CS 107, Probability, Spring 2019
Lecture 33

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AUA

10 April 2019

Content

- Joint Distribution of two R.V.s, Continuous Case

Example:

Example: Assume (X, Y) is a continuous r.vector with a Joint PDF

$$f(x, y) = \begin{cases} K \cdot (x + y), & x, y \in [0, 2] \\ 0, & \textit{otherwise} \end{cases}$$

- Find K
- Find the Marginal PDFs of X and Y
- Find the CDFs of (X, Y), and Marginal CDFs
- Calculate $\mathbb{P}(X^2 + Y^2 \leq 1)$
- Calculate the PDF of the r.v. $Z = Y - X$

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- And if X, Y, Z are Jointly Continuous, then their Joint PDF $f(x, y, z)$ and Joint CDF $F(x, y, z)$ satisfy

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Exercise: Can you express the Joint PDF of, say, X and Z, in terms of $f(x, y, z)$?

Important Multivariate Distributions

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Assume $n, m \in N$, and $p_k \in [0,1]$, $k = 1, 2, ..., m$ with $p_1 + ... + p_m = 1$. We say that the r.vector $\mathbf{X} = (X_1, X_2, ..., X_m)$ has a Multinomial Distribution with probabilities $\mathbf{p} = (p_1, p_2, ..., p_m)$, and we write

$$\mathbf{X} = (X_1, X_2, ..., X_m) \sim Multinomial(n, p_1, p_2, ..., p_m),$$

if its PMF is given by:

$$\mathbb{P}(X_1 = k_1, X_2 = k_2, ..., X_m = k_m) = \binom{n}{k_1, k_2, ..., k_m} \cdot p_1^{k_1} \cdot p_2^{k_2} \cdot \ldots \cdot p_m^{k_m}$$

for any $k_1, ..., k_m \in \mathbb{N} \cup \{0\}$, with $k_1 + k_2 + ... + k_m = n$.

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Exercise: Find the Joint Distribution of, say, (X_1, X_2).

Example

Example: 10 AUA instructors are choosing (independently) at random one AUA student for some committee. We know that the relationship between the number of Bus/CSE/EC students is $8 : 3 : 1$. What is the Probability that among 10 chosen students, we will have exactly 3 Bus, 5 CSE and 2 EC students?