

TABLE I
COMPARISON OF COMPUTERIZED ULTRASONIC ARTERIOGRAPHY (CUA)
RESULTS WITH X-RAYS AS THE STANDARD

| X - RAY STENOSIS | |
|---|------------------|
| | < 40% ≥ 40% |
| CUA < 40% | 52 0 |
| CUA ≥ 40% | 9 33 |
| Sensitivity = 100% Specificity = 85% | |
| Accuracy = 90% False + = 21% False - = 0% | |

reduced the lumen diameter by 40 percent or more. This compares to a sensitivity of 86 percent and a specificity of 90 percent obtained in a previous study with the standard arteriograph. Thus, in this study, the new technique was 14 percent more sensitive and 5 percent less specific than the standard arteriograph. This represents a significant improvement in sensitivity.

Since computer programs are stored on floppy diskette, updating of the instrument with new diagnostic programs is easy. Further efforts are now underway to use the microcomputer to improve diagnostic accuracy through calculation of percent stenosis. Other additions include interfacing the same microcomputer to a fast Fourier transform (FFT) spectrum analyzer for spectral pattern recognition, in addition to multiplanar vessel imaging.

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Analysis and Control of the Current Distribution under Circular Dispersive Electrodes

J. D. WILEY AND J. G. WEBSTER

Abstract—An idealized model for a circular electrosurgical dispersive electrode is analyzed by solving Laplace's equation for the potential distribution. The resulting analytical solution shows the origin of the perimetrical burn problem, and, simultaneously, points the way toward

improved electrode designs. One possible improved electrode design, a segmented circular electrode in which the annular segments are connected to the RF generator through a series of current-leveling resistors, is proposed and discussed. Extensions of this concept and future areas for research are also discussed.

I. INTRODUCTION

It is well documented that patients undergoing electro-surgery sometimes suffer burns around the perimeter of the dispersive electrode [1], [2]. Overmyer *et al.* [2] have modeled this phenomenon by considering the potential distribution under a circular disk electrode on a finite slab of conductive material. Using numerical integration of Laplace's equation, they showed that the current density under the electrode is highly nonuniform, exhibiting a strong enhancement around the electrode perimeter. The purpose of the present note is twofold 1) to point out that models very similar to the one used by Overmyer *et al.* can be solved analytically, eliminating the need for numerical integration and simultaneously providing useful insight into the problem; and 2) to propose a new class of circular dispersive electrodes which achieve more uniform current distributions by forcing a more effective utilization of the entire area.¹

II. THEORY

A. Continuous Circular Disk Electrode

Fig. 1 shows the geometry to be analyzed. We assume a circular disk electrode of radius a on the surface of a semi-infinite slab of conducting material. The disk is assumed to be at a quasi-static potential V_0 relative to a ground electrode at infinity. In order to determine the current density $J(r, z)$ in the conductive medium, it is necessary to solve Laplace's equation for the potential distribution

$$\nabla^2 V = 0 \quad (1)$$

subject to the boundary conditions

$$V = V_0 \quad \text{for } z = 0, r \leq a, \quad (2a)$$

$$\frac{\partial V}{\partial z} = 0 \quad \text{for } z = 0, r > a, \quad (2b)$$

and

$$V \rightarrow 0 \quad \text{for } \begin{cases} r \rightarrow +\infty \\ z \rightarrow -\infty \end{cases}. \quad (2c)$$

Equations (1) and (2) define a classic mixed boundary-value problem for which the exact solution is known [3], [4]. Briefly, the solution may be obtained as follows. By symmetry, the solution must be expressible as a superposition of terms of the form

$$e^{-k|z|} J_0(kr) \quad (3)$$

where J_0 is a Bessel function and k is a parameter having dimensions of inverse length. The semi-infinite domain of $V(r, z)$ requires the use of a Fourier-Bessel transform for superposition. Thus, the solution is of the form

$$V(r, z) = \int_0^\infty A(k) e^{-k|z|} J_0(kr) dk. \quad (4)$$

¹A preliminary account of this work was given in three papers presented at the 15th Annual AAMI Meeting, San Francisco, April 13-17, 1980 (Session II, Papers 4, 5, and 6).

