Michael Rizzo

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Continuous Methods

Lab 5 Part 2 Question 3,4,5

(a) For the general two-point BVP with $p(x) = 1$ and $q(x) = 0$ we know the exact entries of the coefficient matrix for a uniform grid; these are given in the notes. Output your

2

matrix for your uniform grid in (1) and compare with the exact value. Are they the same? Why or why not?

Exact Matrix

$$S = \frac{1}{h}\begin{pmatrix} 2 & -1 & 0 & & \cdots & 0 \\ -1 & 2 & -1 & 0 & \cdots & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ & & \ddots & \ddots & \ddots & \\ 0 & \cdots & 0 & -1 & 2 & -1 \\ 0 & \cdots & & 0 & -1 & 2 \end{pmatrix}$$

Matrix I got

```
[[ 8. -4.  0.  0.  0.  0.  0.]
 [-4.  8. -4.  0.  0.  0.  0.]
 [ 0. -4.  8. -4.  0.  0.  0.]
 [ 0.  0. -4.  8. -4.  0.  0.]
 [ 0.  0.  0. -4.  8. -4.  0.]
 [ 0.  0.  0.  0. -4.  8. -4.]
 [ 0.  0.  0.  0.  0. -4.  8.]]
```

let h= $\frac{1}{4}$ and the exact matrix will be the matrix I got. The stiffness matrix depends on the values of the derivative basis function. The results are the same because the derivative value of the basis function is $\frac{1}{h}$ or $\frac{-1}{h}$

(b) For the general two-point BVP with $p(x) = 1$ and $q(x) = 1$ we know the exact entries of the coefficient matrix for a uniform grid; these are given in the notes. Output your matrix for your uniform grid in (1) and compare with the exact value. Are they the same? Why or why not?

Exact Matrix

8.16667	-4.04167	0.00000	0.00000	0.00000	0.00000	0.00000
-4.04167	8.16667	-4.04167	0.00000	0.00000	0.00000	0.00000
0.00000	-4.04167	8.16667	-4.04167	0.00000	0.00000	0.00000
0.00000	0.00000	-4.04167	8.16667	-4.04167	0.00000	0.00000
0.00000	0.00000	0.00000	-4.04167	8.16667	-4.04167	0.00000
0.00000	0.00000	0.00000	0.00000	-4.04167	8.16667	-4.04167
0.00000	0.00000	0.00000	0.00000	0.00000	-4.04167	8.16667

The matrix I got

```
[[ 8.125  -3.9375  0.      0.      0.      0.      0.    ]
 [-3.9375  8.125  -3.9375  0.      0.      0.      0.    ]
 [ 0.     -3.9375  8.125  -3.9375  0.      0.      0.    ]
 [ 0.      0.     -3.9375  8.125  -3.9375  0.      0.    ]
 [ 0.      0.      0.     -3.9375  8.125  -3.9375  0.    ]
 [ 0.      0.      0.      0.     -3.9375  8.125  -3.9375]
 [ 0.      0.      0.      0.      0.     -3.9375  8.125 ]]
```

The values are not the same because the mass matrix used was approximated by a one point gauss quadrature rule.

4.

4. The right hand side of our linear system can be assembled in an analogous way to the matrix. Modify your assembly routine in (3) to assemble the right hand side. The routine should now output your right hand side and matrix. Print out your right hand side vector for $f(x) = 1$ for your uniform grid in (1) and compare with the exact answer.

Right hand side I Got

```
[[ 0.25]
 [ 0.25]
 [ 0.25]
 [ 0.25]
 [ 0.25]
 [ 0.25]
 [ 0.25]]
```

Exact Matrix

rhs =

0.25000 0.25000 0.25000 0.25000 0.25000 0.25000 0.25000

Both right hand sides are exactly the same.

5.

5. Solve the following two-point BVP

$$-u''(x) = \pi^2 \sin(\pi x) \quad 0 < x < 1$$

$$u(0) = u(1) = 0$$

whose exact solution is $\sin(\pi x)$ with $h = 0.25$. You haven't written an error routine yet but you should be able to check that your answer is close to the exact solution at the nodes. Output your approximation.

