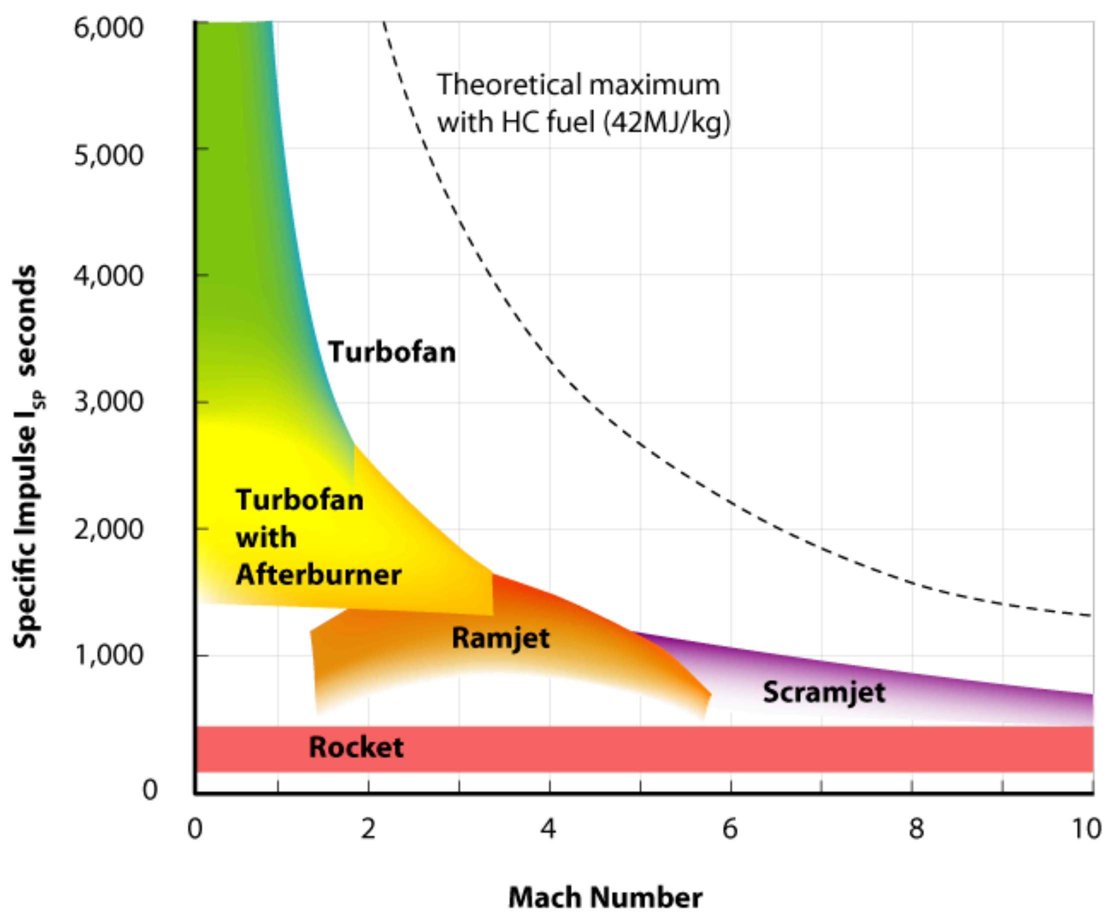
2.4 Air-breathing vehicles

- Types of air-breathers:
  - <u>fully expendable</u>: missiles
    - rocket launched, ballistic trajectories
  - <u>partly reusable</u>: shuttle
    - orbiter + booster tanks re-usable
    - main propellant tank is expended
    - rocket-launched, enters on equilibrium glide
  - <u>fully reusable</u>: no vehicle yet
    - non-rocket launched, powered entry
    - most reusable vehicle systems will use air-breathing propulsion
      - ingest air at engine inlet
      - mix with fuel and burn via combustor
      - accelerate hot gas through nozzle to create thrust

2.4 Air-breathing vehicles

- Propulsion options for re-usable vehicle discussed separately
- However, the only viable class of propulsion uses air-breathing systems to greatly reduce the required propellent weight



