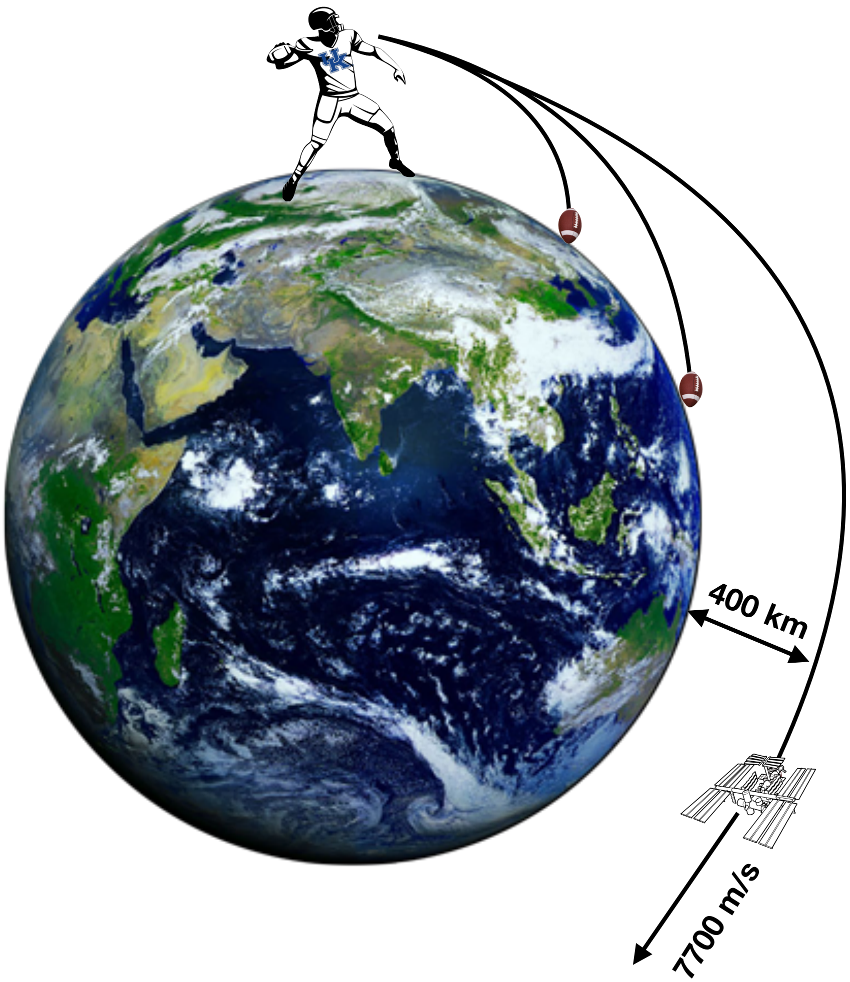
1.2 Flight regimes

Orbital mechanics



Entry, Descent, Landing

$$E_k = \frac{1}{2}mv^2$$

$$E_k = 0$$

$$H_o = c_p\Delta T + \frac{1}{2}mv^2$$

- Since H_o is conserved, most of E_k will be going into $c_p\Delta T$
- The challenge is to prevent that thermal energy to reach the vehicle

Thermal Barrier

- This can be re-arranged as

$$Q = E_k = \frac{1}{2}\frac{W}{g}V^2$$

$$Q/W = \frac{V^2}{2g}$$

- For a velocity of 26,000 ft/s (~8 km/s), the Q/W = 13,500 Btu/lb

Table 1.1 Energy Required to Vaporize Some Typical Materials

Material	Energy to vaporize, Btu/lb	Melting temperature, °R
Tungsten	1,870	6500
Titanium	3,865	3700
Beryllium oxide	13,400	2900
Graphite	28,700	6800

Thermal Barrier

- If everything is transferred to thermal energy

$$E_T = C_p(T_o - T_\infty) \longrightarrow T_o = \frac{V_\infty^2}{2Cp} + T_\infty$$

- This would be the case from a stagnation line
- The temperature would become completely unsustainable
- However:
  - all the energy is not transferred to thermal energy
  - all the energy is not transferred to the vehicle

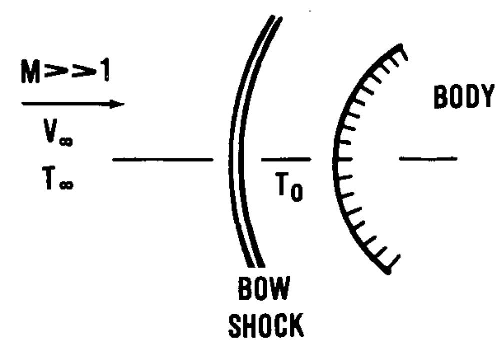


Fig. 1.1 Stagnation temperature on the nose of a re-entry vehicle.

Table 1.2 Stagnation Temperatures

V_∞, ft/s	T_0, °R
10,000	8,325
20,000	33,300
26,000	56,277

Table 1.1 Energy Required to Vaporize Some Typical Materials

Material	Energy to vaporize, Btu/lb	Melting temperature, °R
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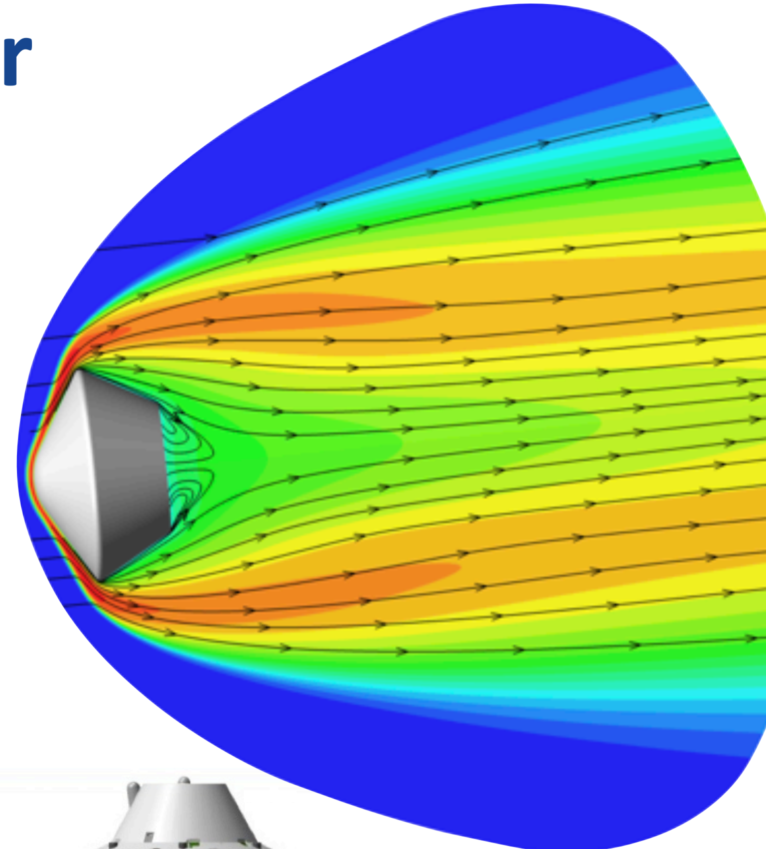
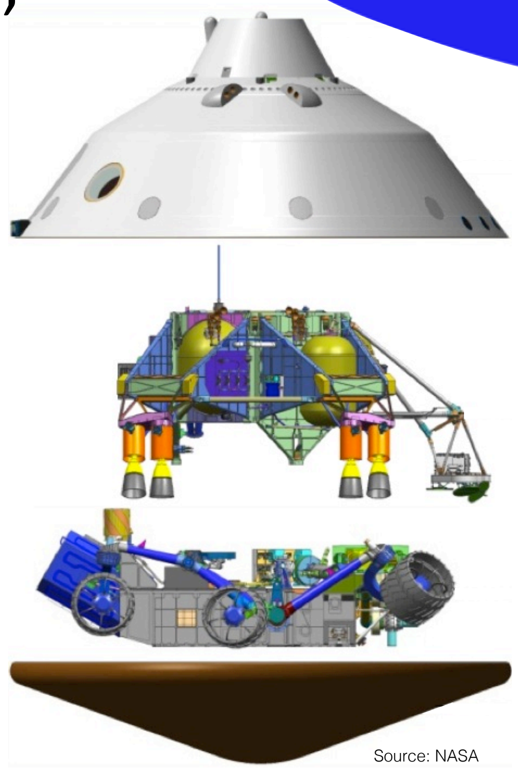
Thermal Barrier



