Spectral Analysis and POD of Turbulent Pipe

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- Part 1. Spectral Analysis
  - take fft azimuthally
    - use half of θ data to avoid aliasing
  - find correlation in $ t'$ described in Smits2017.below.eq.2.4.

$$\mathsf{R}\left(km; t, t'\right) = \int_r \mathsf{u}(k; m; r, t)\mathsf{u}^*\left(k; m; r, t'\right) r \, \mathrm{d}r \tag{1}$$

  - take fft in x of th above correlation to get k modes.
- Part 2. Snapshot POD
  - the crossspectra for the kernal of the pod

$$\lim_{\tau\to\infty} \frac{1}{\tau} \int_0^\tau \mathsf{R}\left(k; m; t, t'\right) \alpha^{(n)}\left(k; m; t'\right) \mathrm{d}t' = \lambda^{(n)}(k; m)\alpha^{(n)}(k; m; t) \tag{2}$$

  - Find the (sorted) eigenvalues $\alpha^{(n)}$ found in (2) to solve for $\Phi^{(n)}$,

$$\lim_{\tau\to\infty} \frac{1}{\tau} \int_0^\tau \mathsf{u}_{\mathrm{T}}(k; m; r, t)\alpha^{(n)^*}(k; m; t)\mathrm{d}t = \Phi_{\mathrm{T}}^{(n)}(k; m; r)\lambda^{(n)}(k; m)$$

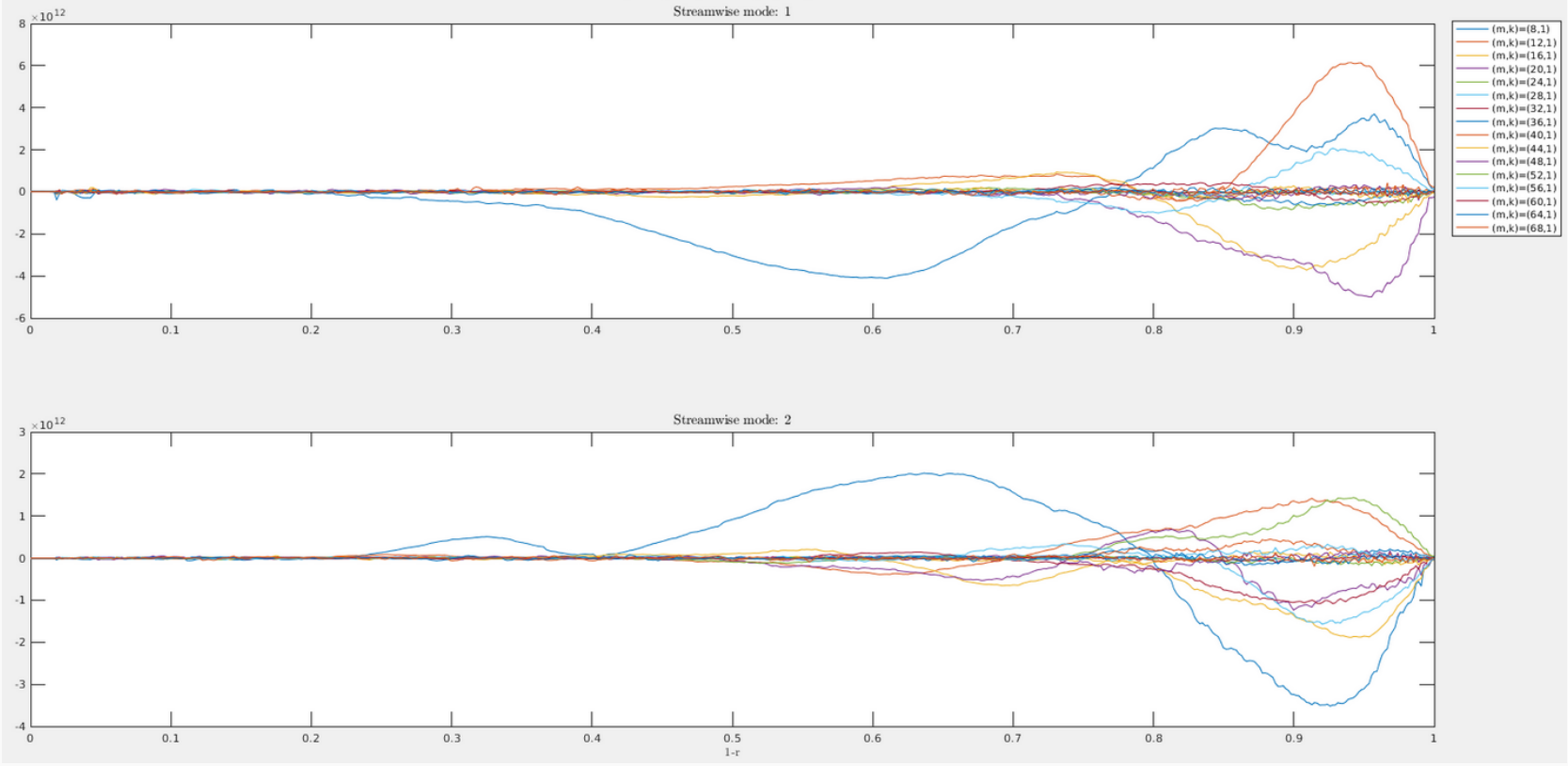

Figure 1: Shows snapshot POD for differen k modes.