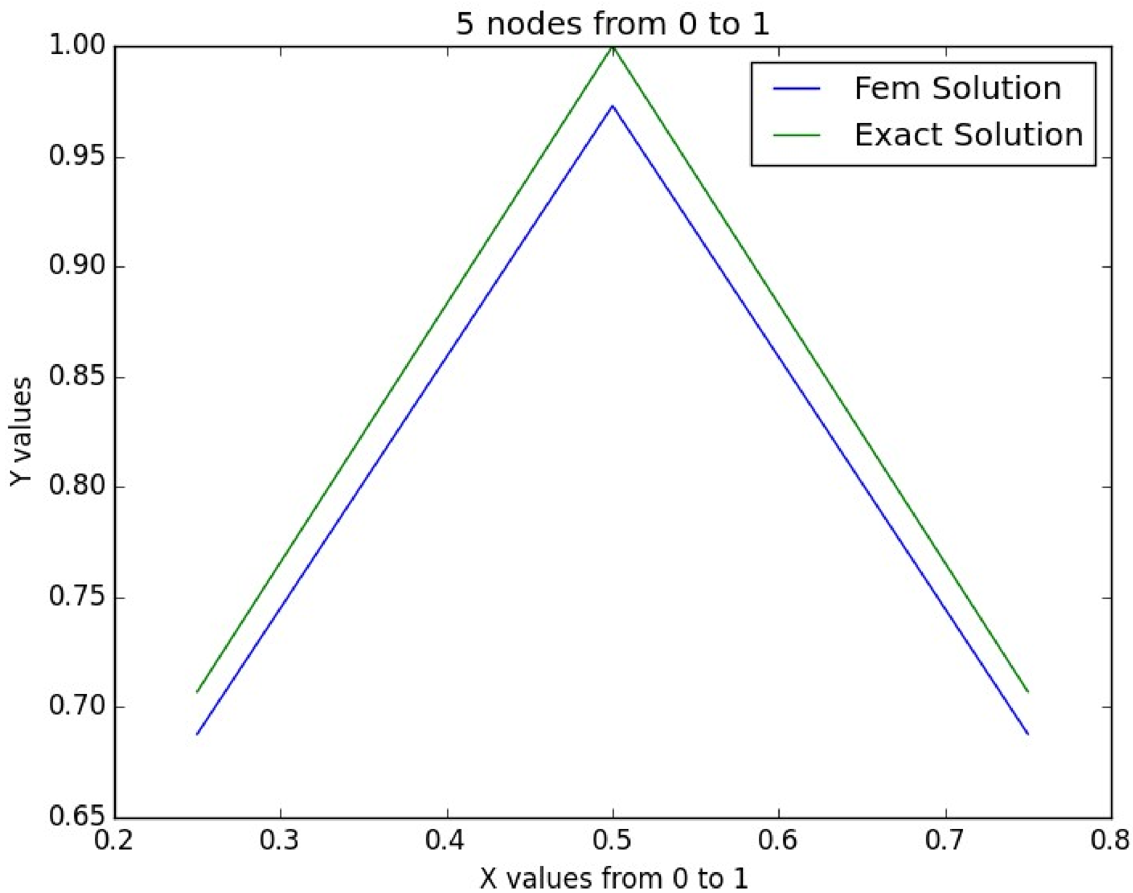
My Solution

