3.4 Fundamentals of Aerothermodynamics

In this section, we summarize several basic concepts and results that will be needed in our analysis of the aerothermodynamics of hypersonic vehicles. In many cases, the results are the same as for lower speed vehicles, but there are some new concepts too.

3.4.1 Aerodynamic forces

Forces acting on a hypersonic vehicle viewed from the side (the pitch plan), at an angle of attack α

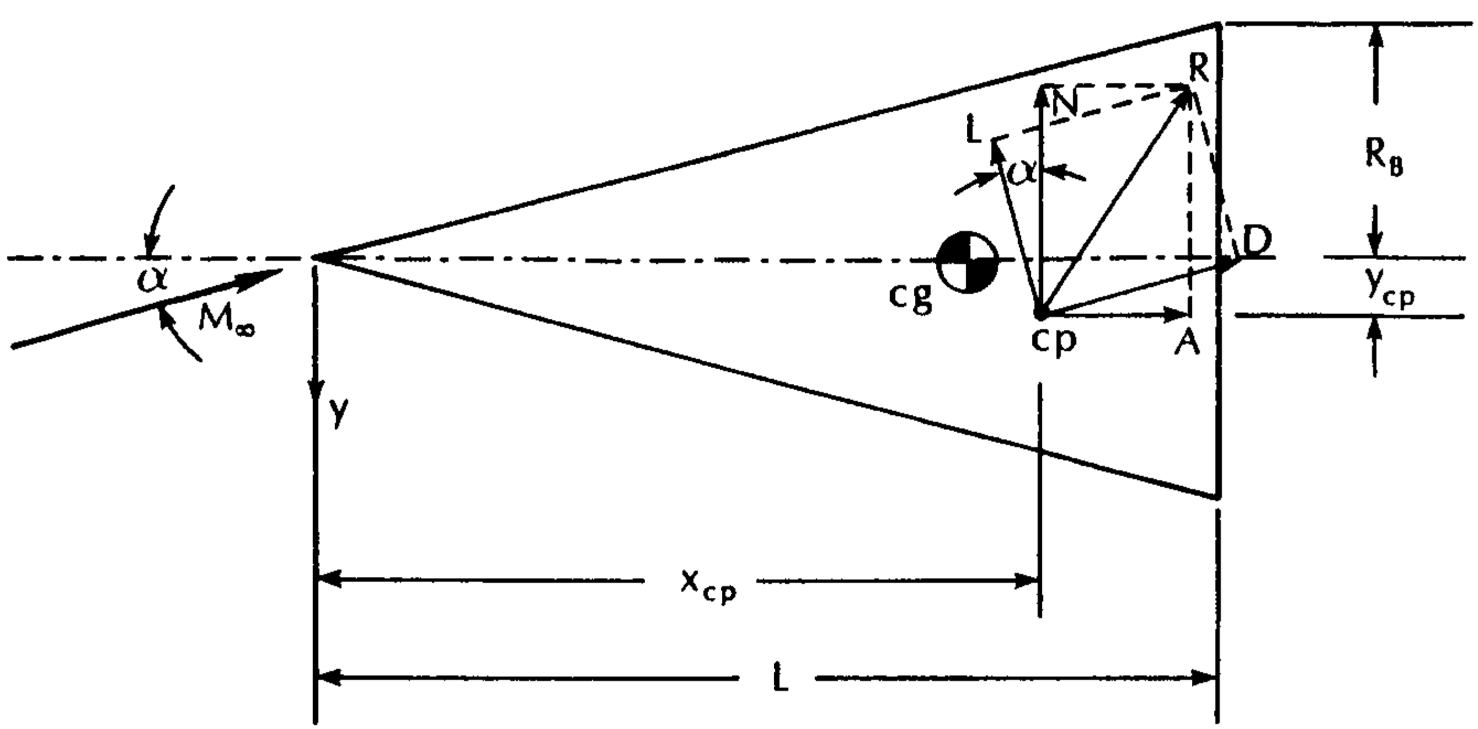


Fig. 8.1 Nomenclature for aerodynamic forces in the pitch plane.