- Heat transfer is the transport of energy

$$\dot{q} = u \cdot \frac{1}{2}\rho u^2 = \frac{1}{2}\rho u^3$$

- Most of the thermal energy is advected away in the atmosphere, around the vehicle
- A fraction (< 5%) still reaches the surface, and convectively heat the vehicle
- That amount is still significant!
  - Mars Science Lab: 226 W/cm^2
  - Apollo 4 (11 km/s): 450 W/cm^2
  - Stardust Return Capsule (14 km/s): 950 W/cm^2
  - Small appliance (Iron, toaster): 0.17—0.44 W/cm^2
- **A Heat Shield, or Thermal Protection System (TPS) is needed!**

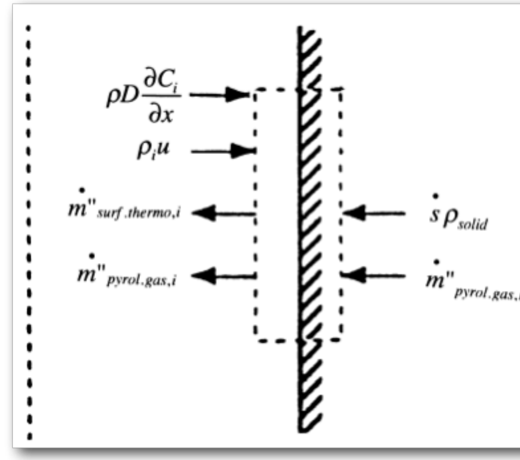


Thermal Barrier

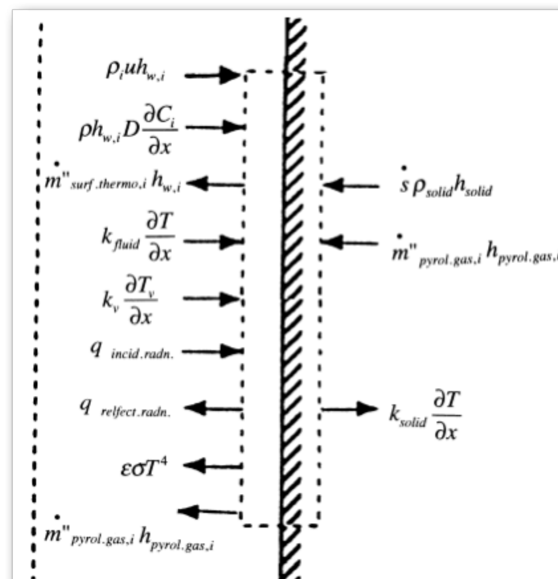
- A lot of the energy is transferred into other modes
  - chemical reaction (dissociation)
  - Ionization
- A lot of the absorber energy is not transferred to the structure
  - ablation
  - surface radiative emission
- That being said, the velocity remains the most important parameter to both total enthalpy (energy) and heat transfer

$$H_o = c_p \Delta T + \frac{1}{2} m v^2$$

$$\dot{q} = u \cdot \frac{1}{2}\rho u^2 = \frac{1}{2}\rho u^3$$



Mass balance (Havstad, 2001)



Energy balance (Havstad, 2001)

Source: NASA

1.3 Hypersonic flight conditions

- Hypersonic vehicles fly at very high speed
- This dominates **<u>all aspects</u>** of vehicle design
- Two main properties to consider:
  - pressure $\approx \frac{1}{2}\rho_\infty u_\infty^2$
  - heat transfer $\approx \frac{1}{2}\rho_\infty u_\infty^3$

Pressure

- Momentum flux (dynamic pressure)
- <u>Surface pressure</u> $\approx \frac{1}{2}\rho_\infty u_\infty^2$
  - Defines the the basic aerodynamic forces (lift, drag) and moments (yaw, pitch, roll)
  - Very high surface pressure may affect vehicle survivability
- <u>Air breathing propulsion</u>
  - Efficient combustion demands a pressure of about 1 atm inside the engine
  - This determines the flight paths of the vehicle

Heat transfer

- Kinetic energy flux $\approx \frac{1}{2}\rho_\infty u_\infty^3$
- Hypersonic vehicle experience high heat transfer
- Many approaches to address this problem
  - surface cooling
  - high temperature materials (Shuttle tiles)
  - ablative heat shield (Apollo return capsule)

Parameters

- Clearly, velocity is high
- Also (linearly) dependent on the mass density, which is a function of altitude
- Hypersonic vehicle thus fly at high altitude to reduce heating (several 100 000 ft.)
- Accurate description of atmospheric properties (density and temperature) is required

1976 U.S. Standard Atmosphere

- One of many model of the atmosphere
- Using tabulated temperature data at various locations
- 7 layers between sea level and ~85 km
- 2 layer are isothermal, the other have linear variation
- Table 1.1 from Bertin

