4. Pressure distributions

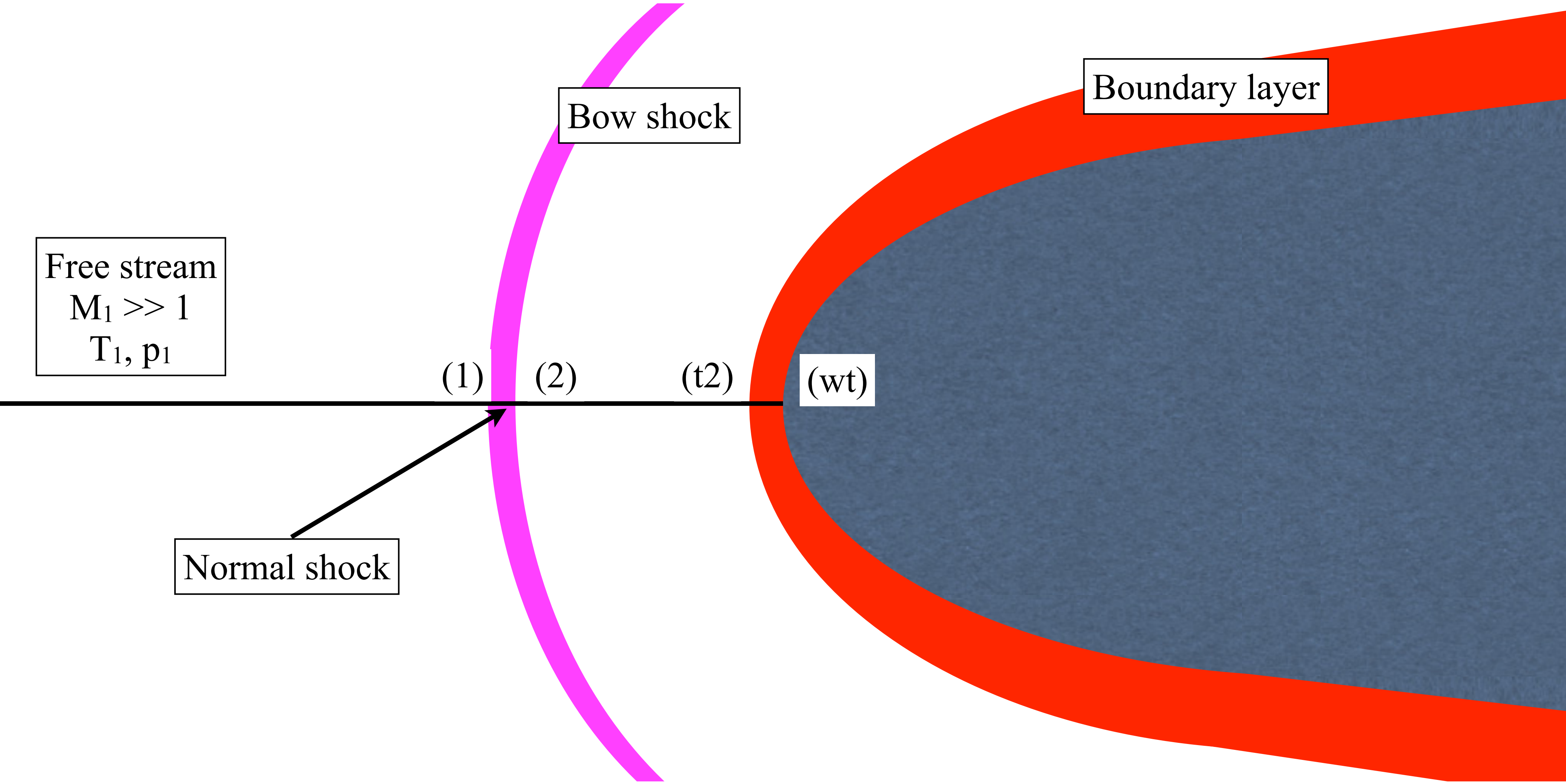
We know that blunt hypersonic vehicles generate bow shock waves, with a normal shock formed on stagnation streamline. The stagnation point is location of maximum pressure and heat transfer. In this chapter we study this important region.

Surface pressure on a hypersonic vehicle can be estimated using relatively simple models:

- Stagnation point
- Newtonian models
- Slender bodies

4.1 Stagnation streamline processes

For a blunt body at zero angle of attack in a hypersonic flow, the stagnation region is as follow



4.1 Stagnation streamline

Phenomena along stagnation streamline:

- (*1*) hypersonic freestream is isentropic
- (*1*) → (*2*): across NS, T and p increase, flow becomes subsonic ($M_2 < 1$)
- (*2*) → (*t2*): for a perfect gas (no chemistry), air is compressed isentropically to almost stagnation conditions at the edge of the boundary layer
- (*t2*) → (*wt*): pressure is almost constant, temperature usually decreases across the viscous and thermal BL

4.1 Stagnation streamline

Example 4.1

Sketch profiles of p and T along stagnation streamline of Space Shuttle at 70 km altitude and 6.246 km/s speed. Assume air is calorically perfect and T_{wt} = 1500 K (achieved using Reinforced Carbon-Carbon (RCC) thermal blanket).