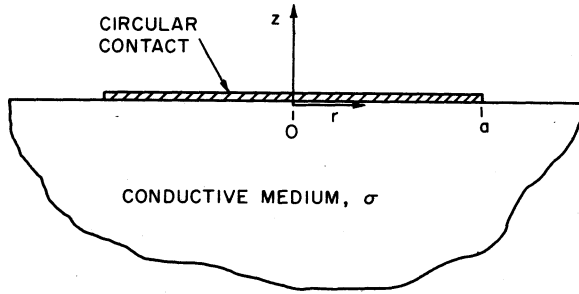


Fig. 1. Geometry of the electrode to be analyzed. A circular disk contact of radius a on the planar surface of a semi-infinite medium of conductivity σ . The counter-electrode is assumed to be infinitely removed in order to preserve azimuthal symmetry.

This expression automatically satisfies boundary condition (2c). Applying boundary conditions (2a) and (2b) gives

$$V_0 = \int_0^\infty A(k) J_0(kr) dk \quad r \leq a \quad (5)$$

and

$$0 = \int_0^\infty k A(k) J_0(kr) dk \quad r > a. \quad (6)$$

Equations (5) and (6) constitute a set of dual integral equations for $A(k)$, having the solution [3], [4]

$$A(k) = \frac{2aV_0}{\pi} \frac{\sin ka}{ka}. \quad (7)$$

Thus, $V(r, z)$ is given formally by

$$V(r, z) = \frac{2aV_0}{\pi} \int_0^\infty \frac{\sin ka}{ka} e^{-kz} J_0(kr) dk. \quad (8)$$

This is a discontinuous integral which reduces to

$$V(r, 0) = \begin{cases} \frac{2V_0}{\pi} \sin^{-1} \frac{a}{r} & \text{for } r \geq a \\ V_0 & \text{for } r \leq a \end{cases} \quad (9)$$

on the $z = 0$ surface, and

$$V(r, z) = \frac{2V_0}{\pi} \sin^{-1} \left\{ \frac{2a}{[(r-a)^2 + z^2]^{1/2} + [(r+a)^2 + z^2]^{1/2}} \right\} \quad (10)$$

for $z \neq 0$. Fig. 2 shows a few equipotential contours obtained from (10).

In principle, it is now straightforward to obtain the electric field \vec{E} and current density \vec{J} from V :

$$\vec{E}(r, z) = -\nabla V(r, z), \quad (11)$$

and

$$\vec{J}(r, z) = \sigma \vec{E}(r, z) \quad (12)$$

where σ is the conductivity of the medium. In practice, the expressions which result from a mechanical application of (11) and (12) in cylindrical coordinates are unwieldy. Fortunately, there is an easier way to obtain the current density at the electrode surface. Note that \vec{J} must be parallel to \vec{E} (12), that \vec{E} must be perpendicular to the metal electrode at the electrode/tissue interface (metallic boundary condition, together with the quasi-static assumption), and that the lines of \vec{E} must terminate on surface charge at the electrode (Gauss'

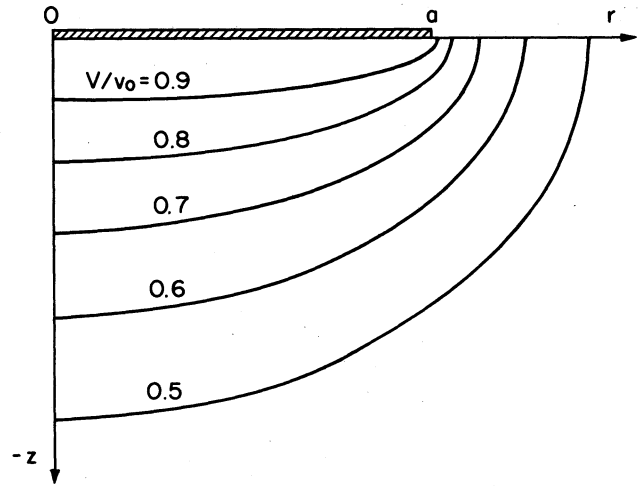


Fig. 2. Equipotentials in the vicinity of the electrode shown in Fig. 1 [see (10) and associated discussion].

