Classical Physics Summary

Mathematical Notes

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Contents

1 Classical Mechanics

1.1 Newton's Laws

1. **First Law**: A body at rest remains at rest, and a body in motion continues in uniform motion, unless acted upon by an external force.
2. **Second Law**: $\vec{F} = m\vec{a}$ or $\vec{F} = \frac{d\vec{p}}{dt}$ where $\vec{p} = m\vec{v}$ is momentum.
3. **Third Law**: For every action, there is an equal and opposite reaction.

1.2 Kinematics

For constant acceleration:

$$\vec{v} = \vec{v}_0 + \vec{a}t \tag{1}$$

$$\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}t^2 \tag{2}$$

$$v^2 = v_0^2 + 2\vec{a} \cdot (\vec{r} - \vec{r}_0) \tag{3}$$

1.3 Work and Energy

Definition 1.1. The **work** done by a force $\vec{F}$ over a displacement $d\vec{r}$ is:

$$W = \int \vec{F} \cdot d\vec{r}$$

Definition 1.2. The **kinetic energy** is $T = \frac{1}{2}mv^2$.

Definition 1.3. The **potential energy** U is defined such that $\vec{F} = -\nabla U$ for conservative forces.

Theorem 1.1 (Work-Energy Theorem)**.**

$$W = \Delta T = T_f - T_i$$

Theorem 1.2 (Conservation of Energy)**.** For conservative forces: $T + U = \text{constant}$.

1.4 Angular Motion

- Angular velocity: $\vec{\omega} = \frac{d\theta}{dt}\hat{n}$
- Angular acceleration: $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$
- Torque: $\vec{\tau} = \vec{r} \times \vec{F}$
- Angular momentum: $\vec{L} = \vec{r} \times \vec{p}$
- Moment of inertia: $I = \sum_i m_i r_i^2$ (discrete) or $I = \int r^2 dm$ (continuous)

1.5 Rotational Dynamics

- $\vec{\tau} = I\vec{\alpha}$
- $\vec{L} = I\vec{\omega}$
- Rotational kinetic energy: $T_{\text{rot}} = \frac{1}{2}I\omega^2$

1.6 Lagrangian Mechanics

Definition 1.4. The **Lagrangian** is $L = T - U$ where T is kinetic energy and U is potential energy.

Theorem 1.3 (Euler-Lagrange Equations)**.** For generalized coordinates q_i:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

1.7 Hamiltonian Mechanics

Definition 1.5. The **generalized momentum** is $p_i = \frac{\partial L}{\partial \dot{q}_i}$.

Definition 1.6. The **Hamiltonian** is $H = \sum_i p_i \dot{q}_i - L$.

Theorem 1.4 (Hamilton's Equations)**.**

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q_i}$$

2 Thermodynamics

2.1 Zeroth Law

Definition 2.1. If two systems are each in thermal equilibrium with a third system, they are in thermal equilibrium with each other.

2.2 First Law

Theorem 2.1 (First Law of Thermodynamics)**.**

$$\Delta U = Q - W$$

where U is internal energy, Q is heat added, and W is work done by the system.

2.3 Second Law

Theorem 2.2 (Second Law of Thermodynamics)**.** Heat cannot spontaneously flow from a colder body to a hotter body. In terms of entropy:

$$\Delta S \geq \frac{Q}{T}$$

with equality for reversible processes.

2.4 Entropy

Definition 2.2. The **entropy** change for a reversible process is:

$$\Delta S = \int \frac{dQ_{\text{rev}}}{T}$$

2.5 Thermodynamic Potentials

- **Internal Energy**: $U = TS - PV + \mu N$
- **Helmholtz Free Energy**: $F = U - TS$
- **Gibbs Free Energy**: $G = H - TS = U + PV - TS$
- **Enthalpy**: $H = U + PV$

2.6 Ideal Gas Law

$$PV = nRT = Nk_BT$$

where $R = 8.314$ J/mol·K is the gas constant and $k_B = 1.381 \times 10^{-23}$ J/K is Boltzmann's constant.

2.7 Kinetic Theory

For an ideal gas:

- Average kinetic energy per molecule: $\langle K \rangle = \frac{3}{2}k_BT$
- Root-mean-square speed: $v_{\text{rms}} = \sqrt{\frac{3k_BT}{m}}$
- Mean free path: $\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$

3 Electromagnetism

3.1 Coulomb's Law

Theorem 3.1 (Coulomb's Law)**.** The force between two point charges is:

$$\vec{F} = k_e \frac{q_1 q_2}{r^2}\hat{r}$$

where $k_e = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9$ N·m²/C².

3.2 Electric Field

Definition 3.1. The **electric field** is $\vec{E} = \frac{\vec{F}}{q}$.

For a point charge: $\vec{E} = k_e \frac{q}{r^2}\hat{r}$

3.3 Gauss's Law

Theorem 3.2 (Gauss's Law)**.**

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

3.4 Electric Potential

Definition 3.2. The **electric potential** is $V = \frac{U}{q}$ where U is electric potential energy.

$$\vec{E} = -\nabla V$$

3.5 Capacitance

Definition 3.3. The **capacitance** is $C = \frac{Q}{V}$.

For a parallel plate capacitor: $C = \frac{\epsilon_0 A}{d}$

3.6 Current and Resistance

- Current: $I = \frac{dQ}{dt}$
- Current density: $\vec{J} = nq\vec{v}_d$
- Ohm's Law: $V = IR$
- Resistance: $R = \frac{\rho L}{A}$

3.7 Magnetic Field

Definition 3.4. The **magnetic force** on a moving charge is $\vec{F} = q\vec{v} \times \vec{B}$.