4.1 Stagnation streamline

While the RCC greatly reduces the gas temperature at the vehicle surface, the resulting temperature gradient leads to significant conductive heat flux $\dot{q} = -\kappa \nabla T$

The previous example omits several important phenomena:

- **1- Vibrational activation**:
  Requires many molecular collisions and requires a finite time or distance to occur. We can place a limit on these effects by performing another calorifically perfect NS analysis for air with full vibrational activation, i.e. for γ = 9/7. Thus, we obtain:

$$p_{wt} = p_{t2} = 3\,633 \text{ N/m}^2 \qquad T_{t2} = 18\,290 \text{ K}$$

  Thus, vibrational activation serves to weaken the effects of compression.

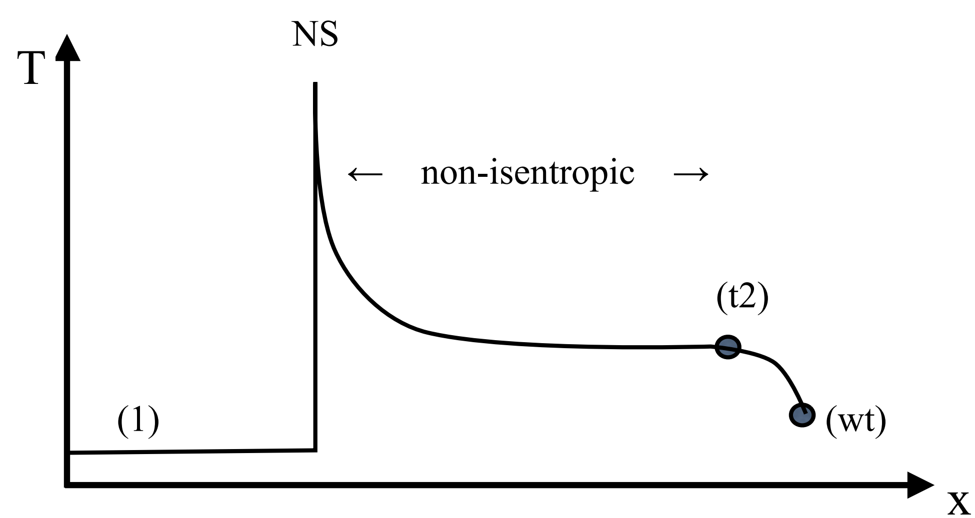
4.1 Stagnation streamline

- **2- Air chemistry:**
  Air will dissociate at this flight condition and this drains energy from the flow, thus reducing temperature. Our earlier equilibrium, real-gas analysis (vibrational and chemistry) for this same condition gave:

$$T_2 = 6,000\ K \qquad\qquad p_2 = 3,835\ {N}/{m^2}$$

- temperature greatly reduced
- pressure relatively unchanged

- As with vibration, a finite distance (or time) is required to reach chemical equilibrium and these processes are often mixed with post-shock compression



4.1 Stagnation streamline

- **3- Non-equilibrium**:
  At high altitude, density is low, and therefore, the collision rate is low (mean free path is large). At this condition, there might be insufficient space behind the NS for the gas to reach chemical or even thermal equilibrium. Such conditions require detailed computational analysis.

Investigating hypersonic, high-temperature air flows in a wind-tunnel is very difficult and expensive. Density ratios ρ_2/ρ_1 created in vibrationally-activated + chemically-reacting air can be reproduced using exotic gases (CF_4, C_2F_6) at lower Mach numbers. Wind tunnel experiments using these gases have been conducted at NASA.

4.2 Stagnation Point Pressure

Introducing the pressure coefficient

$$C_p \equiv \frac{p - p_1}{\frac{1}{2}\rho_1 u_1^2} \tag{4.1}$$

So, at a stagnation point:

$$(C_p)_{wt} = \frac{p_{wt} - p_1}{\frac{1}{2}\rho_1 u_1^2} \tag{4.2}$$

From prior discussions, we know that the pressure is not very sensitive to real-gas effects. Thus:

$$p_{wt} \approx p_{t2} = p_{o2} \text{ (for perfect gas)}$$

Therefore, re-writing the pressure coefficient:

$$(C_p)_{wt} = \frac{2p_1}{\rho_1 u_1^2}\left(\frac{p_{o2}}{p_1} - 1\right) \tag{4.3}$$

