CS 107, Probability, Spring 2019
Lecture 31

Michael Poghosyan

AUA

5 April 2019

Content

- Joint Distribution of two R.V.s

LZ

LZ

Fact: There are exactly 10! seconds in six weeks.

LZ

Fact: There are exactly 10! seconds in six weeks.
Another Fact: There is a 3D figure having the following property: you can fill it by a paint, but you can't paint the inner surface.

LZ

Fact: There are exactly 10! seconds in six weeks.
Another Fact: There is a 3D figure having the following property: you can fill it by a paint, but you can't paint the inner surface.
See `https://en.m.wikipedia.org/wiki/Gabriel's_Horn`

Joint CDF of 2 RVs, aka bivariate case

Recall that in our last lecture we were talking about the Joint Distribution of 2 r.v.s, X and Y.

Joint CDF of 2 RVs, aka bivariate case

Recall that in our last lecture we were talking about the Joint Distribution of 2 r.v.s, X and Y.
One of the methods to describe their Joint Distribution was to give their Joint CDF:

$$F(x, y) =$$

Joint CDF of 2 RVs, aka bivariate case

Recall that in our last lecture we were talking about the Joint Distribution of 2 r.v.s, X and Y.
One of the methods to describe their Joint Distribution was to give their Joint CDF:

$$F(x, y) = \mathbb{P}(X \leq x, Y \leq y), \qquad \forall (x, y) \in \mathbb{R}^2.$$

Joint CDF of 2 RVs, aka bivariate case

Recall that in our last lecture we were talking about the Joint Distribution of 2 r.v.s, X and Y.
One of the methods to describe their Joint Distribution was to give their Joint CDF:

$$F(x, y) = \mathbb{P}(X \leq x, Y \leq y), \qquad \forall (x, y) \in \mathbb{R}^2.$$

As in 1D case, usually it is more comfortable to work with PMF/PDFs, so we want to describe our 2D Random Vectors through their Joint PMF/PDFs.

Discrete R. Vectors, Joint PMF

Assume that the r.v.s X and Y, defined on the same Experiment, are both discrete.

Discrete R. Vectors, Joint PMF

Assume that the r.v.s X and Y, defined on the same Experiment, are both discrete.

Let $x_1, x_2, \ldots, x_n, \ldots$ be the values of X and $y_1, y_2, \ldots, y_m, \ldots$ be the values of Y (not necessary of the same size: say, the range of X can be finite, and the range of Y can be infinite).

Discrete R. Vectors, Joint PMF

Assume that the r.v.s X and Y, defined on the same Experiment, are both discrete.

Let $x_1, x_2, ..., x_n, ...$ be the values of X and $y_1, y_2, ..., y_m, ...$ be the values of Y (not necessary of the same size: say, the range of X can be finite, and the range of Y can be infinite).

The function

$$p(x, y) = \mathbb{P}(X = x, Y = y), \qquad (x, y) \in \mathbb{R}^2$$

is called **the Joint PMF** of X and Y (or, of the Random Vector (X, Y)).

Discrete R. Vectors, Joint PMF

Assume that the r.v.s X and Y, defined on the same Experiment, are both discrete.

Let $x_1, x_2, ..., x_n, ...$ be the values of X and $y_1, y_2, ..., y_m, ...$ be the values of Y (not necessary of the same size: say, the range of X can be finite, and the range of Y can be infinite).

The function

$$p(x, y) = \mathbb{P}(X = x, Y = y), \qquad (x, y) \in \mathbb{R}^2$$

is called **the Joint PMF** of X and Y (or, of the Random Vector (X, Y)).

In fact, it is enough to give only the values of

$$p(x_i, y_j) = \mathbb{P}(X = x_i, Y = y_j)$$

for all x_i and y_j.

Discrete R. Vectors, Joint PMF

Assume that the r.v.s X and Y, defined on the same Experiment, are both discrete.

