

Definition (Full Procedure for Master Branch <2022-05-27 Fri>)

- Spectral procedure follows Smits 2017.
- Pod procedure follows Smits 2017.

- Part 1. Spectral Analysis
 - take fft azimuthally
 - use half of θ data to avoid aliasing
 - **Note:** in my opinion $\sum_{m=0}^M (fft(theta)) (\cos(\theta) + i * \sin(\theta))$ rather than just the fft ought to be used. This is done in (cite).
 - #TODO: **include this in next update**
 - find correlation in t' described in Smits2017.below.eq.2.4.

$$R(km; t, t') = \int_r u(k; m; r, t) u^*(k; m; r, t') r \, dr \tag{1}$$

- Note in particular that the function **xcorr** is not used (when function **m5.m** on master branch <2022-05-27 Fri> is used) – the above equation for R done as a explicitly as $\int uu^*$.
 - take fft in x of th above correlation to get k modes.
- Part 2. Snapshot POD
 - the crossspectra for the kernal of the pod is given by the r -averaged function

$$\lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau R(k; m; t, t') \alpha^{(n)}(k; m; t') \, dt' = \lambda^{(n)}(k; m) \alpha^{(n)}(k; m; t) \tag{2}$$

- Note that $\alpha^{(n)}$ act as the eigenfunctions in the above Second Type Fredholm integral equation. This is simply the formulation of the snapshot POD.
- Find the (sorted) eigenvalues $\alpha^{(n)}$ found in (2) to solve for $\Phi^{(n)}$,

$$\lim_{\tau \rightarrow \infty} \frac{1}{\tau} \int_0^\tau u_T(k; m; r, t) \alpha^{(n)*}(k; m; t) dt = \Phi_T^{(n)}(k; m; r) \lambda^{(n)}(k; m)$$

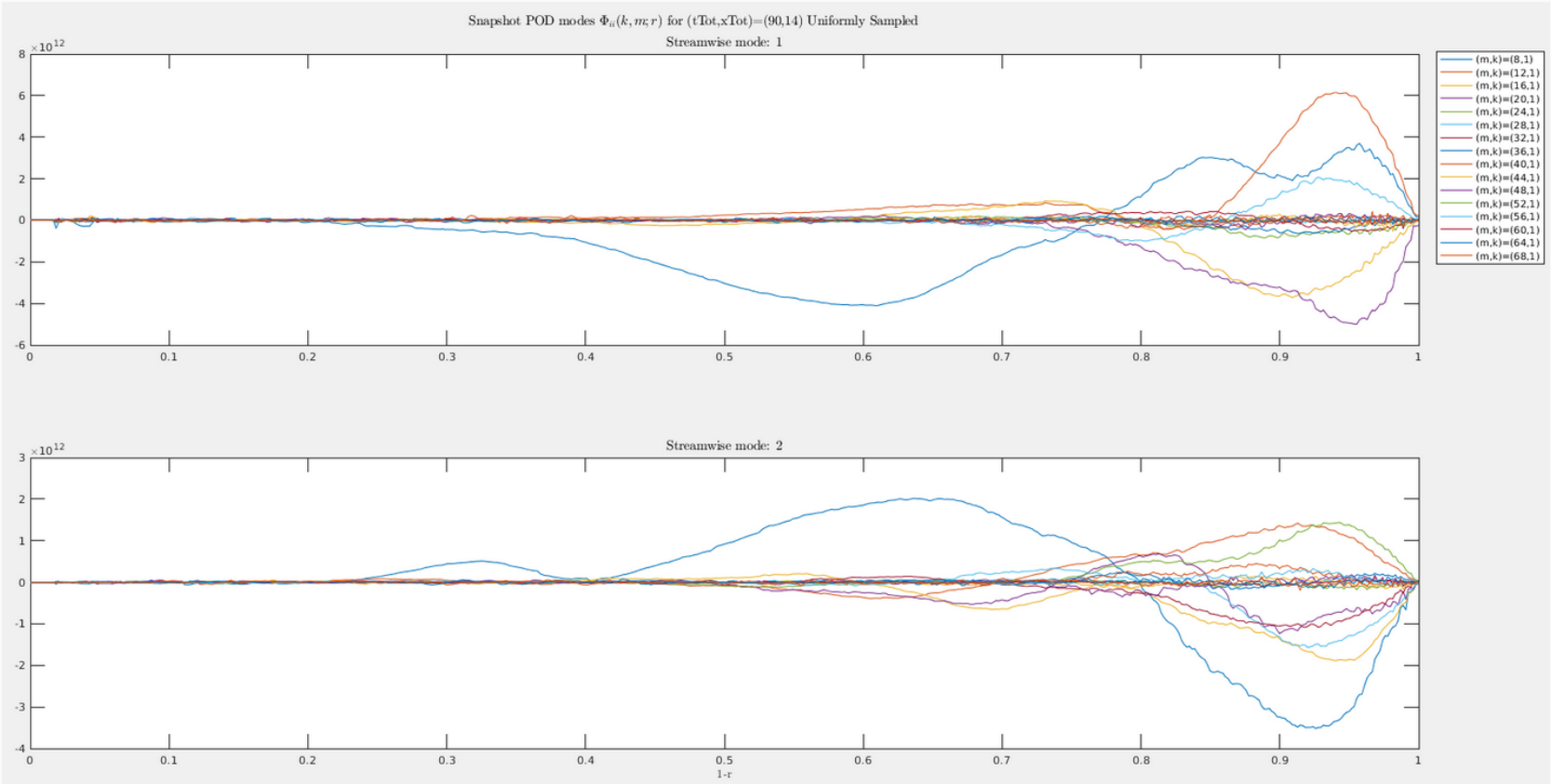


Figure 1: Shows snapshot POD for differen k modes.

- **Example correlation coefficient matrix R .**
 - The maximum values should occur along the diagonal since this is 0 lag occurs (but do not have that)
- #TODO: **Unfortonuatelty, the maximum is not occuring along the diagonal.**
- Here is the integrated correlation tensor with the $\int ruu^* dr$ minimalbeispiel,

$$R(x_1, m_1; t, t') = \begin{bmatrix} -1.9672 & -3.3689 & -3.6159 & -2.7419 & -2.5511 \\ -3.3689 & -5.7692 & -6.1922 & -4.6955 & -4.3688 \\ -3.6159 & -6.1922 & -6.6463 & -5.0398 & -4.6891 \\ -2.7419 & -4.6955 & -5.0398 & -3.8216 & -3.5557 \\ -2.5511 & -4.3688 & -4.6891 & -3.5557 & -3.3083 \end{bmatrix} \quad \text{ntimesteps} = 5 \tag{1}$$

which is indeed symmetic. This is matlabcorrMatSmits(1).dat.