3.8 Biot-Savart Law

Theorem 3.3 (Biot-Savart Law)**.**

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

3.9 Ampère's Law

Theorem 3.4 (Ampère's Law)**.**

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

3.10 Faraday's Law

Theorem 3.5 (Faraday's Law of Induction)**.**

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

where $\Phi_B = \int \vec{B} \cdot d\vec{A}$ is magnetic flux.

3.11 Maxwell's Equations

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \text{(Gauss's Law)} \tag{4}$$

$$\nabla \cdot \vec{B} = 0 \quad \text{(Gauss's Law for Magnetism)} \tag{5}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{(Faraday's Law)} \tag{6}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{(Ampère-Maxwell Law)} \tag{7}$$

4 Waves and Oscillations

4.1 Simple Harmonic Motion

Definition 4.1. A system undergoes **simple harmonic motion** if it satisfies:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

The solution is $x(t) = A\cos(\omega t + \phi)$ where:

- A is amplitude
- $\omega = \sqrt{\frac{k}{m}}$ is angular frequency
- ϕ is phase constant

4.2 Wave Equation

Theorem 4.1 (Wave Equation)**.**

$$\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$$

where v is wave speed.

4.3 Wave Properties

- Wavelength: λ
- Frequency: $f = \frac{\omega}{2\pi}$
- Wave speed: $v = f\lambda = \frac{\omega}{k}$
- Wave number: $k = \frac{2\pi}{\lambda}$

4.4 Standing Waves

For a string fixed at both ends:

$$y(x,t) = A\sin(kx)\cos(\omega t)$$

with boundary conditions $y(0,t) = y(L,t) = 0$ giving:

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

4.5 Sound Waves

- Speed of sound: $v = \sqrt{\frac{B}{\rho}}$ where B is bulk modulus
- Intensity: $I = \frac{P}{A} = \frac{1}{2}\rho v \omega^2 A^2$
- Decibel level: $\beta = 10\log_{10}\left(\frac{I}{I_0}\right)$ where $I_0 = 10^{-12}$ W/m²

4.6 Doppler Effect

For a moving source and stationary observer:

$$f' = f\frac{v}{v \pm v_s}$$

where v_s is source velocity (positive for approaching).

5 Fluid Mechanics

5.1 Fluid Statics

Theorem 5.1 (Pascal's Principle)**.** Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and walls of the container.

Theorem 5.2 (Archimedes' Principle)**.** The buoyant force on a submerged object equals the weight of the displaced fluid.

5.2 Fluid Dynamics

Theorem 5.3 (Continuity Equation)**.** For incompressible flow: $A_1v_1 = A_2v_2$

Theorem 5.4 (Bernoulli's Equation)**.** For steady, incompressible, non-viscous flow:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

5.3 Viscosity

Definition 5.1. The **viscous force** is $F = \eta A\frac{dv}{dy}$ where η is viscosity.

6 Optics

6.1 Geometric Optics

- Law of reflection: $\theta_i = \theta_r$
- Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$
- Critical angle: $\sin\theta_c = \frac{n_2}{n_1}$ (for $n_1 > n_2$)

6.2 Lens Equation

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

where f is focal length, d_o is object distance, and d_i is image distance.

6.3 Magnification

$$m = -\frac{d_i}{d_o} = \frac{h_i}{h_o}$$

6.4 Thin Lens Formula

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

6.5 Wave Optics

- Constructive interference: $\Delta\phi = 2\pi n$
- Destructive interference: $\Delta\phi = \pi(2n+1)$
- Path difference: $\Delta = d\sin\theta$

6.6 Diffraction

For single slit diffraction:

$$\sin\theta = \frac{m\lambda}{a}$$

where a is slit width and m is order number.

7 Special Relativity

7.1 Postulates

1. The laws of physics are the same in all inertial reference frames.
2. The speed of light in vacuum is constant in all inertial frames.

7.2 Lorentz Transformations

For frames moving with relative velocity v along x-axis:

$$x' = \gamma(x - vt) \tag{8}$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \tag{9}$$
$$y' = y \tag{10}$$
$$z' = z \tag{11}$$

where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$.

7.3 Time Dilation and Length Contraction

- Time dilation: $\Delta t = \gamma \Delta t_0$
- Length contraction: $L = \frac{L_0}{\gamma}$

7.4 Relativistic Energy and Momentum

- Relativistic momentum: $\vec{p} = \gamma m \vec{v}$
- Total energy: $E = \gamma mc^2$
- Rest energy: $E_0 = mc^2$
- Kinetic energy: $K = (\gamma - 1)mc^2$
- Energy-momentum relation: $E^2 = (pc)^2 + (mc^2)^2$

8 Applications

8.1 Mechanics Applications

- Planetary motion and Kepler's laws
- Rigid body dynamics
- Collision analysis
- Central force problems

8.2 Thermodynamics Applications

- Heat engines and refrigerators
- Phase transitions
- Statistical mechanics foundations
- Entropy and information theory

8.3 Electromagnetism Applications

- Circuit analysis
- Electromagnetic waves
- Antenna theory
- Plasma physics

9 Important Constants

- Speed of light: $c = 2.998 \times 10^8$ m/s
- Gravitational constant: $G = 6.674 \times 10^{-11}$ N·m²/kg²
- Electron charge: $e = 1.602 \times 10^{-19}$ C
- Electron mass: $m_e = 9.109 \times 10^{-31}$ kg
- Proton mass: $m_p = 1.673 \times 10^{-27}$ kg

- Permittivity of free space: $\epsilon_0 = 8.854 \times 10^{-12}$ F/m
- Permeability of free space: $\mu_0 = 4\pi \times 10^{-7}$ H/m
- Boltzmann constant: $k_B = 1.381 \times 10^{-23}$ J/K
- Avogadro's number: $N_A = 6.022 \times 10^{23}$ mol^{-1}