Linear Algebra Transformations

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Normal Vector

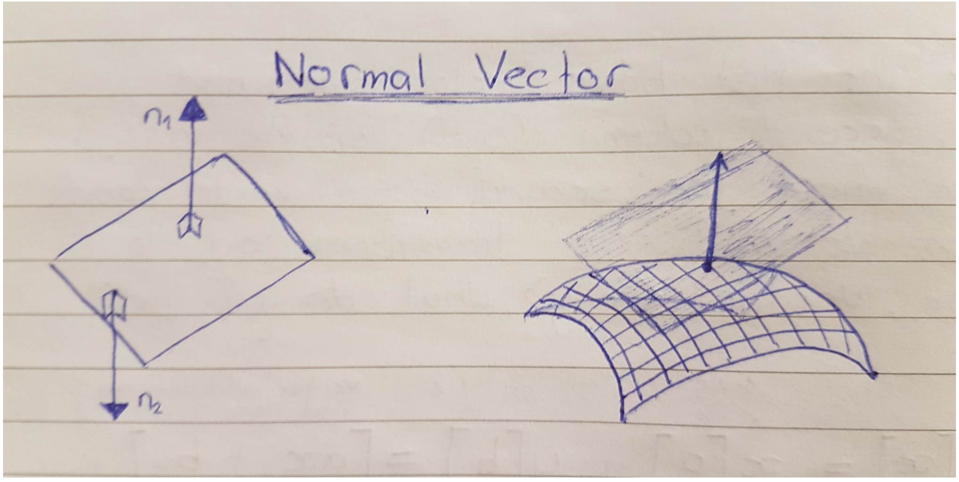

Figure 1: Normal Vector

The normal vector, often simply called the *normal* to a surface is a vector which is perpendicular to the surface at a given point. When normals are considered on closed surfaces, the inward-pointing normal (pointing towards the interior of the surface) and outward-pointing normal are usually distinguished.

The unit vector obtained by normalizing the normal vector (i.e. dividing a nonzero normal vector by its vector norm) is the unit normal vector, often known simply as the *unit normal.* Care should be taken to not confuse the terms *vector norm* (Length of vector), *normal vector* (Perpendicular vector) and *normalized vector* (Unit-length vector).

The normal vector is commonly denoted N, with a hat sometimes (but not always) added ($\hat{N}$) to explicitly indicate a unit normal vector.

Normalized Vector

The normalized vector of X is a vector in the same direction but with norm (length) of 1. It is denoted $\hat{X}$ and given by;

$$\hat{X} = \frac{X}{|X|}$$

Where $|X|$ is the norm of X. It is also called a **unit vector**.

Vector Norm

In mathematics, the *norm* of a vector is its **length**. A vector is a mathematical object that has a size, called the **magnitude** and a direction. For the real numbers, the only norm is the absolute value. For spaces with more dimensions, the norm can be any function p with the following three properties:

1. Scales for real numbers a, that is, $p(ax) = |a|p(x)$
2. Function of sum is less than sum of functions, that is, $p(x + y) \leq p(x) + p(y)$ (also known as the triangle equality)
3. $p(x) = 0$ if and only if $x = 0$

For a vector x, the associated norm is written as $\|x\|_p$ or L_p where p is some value. The value of the norm of x with some length N is as follows:

$$\|x\|_p = \sqrt[p]{x_1^p + x_2^p + ... + x_N^p}$$

The most common usage of this is the Euclidean norm, also called the standard distance formula.

e.g.

1. The one-norm is the sum of absolute values: $\|X\|_1 = |x_1| + |x_2| + ... + |x_N|$. This is like finding the distance from one place on a grid to another by summing together the distances in all directions the grid goes; see Manhatten Distance.