4.2 Stagnation Point Pressure

From compressible gas dynamics, supersonic pitot tube (Raleigh) formula:

$$\frac{p_{o2}}{p_1} = \frac{\gamma+1}{2} M_1^2 \left[\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma-1)}\right]^{\frac{1}{\gamma-1}} \quad \boxed{4.4}$$

For a perfect gas:

$$\begin{aligned} \frac{p_1}{\rho_1} &= RT_1 \quad \boxed{4.5} \\ u_1^2 &= M_1^2 a_1^2 = M_1^2 \gamma R T_1 \end{aligned}$$

So Eq. 4.3 becomes:

$$\begin{aligned} (C_p)_{wt} &= \frac{2}{\gamma M_1^2}\left(\frac{\gamma+1}{2} M_1^2 \left[\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma-1)}\right]^{\frac{1}{\gamma-1}} - 1\right) \\ &= \frac{\gamma+1}{\gamma}\left[\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma-1)}\right]^{\frac{1}{\gamma-1}} - \frac{2}{\gamma M_1^2} \quad \boxed{4.6} \end{aligned}$$

4.2 Stagnation Point Pressure

And if the high Mach number limit is evaluated, we obtain:

$$\lim_{M_1 \to \infty} (C_p)_{wt} = \frac{\gamma+1}{\gamma}\left[\frac{(\gamma+1)^2}{4\gamma}\right]^{\frac{1}{\gamma-1}} \qquad \boxed{4.7}$$

The following plot shows variation of $(C_p)_{wt}$ for two difference γ as a function of M_1. We note that:

- $C_p < 2$ in all cases
- not a significant difference for each γ

From this, a model is devised to describe the pressure away from stagnation point.