1976 U.S. Standard Atmosphere

Table 1.1a U.S. Standard Atmosphere, 1976: Metric Units

Geometric Altitude (km)	Pressure (p/p_{SL})	Temperature (K)	Density (ρ/ρ_{SL})	Viscosity (μ/μ_{SL})	Speed of Sound (m/s)
0	1.0000 E+00	288.150	1.0000 E+00	1.00000	340.29
1	8.8700 E−01	281.651	9.0748 E−01	0.98237	336.43
2	7.8461 E−01	275.154	8.2168 E−01	0.96456	332.53
3	6.9204 E−01	268.659	7.4225 E−01	0.94656	328.58
4	6.0854 E−01	262.166	6.6885 E−01	0.92836	324.59
5	5.3341 E−01	255.676	6.0117 E−01	0.90995	320.55
6	4.6600 E−01	249.187	5.3887 E−01	0.89133	316.45
7	4.0567 E−01	242.700	4.8165 E−01	0.87249	312.31
8	3.5185 E−01	236.215	4.2921 E−01	0.85343	308.11
9	3.0397 E−01	229.733	3.8128 E−01	0.83414	303.85
10	2.6153 E−01	223.252	3.3756 E−01	0.81461	299.53
11	2.2403 E−01	216.774	2.9780 E−01	0.79485	295.15
12	1.9145 E−01	216.650	2.5464 E−01	0.79447	295.07
13	1.6362 E−01	216.650	2.1763 E−01	0.79447	295.07
14	1.3985 E−01	216.650	1.8601 E−01	0.79447	295.07
15	1.1953 E−01	216.650	1.5898 E−01	0.79447	295.07
16	1.0217 E−01	216.650	1.3589 E−01	0.79447	295.07
17	8.7340 E−02	216.650	1.1616 E−01	0.79447	295.07
18	7.4663 E−02	216.650	9.9304 E−02	0.79447	295.07
19	6.3829 E−02	216.650	8.4894 E−02	0.79447	295.07
20	5.4570 E−02	216.650	7.2580 E−02	0.79447	295.07
21	4.6671 E−02	217.581	6.1808 E−02	0.79732	295.70
22	3.9945 E−02	218.574	5.2661 E−02	0.80037	296.38
23	3.4215 E−02	219.567	4.4903 E−02	0.80340	297.05
24	2.9328 E−02	220.560	3.8317 E−02	0.80643	297.72
25	2.5158 E−02	221.552	3.2722 E−02	0.80945	298.39
26	2.1597 E−02	222.544	2.7965 E−02	0.81247	299.06
27	1.8553 E−02	223.536	2.3917 E−02	0.81547	299.72
28	1.5950 E−02	224.527	2.0470 E−02	0.81847	300.39
29	1.3722 E−02	225.518	1.7533 E−02	0.82147	301.05
30	1.1813 E−02	226.509	1.5029 E−02	0.82446	301.71
31	1.0177 E−02	227.500	1.2891 E−02	0.82744	302.37
32	8.7743 E−03	228.490	1.1065 E−02	0.83041	303.02
33	7.5727 E−03	230.973	9.4474 E−03	0.83785	304.67
34	6.5473 E−03	233.743	8.0714 E−03	0.84610	306.49
35	5.6708 E−03	236.513	6.9089 E−03	0.85431	308.30
36	4.9200 E−03	239.282	5.9248 E−03	0.86247	310.10
37	4.2758 E−03	242.050	5.0902 E−03	0.87059	311.89
38	3.7220 E−03	244.818	4.3809 E−03	0.87866	313.67
39	3.2452 E−03	247.584	3.7769 E−03	0.88669	315.43
40	2.8338 E−03	250.350	3.2618 E−03	0.89468	317.19
41	2.4784 E−03	253.114	2.8216 E−03	0.90262	318.94
42	2.1709 E−03	255.878	2.4447 E−03	0.91052	320.67
43	1.9042 E−03	258.641	2.1216 E−03	0.91838	322.40
44	1.6728 E−03	261.403	1.8440 E−03	0.92620	324.12
45	1.4715 E−03	264.164	1.6051 E−03	0.93398	325.82
46	1.2962 E−03	266.925	1.3993 E−03	0.94172	327.52
47	1.1433 E−03	269.684	1.2217 E−03	0.94941	329.21
48	1.0095 E−03	270.650	1.0749 E−03	0.95210	329.80
49	8.9155 E−04	270.650	9.4920 E−04	0.95210	329.80
50	7.8735 E−04	270.650	8.3827 E−04	0.95210	329.80
55	4.1969 E−04	260.771	4.6376 E−04	0.92442	323.72
60	2.1671 E−04	247.021	2.5280 E−04	0.88506	315.07
65	1.0786 E−04	233.292	1.3323 E−04	0.84476	306.19
70	5.1526 E−05	219.585	6.7616 E−05	0.80346	297.06
75	2.3569 E−05	208.399	3.2589 E−05	0.76892	289.40
80	1.0387 E−05	198.639	1.5068 E−05	0.73813	282.54
85	4.3985 E−06	188.893	6.7099 E−06	0.70677	275.52

Reference values: $p_{SL} = 1.01325 \times 10^5$ N/m^2; $T_{SL} = 288.150$ K
$\rho_{SL} = 1.2250$ kg/m^3; $\mu_{SL} = 1.7894 \times 10^{-5}$ kg/s·m

Exponential atmosphere

Let us assume that we can model atmospheric density by:

$$\rho = \rho_0 e^{-xh}$$

where,

ρ_0 - atmospheric density at surface (sea level for Earth)
1/x - scale height, a measure of rate of density decrease with altitude

Planet	ρ_0 (kg/m^3)	x (m^{-1})
Venus	16.02	1.606×10^{-4}
Earth	1.225	1.378×10^{-4}
Mars	0.057	1.275×10^{-4}

Flight trajectories of re-entry-vehicles (RV)

- 3 classes of trajectories (Fig. 1.1, from Bertin)
  - high-altitude deceleration
    - high drag
    - no propulsion
    - Shuttle, Apollo
  - mid-altitude deceleration
    - low drag
    - air-breating prop
    - X-43, Orient Express
  - low-altitude deceleration
    - high velocity at sea level
    - ICBM

Flight trajectories of re-entry-vehicles (RV)

