AUA CS 108, Statistics, Fall 2019

Lecture 39

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Contents

Last Lecture ReCap

- What are we testing when using a (one-sample) t-Test?

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- What are we testing when using a (one-sample) t-Test?
- Describe the t-Test.

Note

Note: In **R** `t.test` command, the default values for parameters are:

- `mu = 0`
- `alternative = "two.sided"`
- `conf.level = 0.95`

Note

Note: In many textbooks, you will find the Critical Values and quantiles, calculated using areas of the Right-Tail. So you can meet in textbooks t-Test with the Rejection Region

$$|t| > t_{n-1,\alpha/2}.$$

In fact, here $t_{n-1,\alpha/2}$ is the point such that the area under the PDF of $t(n-1)$ **right to that point** is $\alpha/2$. This coincides with our standard quantile $t_{n-1,1-\alpha/2}$, where we are calculating the area to the **left**.

R can calculate also these type of quantiles:

```
qt(1-0.05, df = 15)
```

```
## [1] 1.75305
```

```
qt(0.05, df = 15, lower.tail = FALSE)
```

```
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Test for the Normal Variance, μ is known

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Large Sample Hypothesis Testing

aka

Asymptotic Testing

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Asymptotic Distrib of the TS Under $\mathcal{H}_0$: $t \xrightarrow{D} \mathcal{N}(0, 1)$;

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Model: $X_1, X_2, ..., X_n$ are IID from some Distribution with Mean μ and Variance σ^2. σ^2 is not known. The Parameter is μ;

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Note: People use this Test only if $n \cdot p_0 > 5$ and $n \cdot (1 - p_0) > 5$.