4.2 Stagnation Point Pressure

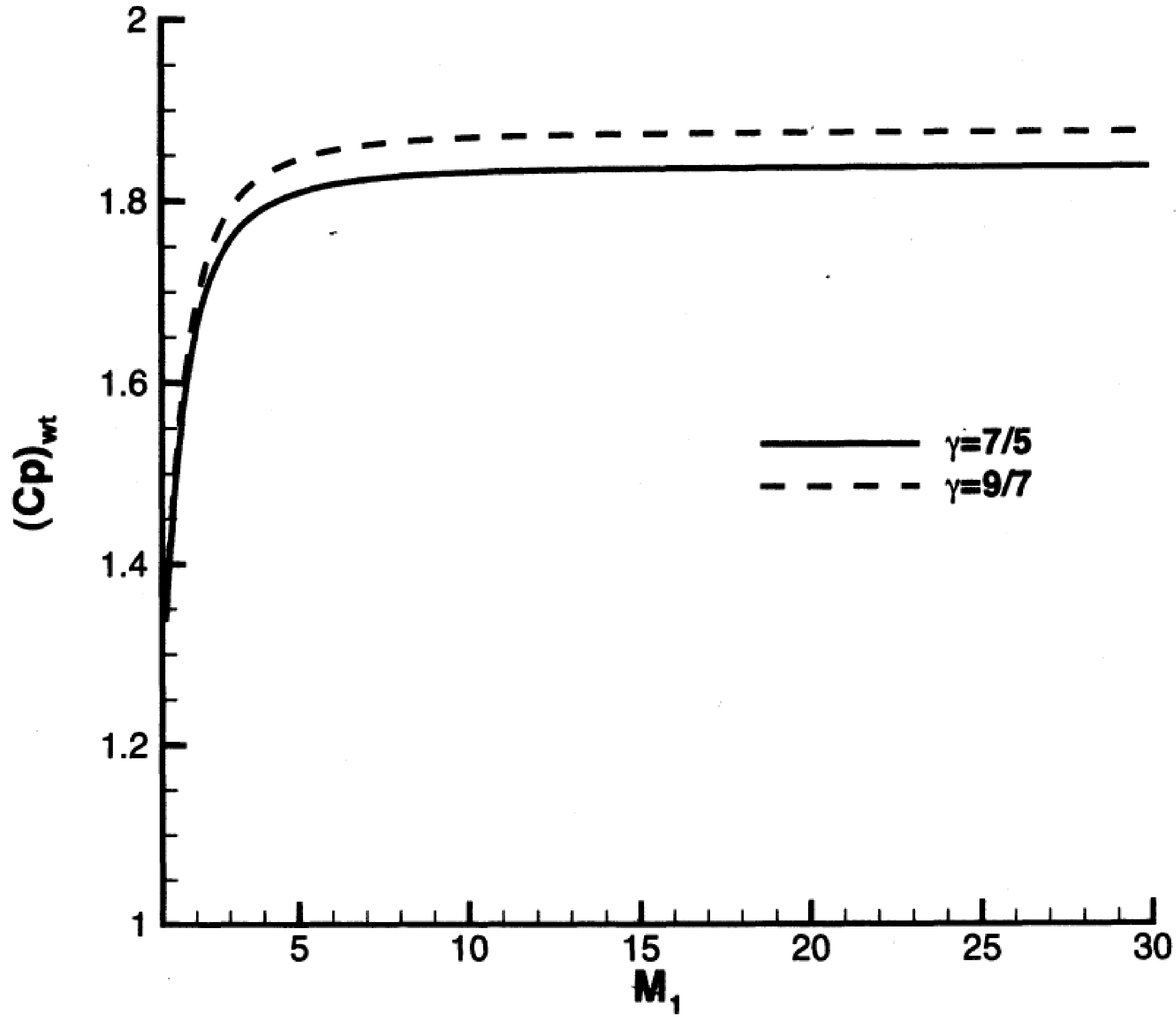


Fig. 4.1

4.3 Newtonian model

Types of drag

Skin friction drag (low speeds)

- viscous forces acting on the surface
- negligible as compared to other sources in most hypersonic flow

Pressure drag (all speeds)

- drag (motion opposing) pressure force due to motion through gas

Base drag (subsonic-supersonic)

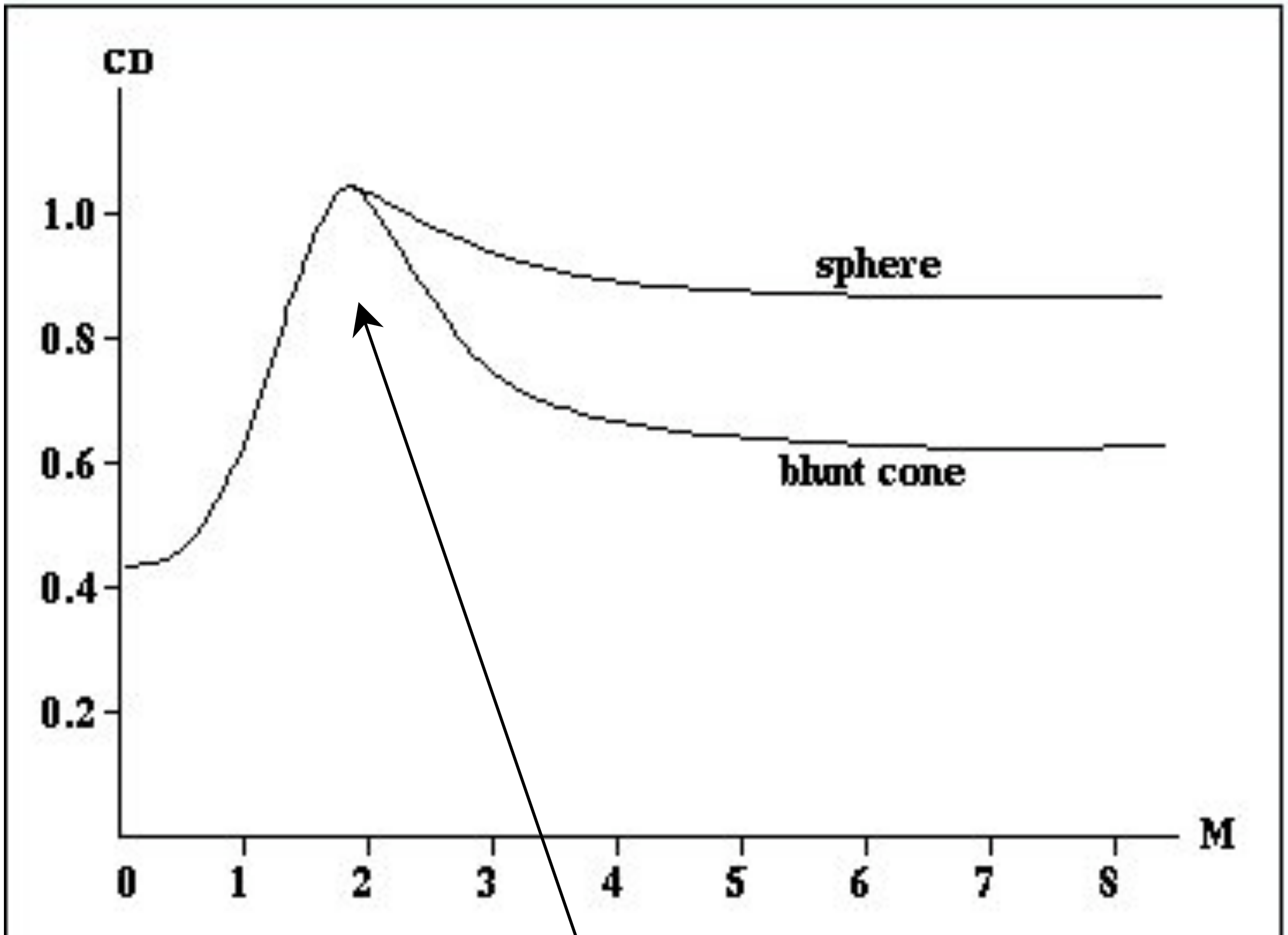
- "suction" force due to low pressure in separated wake
- negligible as compared to other sources in most hypersonic flow

