

Definition (Full Procedure for Master Branch <2022-05-27 Fri>)

- Spectral procedure follows Smits 2017.
- Pod procedure follows Smits 2017.

- Part 1. Spectral Analysis
 - take fft azimuthally
 - use half of θ data to avoid aliasing
 - **Note:** in my opinion $\sum_{m=0}^M (fft(theta)) (cos(\theta) + i * sin(\theta))$ rather than just the fft ought to be used.
 - #TODO: include this in next update
 - find correlation in t' described in Smits2017.below.eq.2.4.

$$\mathbf{R}(km; t, t') = \int_r \mathbf{u}(k; m; r, t) \mathbf{u}^*(k; m; r, t') r \, dr \tag{1}$$

- Note in particular that the function **xcorr** is not used (when function **m5.m** on master branch <2022-05-27 Fri> is used).
 - take fft in x of th above correlation to get k modes.
- Part 2. Snapshot POD
 - the crossspectra for the kernal of the pod is given by the r -averaged function

$$\lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau \mathbf{R}(k; m; t, t') \alpha^{(n)}(k; m; t') \, dt' = \lambda^{(n)}(k; m) \alpha^{(n)}(k; m; t) \tag{2}$$

- Note that $\alpha^{(n)}$ act as the eigenfunctions in the above Second Type Fredholm integral equation. This is simply the formulation of the snapshot POD.
- Find the (sorted) eigenvalues $\alpha^{(n)}$ found in (2) to solve for $\Phi^{(n)}$,

$$\lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau \mathbf{u}_T(k; m; r, t) \alpha^{(n)*}(k; m; t) \, dt = \Phi_T^{(n)}(k; m; r) \lambda^{(n)}(k; m)$$

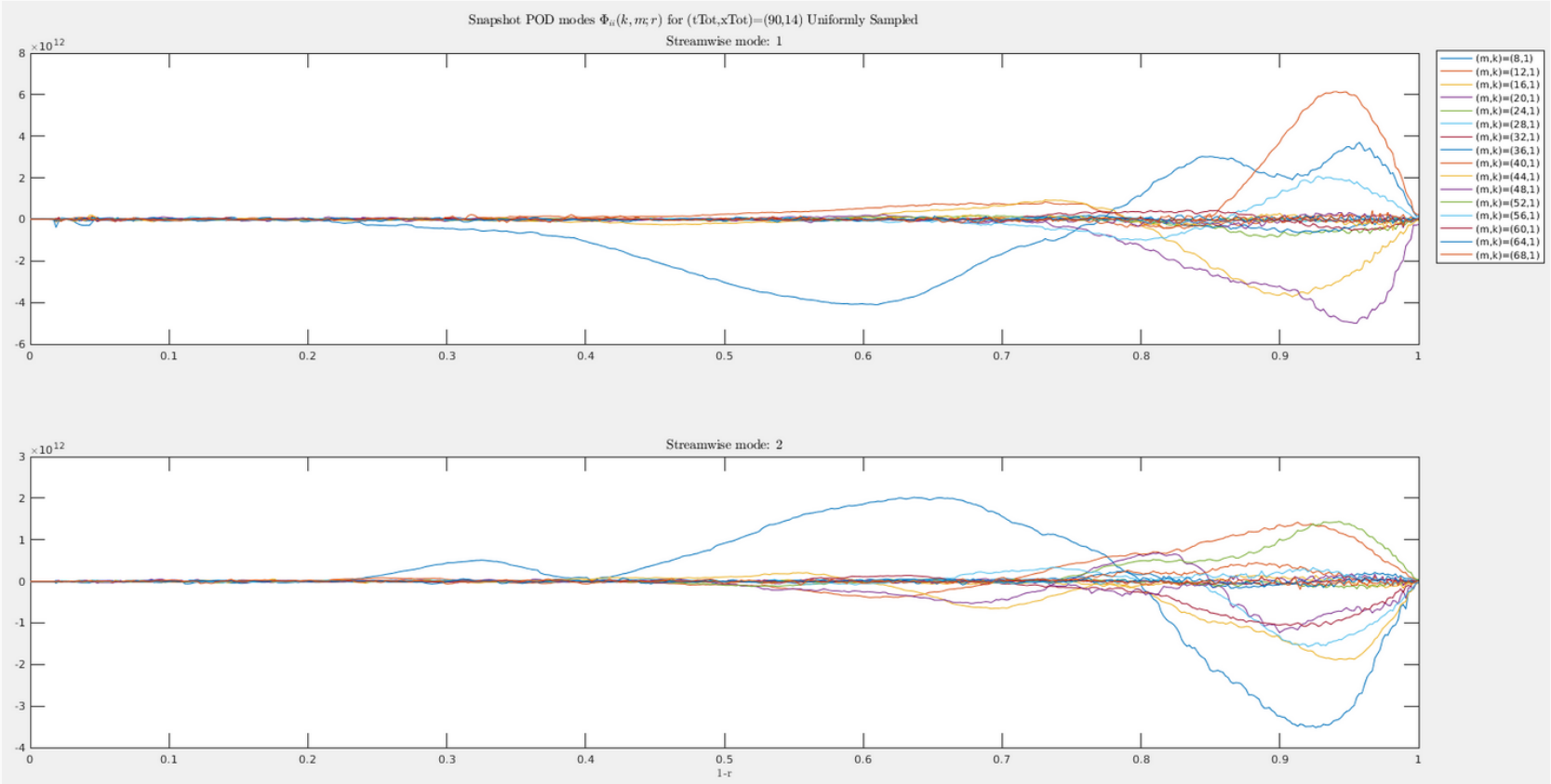


Figure 1: Shows snapshot POD for differen k modes.

- **Example correlation coefficient matrix R .**
 - The maximum values should occur along the diagonal since this is 0 lag occurs (but do not have that)
 - This matrix is symmetric when not multiplied by the weight r , eg $\int uu^* \, dr$. For timestep =5, here is an example matrix without the r weight:

$$\begin{bmatrix} -1.9672 & -3.3689 & -3.6159 & -2.7419 & -2.5511 \\ -3.3689 & -5.7692 & -6.1922 & -4.6955 & -4.3688 \\ -3.6159 & -6.1922 & -6.6463 & -5.0398 & -4.6891 \\ -2.7419 & -4.6955 & -5.0398 & -3.8216 & -3.5557 \\ -2.5511 & -4.3688 & -4.6891 & -3.5557 & -3.3083 \end{bmatrix} \tag{1}$$

- #TODO: Unfortunuatelty, the maximum is not occuring along the diagonal.

- Here is the integrated correlation tensor with the $\int ruu^* \, dr$

$$\begin{bmatrix} -1.9672 & -3.3689 & -3.6159 & -2.7419 & -2.5511 \\ -3.3689 & -5.7692 & -6.1922 & -4.6955 & -4.3688 \\ -3.6159 & -6.1922 & -6.6463 & -5.0398 & -4.6891 \\ -2.7419 & -4.6955 & -5.0398 & -3.8216 & -3.5557 \\ -2.5511 & -4.3688 & -4.6891 & -3.5557 & -3.3083 \end{bmatrix} \tag{2}$$