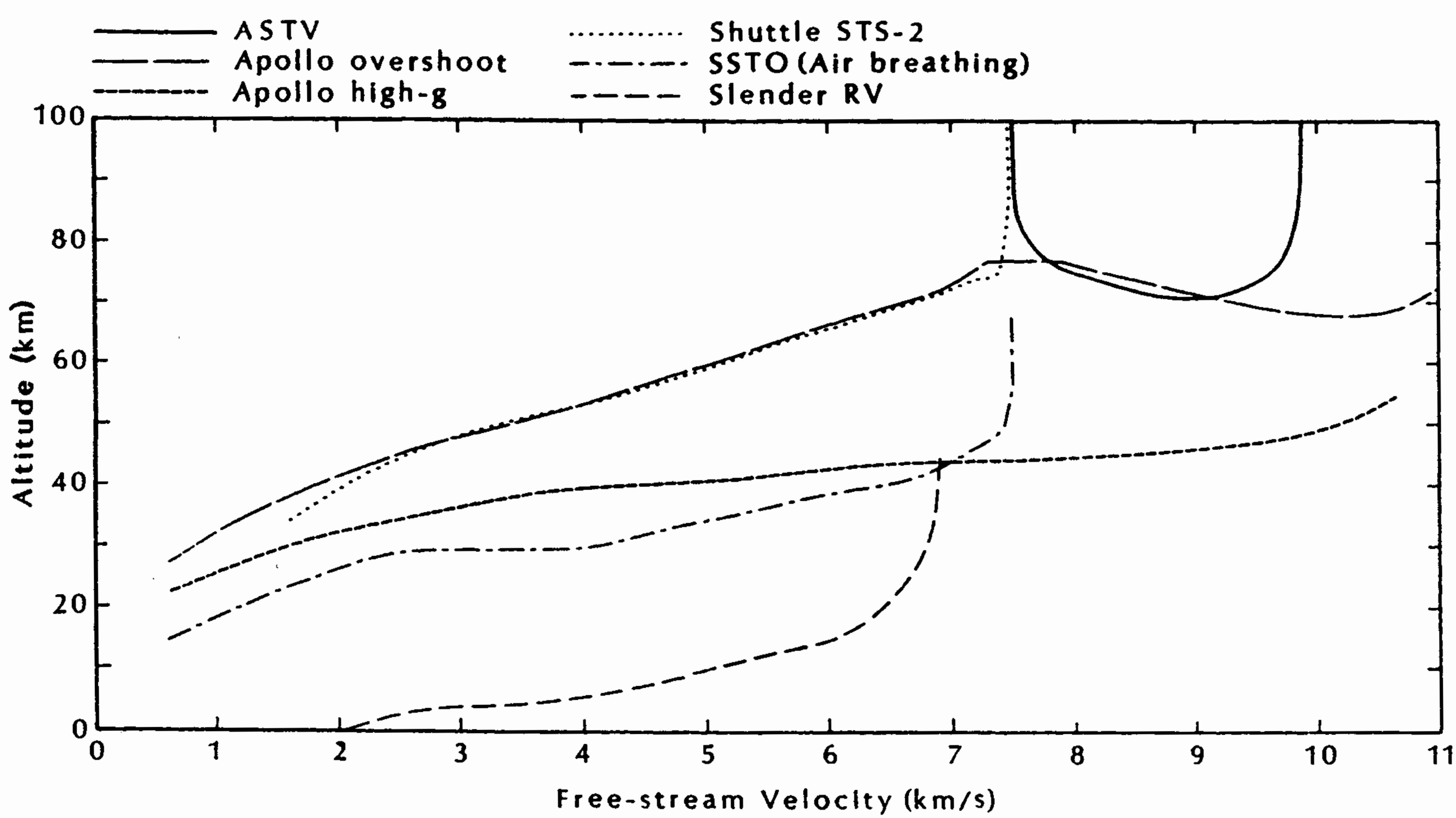


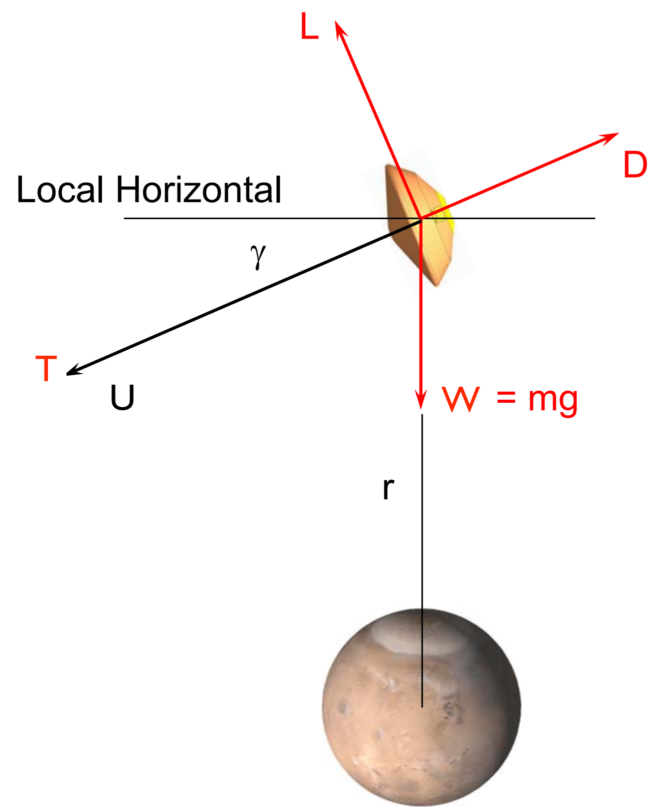
Fig. 1.1 Velocity/altitude parameters for several re-entry vehicles.

Chapter 2: Flight Mechanics

$$-L + W\cos\gamma = m\omega U \quad (2.5a)$$

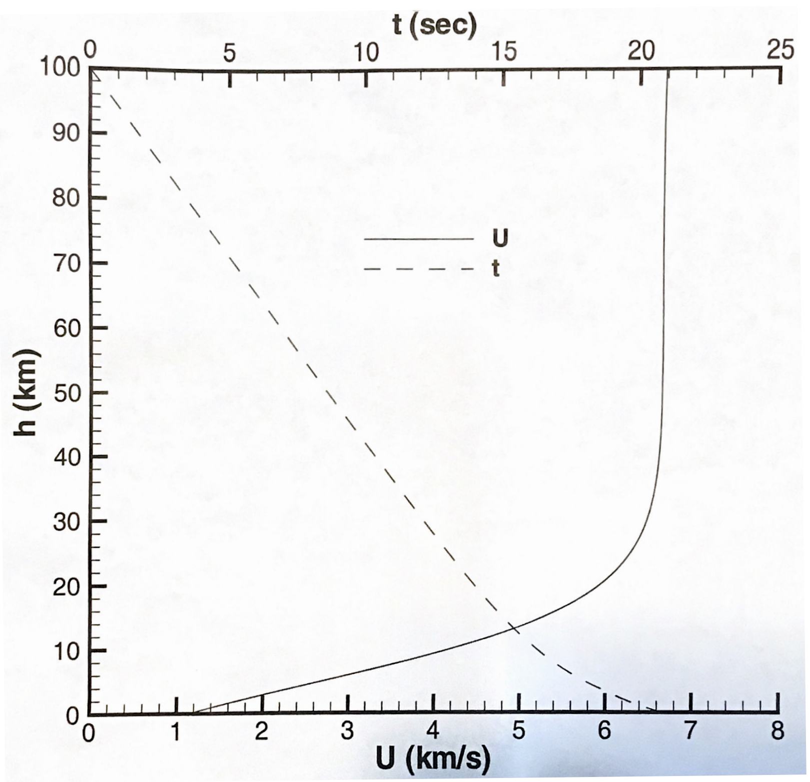
$$T - D - W\sin\gamma = m\dot{U} \quad (2.5b)$$

$$\boxed{\frac{U\dot{\gamma}}{g} = \frac{L}{W} - \left(1 - \frac{U^2}{gR}\right)\cos\gamma \quad (2.8a) \qquad \frac{T}{W} - \frac{\dot{U}}{g} = \frac{D}{W} + \sin\gamma \quad (2.8b)}$$

