

# AUA CS108, Statistics, Fall 2020

## Lecture 06

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# Contents

- ▶ Histogram
- ▶ Stem and Leaf Plot
- ▶ ScatterPlot
- ▶ Numerical Summaries for the Central Tendency

# The Density or Normalized Relative Frequency Histogram

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**Definition:** The **Density Histogram** or the **Normalized Relative Frequency Histogram** of our Data  $x_1, \dots, x_n$  is the piecewise constant function

$$h_{dens}(x) = \frac{n_j}{n} \cdot \frac{1}{length(I_j)}, \quad \forall x \in I_j.$$

## Example

**Example:** Plot the Frequency, Relative Frequency and Density Histograms for

0, 4, 2, 2, 0, 0.5, 1, 3

## Example

To draw the Density Histogram in **R**, we will use the *freq=FALSE* parameter in the *hist* command.

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We use here the *discoveries* Standard Dataset from **R**, which gives us the numbers of “great” inventions and scientific discoveries in each year from 1860 to 1959:

## Example

To draw the Density Histogram in **R**, we will use the *freq=FALSE* parameter in the *hist* command.

We use here the *discoveries* Standard Dataset from **R**, which gives us the numbers of “great” inventions and scientific discoveries in each year from 1860 to 1959:

```
discoveries
```

```
## Time Series:
```

```
## Start = 1860
```

```
## End = 1959
```

```
## Frequency = 1
```

```
## [1] 5 3 0 2 0 3 2 3 6 1 2 1 2 1 3 3 3
```

```
## [26] 12 3 10 9 2 3 7 7 2 3 3 6 2 4 3 5 2
```

```
## [51] 3 6 5 8 3 6 6 0 5 2 2 2 6 3 4 4 2
```

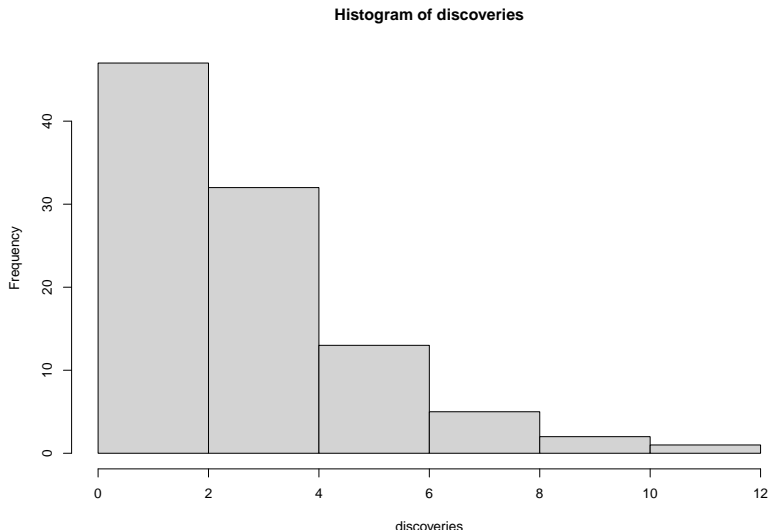
```
## [76] 2 2 1 3 4 2 2 1 1 1 2 1 4 4 3 2 1
```



## Example

First, the Frequency Histogram:

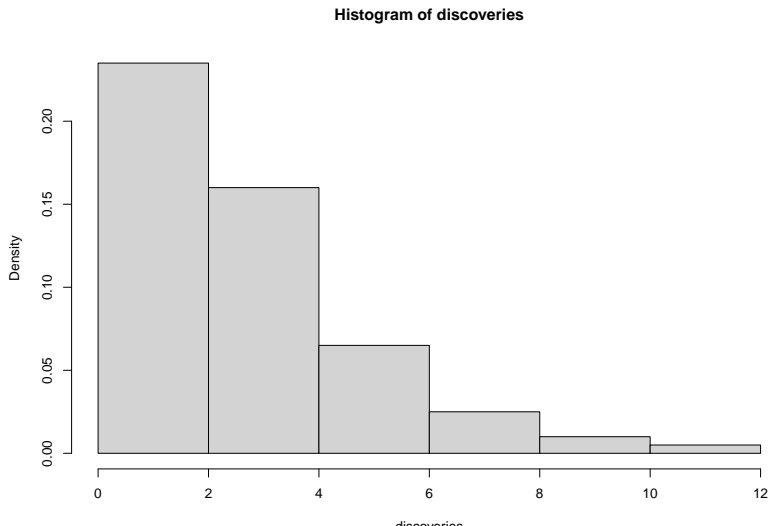
```
hist(discoveries)
```



## Example

Now, the Density Histogram:

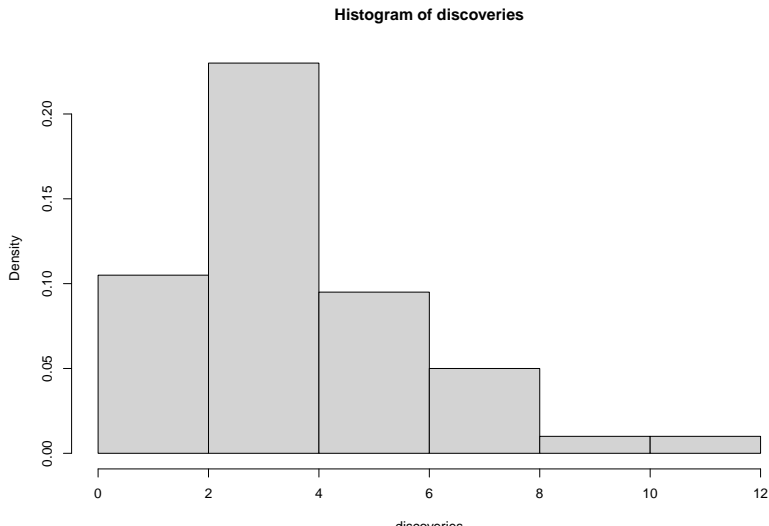
```
hist(discoveries, freq = FALSE)
```



## Example

Finally, the Density Histogram with the Bins left-endpoints included:

```
hist(discoveries, freq = FALSE, right = FALSE)
```



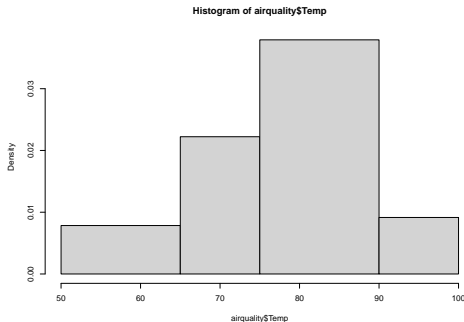
## Example

Now let us change the default bins for a Histogram.

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Now let us change the default bins for a Histogram. We can use the following - first define the vector of our class interval (Bins) endpoints: (note that you need to cover all Datapoints!)

```
bins.endpoints <- c(50, 65, 75, 90, 100)  
hist(airquality$Temp, breaks = bins.endpoints)
```



# Notes

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- ▶ By default, if we give custom bins with non-equal lengths, **R** is plotting the Density Histogram!
- ▶ You can give the *breaks* parameter either the vector of Bins' endpoints or the number of (equal-length) intervals

## Estimation of the PDF through the Density Histogram

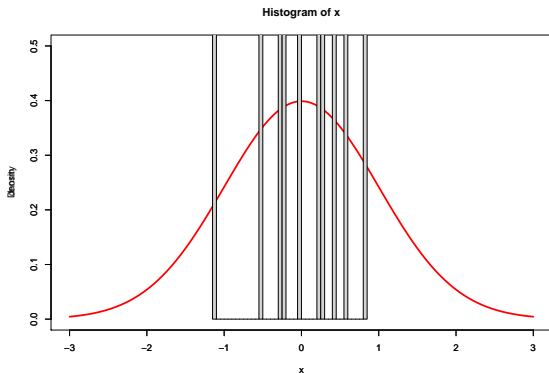
As it was stated above, the Density Histogram is an approximation (estimate) of the PDF of the Data unknown Distribution. To check this, let us take a synthetic Dataset from the Distribution we know:



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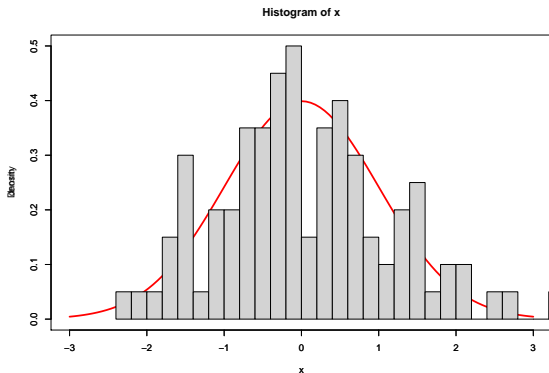
```
plot(dnorm, lwd = 3, col= "red", xlim=c(-3,3), ylim=c(0,0.5))  
x <- rnorm(10)  
par(new = TRUE)  
hist(x, breaks = 40, freq = FALSE, xlim=c(-3,3), ylim=c(0,0.5))
```



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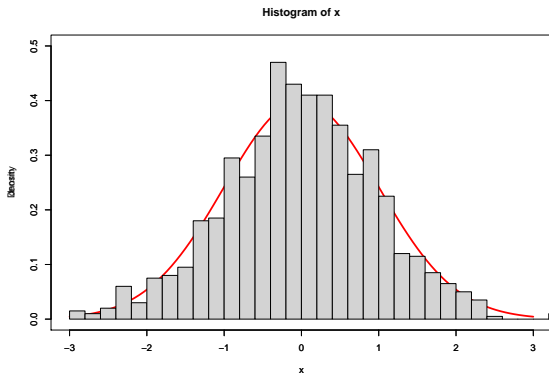
```
plot(dnorm, lwd = 3, col= "red", xlim=c(-3,3), ylim=c(0,0.5))  
x <- rnorm(100)  
par(new = TRUE)  
hist(x, breaks = 40, freq = FALSE, xlim=c(-3,3), ylim=c(0,0.5))
```



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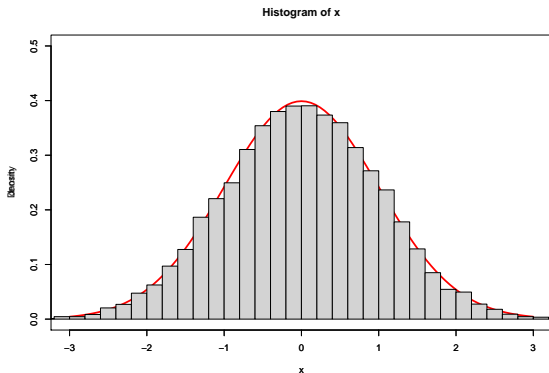
```
plot(dnorm, lwd = 3, col= "red", xlim=c(-3,3), ylim=c(0,0.5))  
x <- rnorm(1000)  
par(new = TRUE)  
hist(x, breaks = 40, freq = FALSE, xlim=c(-3,3), ylim=c(0,0.5))
```



## Estimation of the PDF through the Density Histogram

As it was stated above, the Density Histogram is an approximation (estimate) of the PDF of the Data unknown Distribution. To check this, let us take a synthetic Dataset from the Distribution we know:

```
plot(dnorm, lwd = 3, col= "red", xlim=c(-3,3), ylim=c(0,0.5))  
x <- rnorm(10000)  
par(new = TRUE)  
hist(x, breaks = 40, freq = FALSE, xlim=c(-3,3), ylim=c(0,0.5))
```



## Choosing Bin sizes correctly

It is important to choose the Bin sizes (lengths of the Bin, class, intervals) wisely. Otherwise you will skip some info or you will not get any valuable info.

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Let us use another **R** standard dataset to show the effect of the choice of the bin size: *precip*. This Dataset shows the average amount of precipitation (rainfall) in inches for each of 70 United States (and Puerto Rico) cities.