Let $x_1, x_2, \dots, x_n, \dots$ be the values of X and $y_1, y_2, \dots, y_m, \dots$ be the values of Y (not necessary of the same size: say, the range of X can be finite, and the range of Y can be infinite).

The function

$$p(x, y) = \mathbb{P}(X = x, Y = y), \qquad (x, y) \in \mathbb{R}^2$$

is called **the Joint PMF** of X and Y (or, of the Random Vector (X, Y)).

In fact, it is enough to give only the values of

$$p(x_i, y_j) = \mathbb{P}(X = x_i, Y = y_j)$$

for all x_i and y_j.

We will denote for short $p_{i,j} = \mathbb{P}(X = x_i, Y = y_j)$.

Joint PMF, Properties, and the Table form

Clearly,

- $p_{i,j} \geq 0$;

Joint PMF, Properties, and the Table form

Clearly,

- $p_{i,j} \geq 0;$
- $\displaystyle\sum_{i,j} p_{i,j} =$

Joint PMF, Properties, and the Table form

Clearly,

- $p_{i,j} \geq 0$;
- $\displaystyle\sum_{i,j} p_{i,j} = 1.$

Joint PMF, Properties, and the Table form

Clearly,

- $p_{i,j} \geq 0$;
- $\sum_{i,j} p_{i,j} = 1$.

Usually we write the Joint PMF of X and Y in the table form:

$Y \setminus X$	x_1	x_2	$\dots$
y_1	$p_{1,1} = \mathbb{P}(X = x_1, Y = y_1)$	$p_{2,1} = \mathbb{P}(X = x_2, Y = y_1)$	$\dots$
y_2	$p_{1,2} = \mathbb{P}(X = x_1, Y = y_2)$	$p_{2,2} = \mathbb{P}(X = x_2, Y = y_2)$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Joint PMF, the Table form

We will write, for short

$Y \setminus X$	x_1	x_2	$\ldots$
y_1	$p_{1,1}$	$p_{2,1}$	$\ldots$
y_2	$p_{1,2}$	$p_{2,2}$	$\ldots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Joint PMF, the Table form

We will write, for short

$Y \setminus X$	x_1	x_2	$\dots$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Here, the sum of all $p_{i,j}$-s is 1.

Joint PMF, the Table form

We will write, for short

$Y \setminus X$	x_1	x_2	$\dots$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Here, the sum of all $p_{i,j}$-s is 1.

Having the Joint PMF, we can calculate all probabilities we want: if $A \subset \mathbb{R}^2$, then

$$\mathbb{P}\Big((X,Y) \in A\Big) = \sum_{(x_i,y_j)\in A} p_{i,j}.$$

Joint PMF, the Table form

We will write, for short

$Y \setminus X$	x_1	x_2	$\dots$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Here, the sum of all $p_{i,j}$-s is 1.

Having the Joint PMF, we can calculate all probabilities we want: if $A \subset \mathbb{R}^2$, then

$$\mathbb{P}\Big((X, Y) \in A\Big) = \sum_{(x_i, y_j) \in A} p_{i,j}.$$

Say,

$$\mathbb{P}(a \leq X \leq b, c \leq Y \leq d) =$$

Joint PMF, the Table form

We will write, for short

$Y \setminus X$	x_1	x_2	$\dots$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Here, the sum of all $p_{i,j}$-s is 1.

Having the Joint PMF, we can calculate all probabilities we want: if $A \subset \mathbb{R}^2$, then

$$\mathbb{P}\Big((X, Y) \in A\Big) = \sum_{(x_i, y_j) \in A} p_{i,j}.$$

Say,

$$\mathbb{P}(a \le X \le b, c \le Y \le d) = \sum_{\substack{a \le x_i \le b \\ c \le y_j \le d}} p_{i,j}.$$

Example:

Example: Assume we are rolling a fair die. Let X be the number shown, and Y be 0, if 2 or 5 appears, and $Y = 1$ otherwise.

Example:

Example: Assume we are rolling a fair die. Let X be the number shown, and Y be 0, if 2 or 5 appears, and $Y = 1$ otherwise.

