



# Spectral Analysis and POD of Turbulent Pipe

---

M. Raba

May 27, 2022

Definition (Procedure)

- Spectral procedure follows Smits 2017.
- Pod procedure follows Smits 2017.

- Part 1. Spectral Analysis
  - take fft azimuthally
    - use half of  $\theta$  data to avoid aliasing
  - find correlation in  $t'$  described in Smits2017.below.eq.2.4.

$$\mathbf{R}(km;t,t') = \int_r \mathbf{u}(k;m;r,t) \mathbf{u}^*(k;m;r,t') r \, dr \tag{1}$$

- take fft in x of th above correlation to get  $k$  modes.
- Part 2. Snapshot POD
  - the crossspectra for the kernal of the pod

$$\lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau \mathbf{R}(k;m;t,t') \alpha^{(n)}(k;m;t') \, dt' = \lambda^{(n)}(k;m) \alpha^{(n)}(k;m;t) \tag{2}$$

- Note that  $\alpha^{(n)}$  act as the eigenfunctions in the above Second Type Fredholm integral equation. This is simply the formulation of the snapshot POD.
- Find the (sorted) eigenvalues  $\alpha^{(n)}$  found in (2) to solve for  $\Phi^{(n)}$ ,

$$\lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau \mathbf{u}_T(k;m;r,t) \alpha^{(n)*}(k;m;t) \, dt = \Phi_T^{(n)}(k;m;r) \lambda^{(n)}(k;m)$$

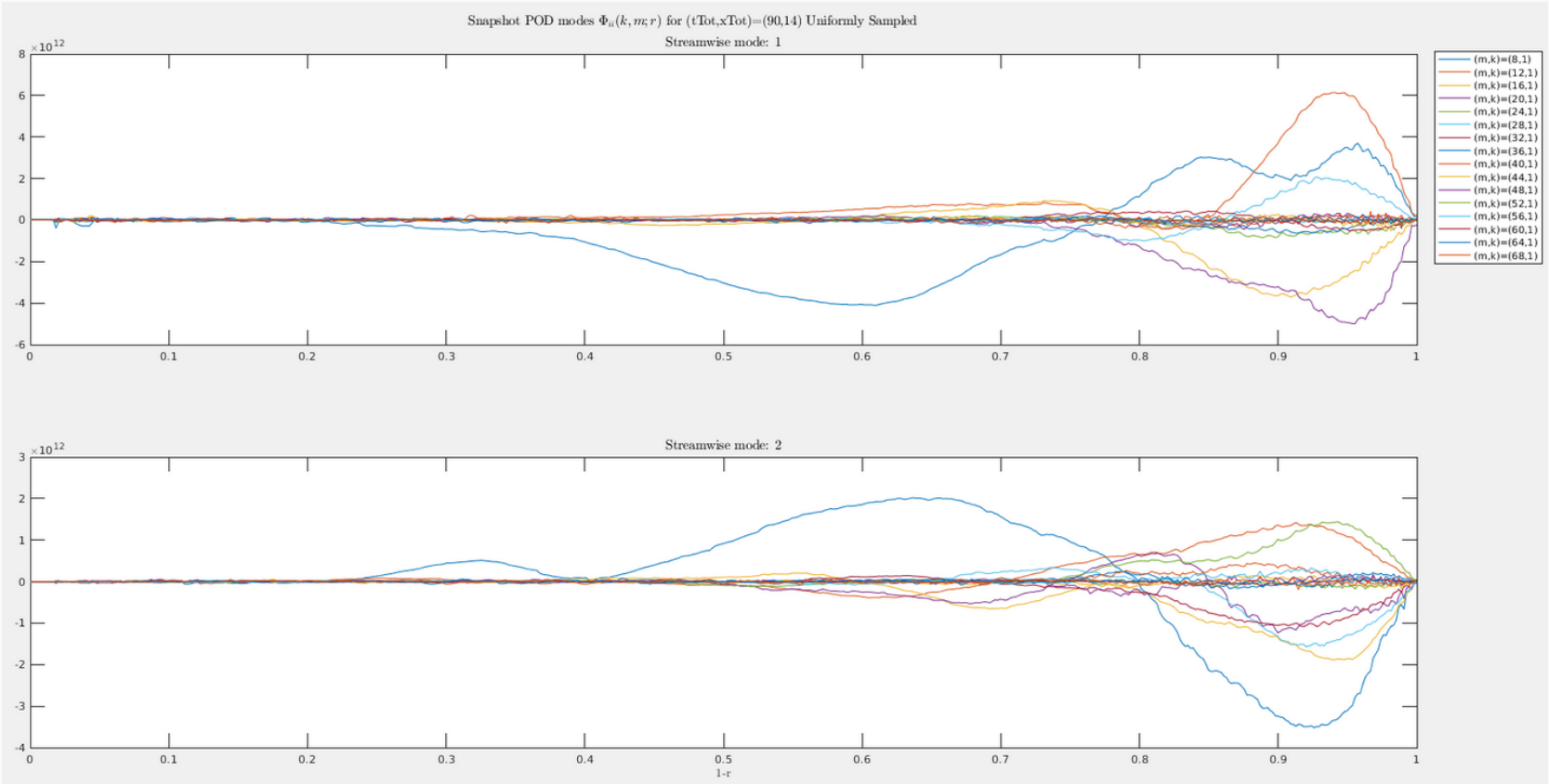


Figure 1: Shows snapshot POD for differen  $k$  modes.

- **Example correlation coefficient matrix  $R$ .**
  - The maximum values should occur along the diagonal since this is 0 lag occurs (but do not have that)
  - This matrix is symmetric when not multiplied by the weight  $r$ , eg  $\int uu^* \, dr$  . For timestep =5, here is an example matrix without the  $r$  weight:

$$\begin{bmatrix} -3.3689 & -3.6159 & -2.7419 & -2.5511 \\ -5.7692 & -6.1922 & -4.6955 & -4.3688 \\ -6.1922 & -6.6463 & -5.0398 & -4.6891 \\ -4.6955 & -5.0398 & -3.8216 & -3.5557 \\ -4.3688 & -4.6891 & -3.5557 & -3.3083 \end{bmatrix} \tag{1}$$

- #TODO: Unfortonuatelty, the maximum is not occuring along the diagonal.
- Here is the integrated correlation tensor with the  $\int ruu^* \, dr$

$$\begin{bmatrix} -3.3689 & -3.6159 & -2.7419 & -2.5511 \\ -5.7692 & -6.1922 & -4.6955 & -4.3688 \\ -6.1922 & -6.6463 & -5.0398 & -4.6891 \\ -4.6955 & -5.0398 & -3.8216 & -3.5557 \\ -4.3688 & -4.6891 & -3.5557 & -3.3083 \end{bmatrix} \tag{2}$$

which is indeed symmetric. This is matlabcorrMatSmits(1).dat.