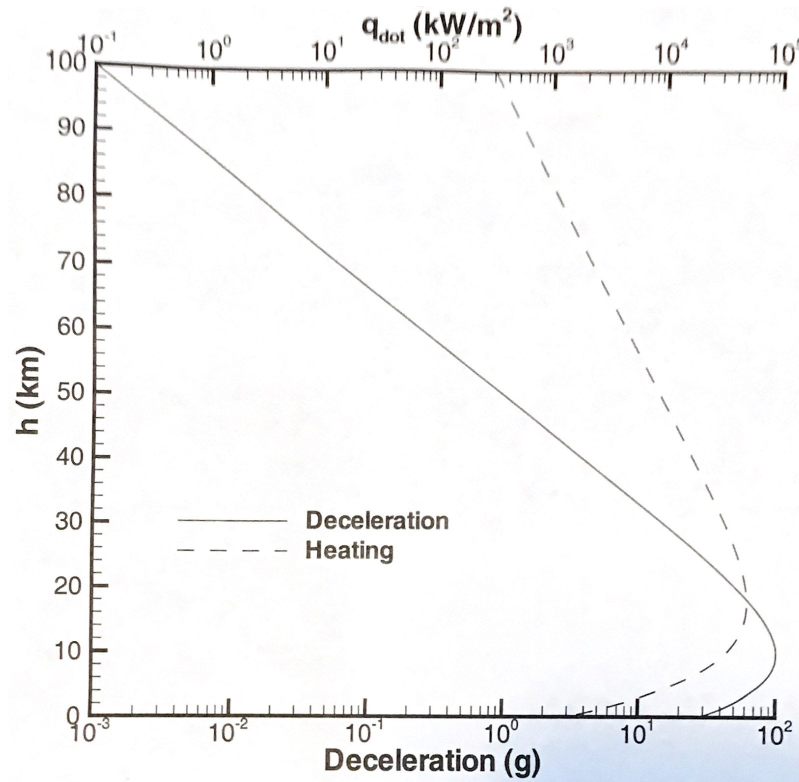


Forces: T, D, L, W = thrust, drag, lift, weight
U = vehicle velocity
γ = flight path angle

Example 2.1



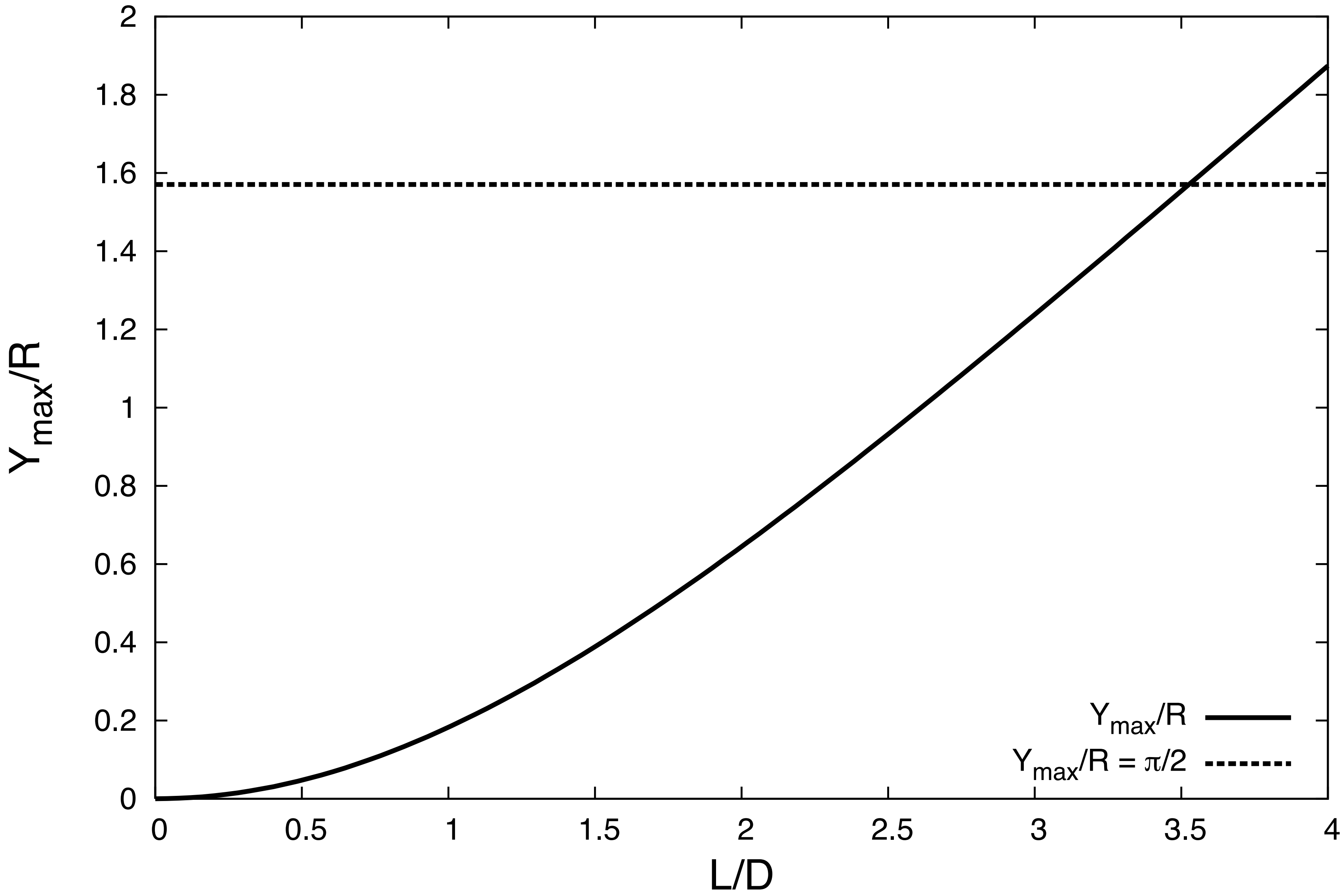
t (sec)
0
5
10
15
20
25
100
90
80
70
60
50
40
30
20
10
0
h (km)
U
t
0
1
2
3
4
5
6
7
8
U (km/s)



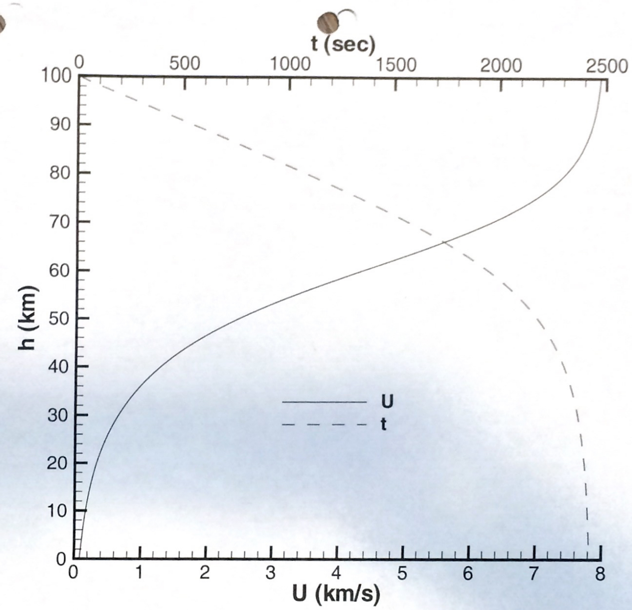
q_{dot} (kW/m^2)
10^{-1}
10^0
10^1
10^2
10^3
10^4
10^5
100
90
80
70
60
50
40
30
20
10
0
h (km)
Deceleration
Heating
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
10^2
Deceleration (g)

