$	p	$p(1-p)$	$p(1-p)^2$	...

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Values of X	1	2	3	...
$\mathbb{P}(X = x)$	p	$p(1-p)$	$p(1-p)^2$	...

- Mean and Variance: $\mathbb{E}(X) = \dfrac{1}{p}$, $Var(X) = \dfrac{1-p}{p^2}$.

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Values of X	1	2	3	...
$\mathbb{P}(X = x)$	p	$p(1-p)$	$p(1-p)^2$	...

- Mean and Variance: $\mathbb{E}(X) = \dfrac{1}{p}$, $Var(X) = \dfrac{1-p}{p^2}$.
- Models: Models the independent repetition of the *Bernoulli*(p) Experiment until the *First Success*.

Geometric Distribution

- **R** name: `geom` with the parameter `prob`

Geometric Distribution

- **R** name: `geom` with the parameter `prob`
- **Note: R** is using the second definition of the Geometric Distribution, with the support $\{0, 1, 2, 3, ...\}$, i.e., in **R**, $X \sim Geom(p)$ shows *the number of Failures before the first Success*
- Example:

```
rgeom(10,prob = 0.3)
```

```
##  [1] 17  0  3  0  1  1  2  4  0  4
```

Poisson Distribution

- Parameter: $\lambda > 0$

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Values of X	0	1	2	...
$\mathbb{P}(X = x)$	$e^{-\lambda}\frac{\lambda^0}{0!}$	$e^{-\lambda}\frac{\lambda^1}{1!}$	$e^{-\lambda}\frac{\lambda^2}{2!}$	...

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$\mathbb{P}(X = x)$	$e^{-\lambda}\frac{\lambda^0}{0!}$	$e^{-\lambda}\frac{\lambda^1}{1!}$	$e^{-\lambda}\frac{\lambda^2}{2!}$	...

- Mean and Variance: $\mathbb{E}(X) = \lambda$, $\mathit{Var}(X) = \lambda$.

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- Mean and Variance: $\mathbb{E}(X) = \lambda$, $Var(X) = \lambda$.
- Models: Models the number of success in a fixed interval. Say, number of calls, number of customers, number of clicks, number of page visits, ...

Poisson Distribution

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$\mathbb{P}(X = x)$	$e^{-\lambda}\frac{\lambda^0}{0!}$	$e^{-\lambda}\frac{\lambda^1}{1!}$	$e^{-\lambda}\frac{\lambda^2}{2!}$	...

- Mean and Variance: $\mathbb{E}(X) = \lambda$, $Var(X) = \lambda$.
- Models: Models the number of success in a fixed interval. Say, number of calls, number of customers, number of clicks, number of page visits, ... λ is the average number of calls, customers, clicks, page visits,...

Poisson Distribution

- **R** name: `pois` with the parameter `lambda`

Poisson Distribution

- **R** name: `pois` with the parameter `lambda`
- Example:

```r
rpois(10, lambda = 2)
```

```
##  [1] 4 2 2 0 1 2 2 4 2 1
```

Important Continuous Distributions

Uniform Distribution

- Parameters: a, b $(a < b)$

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- Support: $[a, b]$
- PDF:

$$f(x) = f(x|a, b) = \begin{cases} \dfrac{1}{b-a}, & x \in [a, b] \\ 0, & otherwise \end{cases}$$

Uniform Distribution

- Parameters: a, b $(a < b)$
- Notation: $X \sim Unif[a, b]$;
- Support: $[a, b]$
- PDF:

$$f(x) = f(x|a, b) = \begin{cases} \dfrac{1}{b-a}, & x \in [a, b] \\ 0, & otherwise \end{cases}$$

- Mean and Variance: $\mathbb{E}(X) = \frac{a+b}{2}$, $Var(X) = \frac{(b-a)^2}{12}$.

Uniform Distribution

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- Notation: $X \sim Unif[a, b]$;
- Support: $[a, b]$
- PDF:

$$f(x) = f(x|a, b) = \begin{cases} \dfrac{1}{b-a}, & x \in [a, b] \\ 0, & otherwise \end{cases}$$

- Mean and Variance: $\mathbb{E}(X) = \frac{a+b}{2}$, $Var(X) = \frac{(b-a)^2}{12}$.
- Models: Usually we think about the Uniform Distribution when talking about *picking a random number from an interval*

Uniform Distribution

- **R** name: `unif` with the parameters `min = 0` and `max = 1`

Uniform Distribution

- **R** name: `unif` with the parameters `min = 0` and `max = 1`
- Example:

```
runif(10, min = 2, max = 5)
```

```
##  [1] 4.336598 4.944355 3.633583 3.639146 2.274558 4.7676
##  [9] 2.691521 3.161340
```