- Construct the Joint PMF of (X, Y);
- Calculate $\mathbb{P}(X \in [0, 3.5], Y \in [-1, 0.5])$;
- Calculate $\mathbb{P}(X^2 + Y^2 > 4)$;
- Calculate $F(3, 2.4)$, where F is the Joint CDF of X and Y.

Example:

Example: Assume we are rolling a fair die. Let X be the number shown, and Y be 0, if 2 or 5 appears, and $Y = 1$ otherwise.

- Construct the Joint PMF of (X, Y);
- Calculate $\mathbb{P}(X \in [0, 3.5], Y \in [-1, 0.5])$;
- Calculate $\mathbb{P}(X^2 + Y^2 > 4)$;
- Calculate $F(3, 2.4)$, where F is the Joint CDF of X and Y.

Solution: Let ω be the outcome of our Experiment. To construct the Joint PMF, we calculate:

- $\mathbb{P}(X = 1, Y = 0) = \mathbb{P}(\omega = 1, \omega \in \{2, 5\}) = 0$;

Example:

Example: Assume we are rolling a fair die. Let X be the number shown, and Y be 0, if 2 or 5 appears, and $Y = 1$ otherwise.

- Construct the Joint PMF of (X, Y);
- Calculate $\mathbb{P}(X \in [0, 3.5], Y \in [-1, 0.5])$;
- Calculate $\mathbb{P}(X^2 + Y^2 > 4)$;
- Calculate $F(3, 2.4)$, where F is the Joint CDF of X and Y.

Solution: Let ω be the outcome of our Experiment. To construct the Joint PMF, we calculate:

- $\mathbb{P}(X = 1, Y = 0) = \mathbb{P}(\omega = 1, \omega \in \{2, 5\}) = 0$;
- $\mathbb{P}(X = 1, Y = 1) = \mathbb{P}(\omega = 1, \omega \in \{1, 3, 4, 6\}) = \frac{1}{6}$;

Example:

Example: Assume we are rolling a fair die. Let X be the number shown, and Y be 0, if 2 or 5 appears, and $Y = 1$ otherwise.

- Construct the Joint PMF of (X, Y);
- Calculate $\mathbb{P}(X \in [0, 3.5], Y \in [-1, 0.5])$;
- Calculate $\mathbb{P}(X^2 + Y^2 > 4)$;
- Calculate $F(3, 2.4)$, where F is the Joint CDF of X and Y.

Solution: Let ω be the outcome of our Experiment. To construct the Joint PMF, we calculate:

- $\mathbb{P}(X = 1, Y = 0) = \mathbb{P}(\omega = 1, \omega \in \{2, 5\}) = 0$;
- $\mathbb{P}(X = 1, Y = 1) = \mathbb{P}(\omega = 1, \omega \in \{1, 3, 4, 6\}) = \frac{1}{6}$;
- $\mathbb{P}(X = 2, Y = 0) = \mathbb{P}(\omega = 2, \omega \in \{2, 5\}) = \frac{1}{6}$;

Example:

Example: Assume we are rolling a fair die. Let X be the number shown, and Y be 0, if 2 or 5 appears, and $Y = 1$ otherwise.

- Construct the Joint PMF of (X, Y);
- Calculate $\mathbb{P}(X \in [0, 3.5], Y \in [-1, 0.5])$;
- Calculate $\mathbb{P}(X^2 + Y^2 > 4)$;
- Calculate $F(3, 2.4)$, where F is the Joint CDF of X and Y.