Crossrange

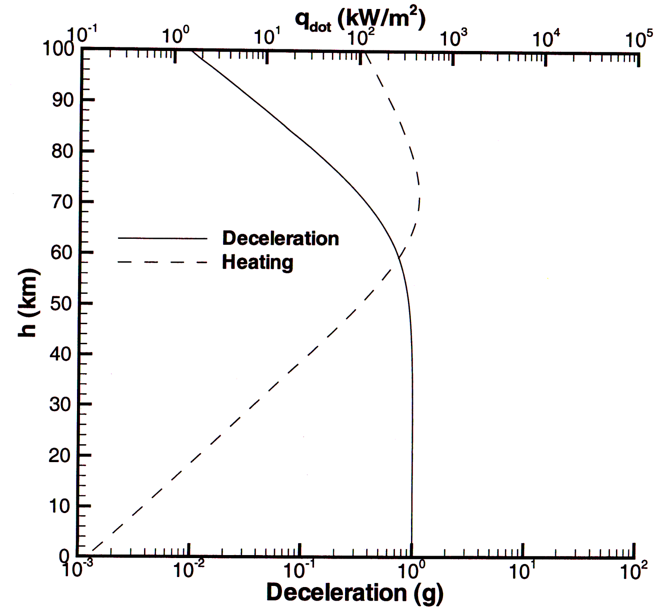
Fig. 2.2.1: Maximum crossrange as a function of L/D



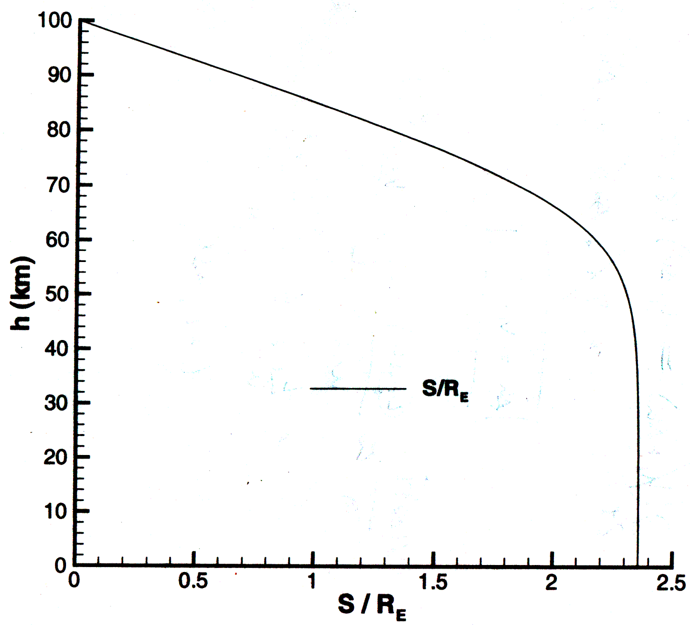
Example 2.2



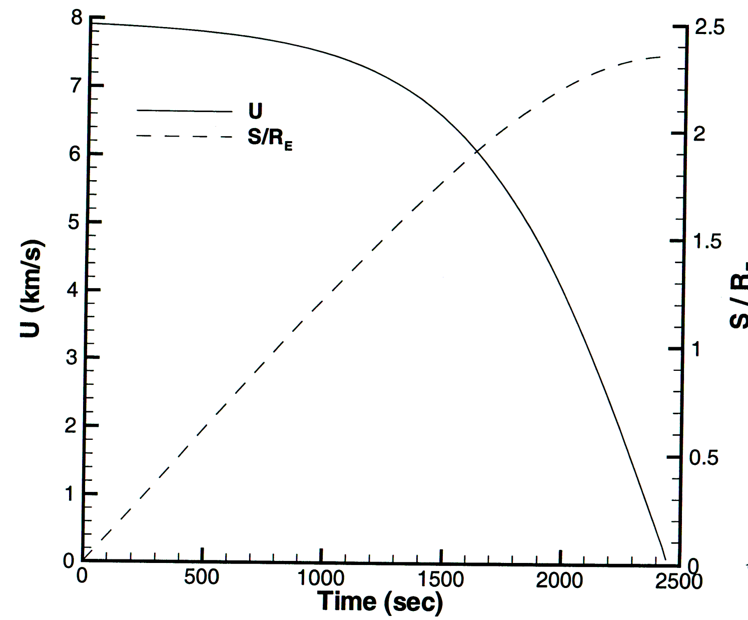
Ex. 2.2 Fig 1



Ex. 2.2 Fig 2.

