Estimating Factor Price Markdowns using Production Models

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Abstract

Factor price markdowns are a key object of interest when studying monopsony power. In this article, we test the performance of production-function-based estimators of factor price markdowns, which have been used increasingly often in the literature. We evaluate the performance of these estimators under various data generating processes using Monte Carlo simulations. We discuss the commonly-made assumptions in this class of estimators, and address the methodological challenges involved with relaxing these assumptions, such as departing from Hicks neutrality, allowing for nonsubstitutable inputs, and allowing for various types of labor market conduct.

Keywords: Monopsony, Markdowns, Factor-Biased Technological Change, Production Function Estimation
JEL Codes: L11, J42, O33

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1 Introduction

There is an increased interest in the monopsony power of firms on their labor and other factor markets. Production functions are increasingly used to estimate ‘factor price markdowns’, which are a key object of interest when studying monopsony power.[1] However, there is a lack of evidence on the performance of these estimators. To fill this gap, we conduct Monte Carlo simulations in which we generate oligopsonistic labor market equilibria. We use these simulated data to estimate factor price markdowns using existing production function-based estimators. Comparing these estimates to the true markdown distribution enables us how well these ‘cost-based’ estimators succeed at recovering true markdowns under a variety of data generating processes.

We start the paper by discussing existing production approaches to markdown estimation, which extend the markup estimation approach of De Loecker and Warzynski (2012) and Hall (1988) to allow for endogenous factor prices (Morlacco, 2017; Brooks, Kaboski, Li, & Qian, 2021; Yeh, Hershbein, & Macaluso, 2022; Mertens, 2019; Rubens, 2023; Delabastita & Rubens, in press; Mertens & Schoefer, 2024). These approaches have in common that they assume that the price of at least one variable input is exogenous, and normalize the markup expressions from the cost minimization first order conditions of all other variable inputs compared to the variable input with the exogenous input price.[2] This allows recovering the markdowns for all other inputs.

We test this class of markdown estimators by simulating data that are generated under a discrete-choice labor supply model in the spirit of Berry (1994) and Card, Cardoso, Heinig, and Kline (2018), in which we let firms compete oligopsonistically à la Nash-Bertrand.[3] On the labor demand side, we assume cost-minimizing firms that produce following a Cobb-Douglas production function with two variable inputs, labor and materials. Given the oligopsonistic labor supply side, labor wages are endogenous to individual firms, meaning that the residual labor supply curves are upward-sloping. In contrast, we assume that material prices are taken as given by the individual firms. We run a Monte Carlo simulation in which we sample 250 firms that operate in 50 different markets during 10 years, and solve for labor market equilibrium in each of these markets. We resample this simulation exercise for

[1] See Syverson (2024) for an excellent survey of the markups and markdowns literature.
[2] This identification approach was proposed in Appendix D of De Loecker, Goldberg, Khandelwal, and Pavcnik (2016).
[3] In contrast to Card et al. (2018), which imposes monopsonistic competition, we allow for granular employers, similarly to Berger, Herkenhoff, and Mongey (2022) and Azar, Berry, and Marinescu (2022).

200 random draws. We find that under the Hicks-neutral data generating process (DGP), the production-based markdown estimator delivers consistent and precise estimates of wage markdowns and of the production function coefficients.

Next, we argue that Hicks neutrality is the key assumption needed for markdown identification, as discussed in more detail in Rubens, Wu, and Xu (2024). We relax this assumption by allowing for random coefficients in the production function. Such random coefficients arise if there is unobserved technological heterogeneity between firms, as is likely to occur in many applications. We re-run our Monte Carlo simulation under an identical labor supply side, but with the non-Hicks-neutral production function on the labor demand side. We find that in contrast to the Hicks-neutral DGP, both the production function coefficients and the wage markdowns are estimated with considerable bias. The main reason for this bias is that latent technological heterogeneity and wage markdowns are not separately identified using the cost-minimization first order conditions.

We test an alternative production function estimator, proposed by Rubens et al. (2024), which adapts the production function estimator of Doraszelski and Jaumandreu (2018) to allow for imperfect factor market competition. We find that this estimator delivers consistent estimates of the production function and of markdowns when there is unobserved heterogeneity in the production function coefficients. We also simulate the performance of this estimator when the true production function is Hicks-neutral, and find that the estimator of Rubens et al. (2024) is less efficient than the Hicks-neutral markdown estimator in this case, but still recovers the true production function coefficients with a minimal bias.