2.4 Air-breathing vehicles

- Simple view of an air-breathing system
  - air is mixed with propellant to generate combustion
  - the combustion creates hot gas
  - the hot gas is accelerated through a nozzle to produce thrust
- The flight trajectory of an air-breathing system is set by
  - aerodynamic loads (like all hypersonic vehicles)
  - need to maintain pressure inside the air-breathing propulsion system close to atmospheric in order to achieve efficient combustion

2.4 Air-breathing vehicles

- The second requirement is usually expressed in terms of the dynamic pressure

$$\frac{1}{2}\rho u^2 = \phi \cdot 101325 \qquad (2.31)$$

where

$$\phi = \mathcal{O}(1)$$

2.4 Air-breathing vehicles

Exercise 2.4
Plot the variation in velocity and Mach number for an air-breathing hypersonic vehicle subject to Eq. 2.31 with $\phi = 0.5$

Re-writing Eq. 2.31, with the exponential atmosphere:

$$u(h) = \sqrt{\frac{2\phi \cdot 101325}{\rho_{SL} \exp(-\alpha h)}}$$

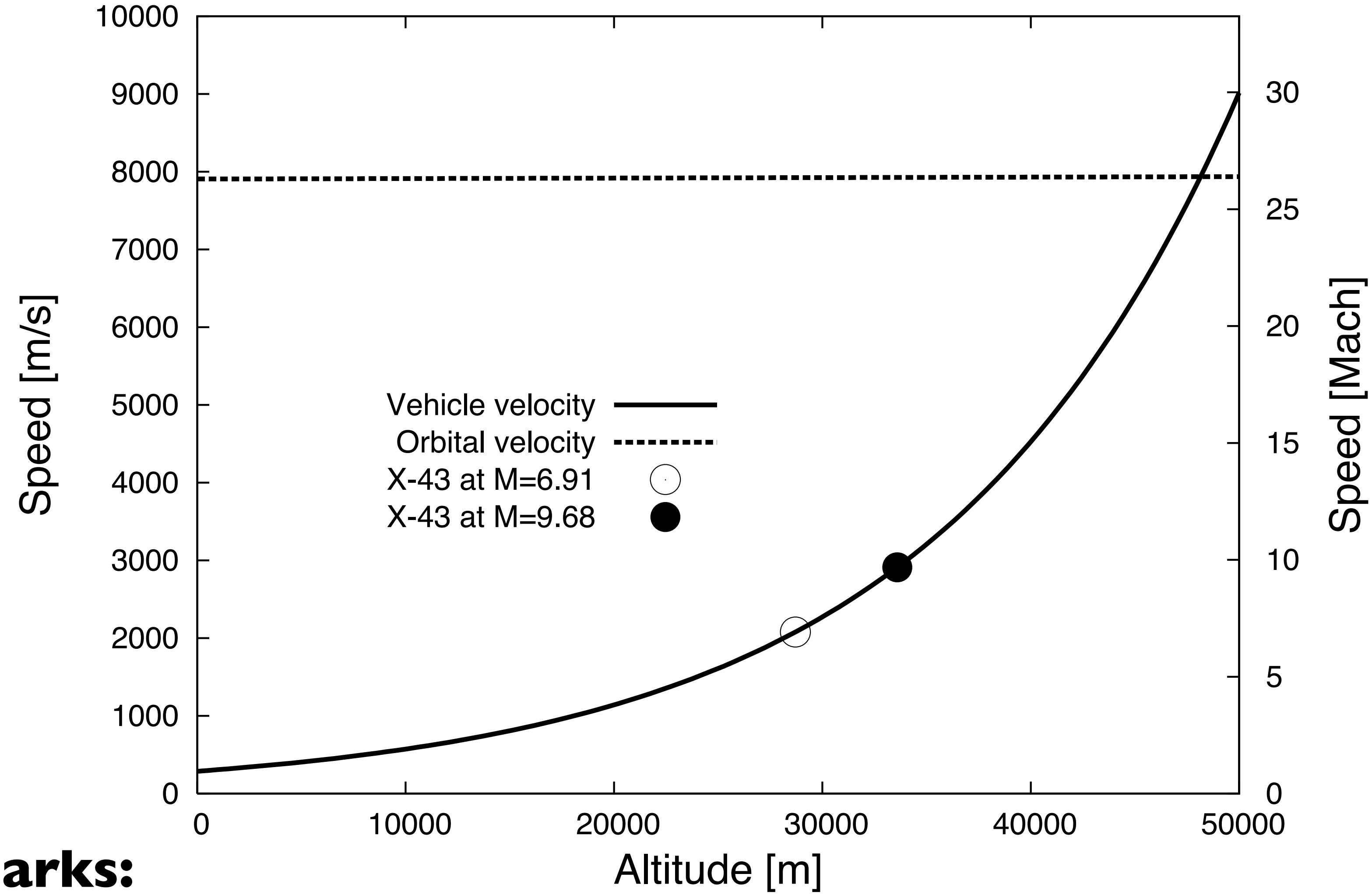
And using:

