Definition (Full Procedure for Master Branch *<2022-05-27 Fri>*)

- Spectral procedure follows Smits 2017.
- Pod procedure follows Smits 2017.

- Part 1. Spectral Analysis
  - take fft azimuthally
    - use half of θ data to avoid aliasing
    - Note: in my opinion $\sum_{m=0}^{M}(fft(theta))\,(cos(\theta)+i*sin(\theta))$ rather than just the fft ought to be used. This is done in (cite).
    - #TODO: include this in next update
  - find correlation in t' described in Smits2017.below.eq.2.4.

$$\mathrm{R}\left(km;t,t'\right)=\int_r \mathbf{u}(k;m;r,t)\mathbf{u}^*\left(k;m;r,t'\right)r\,\mathrm{d}r \tag{1}$$

    - Note in particular that the function xcorr is not used (when function m5.m on master branch *<2022-05-27 Fri>* is used) — the above equation for R done as a explicitly as $\int uu^*$.
  - take fft in x of th above correlation to get k modes.
- Part 2. Snapshot POD
  - the crossspectra for the kernal of the pod is given by the r -averaged function

$$\lim_{\tau\to\infty}\frac{1}{\tau}\int_0^{\tau}\mathrm{R}\left(k;m;t,t'\right)\alpha^{(n)}\left(k;m;t'\right)\mathrm{d}t'=\lambda^{(n)}(k;m)\alpha^{(n)}(k;m;t) \tag{2}$$

  - Note that $\alpha^{(n)}$ act as the eigenfunctions in the above Second Type Fredholm integral equation. This is simply the formulation of the snapshot POD.
  - Find the (sorted) eigenvalues $\alpha^{(n)}$ found in (2) to solve for $\Phi^{(n)}$,

$$\lim_{\tau\to\infty}\frac{1}{\tau}\int_0^{\tau}\mathbf{u}_{\mathrm{T}}(k;m;r,t)\alpha^{(n)^*}(k;m;t)\mathrm{d}t=\Phi_{\mathrm{T}}^{(n)}(k;m;r)\lambda^{(n)}(k;m)$$

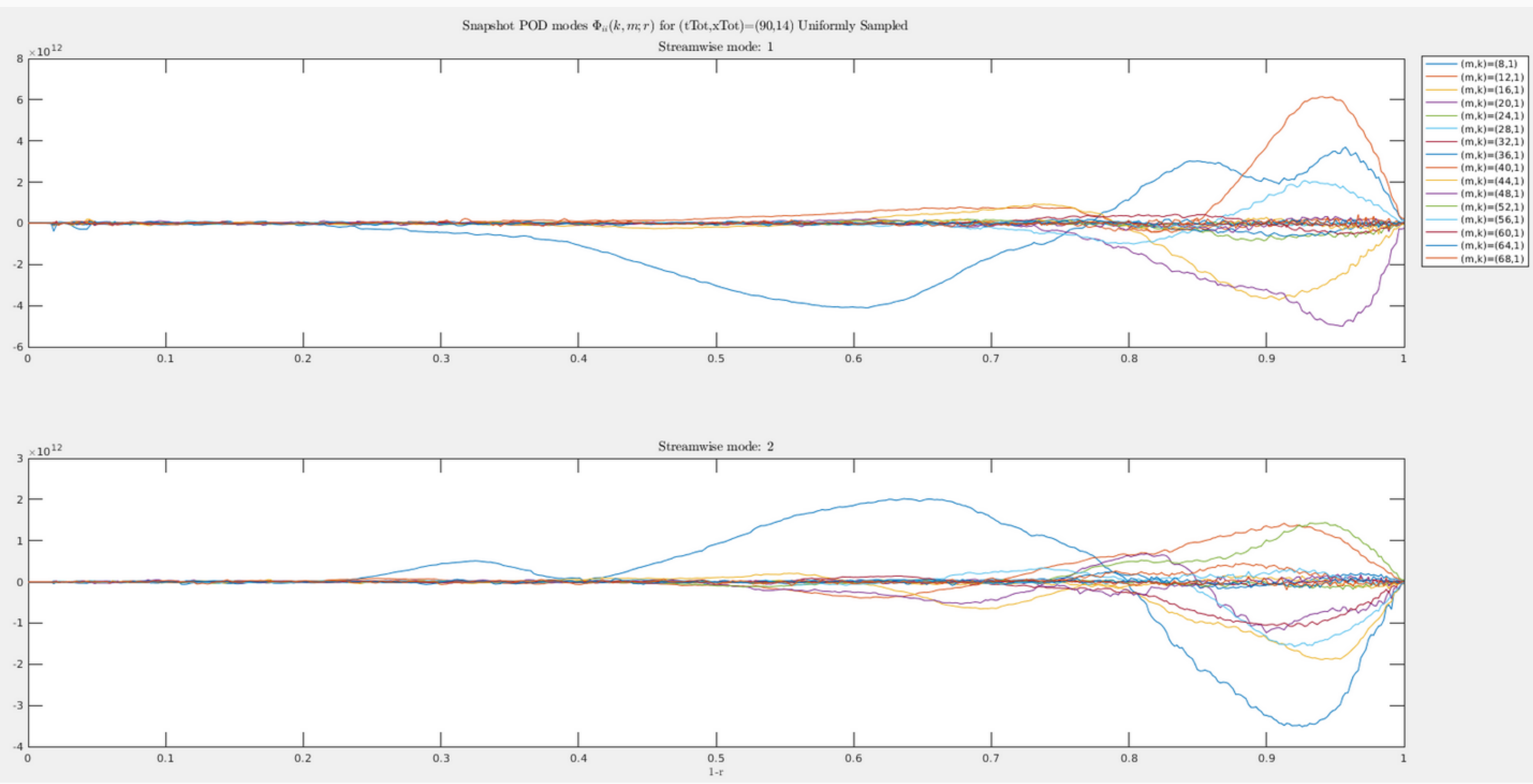

Figure 1: Shows snapshot POD for differen k modes.

- **Example correlation coefficient matrix** R.
  - The maximum values should occur along the diagonal since this is 0 lag occurs (but do not have that)

- #TODO: Unfortonuatelty, the maximum is not occuring along the diagonal.

- Here is the integrated correlation tensor with the $\int ruu^*\,dr$ minimalbeispiel,

$$\mathrm{R}\left(x_1,m_1;t,t'\right)=\begin{bmatrix}-1.9672 & -3.3689 & -3.6159 & -2.7419 & -2.5511\\ -3.3689 & -5.7692 & -6.1922 & -4.6955 & -4.3688\\ -3.6159 & -6.1922 & -6.6463 & -5.0398 & -4.6891\\ -2.7419 & -4.6955 & -5.0398 & -3.8216 & -3.5557\\ -2.5511 & -4.3688 & -4.6891 & -3.5557 & -3.3083\end{bmatrix}\quad ,\mathrm{ntimesteps}=5 \tag{1}$$

which is indeed symmetic. This is matlabcorrMatSmits(1).dat.