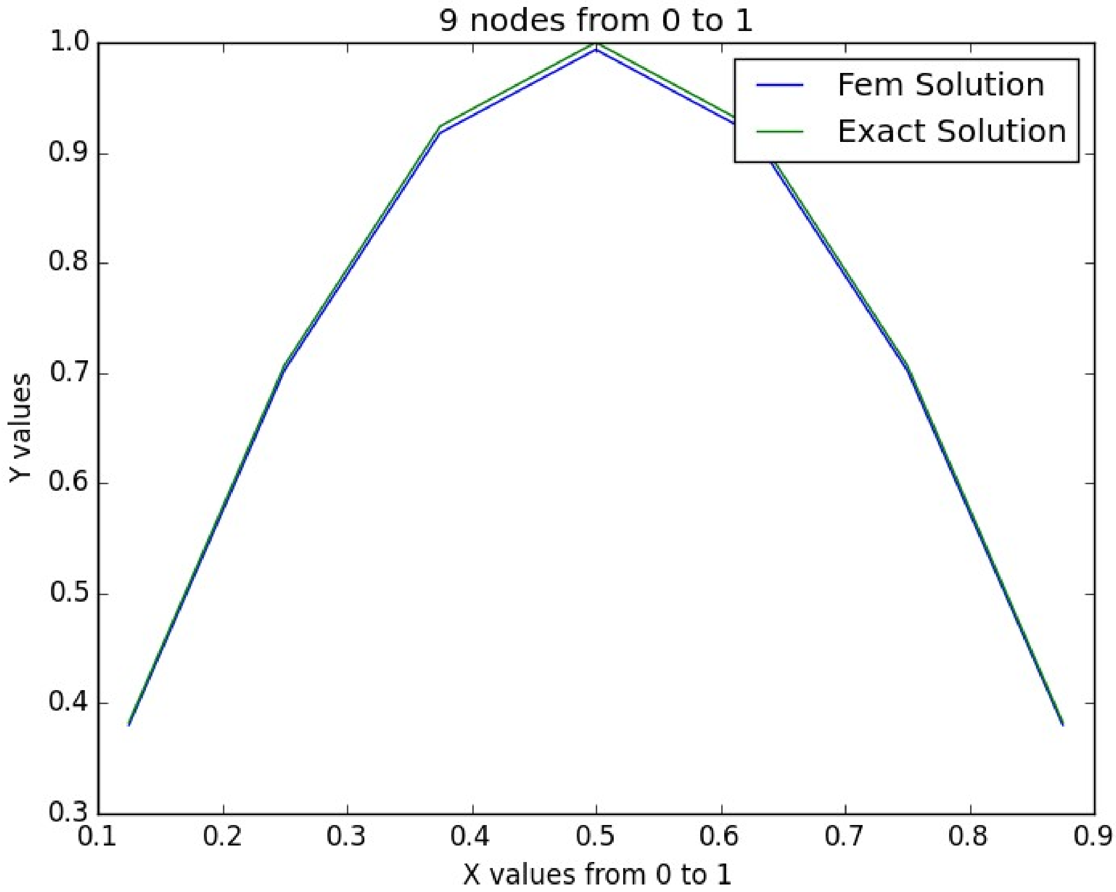
[[0.68792453]
[0.97287221]
[0.68792453]]

