LaTeX example question answers

Michael Manansala

October 18, 2018

Contents

1	Questions	2
2	Answers	2
3	Random Features	4

1 Questions

1. Solve the following for x:

$$(x-3)^2 + 4x = 3x^2 + 7$$

2. Find the product of the following matrices A and B:

$$A = \begin{pmatrix} 3 & 4 & 2 \\ 1 & -1 & 0 \end{pmatrix} B = \begin{pmatrix} 2 & -1 \\ 4 & 0 \\ 6 & -2 \end{pmatrix}$$

3. It is given that:

$$\frac{dx}{dt} = 10 - 4x$$

Find an expression for x(t) and plot the value for $0 \le t \le 5$ if x(0) = 1.

4. Write a function to calculate the nth Fibonacci Number using the programming language of your choice.

2 Answers

1.

$$(x-3)^{2} + 4x = 3x^{2} + 7$$

$$x^{2} - 6x + 9 + 4x = 3x^{2} + 7$$

$$2x^{2} + 2x - 2 = 0$$

$$x^{2} + x - 1 = 0$$

$$x = \frac{-1 \pm \sqrt{1^{2} - 4 \times 1 \times -1}}{2}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

2.

$$A = \begin{pmatrix} 3 & 4 & 2 \\ 1 & -1 & 0 \end{pmatrix} B = \begin{pmatrix} 2 & -1 \\ 4 & 0 \\ 6 & -2 \end{pmatrix}$$

$$AB = \begin{pmatrix} 3 \times 2 + 4 \times 0 + 2 \times 6 & 3 \times -1 + 4 \times 0 + 2 \times -2 \\ 1 \times 2 - 1 \times 4 + 0 \times 6 & 1 \times -1 - 1 \times 0 + 0 \times -2 \end{pmatrix}$$
$$= \begin{pmatrix} 34 & -7 \\ -2 & -1 \end{pmatrix}$$

3.

$$\frac{dx}{dt} = 10 - 4x$$

$$\frac{dx}{10 - 4x} = dt$$

$$\int \frac{dx}{10 - 4x} = \int dt$$

$$-\frac{1}{4}\ln(10 - 4x) = t + C$$

$$\ln 10 - 4x = -4(t + C)$$

$$10 - 4x = e^{-4(t + C)}$$

$$4x = 10 - e^{-4(t + C)}$$

$$x = 2.5 - \frac{1}{4}e^{-4(t + C)}$$

$$= 2.5 - Ae^{-4t}$$

When t = 0, x = 1.

1 = 2.5 -
$$Ae^0$$
 = 2.5 - A
∴ A = 1.5
∴ x = 2.5 - 1.5 e^{-4t}

This is shown in figure 1:

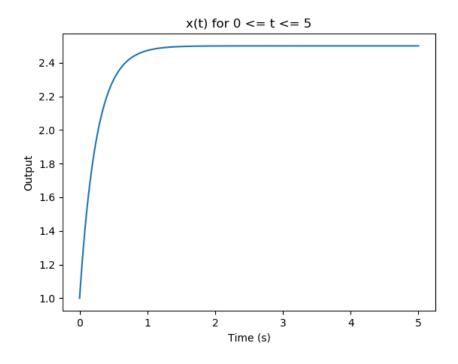


Figure 1: Plot of x

4. The Fibonacci sequence is defined by $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ (Wikipedia (2018)):

```
\begin{array}{lll} def & fib\,(n\,)\colon \\ & if & n <= \,0\colon \\ & & return \,\, 0 \\ & elif & n == \,1\colon \\ & & return \,\, 1 \\ & else \colon \\ & & return \,\, fib\,(n-1) \,\, + \,\, fib\,(n-2) \end{array}
```

3 Random Features

Things to do today:

- Run workshop
- Eat lunch
- \bullet Finish assignment

References

Wikipedia (2018), 'Fibonacci number'.

 $\mathbf{URL:}\ https://en.wikipedia.org/wiki/Fibonacci_number$