Procedure Outline

- Part 1. Spectral Analysis
  - take fft azimuthally
    - use half of θ data to avoid aliasing
  - find correlation in \$ t'\$ described in Smits2017.below.eq.2.4.

$$\mathsf{R}\,(km; t, t') = \int_r \mathbf{u}(k; m; r, t)\mathbf{u}^*\,(k; m; r, t')\,r\,\mathrm{d}r \tag{1}$$

  - take fft in x of th above correlation to get k modes.
- Part 2. Snapshot POD
  - the crossspectra for the kernal of the pod

$$\lim_{\tau\to\infty} \frac{1}{\tau} \int_0^\tau \mathsf{R}\left(k; m; t, t'\right) \alpha^{(n)}\left(k; m; t'\right) \mathrm{d}t' = \lambda^{(n)}(k; m)\alpha^{(n)}(k; m; t) \tag{2}$$

  - Find the (sorted) eigenvalues $\alpha^{(n)}$ found in (2) to solve for $\Phi^{(n)}$,

$$\lim_{\tau\to\infty} \frac{1}{\tau} \int_0^\tau \mathrm{u}_{\mathrm{T}}(k; m; r, t)\alpha^{(n)^*}(k; m; t)\mathrm{d}t = \Phi_{\mathrm{T}}^{(n)}(k; m; r)\lambda^{(n)}(k; m)$$

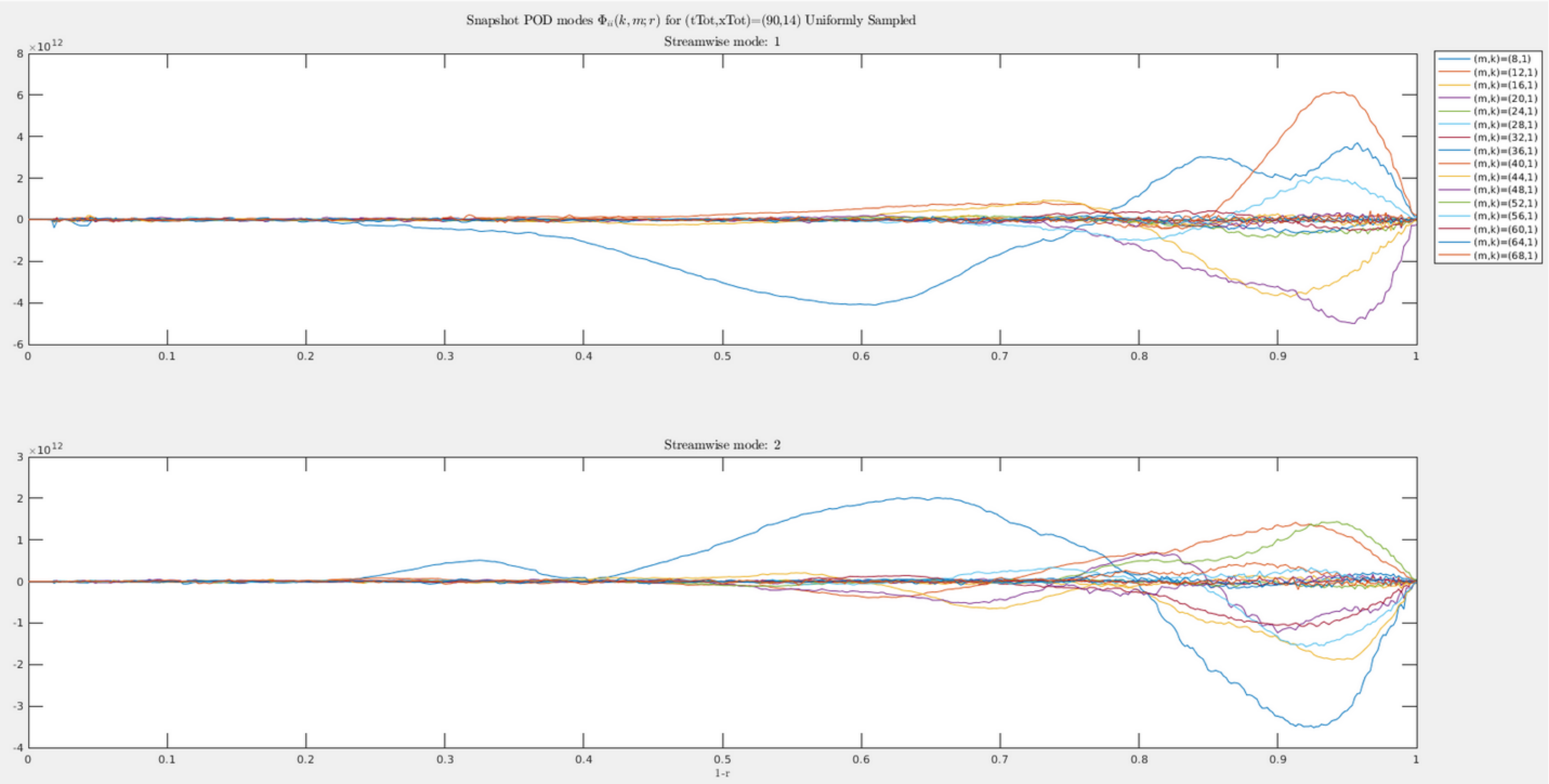

Figure 1: Shows snapshot POD for differen k modes.

- **Example correlation coefficient matrix** R.
  - The maximum values should occur along the diagonal since this is 0 lag occurs (but do not have that)
  - This matrix is symmetric when not multiplied by the weight r, eg $\int uu^*\,dr$. For timestep =5, here is an example matrix without the r weight:

$$\begin{bmatrix} -3.3689 & -3.6159 & -2.7419 & -2.5511 \\ -5.7692 & -6.1922 & -4.6955 & -4.3688 \\ -6.1922 & -6.6463 & -5.0398 & -4.6891 \\ -4.6955 & -5.0398 & -3.8216 & -3.5557 \\ -4.3688 & -4.6891 & -3.5557 & -3.3083 \end{bmatrix} \tag{1}$$

- #TODO: Unfortonuatelty, the maximum is not occuring along the diagonal.
- Here is the integrated correlation tensor with the $\int ruu^*\,dr$

$$\begin{bmatrix} -3.3689 & -3.6159 & -2.7419 & -2.5511 \\ -5.7692 & -6.1922 & -4.6955 & -4.3688 \\ -6.1922 & -6.6463 & -5.0398 & -4.6891 \\ -4.6955 & -5.0398 & -3.8216 & -3.5557 \\ -4.3688 & -4.6891 & -3.5557 & -3.3083 \end{bmatrix} \tag{2}$$

which is indeed symmetic. This is matlabcorrMatSmits(1).dat.