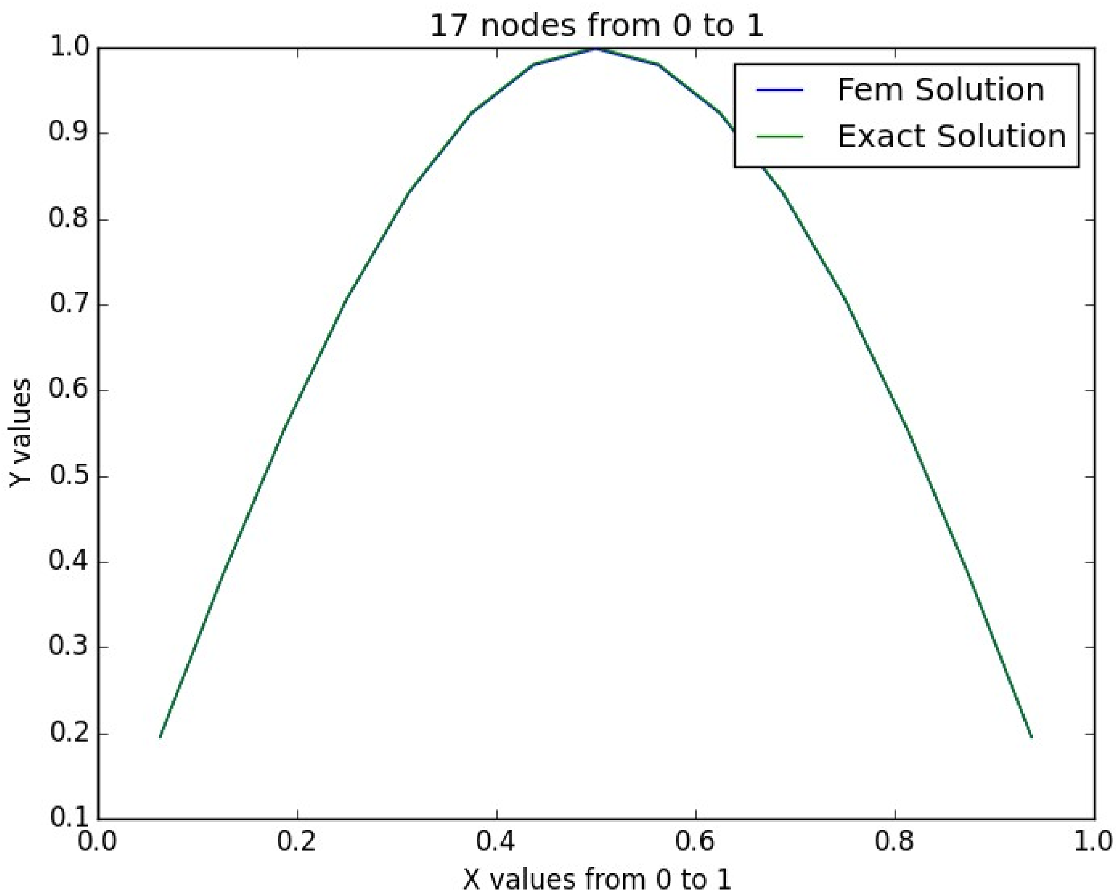
Exact_solution

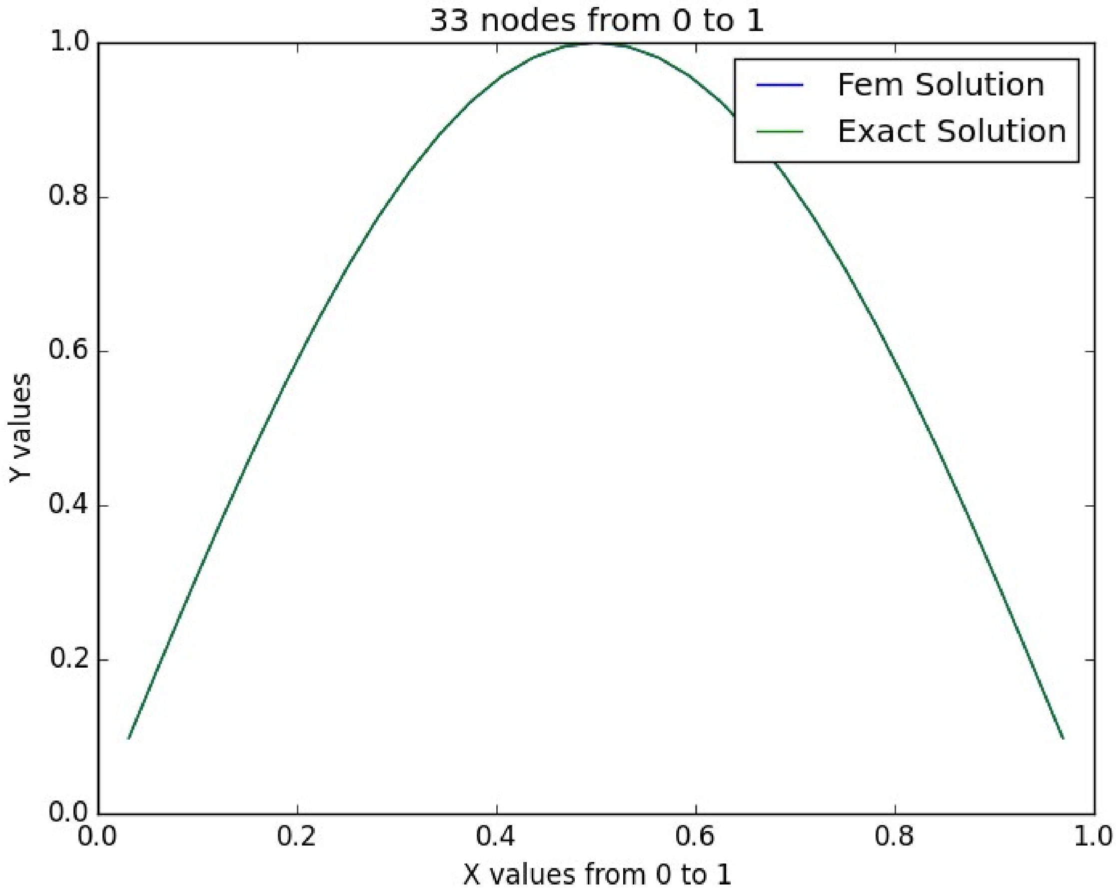
0.70711   1.00000   0.70711

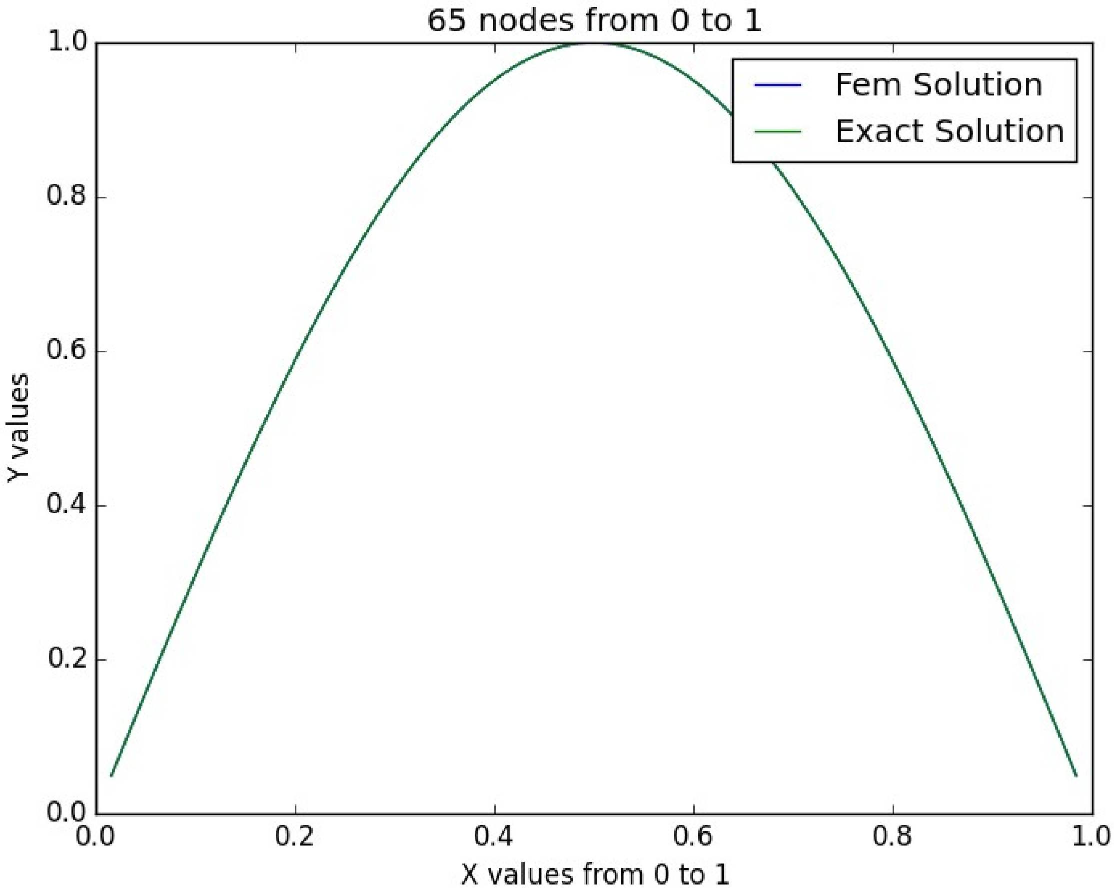
Graphs for 5,9,17,33, and 65 nodes




9 nodes from 0 to 1
Fem Solution
Exact Solution
Y values
X values from 0 to 1


17 nodes from 0 to 1
Fem Solution
Exact Solution
Y values
X values from 0 to 1


33 nodes from 0 to 1
Fem Solution
Exact Solution
Y values
X values from 0 to 1
0.0
0.2
0.4
0.6
0.8
1.0



Convergence Rates and errors for 5,9,17,33, and 65 nodes

Number Of Nodes	Error	Convergence Rate
5	5.39018339046*10^-2	0
9	1.35922704922*10^-2	1.9875478905032724
17	3.40526054443*10^-3	1.9969493766544835
33	8.51762997571*10^-4	1.9992412238568733
65	2.12968714201*10^-4	1.999810548041891