Wave drag (transonic to hypersonic speeds)

- due to the formation of shock waves
- dominant contributor at hypersonic speeds

Source: Michael J. Wright, NASA Ames

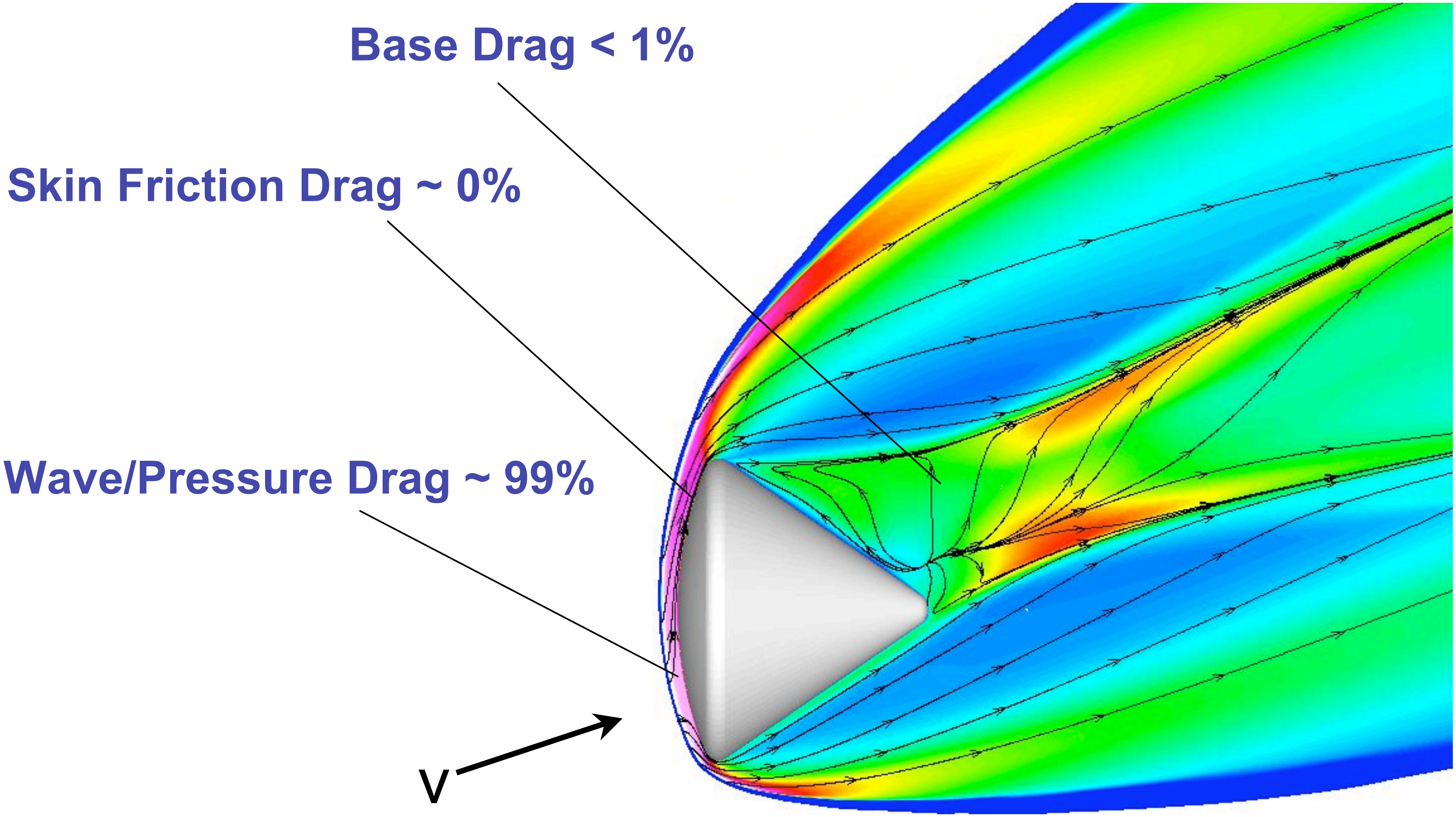
4.3 Newtonian model



Rapid rise as shocks begin to form and wave drag takes over

Source: Michael J. Wright, NASA Ames

4.3 Newtonian model



Source: Michael J. Wright, NASA Ames

4.3 Newtonian model

For blunt body, and unlike slender body (hypersonic cruise), skin friction drag is negligible

- state of the boundary layer (laminar vs. turbulent) does not matter

Base drag becomes negligible at Mach number increases

Analysis of aerodynamics becomes a simple matter of predicting surface pressure distribution

Pressure is notoriously insensitive to real gas dynamics, so this should be easy, right?