Solution: Let ω be the outcome of our Experiment. To construct the Joint PMF, we calculate:

- $\mathbb{P}(X = 1, Y = 0) = \mathbb{P}(\omega = 1, \omega \in \{2, 5\}) = 0$;
- $\mathbb{P}(X = 1, Y = 1) = \mathbb{P}(\omega = 1, \omega \in \{1, 3, 4, 6\}) = \frac{1}{6}$;
- $\mathbb{P}(X = 2, Y = 0) = \mathbb{P}(\omega = 2, \omega \in \{2, 5\}) = \frac{1}{6}$;
- $\mathbb{P}(X = 2, Y = 1) = \mathbb{P}(\omega = 2, \omega \in \{1, 3, 4, 6\}) = 0$

Example:

Example: Assume we are rolling a fair die. Let X be the number shown, and Y be 0, if 2 or 5 appears, and $Y = 1$ otherwise.

- Construct the Joint PMF of (X, Y);
- Calculate $\mathbb{P}(X \in [0, 3.5], Y \in [-1, 0.5])$;
- Calculate $\mathbb{P}(X^2 + Y^2 > 4)$;
- Calculate $F(3, 2.4)$, where F is the Joint CDF of X and Y.

Solution: Let ω be the outcome of our Experiment. To construct the Joint PMF, we calculate:

- $\mathbb{P}(X = 1, Y = 0) = \mathbb{P}(\omega = 1, \omega \in \{2, 5\}) = 0$;
- $\mathbb{P}(X = 1, Y = 1) = \mathbb{P}(\omega = 1, \omega \in \{1, 3, 4, 6\}) = \frac{1}{6}$;
- $\mathbb{P}(X = 2, Y = 0) = \mathbb{P}(\omega = 2, \omega \in \{2, 5\}) = \frac{1}{6}$;
- $\mathbb{P}(X = 2, Y = 1) = \mathbb{P}(\omega = 2, \omega \in \{1, 3, 4, 6\}) = 0$

The rest OTB.

Marginal PMFs

Assume that the Joint PMF of Discrete r.v.s X and Y is given by:

$Y \setminus X$	x_1	x_2	$\dots$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Marginal PMFs

Assume that the Joint PMF of Discrete r.v.s X and Y is given by:

$Y \setminus X$	x_1	x_2	$\dots$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

We want to construct the individual PMFs of X and Y.

Marginal PMFs

Assume that the Joint PMF of Discrete r.v.s X and Y is given by:

$Y \setminus X$	x_1	x_2	$\dots$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

We want to construct the individual PMFs of X and Y. The process described below is called **Marginalization**.

Marginal PMFs

Assume that the Joint PMF of Discrete r.v.s X and Y is given by:

$Y \setminus X$	x_1	x_2	$\dots$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

We want to construct the individual PMFs of X and Y. The process described below is called **Marginalization**.
To find the PMF of X, we need to calculate $\mathbb{P}(X = x_i)$. And we need to use the Probabilities $\mathbb{P}(X = x_i, Y = y_j)$:

$$\mathbb{P}(X = x_i) =$$

Marginal PMFs

Assume that the Joint PMF of Discrete r.v.s X and Y is given by:

$Y \setminus X$	x_1	x_2	$\dots$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

We want to construct the individual PMFs of X and Y. The process described below is called **Marginalization**.
To find the PMF of X, we need to calculate $\mathbb{P}(X = x_i)$. And we need to use the Probabilities $\mathbb{P}(X = x_i, Y = y_j)$:

$$\mathbb{P}(X = x_i) = \mathbb{P}(X = x_i, Y = y_1) + \mathbb{P}(X = x_i, Y = y_2) + \dots$$

so

$$\mathbb{P}(X = x_i) = \text{The sum of the column under } x_i$$

Marginal PMFs

Assume that the Joint PMF of Discrete r.v.s X and Y is given by:

$Y \setminus X$	x_1	x_2	$\dots$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Similarly, for the PMF of Y,

$$\mathbb{P}(Y = y_j) = \text{The sum of the row right to } y_j$$

Marginal PMFs

Again, if the Joint Distribution of X and Y is

$Y \setminus X$	x_1	x_2	$\dots$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

then

Marginal PMFs

Again, if the Joint Distribution of X and Y is

$Y \setminus X$	x_1	x_2	$\dots$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

then

- $\mathbb{P}(X = x_i) = \sum_j \mathbb{P}(X = x_i, Y = y_j)$

Marginal PMFs

Again, if the Joint Distribution of X and Y is

$Y \setminus X$	x_1	x_2	$\dots$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

then

- $\mathbb{P}(X = x_i) = \sum_j \mathbb{P}(X = x_i, Y = y_j)$
- $\mathbb{P}(Y = y_j) = \sum_i \mathbb{P}(X = x_i, Y = y_j)$

