Appendix B

The current through each component can be described by each of the following:[1]

$$I_R = \frac{1}{R}V_R \qquad (1)$$

$$I_C = C\frac{dV_c}{dt} \qquad (2)$$

$$I_L = \frac{1}{L}\int_{-\infty}^{t} V_L(\tau)d\tau \qquad (3)$$

For our circuit, Kirchhoff's Laws give the following equations:

$$I_L = I_S \qquad (4) \qquad\qquad I_L = I_C + I_R \qquad (5)$$

$$V_S = V_L + V_{RC} \qquad (6)$$

Plug (1), (2), and (3) into (5).

$$\frac{1}{L}\int_{-\infty}^{t} V_L(\tau)d\tau = C\frac{dV_c}{dt} + \frac{1}{R}V_R \qquad (7)$$

Take the derivative of both sides to get rid of the integral.

$$\frac{V_L}{L} = C\frac{d^2V_{RC}}{dt^2} + \frac{1}{R}\frac{dV_{RC}}{dt} \qquad (8)$$

Multiply by L.

$$V_L = CL\frac{d^2V_{RC}}{dt^2} + \frac{L}{R}\frac{dV_{RC}}{dt} \qquad (9)$$

Substitute V_L from (9) into (6).

$$V_S = CL\frac{d^2V_{RC}}{dt^2} + \frac{L}{R}\frac{dV_{RC}}{dt} + V_{RC} \qquad (10)$$

Solve (10) for $\frac{d^2V_{RC}}{dt^2}$ and $\frac{dV_{RC}}{dt}$ to give equations for the slope function.

$$\frac{d^2V_{RC}}{dt^2} = \frac{1}{LC}\left[V_S - \frac{L}{R}\frac{dV_{RC}}{dt} - V_{RC}\right] \qquad (11) \qquad\qquad \frac{dV_{RC}}{dt} = \frac{R}{L}\left[V_S - CL\frac{d^2V_{RC}}{dt^2} - V_{RC}\right] \qquad (12)$$

[1] Retrieved from http://uhaweb.hartford.edu/ltownsend/Series_and_Parallel_Equations_from_a_DE_perspective.pdf on October 29, 2018