Source: Bertin, 1994

3.4.1 Aerodynamic forces

Definitions:

- D = Drag force
  - components of all aerodynamic forces action opposite to direction of motion
- L = Lift force
  - components of all aerodynamic forces acting normal to direction of motion
- A = Axial force
  - components of all aerodynamic forces acting along vehicle axis
  - $A = D \cos\alpha - L \sin\alpha$
- N = Normal force
  - component of all aerodynamic forces acting normal to vehicle axis
  - $A = D \sin\alpha + L \cos\alpha$

D and L are used for trajectory analysis, A and N are used for aerodynamic and structural analysis

3.4.1 Aerodynamic forces

Using the definition for dynamic pressure

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$$

We can define coefficient for all these forces

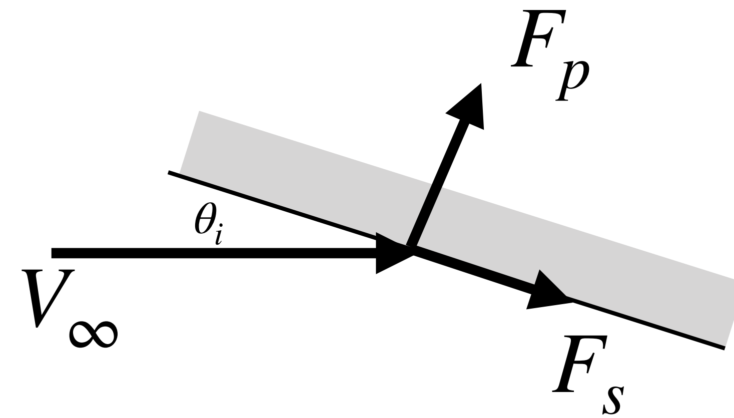
$$C_F = \frac{F}{q_\infty S} \quad \text{where} \quad F = L, D, N, A$$

where S is the cross section.

The resultant force $\boldsymbol{R}$ can generate a moment M at any point in the body, which would also generate a coefficient

These sets of forces represent summations of force vectors on all surface elements of the vehicle that arise from **fluxes of momentum** normal and tangent to each element.

3.4.1 Aerodynamic forces



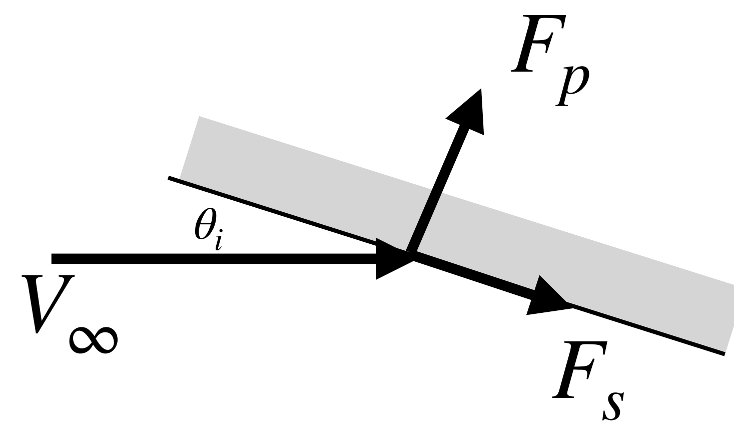
Consider the momentum fluxes for a general surface elements i:

Pressure force acts normal to the surface element due to momentum flux in this direction. For element i:

$$(\delta F_p)_i = (p_i - p_\infty)\delta A_i \qquad \boxed{3.39}$$

where p_i is the gas pressure at the surface and p_∞ is the free stream pressure

3.4.1 Aerodynamic forces



Still considering the same surface element

Skin friction acts tangent to the surface element due to momentum flux in this direction

$$(\delta F_s)_i = \tau_i \delta A_i \qquad \boxed{3.40}$$

where τ_i is the shear stress at the surface $= \left(\mu \dfrac{du}{dn}\right)_i$

3.4.1 Aerodynamic forces

The net forces are then obtained as:

$$\begin{aligned} D &= \sum_i \left(p_i \sin\theta_i + \tau_i \cos\theta_i\right) \delta A_i \\ L &= \sum_i \left(p_i \cos\theta_i - \tau_i \sin\theta_i\right) \delta A_i \end{aligned} \tag{3.41}$$