$$M = \frac{u}{\sqrt{\gamma R \bar{T}}}$$

where

$$\begin{aligned} \bar{T} &= 225 \text{ K} \\ \gamma &= 1.4 \\ R &= 287.05 \text{ J kg}^{-1} \text{ k}^{-1} \end{aligned}$$

Fig. 2.4.1 Velocity constraint for an airbreathing vehicle



Remarks:

Condition 2.31 can only be achieved below 50 km without flying at super-orbital speed

Air-breathing systems are limited to producing Mach number of 6 to 15 (corresponding to 30 ~ 40 km); we'll see why later

2.4 Air-breathing vehicles

- **Heat transfer**
  - The heating to the vehicle may be written as

$$\dot{q} = k\sqrt{\rho}u^3$$

  - Using Eq. 2.31:

$$\dot{q} = k\sqrt{\frac{2\phi \cdot 101325}{u^2}}u^3 \propto u^2$$

  - For the trajectory plotted, where we consider $u_{max} = u_c$, the point of maximum heating is the lowest altitude where $u = u_c$, which is about 48.5 km

2.4 Air-breathing vehicles

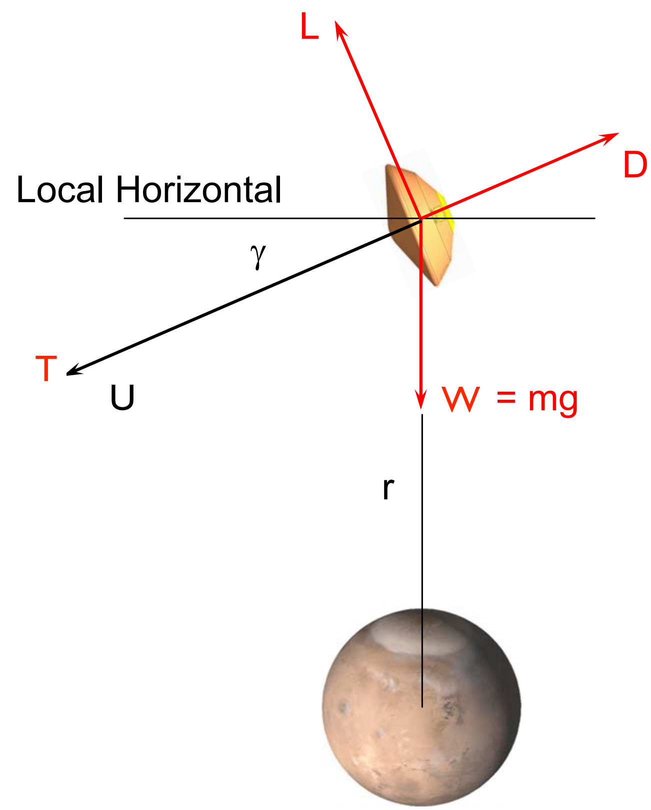
Trajectory

$$-L + W\cos\gamma = m\omega U \qquad (2.5a)$$

$$T - D - W\sin\gamma = m\dot{U} \qquad (2.5b)$$

$$\downarrow$$

$$\boxed{\begin{aligned} \frac{U\dot{\gamma}}{g} &= \frac{L}{W} - \left(1 - \frac{U^2}{g\,R}\right)\cos\gamma \qquad (2.8a) \\ \frac{T}{W} - \frac{\dot{U}}{g} &= \frac{D}{W} + \sin\gamma \qquad (2.8b) \end{aligned}}$$



