CS 107, Probability, Spring 2019
Lecture 41

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AUA

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Content

- Covariance and correlation of R.V.s
- The Law of Large Numbers
- The Central Limit Theorem

Covariance of Random Variables

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- Covariance measures, in fact, the linear relationship between X and Y, it shows how much X and Y relate linearly to each other
- We say that X and Y are **uncorrelated**, if $Cov(X, Y) = 0$

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But we have another very useful formula for the Covariance calculation, fortunately.

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Calculation of Covariance

$$Cov(X, Y) = \mathbb{E}(X \cdot Y) - \mathbb{E}(X) \cdot \mathbb{E}(Y).$$

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- The Joint PMF of (X, Y) is given by

$Y \setminus X$	-1	5
0	0.1	0.2
4	0.3	0.4

- $(X, Y) \sim Unif([0, 2] \times [3, 9])$.

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- If $X \perp\!\!\!\perp Y$, then $Cov(X, Y) = 0$. The inverse is **NOT TRUE**, in general;

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- $Var(X+Y) = Var(X) + 2Cov(X,Y) + Var(Y)$ and $Var(X-Y) = Var(X) - 2Cov(X,Y) + Var(Y)$.

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- $Var(X_1 + X_2 + ... + X_n) = \sum_{i=1}^{n} Var(X_i) + 2 \cdot \sum_{i<j} Cov(X_i, X_j) = \sum_{i,j} Cov(X_i, X_j)$

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- $Var(2X - 5Y + Z)$, if $X \perp\!\!\!\perp Z$, $Y \perp\!\!\!\perp Z$, $Var(X) = 1$, $Var(Y) = 2$, $Var(Z) = 3$ and $Cov(X, Y) = 1$.

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- $Var(2X - 5Y + Z)$, if $X \perp\!\!\!\perp Z$, $Y \perp\!\!\!\perp Z$,
  $Var(X) = 1, Var(Y) = 2$, $Var(Z) = 3$ and $Cov(X, Y) = 1$.
- $Cov(2X, 3X - Y)$, if $SD(X) = 2$, $Cov(Y, X) = 0.5$.

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  So if you are reporting your research using the Covariance, you need to give the units also.

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So we need to introduce another measure for the relationship between r.v.s.

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If X and Y are non-constant r.v.s, then the **Correlation Coefficient** of X and Y is defined as

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Note: Like in the case of Variance-Standard Deviation pair, here the situation is the same: Covariance is easy to handle with, and the Correlation Coefficient is used for reporting.

Properties of a Correlation

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Note: The inverse implications are also true in these two cases, and are much simpler: If there is an exact linear relationship between X and Y, $Y = \alpha X + \beta$, with $\alpha \neq 0$, then

$$Cov(X, Y) = sgn(\alpha).$$

Covariance Matrix

If we have r.v.s $X_1, X_2, ..., X_n$, then we can make the Covariance Matrix:

Covariance Matrix

The following Matrix is called the Covariance Matrix of $X_1, X_2, ..., X_n$:

$$\Sigma = \begin{bmatrix} Cov(X_1, X_1) & \dots & Cov(X_1, X_n) \\ \vdots & \ddots & \vdots \\ Cov(X_n, X_1) & \dots & Cov(X_n, X_n) \end{bmatrix}$$

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Note: It can be proved that the Covariance Matrix is always Symmetric and Positive Semi-Definite (or Non-Negative Definite).

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we have called Σ a Covariance Matrix: it turns out that Σ is exactly the Covariance Matrix of $X_1, ..., X_n$. In particular,

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- If
$$\begin{bmatrix} X \\ Y \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 & 2 \\ 2 & 5 \end{bmatrix} \right),$$
then $Var(X) = 4$, $Var(Y) = 5$ and $Cov(X, Y) = 2$

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- If (X, Y) are Jointly Normally Distributed, $\mathbb{E}(X) = 3, \mathbb{E}(Y) = -1$, and $Var(X) = 8$, $Var(Y) = 5$ and $Cov(X, Y) = 1$, then
$$\begin{bmatrix} X \\ Y \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 8 & 1 \\ 1 & 5 \end{bmatrix}\right),$$