Thus, the aerodynamic forces are governed by pressure **and** skin friction

3.4.1 Aerodynamic forces

Exercise 3.3

Calculate the drag, lift and weight forces on the Space Shuttle at 70 km altitude in Earth's atmosphere given

- $C_D = 0.84$
- $L/D = 1$
- $A = 250 \text{ m}^2$
- $B = W/C_D A = 4220 \text{ N/m}^2$

V = 6246 m/s
at 70km -> rho = 8.283E-5 kg/m3
D = C_D 1/2 rho V^2 A = 339 kN
L/D = 1 -> L = D = 339 kN
W = B C_D A = 4220*250*0.84
= 886 kN

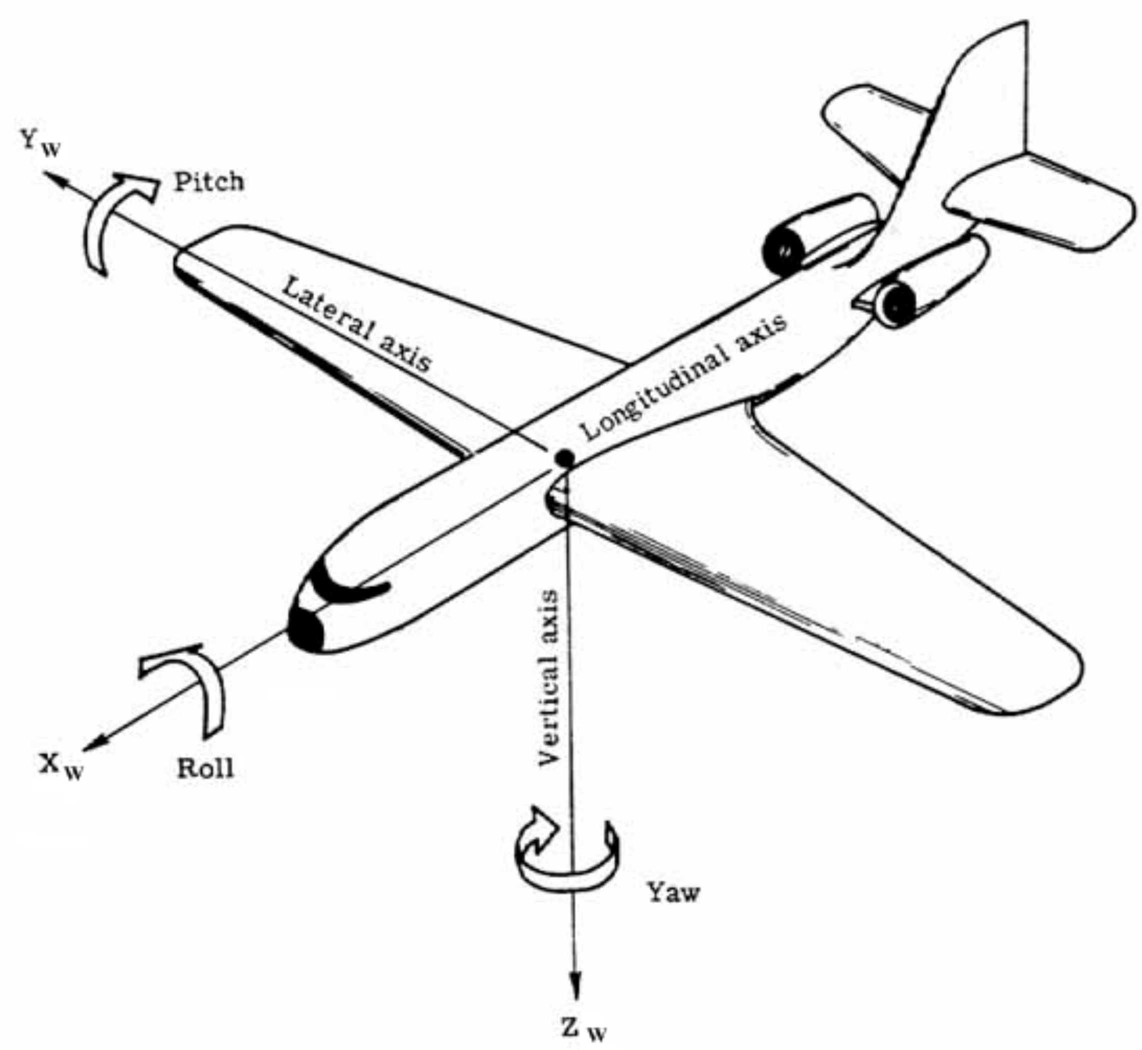
3.4.2 Moments

In aerodynamic analysis, the net forces are considered to act through a single point of the vehicle called the center of pressure (cp). The location of cp depends on