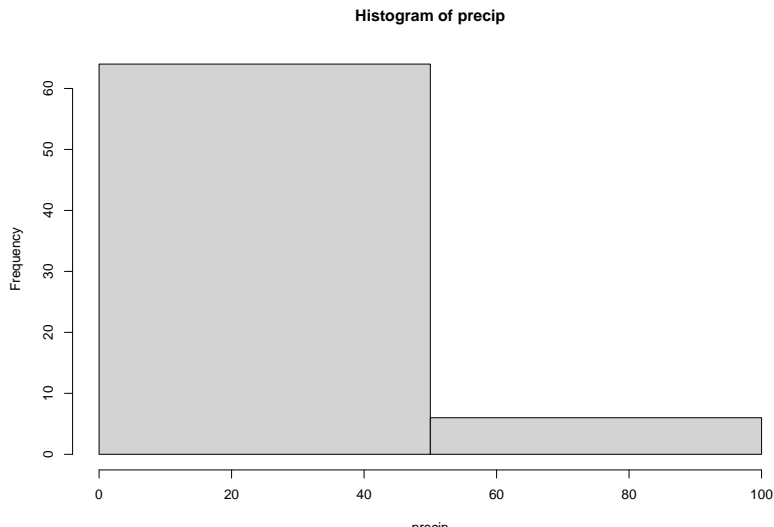
```
head(precip)
```

##	Mobile	Juneau	Phoenix	Little Rock	Los Angeles
##	67.0	54.7	7.0	48.5	1

## Version 1, Small bins

Here, we just use 2 bins:

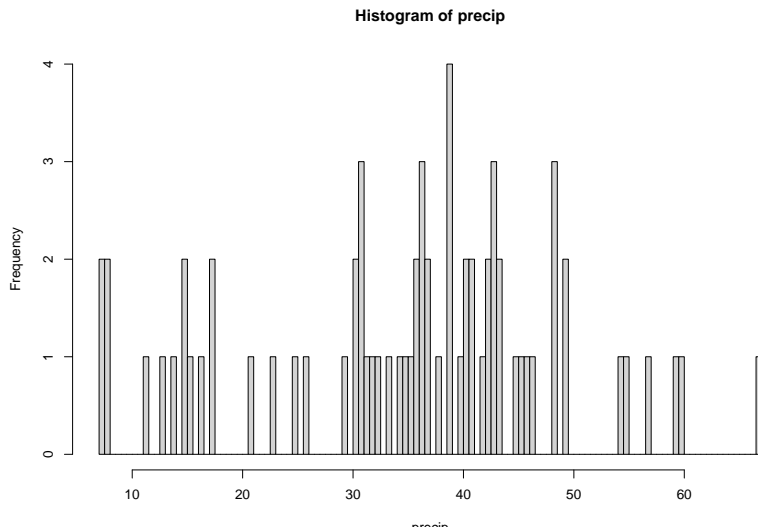
```
hist(precip, breaks = 2)
```



## Version 2, large bins

Here, we use 200 bins:

```
hist(precip, breaks = 200)
```

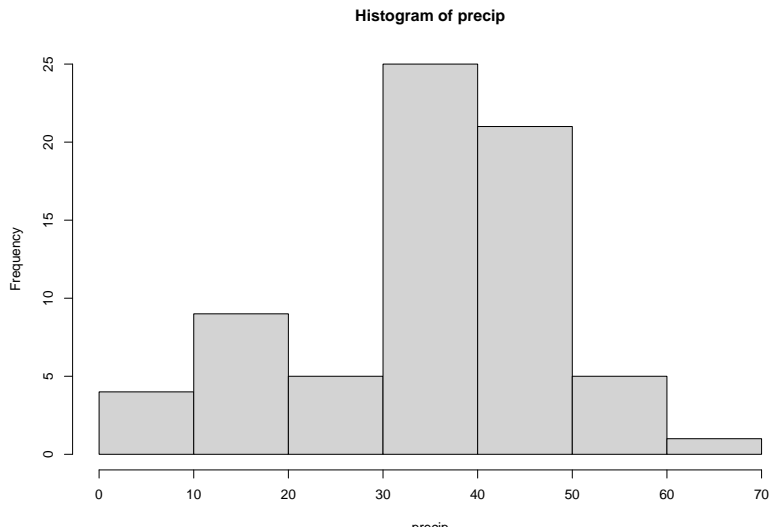




## Version 2, large bins

Now, the default:

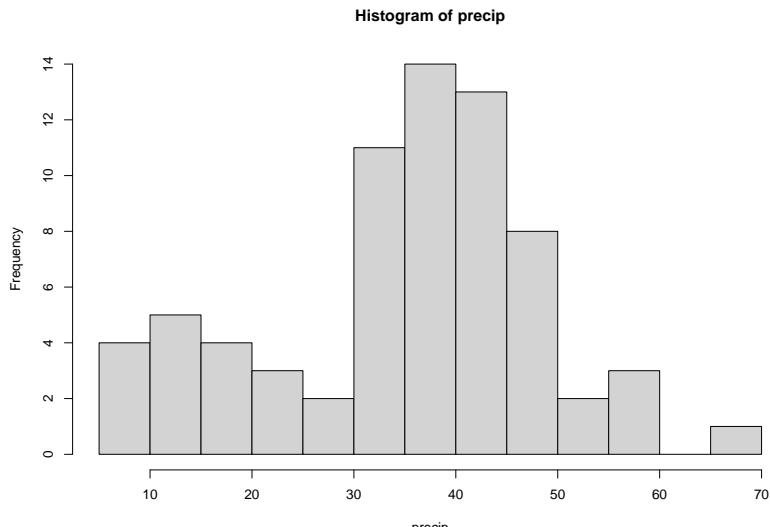
```
hist(precip)
```



## Version 3

Now, let us change to 20 bin intervals:

```
hist(precip, breaks = 20)
```



# Choosing the Bin Length

In fact, choosing the correct Bin width is not an easy job. See, for example, [the Histogram Wiki page](#).

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- ▶ *Barplot* is for a categorical or Discrete Data, *Histogram* is for both Discrete and Continuous
- ▶ We can exactly reconstruct the Dataset from the *Barplot*, but not the *Histogram*

## Addition to the Histogram

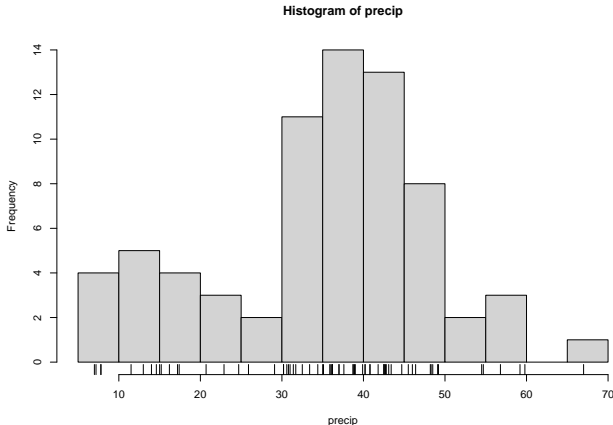
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## Addition to the Histogram

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```
hist(precip, breaks = 20)  
rug(precip)
```



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- ▶ has some gaps
- ▶ has values far apart from others, has outliers (anomalies)
- ▶ is unimodal, bimodal or multimodal

# KDE

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Another estimate for the unknown Distribution PDF is the **Kernel Density Estimator**, KDE. It is, in some sense, the smoothed version of the Histogram: Histogram is a piecewise-constant function, with jumps, so it is not a smooth function.

You will find the Definition of the KDE in the Lecture Notes (and in different books), and here I will give the **R** code to construct the KDE:

## KDE Example

```
x <- precip; d <- density(x)
hist(x, freq = FALSE, xlim = c(0, 80), ylim = c(0,0.04),
     col = "blue", main = "")
rug(x); par(new = TRUE)
plot(d, lwd = 3, col = "red", xlim = c(0,80), ylim = c(0,0.04),
     main = "")
```