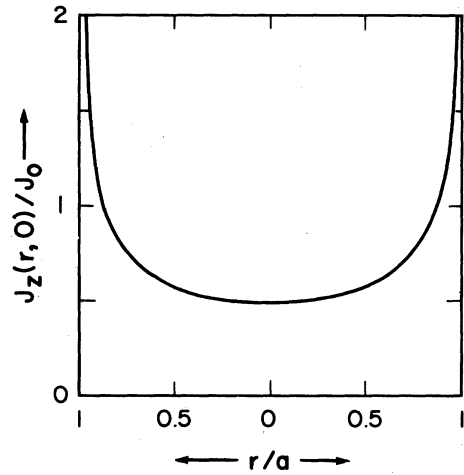


Fig. 3. Normalized current density as a function of radius for the electrode shown in Fig. 1 [see (15) and associated discussion].

law). Therefore, $J_z(r, 0)$ at the electrode may be related directly to the surface-charge density ρ_s , which is given by [3], [4]

$$\rho_s = \frac{2V_0\epsilon}{\pi} \frac{1}{(a^2 - r^2)^{1/2}} \quad (13)$$

where ϵ is the dielectric constant of the medium. Applying Gauss' law and (12)

$$J_z(r, 0) = \frac{2\sigma V_0}{\pi} \frac{1}{(a^2 - r^2)^{1/2}}. \quad (14)$$

This expression can be rewritten as

$$J_z(r, 0) = \frac{J_0}{2[1 - (r/a)^2]^{1/2}} \quad (15)$$

where

$$J_0 \equiv \frac{I_0}{\pi a^2} \quad (16)$$

and I_0 is the total current into the electrode. Note that J_0 is a hypothetical "uniform" current density. Fig. 3 shows $J_z(r, 0)$ as a function of r/a . This figure shows the origin of the burn problem in a dramatic way—the inner portion of the

electrode is relatively ineffective in collecting current, so that half the total current is collected around the ($r > 0.86a$) perimeter.

In comparing our analytic results with the numerical calculations of Overmyer *et al.* [2], several points should be kept in mind. The most striking difference is that we find a mild divergence of $J_z(r, 0)$ at $r = a$, whereas Overmyer *et al.* show a finite maximum at $r = a$. The divergence is an artifact of the model, attributable to the fact that we have ignored the finite thickness of the electrode metal and assumed a mathematically abrupt edge at $r = a$ [5]. In practice, the electric field (and current density) at the edge will remain finite, achieving some large value which depends sensitively on the microscopic details of the edge itself, the electrode thickness, and the interfacial contact layer. The $z = 0$ boundary conditions used by Overmyer *et al.* are the same as ours. Thus, they too should have found a divergence at $r = a$. The fact that they did not is simply a reflection of the finite grid size used in numerical calculations. Had they used a finer grid, they would have obtained a larger (but still finite) electric field at $r = a$.

A more fundamental difference between our model and that of Overmyer *et al.* is that we have assumed a semi-infinite medium and an infinitely distant ground electrode, whereas they have used a medium of finite thickness W , and a nearby annular ground electrode. As long as W and the electrode spacing are both greater than a few contact radii, these differences between our models have no significant effect on $J_z(r, 0)$ for $r < a$.² We have not calculated $J_z(r, 0)$ for $r > a$, and can make no comparisons with Overmyer's results in this region.

B. Segmented Circular Disk Electrode

The method used for solving the problem posed in Section II-A focuses attention on the following points. As long as the dispersive electrode is an equipotential surface, the interface-charge density [and, therefore, the current density $J_z(r, 0)$] will necessarily be nonuniform. This is true, independent of the specific shape of the electrode perimeter. From this we conclude that, in order to design an electrode having a uniform current density, it is necessary to achieve an appropriate non-uniform radial potential distribution across the electrode. The desired potential distribution is that of a uniformly charged disk, and can be calculated as follows.