- Flight conditions (velocity, altitude)
- Vehicle configuration (orientation, control surfaces)

The weight force W always acts though the center of gravity (cg) of the vehicle (in the direction opposite to L). As illustrated in our diagram, cp and cg are generally not co-located (for stability reasons) and so the aerodynamic forces can generate turning moments of the vehicle. These moments are termed pitch (y), roll (x) and yaw (z), respectively

3.4.2 Moments



Source: Teknillinen korkeakoulu, Tekniska Hogskolan

While roll and yaw certainly have to be accounted for, of particular interest in hypersonic vehicle design is the pitching moment. Note that nose up is considered a positive pitching moment

3.4.2 Moments

In general, the moment vector $\mathbf{M}$ induced by a force $\mathbf{F}$ acting at a distance $\mathbf{r}_{\mathrm{gp}}$ from the cg is

$$\mathbf{M} = \mathbf{r}_{gp} \times \mathbf{F} \qquad \boxed{3.42}$$

And we introduce a moment coefficient

$$C_M = \frac{|\mathbf{M}|}{\frac{1}{2}\rho_\infty u_\infty^2 A r'} \qquad \boxed{3.43}$$

where r' is a characteristic length. Referring to our earlier diagram, the pitching moment about cg due to aerodynamic forces acting at cp with distance between cg and cp, $\mathbf{r}_{gp} = (x_{gp}, y_{gp})$ is given by:

$$\mathbf{M_p} = x_{gp}N + y_{gp}A$$

A vehicle is said to be trimmed when it is flown such that the pitching moment is zero.

3.4.3 Surface heating

Unlike lower speed aircraft, vehicle surface heating is an important design driver for hypersonic vehicles. The heating is caused by a net flux of energy to the vehicle surface that is expressed as:

$$\dot{q} = \text{heat flux per unit area in W/m}^2$$

We can introduce a heat transfer coefficient

$$C_h = \mathrm{St} = \frac{\dot{q}}{\frac{1}{2}\rho_\infty u_\infty^3} \tag{3.44}$$

Which is also called the Stanton number.

3.4. Surface heating

It's seems like a good idea to relate skin friction to heating

It is, after all, what most people think is the main cause of heating for re-entry vehicles

A simple analysis gives interesting results

- Assuming a parallel flow over a flat plate, the heat transfer is:

$$\dot{q} = -\kappa \frac{\partial T}{\partial \eta}$$

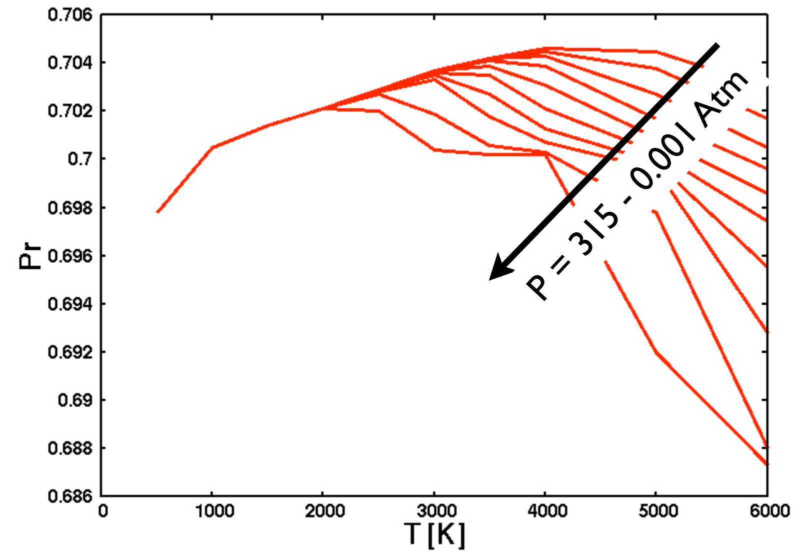
- and the shear stress:

$$\tau = \mu \frac{\partial v}{\partial \eta}$$

- Putting those together:

$$\dot{q} = -\tau \frac{\kappa}{\mu} \frac{\partial T}{\partial v}$$

3.4.3 Reynolds analogy



- The Prandlt number can be introduced, since it has been established that its value is fairly constant over a wide range of temperatures

$$\dot{q} = -\tau \frac{c_p}{\mathrm{Pr}} \frac{\partial T}{\partial v}$$

- Integrating from the surface to the free stream, assuming that the ratio q_w/τ_w is constant

$$h_w - h_\infty = \frac{\dot{q}_w}{\tau_w} v_\infty \mathrm{Pr}$$

- And since $h = e + v^2/2 + RT$, and v^2 is dominant in hypersonics

$$\frac{\tau_w}{\frac{1}{2}\rho_\infty v_\infty} = \frac{2\dot{q}_w}{\frac{1}{2}\rho_\infty v_\infty^3} \mathrm{Pr}$$

3.4.3 Skin Friction

Like other quantities, it is possible to introduce a local skin friction coefficient using the shear stress at the wall:

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho_\infty u_\infty^2} \tag{3.45}$$

3.4.3 Reynolds analogy

Which gives:

$$\mathrm{St} = \frac{1}{2} C_f \mathrm{Pr}^{-1}$$

Which is close enough to the usual Reynolds analogy:

$$\mathrm{St} = \frac{1}{2} C_f \mathrm{Pr}^{-2/3} \qquad \boxed{3.46}$$

This is a very important result in aerothermodynamics; it provides knowledge of skin friction when heat transfer is know, and vice versa

The analogy terminology is due to the fact that, for a Prandlt number of 1, the skin friction behaves in the same manners as as heat transfer in a small layer near the boundary

$$\tau = \mu \frac{\partial v}{\partial \eta} \qquad\qquad \dot{q} = -\kappa \frac{\partial T}{\partial \eta}$$

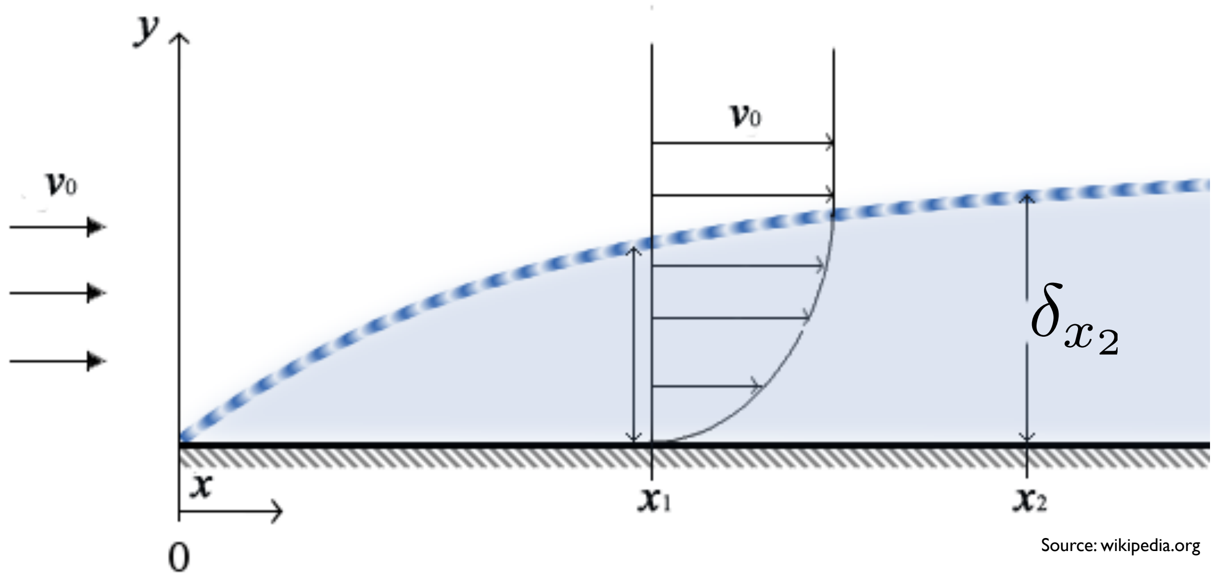
3.4.3 Surface heating

Exercise 3.3

Calculate the heat flux to the nose of the Space Shuttle at 70 km of altitude given that $\mathrm{St} = 0.02$ at this condition.