Marginal PMFs

Again, if the Joint Distribution of X and Y is

$Y \setminus X$	x_1	x_2	$\dots$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

then

- $\mathbb{P}(X = x_i) = \sum_j \mathbb{P}(X = x_i, Y = y_j)$
- $\mathbb{P}(Y = y_j) = \sum_i \mathbb{P}(X = x_i, Y = y_j)$

The description is, say, for the first formula, that we are "summing out Y" (or "integrating out Y" in the continuous case).

Marginal PMFs

We can write this in the table form:

$Y \setminus X$	x_1	x_2	$\dots$	$\mathbb{P}(Y = y_j)$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$	$p_{1,1} + p_{2,1} + \dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$	$p_{1,2} + p_{2,2} + \dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathbb{P}(X = x_i)$	$p_{1,1} + p_{1,2} + \vdots$	$p_{2,1} + p_{2,2} + \vdots$	$\vdots$	

Marginal PMFs

We can write this in the table form:

$Y \setminus X$	x_1	x_2	$\dots$	$\mathbb{P}(Y = y_j)$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$	$p_{1,1} + p_{2,1} + \dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$	$p_{1,2} + p_{2,2} + \dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathbb{P}(X = x_i)$	$p_{1,1} + p_{1,2} + \vdots$	$p_{2,1} + p_{2,2} + \vdots$	$\vdots$	

The red-colored parts are the PMF's of X and Y, respectively.

Marginal PMFs

So, having the Joint PMF of X and Y,

$Y \setminus X$	x_1	x_2	$\dots$	$\mathbb{P}(Y = y_j)$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$	$p_{1,1} + p_{2,1} + \dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$	$p_{1,2} + p_{2,2} + \dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathbb{P}(X = x_i)$	$p_{1,1} + p_{1,2} + \vdots$	$p_{2,1} + p_{2,2} + \vdots$	$\vdots$	

Marginal PMFs

So, having the Joint PMF of X and Y,

$Y \setminus X$	x_1	x_2	$\dots$	$\mathbb{P}(Y = y_j)$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$	$p_{1,1} + p_{2,1} + \dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$	$p_{1,2} + p_{2,2} + \dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathbb{P}(X = x_i)$	$p_{1,1} + p_{1,2} + \vdots$	$p_{2,1} + p_{2,2} + \vdots$	$\vdots$	

We will obtain the PMF of, say, X:

Values of X

Marginal PMFs

So, having the Joint PMF of X and Y,

$Y \setminus X$	x_1	x_2	$\dots$	$\mathbb{P}(Y = y_j)$
y_1	$p_{1,1}$	$p_{2,1}$	$\dots$	$p_{1,1} + p_{2,1} + \dots$
y_2	$p_{1,2}$	$p_{2,2}$	$\dots$	$p_{1,2} + p_{2,2} + \dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathbb{P}(X = x_i)$	$p_{1,1} + p_{1,2} + \vdots$	$p_{2,1} + p_{2,2} + \vdots$	$\vdots$	

We will obtain the PMF of, say, X:

Values of X	x_1	x_2	$\dots$
$\mathbb{P}(X = x)$			

Marginal PMFs

So, having the Joint PMF of X and Y,

$Y \setminus X$	x_1	x_2	$\ldots$	$\mathbb{P}(Y = y_j)$
y_1	$p_{1,1}$	$p_{2,1}$	$\ldots$	$p_{1,1} + p_{2,1} + \ldots$
y_2	$p_{1,2}$	$p_{2,2}$	$\ldots$	$p_{1,2} + p_{2,2} + \ldots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathbb{P}(X = x_i)$	$p_{1,1} + p_{1,2} + \vdots$	$p_{2,1} + p_{2,2} + \vdots$	$\vdots$	