Consider a uniformly charged disk of radius a lying in the x - y plane. The potential along the z axis is easily found (integrating Coulomb's law) to be

$$V(z) = \frac{\rho_s a}{2\epsilon} \left[\left(1 + \frac{z^2}{a^2} \right)^{1/2} - \frac{z}{a} \right] \quad (17)$$

where ρ_s is the surface-charge density. The off-axis potential can be obtained [3] by expanding $V(z)$ in a power series in z , making the transformation $z \rightarrow r$ where r is the radial coordinate of a spherical coordinate system, and multiplying each term r^l of the expansion by the Legendre polynomial $P_l(\cos \theta)$. Using this method, and converting back to the cylindrical coordinate system of the problem gives the in-plane radial potential distribution

$$V(r, 0) = \frac{\rho_s a}{2\epsilon} \left[1 - \sum_{n=1}^{\infty} C_n \left(\frac{r}{a} \right)^{2n} \right] \quad (18)$$

²An illustration of this claim is provided by the work of R. D. Brooks and H. G. Mattes, *Bell Syst. Tech. J.*, vol. 50, p. 775, 1971. Brooks and Mattes calculate the spreading resistance between a circular disk on one side of a slab of thickness W , and a conducting ground plane which covers the other face of the slab. Their results can be used to show that $J_z(r, 0)$ at the disk is essentially the same as our (15) for $W \gtrsim a$. (At $W = a$ the differences are on the order of a few percent.)

where $C_1 = \frac{1}{4}$ and C_n obeys the recursion relation

$$C_n = \frac{(2n-1)(2n-3)}{4n^2} C_{n-1}. \quad (19)$$

It is straightforward to show, using the ratio test and continuity of V , that the infinite series in (18) is absolutely convergent for all $r/a \leq 1$. The first (quadratic) term in the series is strongly dominant so that, to a sufficient approximation, we can write

$$V(r, 0) \approx V_c \left[1 - \frac{1}{4} \left(\frac{r}{a} \right)^2 \right] \quad (20)$$

where $V_c \equiv \rho_s a / 2\epsilon$ is the potential at the center of the disk. It is important to note that a very small change in the potential distribution is sufficient to effect a substantial change in the current density distribution: A uniform potential gives the divergent current density of Section II-A; a potential which falls roughly quadratically with r , (actually to about $0.64 V_c$ at $r = a$) gives a uniform current density. Therefore, anything which can be done to cause the electrode potential to "droop" at the perimeter can be expected to produce a substantial improvement in the current density uniformity.

There are several ways in which the radial potential distribution on the electrode might be tailored to the desired profile. One possible solution involves the use of a nonmetallic electrode having a thickness and/or resistivity profile chosen to provide the correct radial voltage drop. Preliminary calculations based on this approach indicate that the required thickness or resistivity variations (from center to edge) are rather difficult to achieve. Fig. 4 illustrates an alternate solution. The electrode disk is divided into concentric annular zones which are returned separately to ground via individual resistors of appropriately selected values. The ohmic voltage drops in these resistors can then be arranged to provide a lateral voltage profile on the electrode which approximates that given by (18). This, in turn, will assure an approximately uniform current density at the electrode/tissue interface. Although this concept is illustrated in Fig. 4 with only three zones, it is intuitively clear that a larger number of zones would allow a closer approximation to the desired profile. After some preliminary calculations using zones of various relative sizes, we have concluded that the simplest choice involves dividing the total electrode area πa^2 into N zones of equal area $A_n = \pi a^2 / N$. In order to achieve this division of area, the zone radii r_n must obey

$$r_n = (n)^{1/2} r_1 = (n/N)^{1/2} a. \quad (21)$$

This is recognizable as the "Fresnel zone construction" of diffraction theory. It is now straightforward to calculate the required resistances as follows. When the resistances are properly chosen, the voltage on the n th annulus \bar{V}_n must equal the average value given by (18) for that radius range. Thus, we require