3.4.4 Boundary Layer

The aerodynamic forces and moments, and surface heating are all determined by the interaction of the gas with the vehicle surface. The gas in contact with the surface lies within a boundary layer (BL). Hence, the properties of the BL play a significant role in the aerothermodynamics of hypersonic vehicles.



Source: wikipedia.org

There are two important types of BL's that have very different fundamental properties.

3.4.4 Boundary Layer

a) Laminar BL

The flow is smooth and well behaved

Occurs at low Reynolds number ($\mathrm{Re} < 10^6$)

For a laminar flow over a flat plate

$$\delta_x = \frac{5.2x}{\sqrt{\mathrm{Re}_x}} \qquad \boxed{3.47a}$$

where local Reynolds number is $\mathrm{Re}_x = \dfrac{\rho u x}{\mu}$

Total skin friction coefficient: $C_f = \dfrac{1.328}{\sqrt{\mathrm{Re}}}$ $\boxed{3.47b}$

Local skin friction coefficient: $C_{f\,x} = \dfrac{0.664}{\sqrt{\mathrm{Re}_x}}$ $\boxed{3.47c}$

3.4.4 Boundary Layer

b) Turbulent BL

The flow is chaotic and has eddies

Occurs at high Reynolds number ($\mathrm{Re} > 10^7$)