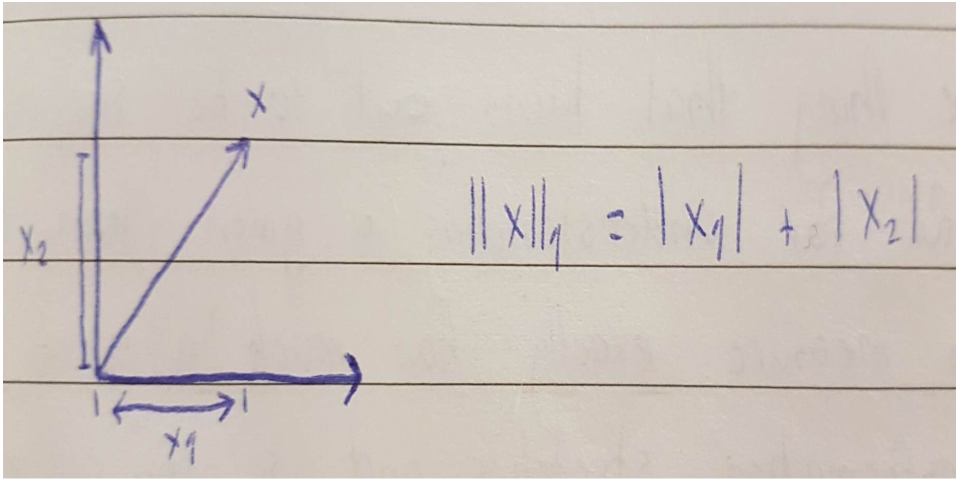

Figure 2: Manhatten Distance

L_1, written as $\|X\|_1$, is defined as $\|X\|_1 = \sum_{i=1}^{n} |X_i|$

2. Euclidean norm (also called L2-norm) is the sum of the squares of the values:

$$\|x\|_2 = \sqrt[p]{x_1^2 + x_2^2 + ... + x_N^2}$$

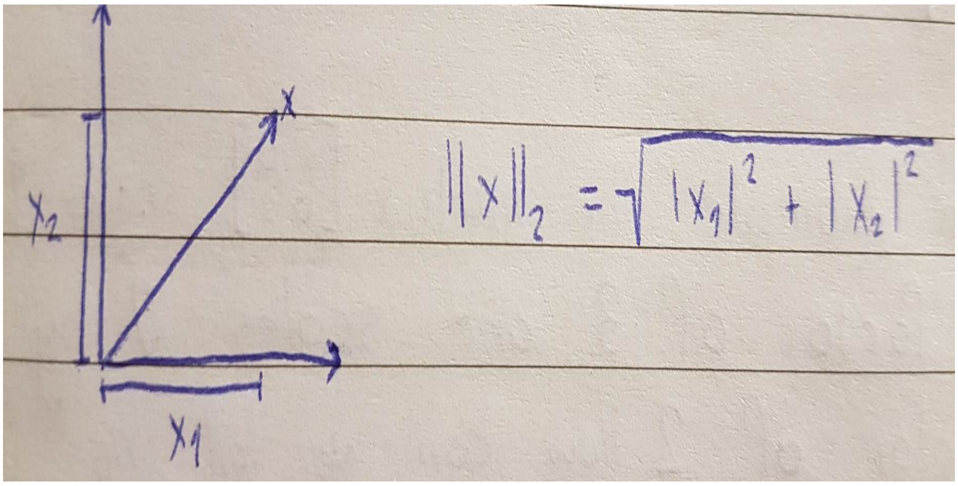

Figure 3: Euclidean Norm

A visual difference between the Euclidean Norm and the Manhatten Norm is shown in figure 4.

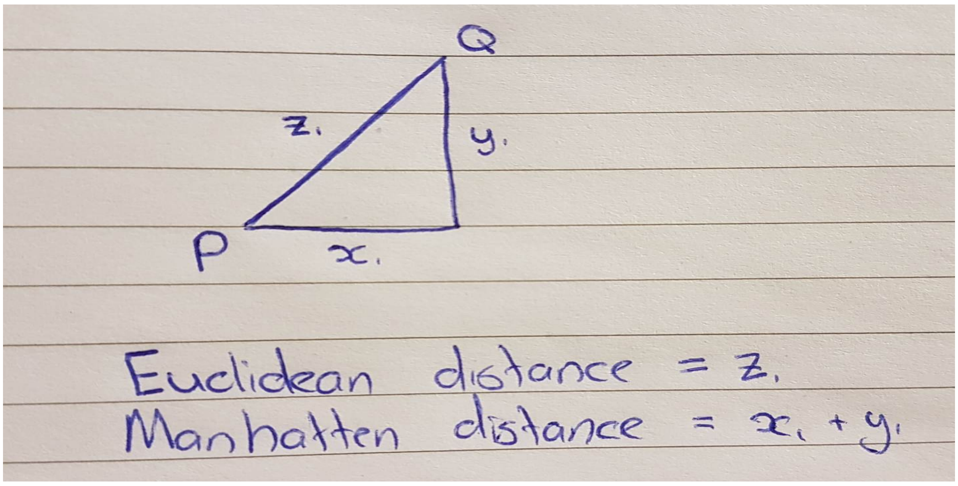

Figure 4: Euclidean vs Manhatten

3. Maximum norm is the maximum absolute value: $\|X\|_\infty = max(|X_1|, |X_2|, ...|X_N|)$