Source: Michael J. Wright, NASA Ames

4.3 Newtonian model

3 centuries ago, Newton postulated a physical model to describe fluid flow over a body.

- Fluid is assumed to have low density so that interactions among the particles is neglected. This has led to the term impact aerodynamics as the presence of the body is not transmitted upstream.
- After impinging on the body, the normal momentum of the particles is entirely lost.
- After impinging on the body, the tangential momentum of the particles is entirely conserved.

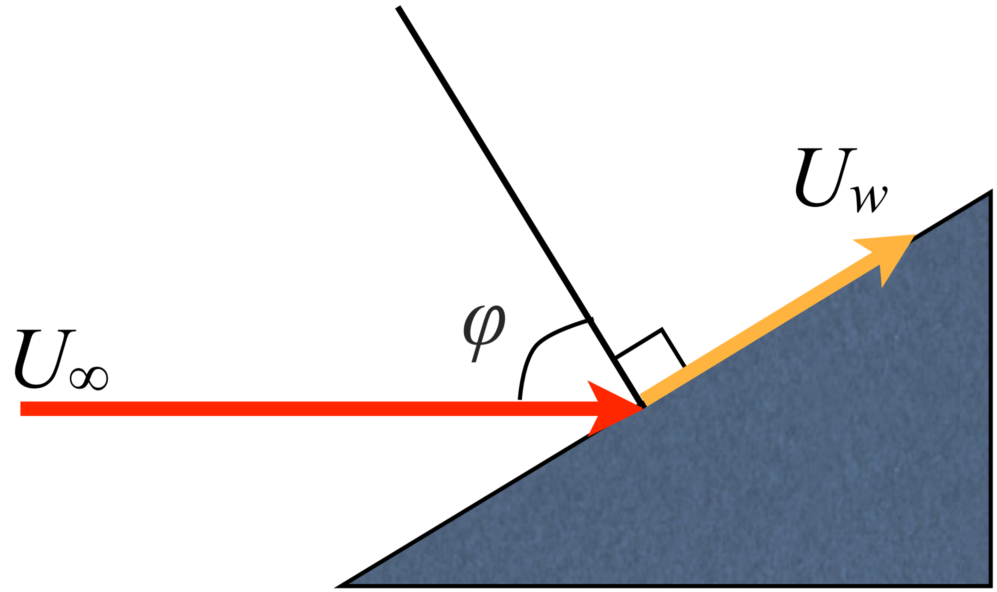
• In general, and for the application of interest to Newton, this theory has been shown to be inaccurate

- With the exception of the hypersonic aerodynamics

Source: Michael J. Wright, NASA Ames

4.3 Newtonian model

Considering a hypersonic air stream that is incident on a flat surface



Applying conservation of momentum normal to surface for steady-flow, ignoring body and viscous forces:

$$p_\infty + \rho_\infty \left(U_\infty\right)_n^2 = p_w + \rho_w \left(U_w\right)_n^2 \quad \boxed{4.8}$$

According to the Newtonian flow model:

$$\left(U_\infty\right)_n = U_\infty \cos\left(\varphi\right) \qquad \left(U_w\right)_n = 0$$

4.3 Newtonian model

Introducing the pressure coefficient:

$$C_p \equiv \frac{p_w - p_\infty}{\frac{1}{2}\rho_\infty U_\infty^2} = \frac{\rho_\infty \left(U_\infty\right)_n^2}{\frac{1}{2}\rho_\infty U_\infty^2} = 2\cos^2(\varphi) \qquad \boxed{4.9a}$$