$$\bar{V}_n = \frac{1}{A_n} \int_{r_{n-1}}^{r_n} V(r, 0) 2\pi r dr. \quad (22)$$

Using (20) as an approximation for $V(r, 0)$ leads to

$$\bar{V}_n = V_c \left[1 - \frac{1}{8N} (2n-1) \right] \quad (23)$$

where V_c is now interpreted as the voltage at the contact terminal (the top of the resistor chain in Fig. 4). The required series resistance for the n th zone is now obtained by forcing a self-consistent solution to the equation

$$V_c - \bar{V}_n = I_n R_n. \quad (24)$$

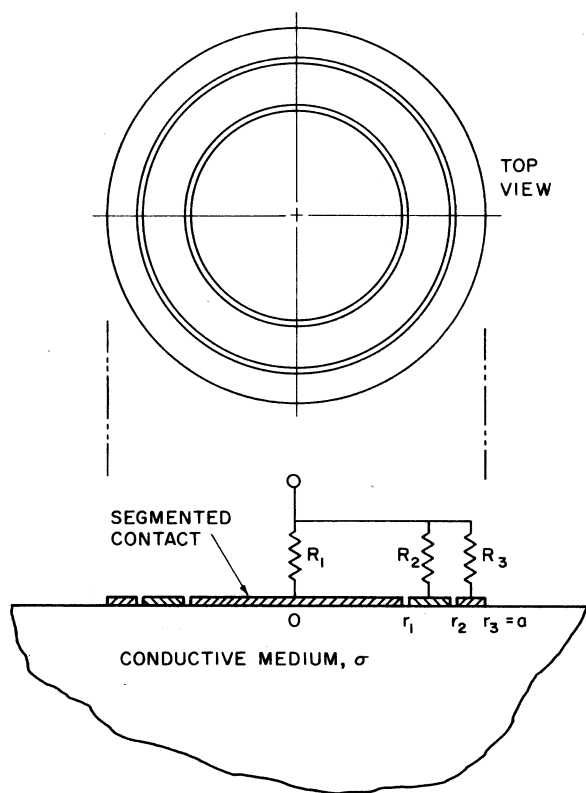


Fig. 4. Geometry of a proposed multiply-connected contact which (for purposes of illustration has been divided into three zones of equal areas. The current collected by the n th zone must pass through a series resistor R_n . The ohmic voltage drop in R_n reduces the potential on the n th zone by an appropriately chosen amount, helping to "level" the current density profile as discussed in the text.

Self-consistency is imposed by recognizing that since the zone areas are equal and the current density is uniform, the individual zones must collect equal currents $I_n = I_t/N$, where I_t is the total current. Using this condition and solving (24) for R_n gives

$$R_n = \frac{V_c}{I_t} \left[\frac{2n-1}{8} \right]. \quad (25)$$

Table I illustrates the application of these ideas to a 10-zone electrode design. Values for r_n/a , \bar{V}_n/V_c , and $R_n/(V_c/I_t)$ were calculated using (21), (23), and (25), respectively. Note that \bar{V}_n falls roughly quadratically from center to edge, as required by (20), and is approximately $0.76 V_c$ at $r=a$. In an exact implementation of (18), $V(r, 0)$ should drop to about $0.63 V_c$ at $r=a$. Thus, some further adjustment of the resistor values may be necessary for optimizing an electrode design. The R_n values given in Table I are normalized to V_c/I_t , so this quantity sets the actual resistance scale. Using the definitions of V_c and I_t , we obtain

$$\frac{V_c}{I_t} \approx \frac{\rho}{2\pi a} \quad (26)$$

where ρ is the resistivity of the conductive medium (tissue) and a is the overall electrode radius. Taking typical values of $\rho \approx 500 \Omega \cdot \text{cm}$ and $a = 5 \text{ cm}$ gives $V_c/I_t \approx 16 \Omega$. Thus, $R_1 \approx 2 \Omega$ and $R_{10} \approx 40 \Omega$ in this example.