Forces: T, D, L, W = thrust, drag, lift, weight

U = vehicle velocity

γ = flight path angle

2.4 Air-breathing vehicles

- The trajectory of air-breathing vehicle, as opposed to the other ones, has thrust
- Assumptions
  - T is not 0 (propulsion)
  - $\gamma = 0$ (shallow entry)
  - $d\gamma/dt = 0$
- The deceleration (on entry) and acceleration (on climb) is controlled by the thrust level

From Eq. 2.8

$$0 = \frac{L}{W} - \left(1 - \frac{U^2}{g\,R}\right) \quad (2.32a)$$

$$-\frac{\dot{U}}{g} = \frac{D-T}{W} + \gamma = \frac{1}{L/D}\left(1 - \frac{U^2}{U_0^2}\right)\left(1 - \frac{T}{D}\right) + \gamma$$

$$(2.32b)$$

2.4 Air-breathing vehicles

$$L = W\left(1 - \frac{V^2}{gR}\right) = \frac{L}{D} T$$

$$\dot{W} = \frac{dW}{dt} = \frac{T}{I_{sp}} = \frac{W}{L/D\, I_{sp}}\left(1 - V^2/gR\right)$$

$$\int \frac{dW}{W} = -\int \frac{1}{L/D} \frac{1}{I_{sp}} \left(1 - \frac{V^2}{gR}\right) dt$$

$$\ln\left(\frac{W_f}{W_i}\right) = \frac{1}{(L/D)} \frac{1}{I_{sp}} \left(1 - \frac{V^2}{gh}\right) t_f$$

$$\text{Endurance} = t_f = I_{sp}\left(\frac{L}{D}\right)\left(1 - \frac{V^2}{gh}\right)^{-1} \ln\left(\frac{W_f}{W_i}\right)$$

$$\text{Range} = E \cdot V = I_{sp} V \frac{L}{D}\left(1 - \frac{V^2}{gR}\right)^{-1} \ln\left(\frac{W_f}{W_i}\right)$$

2.4 Air-breathing vehicles

Cruise operation is described by Breguet-like relations

Range

$$S = Isp\ V\left(1 - \frac{V^2}{V_o^2}\right)^{-1} \frac{L}{D} \ln \frac{W_i}{W_f}$$

Endurance

$$E = S/V$$

Isp: specific impulse of propulsion system

w_i, w_f : initial and final weights of the vehicle

2.4 Air-breathing vehicles

- Both range and endurance are enhanced with
  - high Isp propulsion systems
  - High L/D aerodynamics (slender body with lifting)
  - high w_i/w_f

2.4 Air-breathing vehicles

Exercise 2.5
Calculate range and endurance for a vehicle with Isp = 2000 s (hydrogen scramjet at $M = 12$ or $u = 3.6$ km/s), $L/D = 4$ and $w_f/w_i = 0.3$

Using the Breguet relation, with an ideal efficiency, we obtain:

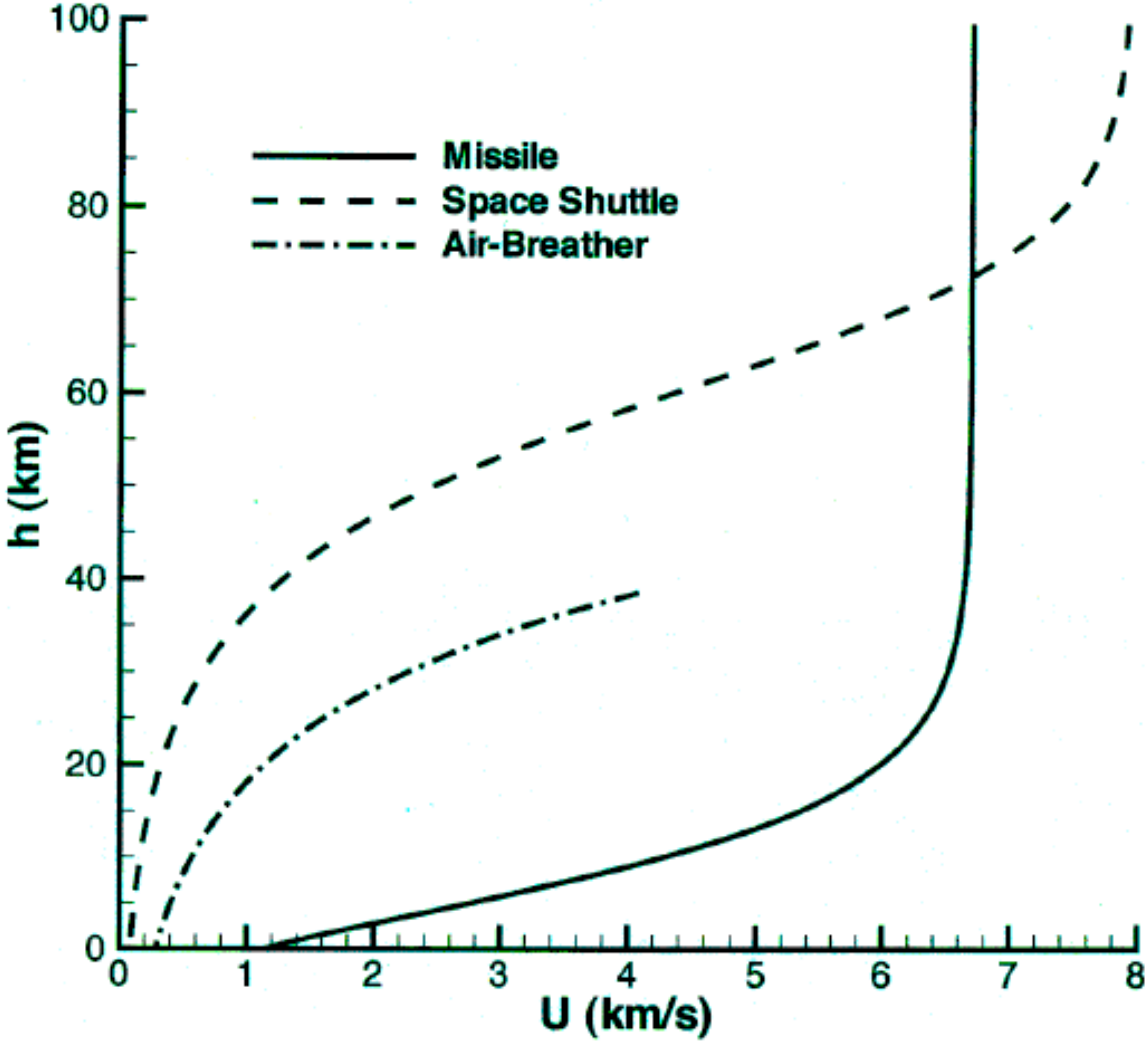
$$S = 34\ 674 \text{ km and } E = 2.7 \text{ hours}$$

Since Earth circumference is a distance of 40 000 km, this illustrates the interest in hypersonic cruise vehicle!

2.5 Trajectory comparisons

- We close this chapter by comparing the trajectories of the 3 hypersonic vehicles we have studied:
  - Peacekeeper ballistic missile (Ex. 2.1)
  - Space Shuttle on equilibrium glide (Ex. 2.2)
  - Air-breather with $\Phi = 0.5$, $L/D = 4$ and $T/D = .5$ (Ex. 2.5)