At stagnation point, $\varphi = 0 \;\rightarrow\; C_{p0} = 2$

$$C_p = C_{p_0} \cos^2(\varphi) \qquad \boxed{4.9b}$$

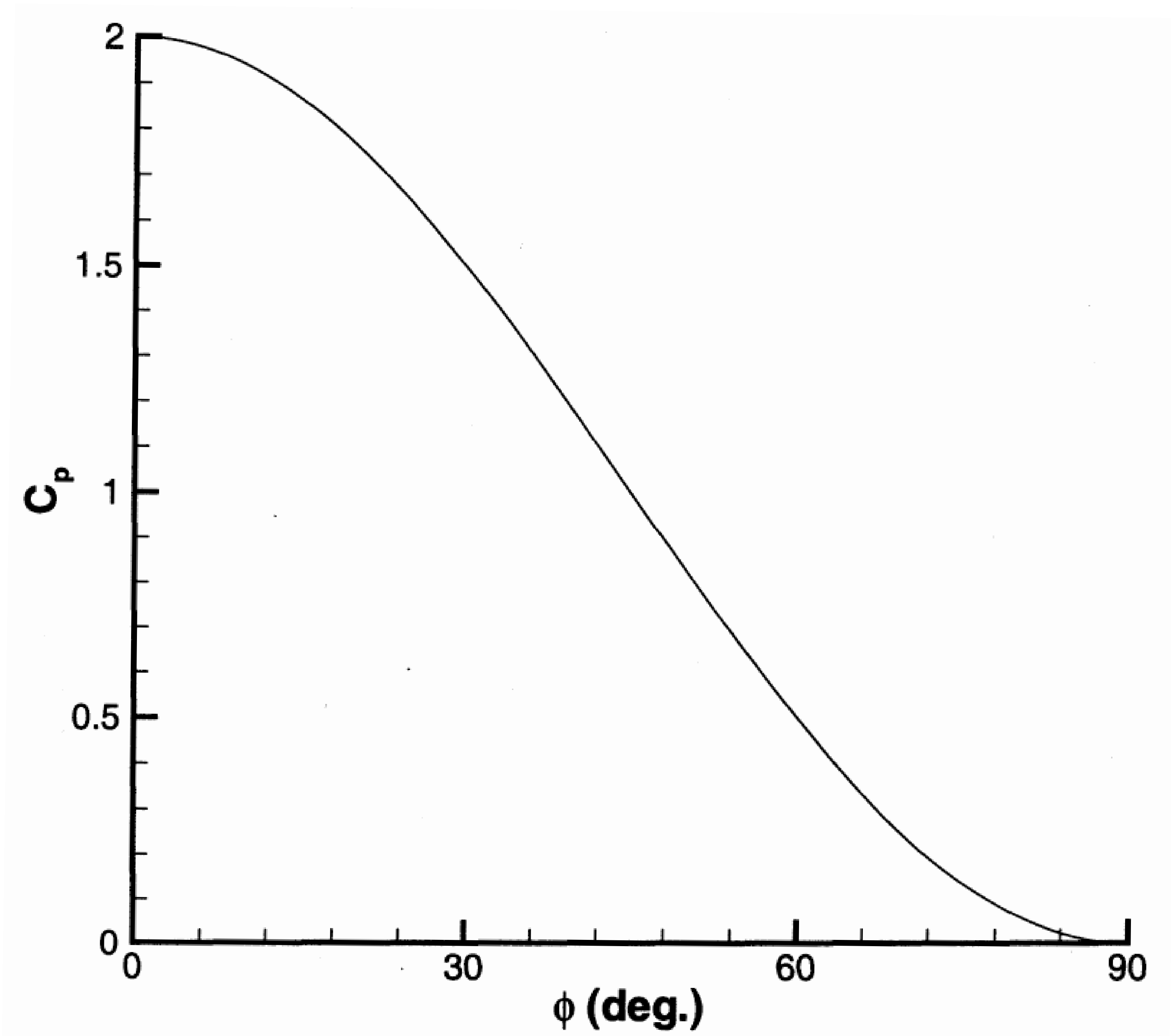
For impact flows, pressure coefficient is solely a function of vehicle geometry, φ. Not a function of Mach, Re or ρ

This theory is generally applicable to continuum hypersonic flow

Model is widely used in early design stages due to its simplicity and relatively good accuracy (especially for blunt bodies)