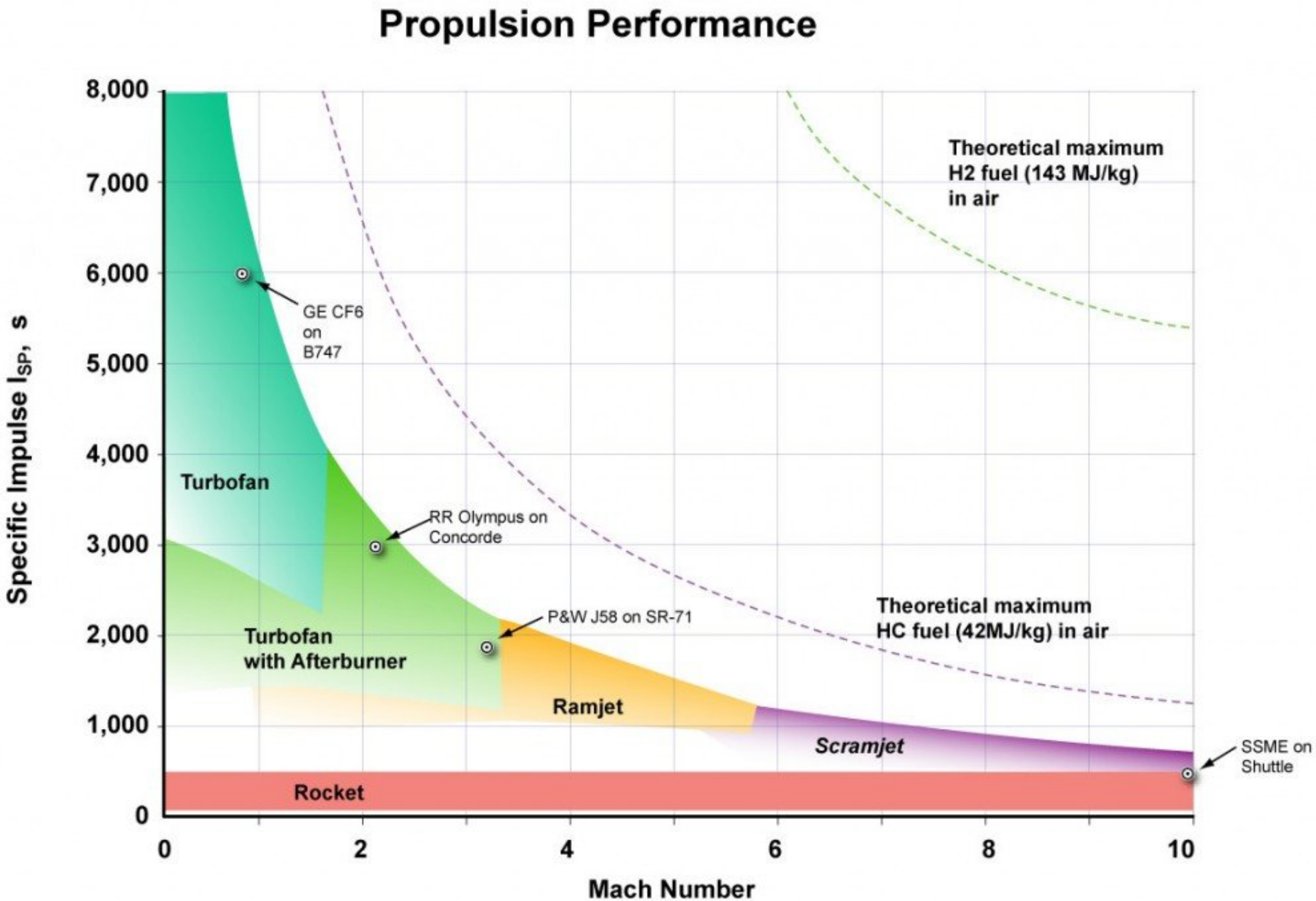


Ex. 2.2 Fig 3.



Ex. 2.2 Fig 4

Air breather



Example 2.3

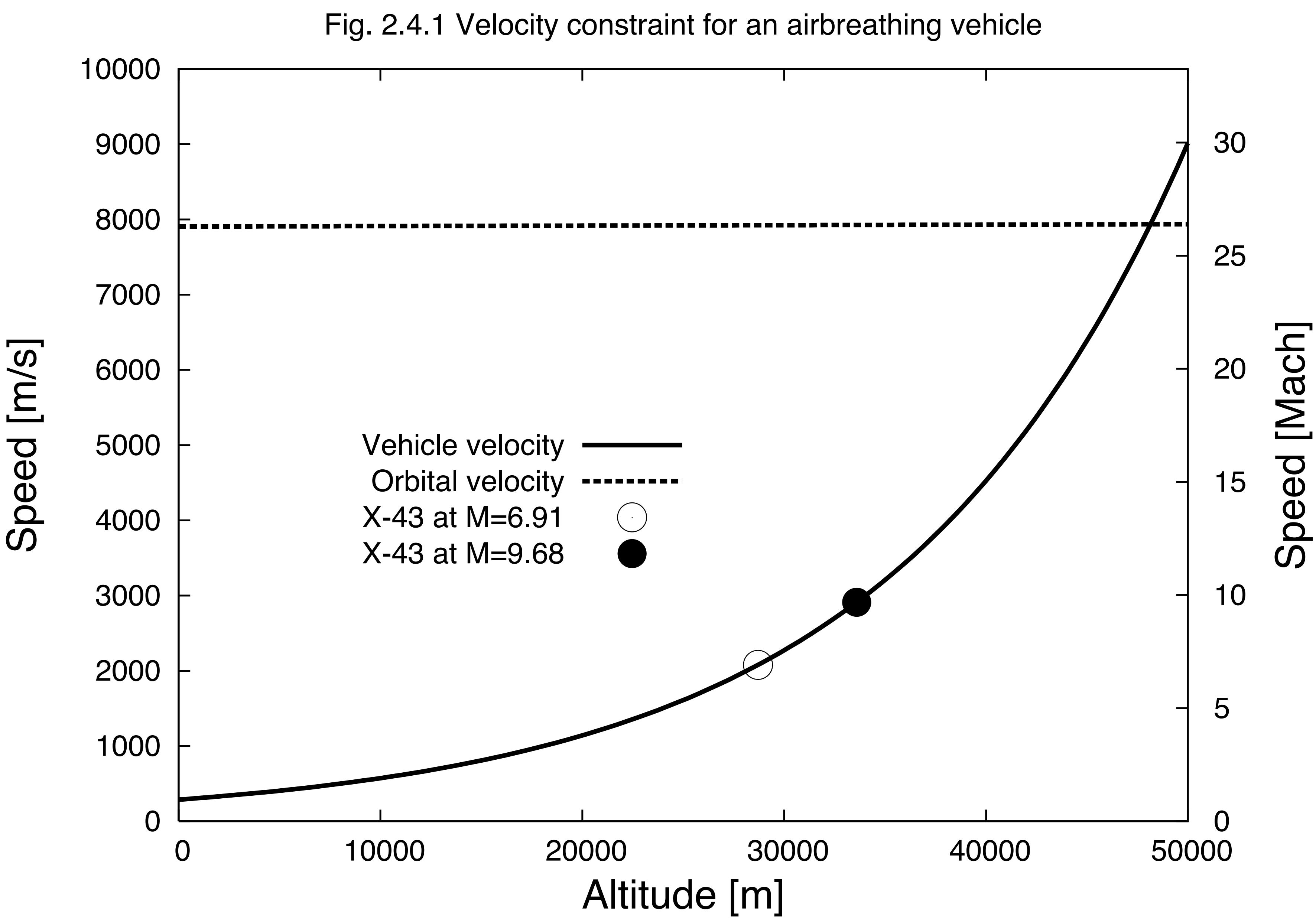
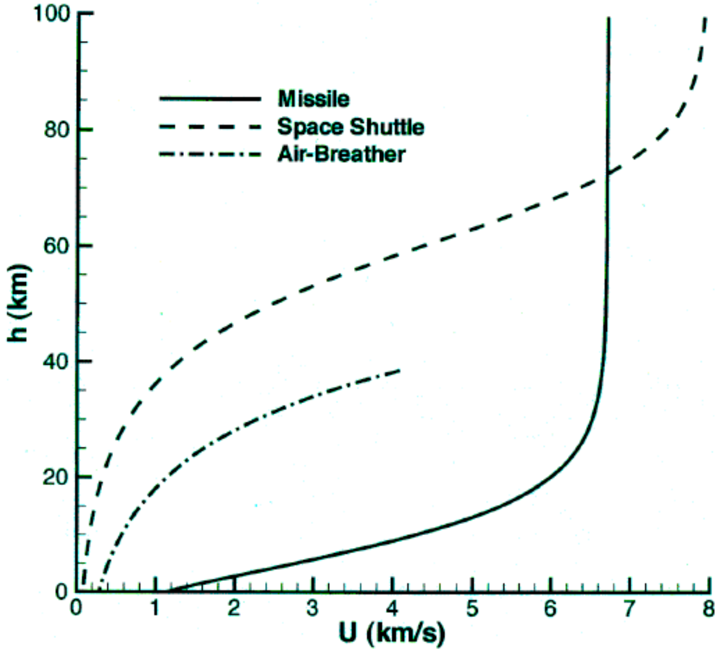
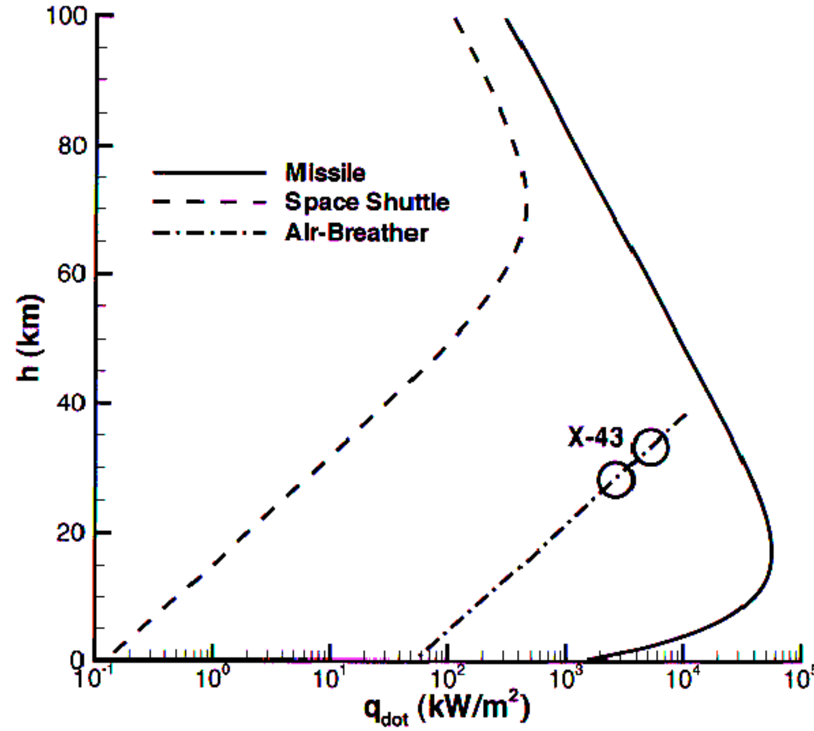