a) Velocity

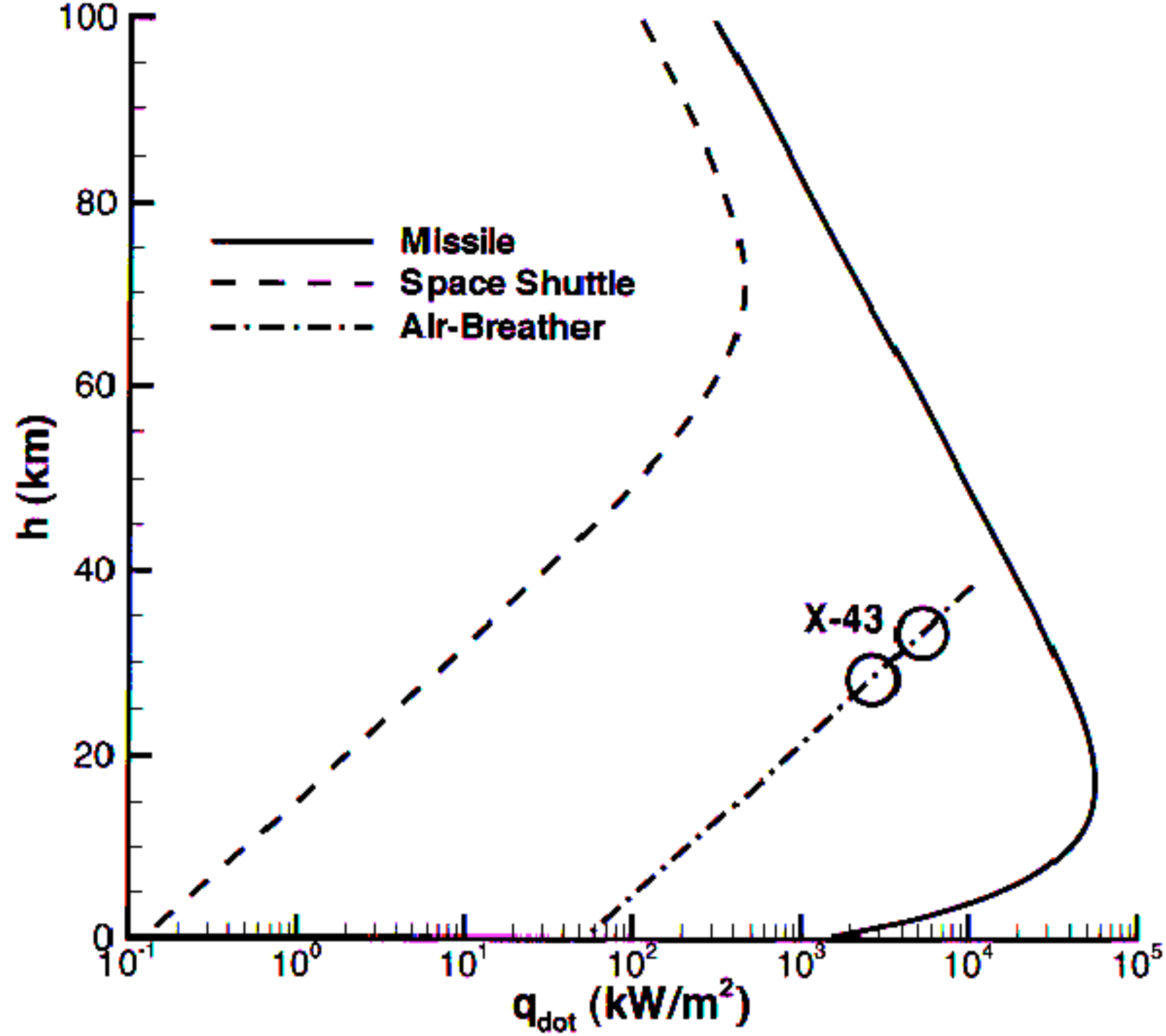


Missile
Space Shuttle
Air-Breather
h (km)
U (km/s)
0
20
40
60
80
100
0
1
2
3
4
5
6
7
8

a) Velocity

- Missile impacts the ground at high velocity and decelerates at low altitude.
- Shuttle decelerates at high altitude to reduce heating
- Air-breather glides at lower altitude than the Shuttle to operate its propulsion system.

