AUA CS 108, Statistics, Fall 2019

Lecture 42

Michael Poghosyan
YSU, AUA
michael@ysu.am, mpoghosyan@aua.am

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Contents

Last Lecture ReCap

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- Give the definition of the p-Value of a Test.

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And, if p-Value is small, then TS_{obs} is very unprobable, very unbelievable, under $\mathcal{H}_0$, so we safely Reject $\mathcal{H}_0$.

To Remember:

- If $p\text{-Value} < \alpha$, then we Reject $\mathcal{H}_0$
- If $p\text{-Value} \geq \alpha$, then we Fail to Reject $\mathcal{H}_0$

Example

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and the value of p-Value is $\ddot\smile$

```
1-(pnorm(1.72,mean=0,sd=1)-pnorm(-1.72,mean=0,sd=1))
```

```
## [1] 0.08543244
```

Linear Regression

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to get some information about θ, in particular,

- to find a good Point Estimator and Estimate;
- to find a CI for θ of given CL;
- to Test a Hypothesis about θ, say, is it likely that $\theta = 3.1415$ or not.

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Recall that, in the Descriptive Statistics part, we considered two Datasets, and defined the Sample Covariance and Correlation Coefficients, to measure the Linear Relationship between that Datasets. That was defined for two **Numerical Dataset**, without any assumptions behind the Process generating that Datasets. Now, if we assume that that Datasets are coming from some Distribution, we are at the stage of doing a Statistical Inference, Statistical Analysis.

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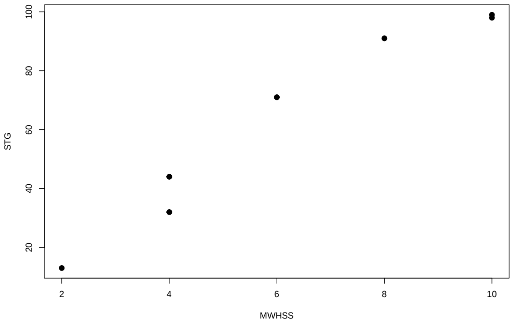

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