Fig. 2.4.1 Velocity constraint for an airbreathing vehicle
Vehicle velocity
Orbital velocity
X-43 at M=6.91
X-43 at M=9.68
Speed [m/s]
Speed [Mach]
Altitude [m]
10000
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
10000
20000
30000
40000
50000
30
25
20
15
10
5

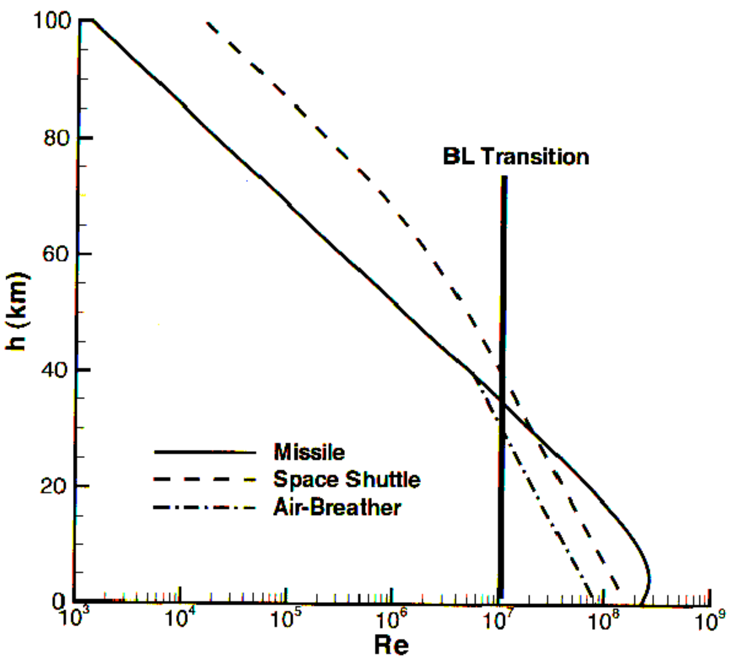
Comparison



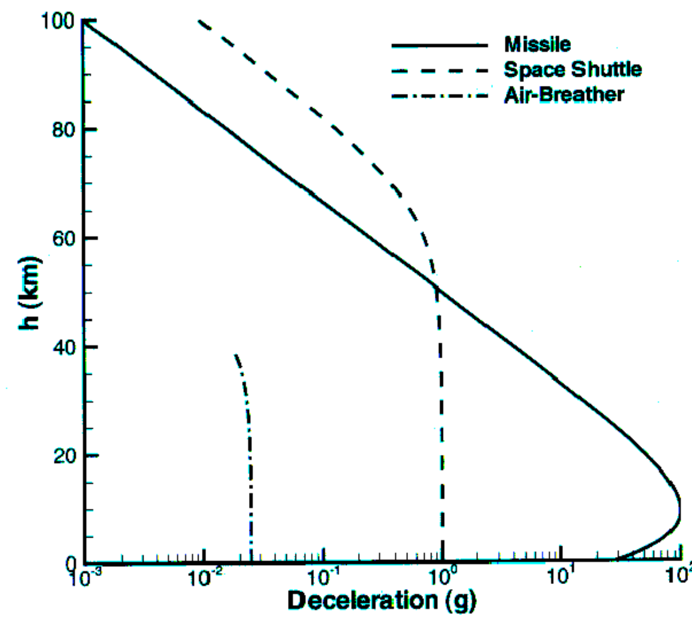
Missile
Space Shuttle
Air-Breather
h (km)
U (km/s)



Missile
Space Shuttle
Air-Breather
X-43
h (km)
q_{dot} (kW/m^2)



BL Transition
Missile
Space Shuttle
Air-Breather
h (km)
Re



Missile
Space Shuttle
Air-Breather
h (km)
Deceleration (g)

2.2 Ballistic Re-entry

- This is the re-entry path of a ballistic missile.
- Assumptions
  - $L \approx 0 = cte$
    (constant and steep)
  - $T = 0$
    (no propulsion)
  - $W \sin(\gamma) = D$
    (gravity force small)

From Eq. 2.8

$$-\frac{\dot{U}}{g} = \frac{D}{W}$$

$$\dot{h} = U \sin \gamma_0$$

Lifting Re-entry

Deceleration:

$$-\frac{\dot{U}}{g} = \frac{D}{W} = \frac{C_D\ A}{W}\frac{\rho\, U^2}{2} = -\frac{\beta}{2}\frac{C_D\ A}{W}U^2 \cdot \ln\left(\frac{U}{U_0}\right) \quad (2.17)$$

Velocity of Maximum deceleration:

$$\frac{U}{U_0} = \exp\left(-\tfrac{1}{2}\right) \quad (2.18)$$

Maximum deceleration:

$$\left[-\frac{\dot{U}}{g}\right]_{\max} = -\frac{\beta}{2}\frac{C_D\ A}{W}U_0^2 \exp\left(-1\right)\times\left(-\tfrac{1}{2}\right) = -\frac{U_0^2 \sin\gamma_0 \times\tilde{\alpha}}{2g}\exp\left(-1\right) \quad (2.19)$$

Peak heating velocity:

$$\frac{U}{U_0} = \exp\left(-\tfrac{1}{6}\right) \quad (2.20)$$