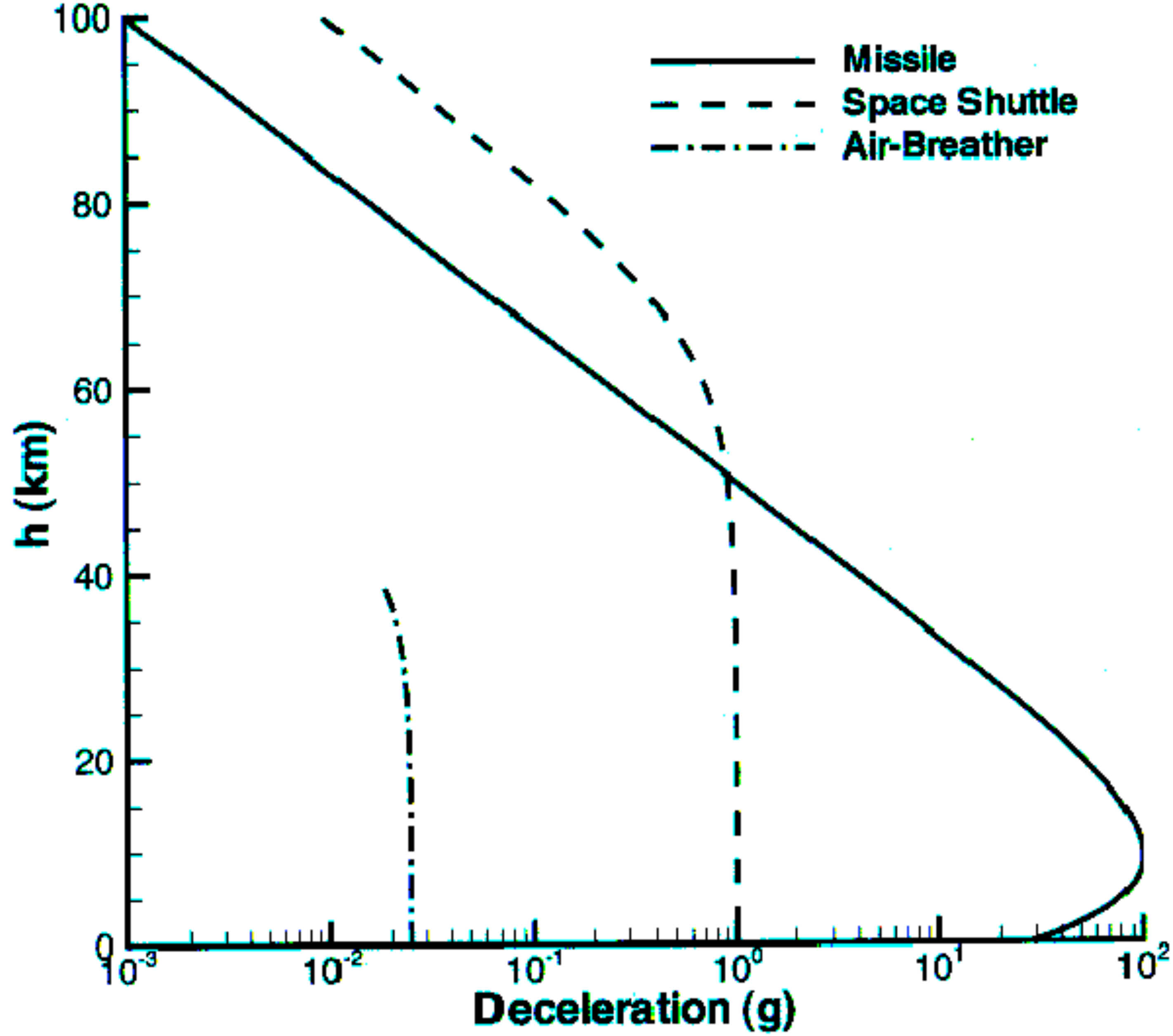
b) Heat transfer



b) Heat transfer

- Uses $\dot{q} = 2 \times 10^{-4} \sqrt{\rho / R_N} u^3 \ \ \mathrm{W/m^2}$
- Missile has easily the highest peak heating and requires a thick heat shield for protection
- Shuttle maximum heating is a factor of 100 lower but special tiles still needed for protection
- Air-breather has much higher heating locally on leading edge ($R_N = 0.01$ m) requiring use of very high-temperature materials