III. DISCUSSION

The calculations presented in Section II are based on a highly idealized model in which the body is treated as a semi-infinite, homogeneous medium having a planar surface, and the current streamlines at the dispersive electrode are forced to be azimuthally symmetric. In practice, of course, dispersive electrodes are used on body surfaces having radii of curvature

TABLE I
CALCULATED PARAMETERS FOR A 10-ZONE SEGMENTED ELECTRODE^a

| n | r_n/a | \bar{V}_n/V_c | $R_n/(V_c/I_t)$ |
|-----|---------|-----------------|-----------------|
| 1 | 0.316 | 0.988 | 0.125 |
| 2 | 0.447 | 0.963 | 0.375 |
| 3 | 0.548 | 0.938 | 0.625 |
| 4 | 0.633 | 0.913 | 0.875 |
| 5 | 0.707 | 0.888 | 1.125 |
| 6 | 0.775 | 0.863 | 1.375 |
| 7 | 0.837 | 0.838 | 1.625 |
| 8 | 0.894 | 0.813 | 1.875 |
| 9 | 0.949 | 0.788 | 2.125 |
| 10 | 1.000 | 0.763 | 2.375 |

^aSee text for discussion.

comparable to the electrode radius; the conductive tissues are stratified (inhomogeneous) and have thicknesses which vary, but are typically less than the electrode radius; and the current streamlines are far from being azimuthally symmetric. Nevertheless, the idealized model is useful in that it has allowed analytical calculations which provide deeper insight into the problem than could have been obtained through purely numerical simulations. In addition to revealing the origin of the burn problem, the calculations presented in Section II-A suggested the strategy for improved contact design which was presented in Section II-B. This strategy consists of dividing the electrode into multiple segments which are individually returned to ground in such a way that the radial potential distribution on the dispersive electrode approximates that of a uniformly charged disk (thereby assuring an essentially uniform current density at the electrode/tissue interface). One possible implementation of this design strategy was proposed in Section II-A. The electrode area was divided into annular zones which were returned to ground through simple series resistors of appropriately chosen values. Because of the differences between the idealized model and the actual conditions which prevail during electrosurgery, it is unlikely that the design example discussed in Section II-B and summarized in Table I will be satisfactory as a practical electrode design. In particular, the approximately quadratic progression of resistor values is optimum for current-leveling in the idealized model, but may require modification for the practical case in which the conductive medium is of finite thickness. The concept of current-leveling through the use of a multiply-connected electrode, however, remains valid. Further development leading to practical electrode designs will require experimentation, perhaps using electrolytic-tank models.

Once the concept of multiply-connected dispersive electrodes is established, it becomes possible to envision more sophisticated modifications which have the potential for providing further improvements in performance and safety. From a circuits point of view, the dispersive electrode and neighboring tissue represent a distributed impedance, and the burn problem results from an inappropriate impedance profile. Viewed in this way, it is clear that there is nothing fundamental about the use of series resistors for current leveling. Other passive components (such as capacitors) could be used as well, to tailor any desired impedance profile. Indeed, it has recently been suggested [6] that dispersive electrodes which are capacitively coupled to the body provide inherently better current uniformity than resistive (gel pad) electrodes. Although we do not agree that capacitive electrodes are necessarily superior to resistive electrodes, it may well be true that capacitive coupling eases the problem of achieving current uniformity in practice. Capacitive coupling certainly has the advantage of blocking any dc or low-frequency components which may result from partial rectification of

the RF current, or from unintentional interaction with other electronic instruments [8]. If simple capacitive electrodes do ultimately prove to be superior in practice, their performance could be further improved by the same zonal division and current-leveling scheme which we have discussed here.

The conditions which prevail during actual electrosurgery are highly variable from one patient or one surgical procedure to another. It would therefore be convenient if the dispersive electrode were *adaptive* in the sense that it could adjust to variable conditions in order to maintain safe current densities. This could be arranged by replacing the passive series resistors with active current-limiting elements. It is very straightforward, for example, to design small, bidirectional, FET current limiters which would allow any zone of the electrode to accept only a selected fraction of the total current. If this approach were combined with azimuthal division of the zones, the well-known "leading-edge" effect [1], [2] could be alleviated or eliminated.

