Definition (Full Procedure for Master Branch *<2022-05-27 Fri>*)

- Spectral procedure follows Smits 2017.
- Pod procedure follows Smits 2017.

- Part 1. Spectral Analysis
  - take fft azimuthally
    - use half of θ data to avoid aliasing
    - Note: in my opinion $\sum_{m=0}^{M}(fft(theta))\,(cos(\theta) + i * sin(\theta))$ rather than just the fft ought to be used. This is done in (cite).
    - #TODO: include this in next update
  - find correlation in t' described in Smits2017.below.eq.2.4.

$$\mathrm{R}\left(km; t, t'\right) = \int_r \mathbf{u}(k; m; r, t)\mathbf{u}^*\left(k; m; r, t'\right) r\, \mathrm{d}r \qquad (1)$$

    - Note in particular that the function xcorr is not used (when function m5.m on master branch *<2022-05-27 Fri>* is used) — the above equation for R done as a explicitly as $\int uu^*$.
  - take fft in x of th above correlation to get k modes.
- Part 2. Snapshot POD
  - the crossspectra for the kernal of the pod is given by the r -averaged function

$$\lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau \mathrm{R}\left(k; m; t, t'\right)\alpha^{(n)}\left(k; m; t'\right)\mathrm{d}t' = \lambda^{(n)}(k; m)\alpha^{(n)}(k; m; t) \qquad (2)$$

  - Note that $\alpha^{(n)}$ act as the eigenfunctions in the above Second Type Fredholm integral equation. This is simply the formulation of the snapshot POD.
  - Find the (sorted) eigenvalues $\alpha^{(n)}$ found in (2) to solve for $\Phi^{(n)}$,

$$\lim_{\tau\to\infty} \frac{1}{\tau}\int_0^\tau \mathrm{u}_{\mathrm{T}}(k; m; r, t)\alpha^{(n)^*}(k; m; t)\mathrm{d}t = \Phi_{\mathrm{T}}^{(n)}(k; m; r)\lambda^{(n)}(k; m)$$

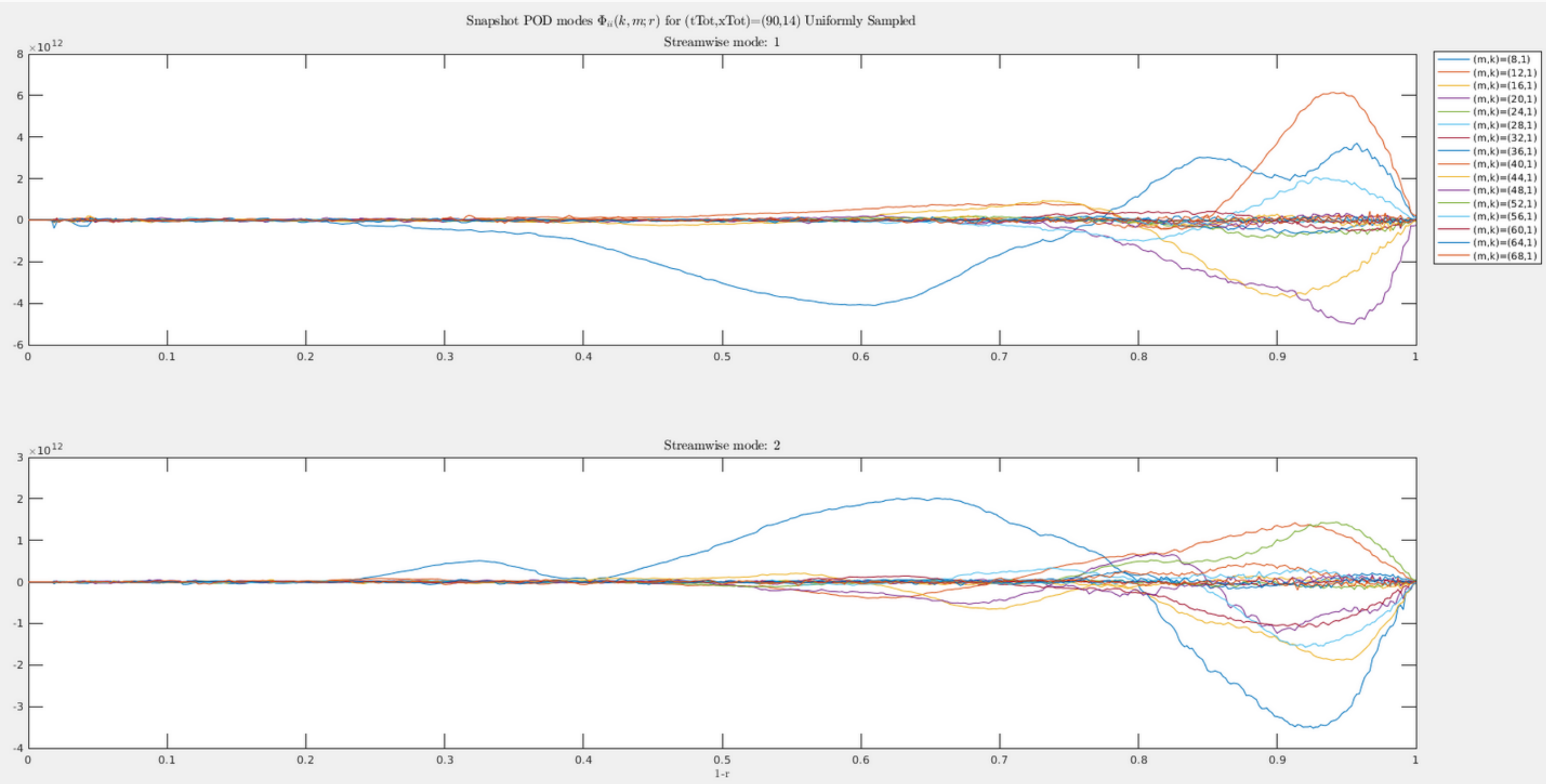

Figure 1: Shows snapshot POD for differen k modes; the timestep and crosssection data is uniformly spaced, with 90 timesteps and 14 crossections used. The small data sample is shown since this code branch must be parallized (in next update). **Note also**, that Smits2017 averages all k-mode graphs.

- **Example correlation coefficient matrix** R.
  - The maximum values should occur along the diagonal since this is 0 lag occurs (but do not have that)

- #TODO: Unfortonuatelty, the maximum is not occuring along the diagonal.

- Here is the integrated correlation tensor with the $\int ruu^*\, dr$ minimalbeispiel,

$$\mathrm{R}\left(x_1, m_1; t, t'\right) = \begin{bmatrix} -1.9672 & -3.3689 & -3.6159 & -2.7419 & -2.5511 \\ -3.3689 & -5.7692 & -6.1922 & -4.6955 & -4.3688 \\ -3.6159 & -6.1922 & -6.6463 & -5.0398 & -4.6891 \\ -2.7419 & -4.6955 & -5.0398 & -3.8216 & -3.5557 \\ -2.5511 & -4.3688 & -4.6891 & -3.5557 & -3.3083 \end{bmatrix} \quad , \text{ntimesteps} = 5 \qquad (1)$$

which is indeed symmetic. This is matlabcorrMatSmits(1).dat.