We will obtain the PMF of, say, X:

Values of X	x_1	x_2	$\ldots$
$\mathbb{P}(X = x)$	$p_{1,1} + p_{1,2} + \ldots$	$p_{2,1} + p_{2,2} + \ldots$	$\ldots$

Example:

Example: Assume The Joint PDF of X and Y is given by:

$Y \setminus X$	-2	0	1
$\frac{1}{3}$	0.1	0.15	0.2
$\sqrt{3}$	0.3	0.1	0.15

Example:

Example: Assume The Joint PDF of X and Y is given by:

$Y \setminus X$	-2	0	1
$\frac{1}{3}$	0.1	0.15	0.2
$\sqrt{3}$	0.3	0.1	0.15

- Check that this is a legitimate (valid) Joint PMF;

Example:

Example: Assume The Joint PDF of X and Y is given by:

$Y \setminus X$	-2	0	1
$\frac{1}{3}$	0.1	0.15	0.2
$\sqrt{3}$	0.3	0.1	0.15

- Check that this is a legitimate (valid) Joint PMF;
- Find the (Marginal) Distributions of X and Y.

Generalization for more than 2 r.v.s

The notions above can be generalized to more than 2 r.v.s.

Generalization for more than 2 r.v.s

The notions above can be generalized to more than 2 r.v.s. Say, if X, Y, Z are r.v. in the same Experiment, then

Generalization for more than 2 r.v.s

The notions above can be generalized to more than 2 r.v.s. Say, if X, Y, Z are r.v. in the same Experiment, then

- Their Joint CDF is:

$$F(x, y, z) = \mathbb{P}(X \leq x, Y \leq y, Z \leq z), \qquad \forall (x, y, z) \in \mathbb{R}^3;$$

Generalization for more than 2 r.v.s

The notions above can be generalized to more than 2 r.v.s. Say, if X, Y, Z are r.v. in the same Experiment, then

- Their Joint CDF is:

$$F(x, y, z) = \mathbb{P}(X \leq x, Y \leq y, Z \leq z), \qquad \forall (x, y, z) \in \mathbb{R}^3;$$

- If X, Y, Z are Discrete, then their Joint PMF is given by

$$p_{i,j,k} = \mathbb{P}(X = x_i, Y = y_j, Z = z_k)$$

Generalization for more than 2 r.v.s

The notions above can be generalized to more than 2 r.v.s. Say, if X, Y, Z are r.v. in the same Experiment, then

- Their Joint CDF is:

$$F(x, y, z) = \mathbb{P}(X \leq x, Y \leq y, Z \leq z), \qquad \forall (x, y, z) \in \mathbb{R}^3;$$

- If X, Y, Z are Discrete, then their Joint PMF is given by

$$p_{i,j,k} = \mathbb{P}(X = x_i, Y = y_j, Z = z_k)$$

- And, in the latter case, say, the Distribution of X can be found by:

$$\mathbb{P}(X = x_i) =$$

Generalization for more than 2 r.v.s

The notions above can be generalized to more than 2 r.v.s. Say, if X, Y, Z are r.v. in the same Experiment, then

- Their Joint CDF is:

$$F(x, y, z) = \mathbb{P}(X \leq x, Y \leq y, Z \leq z), \qquad \forall (x, y, z) \in \mathbb{R}^3;$$

- If X, Y, Z are Discrete, then their Joint PMF is given by

$$p_{i,j,k} = \mathbb{P}(X = x_i, Y = y_j, Z = z_k)$$

- And, in the latter case, say, the Distribution of X can be found by:

$$\mathbb{P}(X = x_i) = \sum_{j,k} \mathbb{P}(X = x_i, Y = y_j, Z = z_k)$$