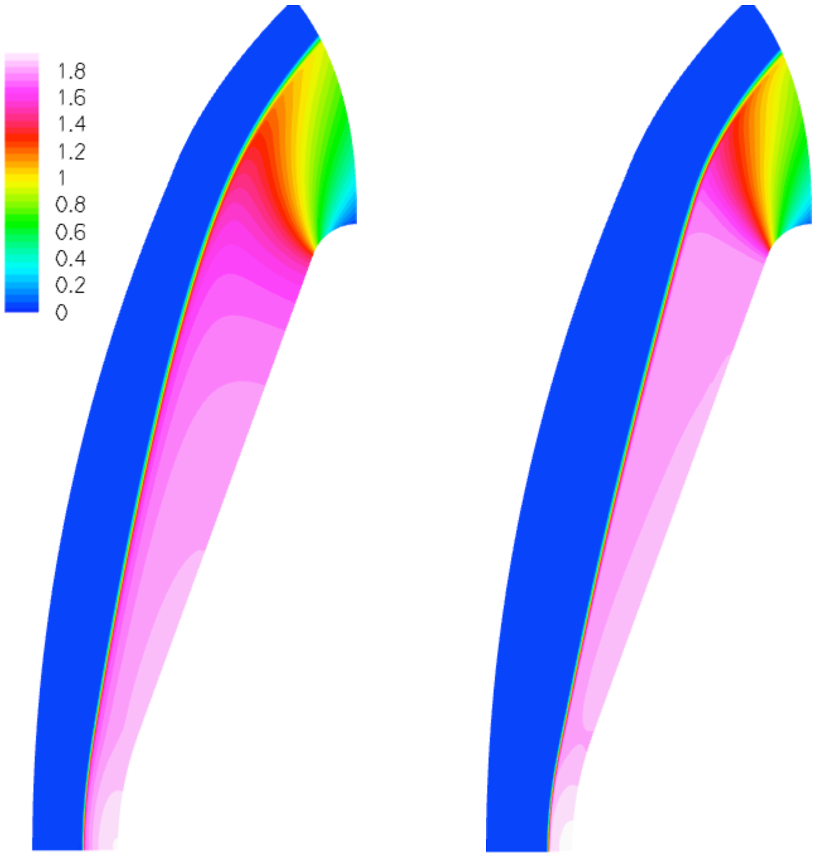
Source: Michael J. Wright, NASA Ames

4.3 Newtonian model

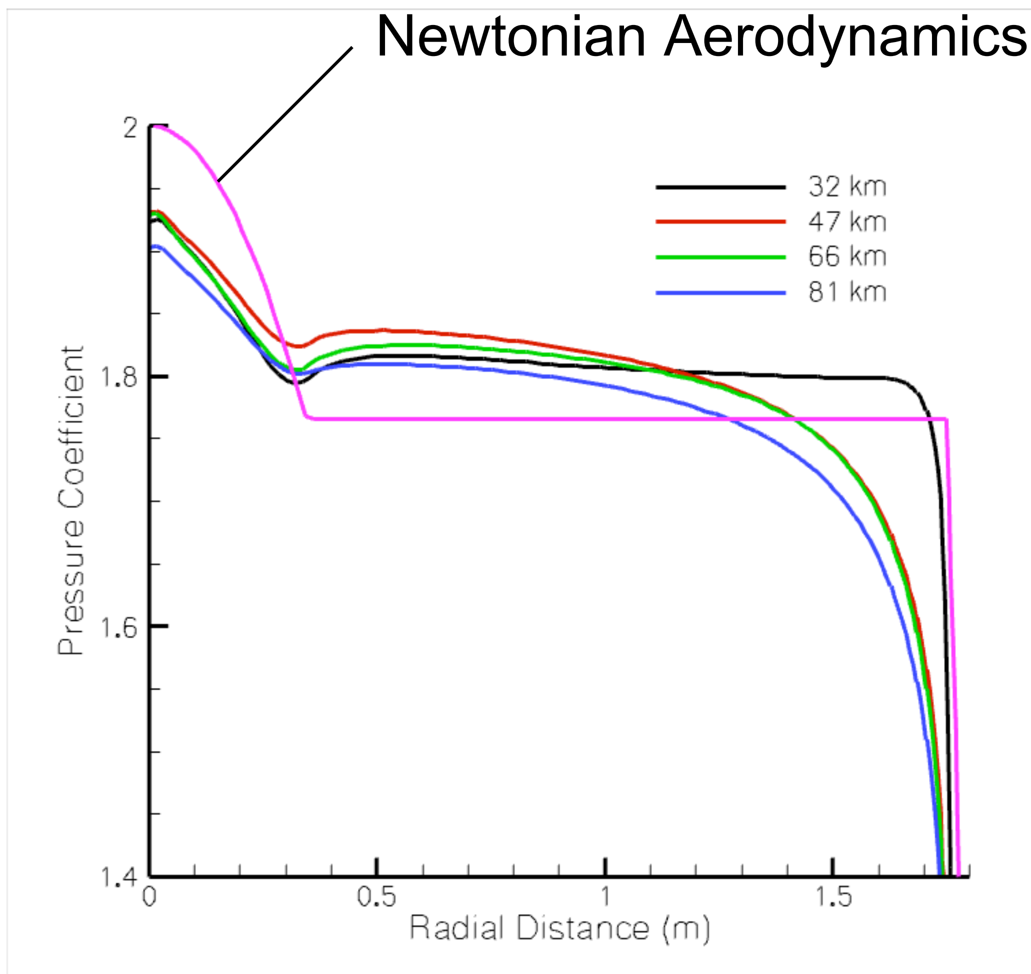


4.3 Newtonian model

Viking-like Aeroshell: sphere-70° cone



CFD at 81 km CFD at 32 km



Source: G.V. Candler (UMN) & I. D. Boyd (UM)