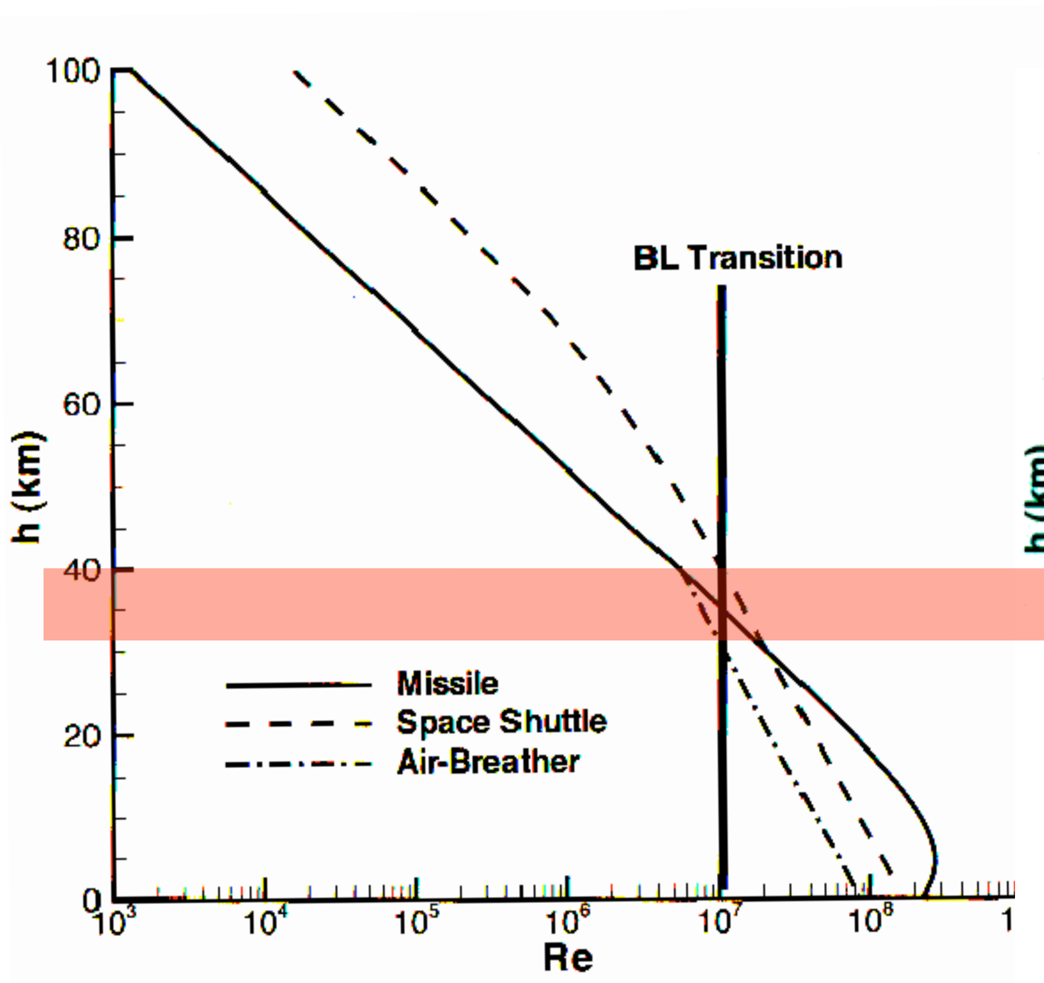
For turbulent flow over a flat plate

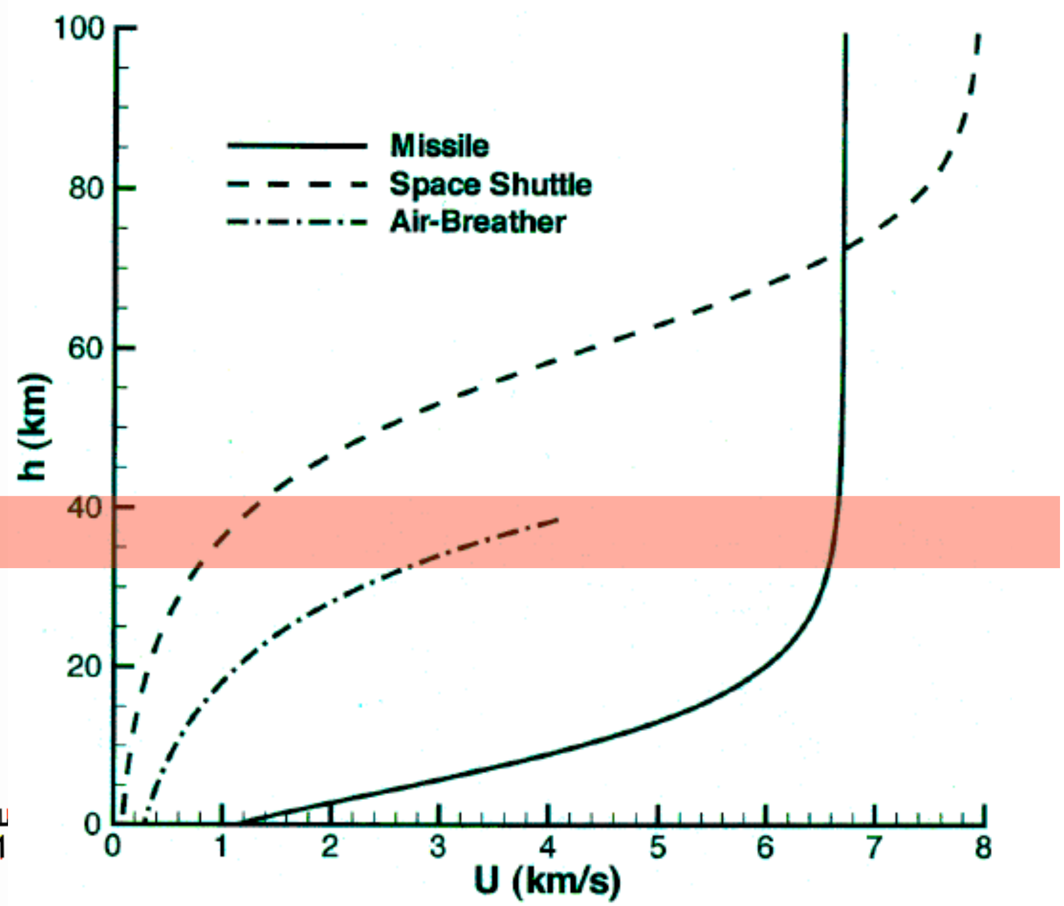
$$\delta_x = \frac{0.37x}{(\mathrm{Re}_x)^{0.2}} \qquad \boxed{3.48a}$$

Total skin friction coefficient:

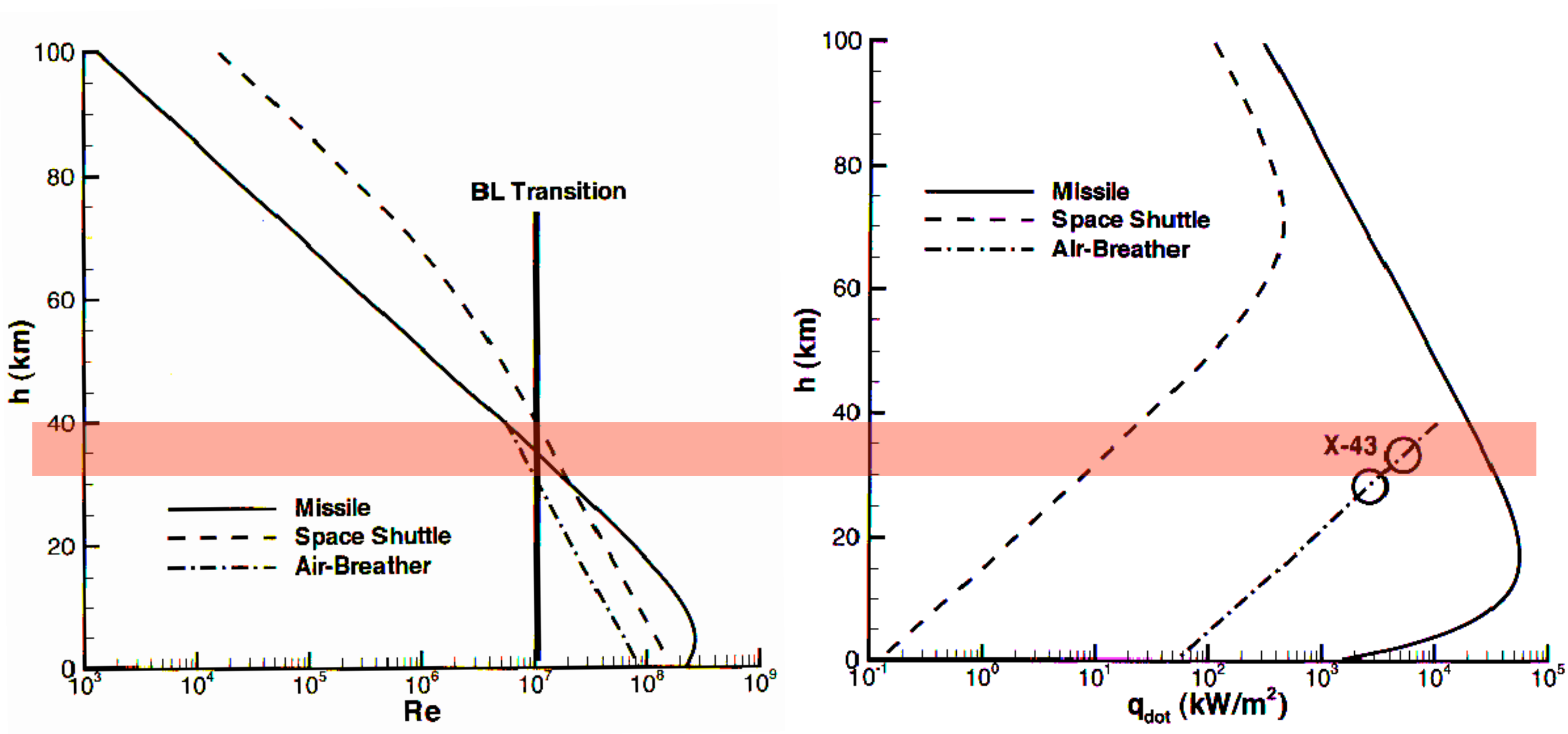
$$C_f = \frac{0.455}{\sqrt{\log_{10} \mathrm{Re}}} \qquad \boxed{3.48b}$$

Boundary Layer





Boundary Layer



3.4.4 Boundary Layer

Comments:

Laminar BL's are thinner, and this has implications for flow separation

Laminar BL's have lower skin friction and hence, by Reynolds analogy, lowest heat transfer than fully turbulent flows (also transitional flows!)

Therefore, the determination of the nature of the BL plays a significant role in hypersonic vehicle analysis