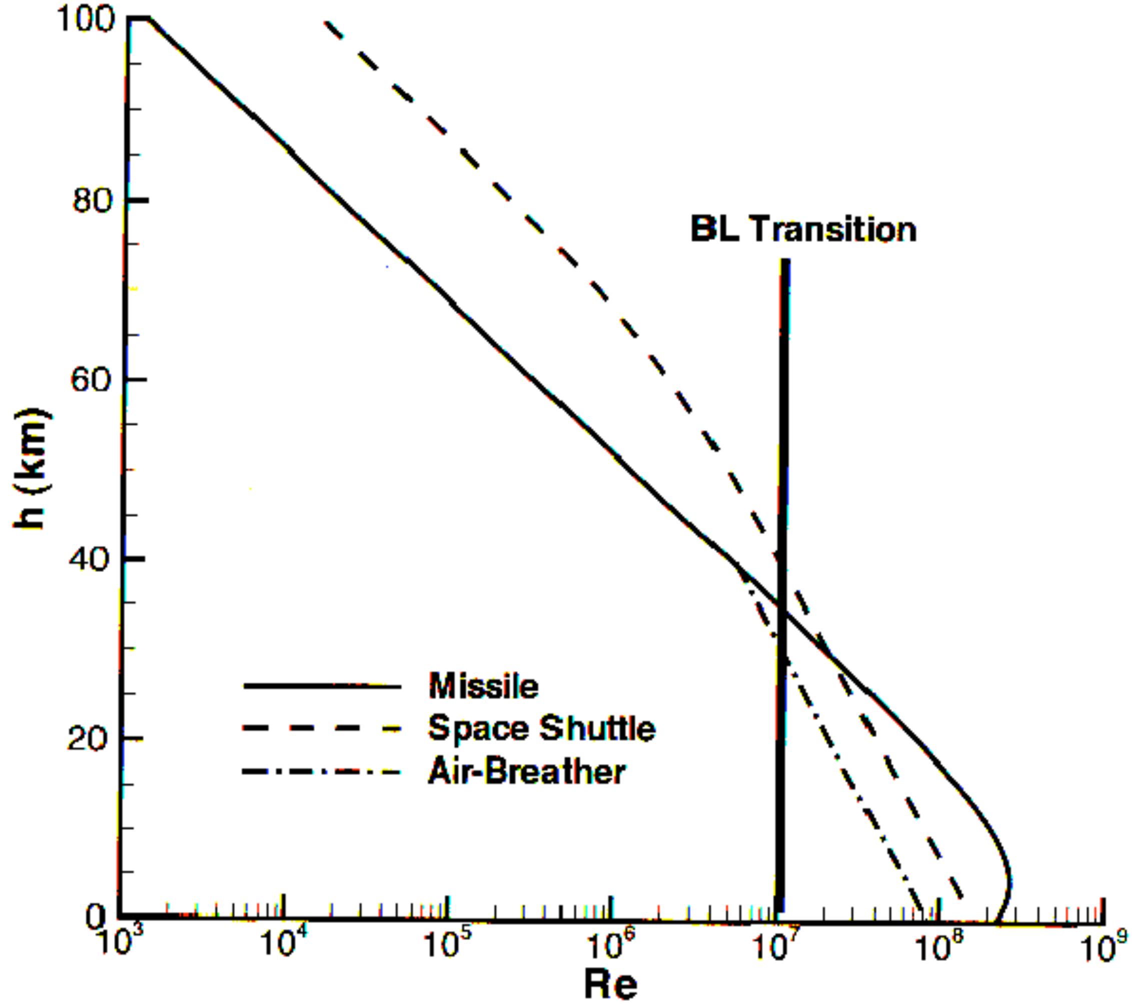
c) Deceleration



c) Deceleration

- The missile experiences huge g-forces that could not be survived by (ordinary) humans!
- The Shuttle has relatively benign loads
- The air-breather can greatly control the loads

d) Reynolds Number



- Transition of boundary layer occurs around 10^7 and occurs on all vehicles

Concluding remarks

- While it must be recognized that our models are approximations, they provide the correct trends in illustrating the basic differences between these very different hypersonic vehicle types
- Next, we will consider the gas dynamics and thermodynamics of hypersonic flows