CS 107, Probability, Spring 2019
Lecture 02

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Content

- Experiment, Outcomes and the Sample Space
- Events, Operations with Events

Assume our mobile phone Weather App says that there is a 50% chance of snow for this Saturday, and also 50% chance of snow this Sunday.
Is it true that it will snow for sure (i.e., with probability 1) this weekend?

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- The set of all Outcomes of an Experiment is called the **Sample Space** of that Experiment:

$$\Omega = \text{the Sample Space of the Experiment} =$$

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Usually in the Probability Theory (and in real-life situations), we are interested not in particular outcomes and their probabilities, but also in some groups of outcomes and their probabilities.
An **Event** in some Experiment is some collection of Outcomes of that Experiment. Mathematically, every subset of the Sample Space is called an **Event**[1].

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Operations with Events

Assume we have an Experiment with Sample Space Ω. And assume A and B are two Events in this experiment, i.e.,

$$A \subset \Omega \qquad \textit{and} \qquad B \subset \Omega.$$

Then we can obtain new Events from A and B:

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And having Events $A_1, A_2, ..., A_k, ...$ in our Experiment, i.e., $A_k \subset \Omega$, we can form

$$\bigcup_k A_k, \qquad \bigcap_k A_k, ...$$

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- If the Sample Space of an Experiment has cardinality n, $\#\Omega = |\Omega| = card(\Omega) = n$ (i.e., the number of elements in Ω is n), how many different Events our Experiment have?