Finally, it should be pointed out that the added complications which are inherent in our proposed multiply-connected electrodes do not necessarily imply that such electrodes would be difficult or expensive to manufacture or use. The passive or active components which are used for current leveling could be incorporated in the RF generator, in the connecting cable, or in the specially-designed connector which would be required for attachment to the electrode. The electrode, itself, could remain relatively simple, inexpensive, and disposable.

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Distributed Equivalent-Circuit Models for Circular Dispersive Electrodes

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Abstract—Analytically solvable distributed equivalent-circuit models have been developed for circular electrosurgical dispersive electrodes which are either resistively or capacitively coupled to the body. Calculations based on these models show that for either electrode type it is possible to define a characteristic length, the magnitude of which governs the current distribution under the electrode. The well-known perimetrical burn problem occurs when the current transfer length is much smaller than the electrode radius: a problem which may arise

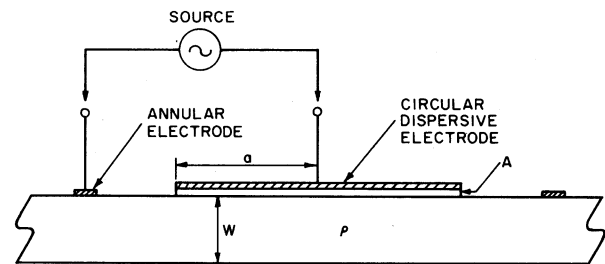


Fig. 1. Cross-sectional view of the structure to be modeled. A circular dispersive electrode of radius "a" on a homogeneous slab of conductive material of thickness W and resistivity ρ . The layer labeled A is a conductive gel in the case of resistivity coupled electrodes or a dielectric film in the case of capacitively coupled electrodes. The RF source drives a total current i_0 between the annular electrode and the circular dispersive electrode.

with either capacitive or resistive electrodes. Design guidelines are given for the optimization of simple circular dispersive electrodes, and suggestions for further improvements are discussed.

I. INTRODUCTION

In electrosurgery, a strong RF current (typically several hundred milliamperes at 1 MHz) is delivered by the active electrode to the surgical site, and returned to the generator via a large-area dispersive electrode. In current practice, the area of the dispersive electrode is selected in accordance with a simple power-density guideline which calls for 1 cm² of electrode area for each 1.5 W of applied RF power [1]. The intent of this guideline is to assure that the current density at the dispersive electrode site is sufficiently low to avoid patient burns. Recent investigations, however, have shown [2]–[4] that the current densities under dispersive electrodes are often highly nonuniform, and that severe burns may occur, even with electrodes which are conservatively within the "area" guidelines. These current nonuniformities depend on a number of factors including electrode placement, quality and uniformity of the electrode/skin interface, and the effective electrical and thermal conductivities of the tissue immediately beneath the contact. Thus, a complete treatment of the problem is expected to be beyond the scope of analytical calculations. Nevertheless, idealized models have already proved useful in explaining some aspects of the burn problem, and may help point the way toward better electrode designs. Numerical [2] and analytical [3] calculations of the electric field pattern under idealized circular electrodes, for example, have shown that the current density is highest near the perimeter, with about half the current being collected by the outer 15 percent of the contact. The results of these calculations have been used to suggest improved electrode designs as reported elsewhere [5]–[7].

Equivalent-circuit models provide a second useful approach for exploring the behavior of complex systems. In the present paper, simple distributed equivalent circuits are proposed for two types of circular dispersive electrodes: 1) conventional resistive (gel-pad) electrodes and 2) capacitive electrodes. Although the models are highly idealized, they provide considerable insight into the problems and limitations of existing electrodes, and should be useful in the development of improved design criteria.

II. EQUIVALENT CIRCUITS

Fig. 1 shows a cross-sectional view of the idealized contact structure to be modeled. We assume a slab of homogeneous conductive material having thickness W and electrical resistivity ρ . The circular dispersive electrode consists of a planar metal disk of radius a separated from the conductive medium by the layer which is labeled A in Fig. 1. This layer is assumed to

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