We end the paper by discussing how some of the other commonly made assumptions have been relaxed in the literature. We discuss allowing for non-substitutable inputs in production, labor market conduct assumptions other than Nash-Bertrand, differentiated goods and inputs, adjustment frictions, and cases in which none of the factor prices can be reasonably assumed to be competitive.

The remainder of this paper is structured as follows. In Section 2, we discuss cost-side markdown estimators when the production function is Hicks-neutral. In Section 3, we introduce technological heterogeneity in the production function, which relaxes the assumption of Hicks-neutrality. Section 4 discusses further extensions, and Section 5 concludes.

2 Markdown Estimation under Hicks Neutrality

2.1 Primitives

In this section, we test the canonical 'production approach' to markdown estimation, which relies on Hicks-neutrality. We start by discussing the model primitives: the production function and the factor supply model.

Production Function

Let firms be indexed by f and time periods by t. We assume firms use two factors of production: labor L_{ft} and materials M_{ft}, which are transformed into a scalar output level Q_{ft} following a production function $H(.)$.

$$Q_{ft} = H(L_{ft}, M_{ft}, \beta)\Omega_{ft} \tag{1a}$$

We start by highlighting three assumptions. As a convention throughout the paper, we list the assumptions that are used to estimate the production function and markdowns, whereas other assumptions that are used to simulate the data but are not key to estimate the production function are simply mentioned, but not listed separately.

***Assumption* 1** *There is a scalar unobservable in production, Ω_{ft}.*

Assumption 1 imposes Hicks neutrality, as it rules out unobserved heterogeneity in the production coefficients β. We relax this assumption in Section 3.

***Assumption* 2** *The production function $H(.)$ is twice differentiable.*

Assumption 2 rules out perfect complementarities between inputs, such as in a Leontief production function. We relax this assumption in Section 4.1.

***Assumption* 3** *Both the good Q_{ft} and the inputs L_{ft} and M_{ft} are homogeneous.*

Assumption 3 assumes that both the produced good and the inputs are undifferentiated. We discuss how to relax this assumption in Section 4.4.

For the simulations, we impose a simple Cobb-Douglas production function, which in logs gives Equation (1b), but any production function that satisfies Assumptions 2 and 1 can be used. The output elasticities of labor and materials are denoted as β^l and β^m.

$$q_{ft} = \beta^l l_{ft} + \beta^m m_{ft} + \omega_{ft} \tag{1b}$$

We impose an AR(1) transition process for Hicks-neutral productivity with serial correlation ρ and i.i.d. productivity shocks e_{ft}. This assumption is useful for estimation using a dynamic panel approach, but not strictly necessary.

$$\omega_{ft} = \rho\omega_{ft-1} + e_{ft} \tag{2}$$

***Assumption* 4** *Labor and materials are variable, static inputs.*

Finally, we assume that both labor and materials are variable, static inputs, meaning that they are not subject to adjustment frictions and fully depreciate during every period.[4]

Labor Supply

We impose a discrete choice model of labor supply with oligopsonistic competition in the spirit of Berry (1994) and Card et al. (2018), to simulate an environment in which markdown vary between firms, and in which firms set wages strategically. Firms pay per-unit wages W^l_{ft} to workers i, who choose their employment between a set of firms, $\mathcal{F}_t$, with $f = 0$ indicating the outside option of being unemployed. We assume that firms are not able to wage-discriminate between their homogeneous workers. We assume that the utility of a worker i that works at firm f depends on the wage W_{ft}, an unobserved amenity ξ_{ft}, and an i.i.d. type-I distributed manufacturer-worker error term υ_{ift}, as shown in Equation (3).

$$U_{ift} = \underbrace{\gamma W_{ft} + \xi_{ft}}_{\equiv \delta_{ft}} + \upsilon_{ift} \tag{3}$$

We denote mean utility as δ_{ft} and normalize the utility of the outside option to zero, as usual: $U_{i0t} = 0$. Using the logit formula, the labor market share $s_{ft} = \frac{L_{ft}}{\sum_{g\in\mathcal{F}_t} L_{gt}}$ is given by:

$$s_{ft} = \frac{\exp(\delta_{ft})}{\sum_{g\in\mathcal{F}_t}\exp(\delta_{gt})}$$

[4] Fixed inputs, such as capital, can be added to the model, but need to be solved using a dynamic investment model, rather than the static cost minimization problem for the variable inputs. However, fixed inputs cannot be used to identify markdowns in this approach, as it requires normalizing first-order conditions from the static cost minimization approach.

Denoting the labor force as $\overline{L}$, the labor supply function $H(.)$ is given by:

$$L_{ft} = \frac{\exp(\gamma \ln(W_{ft}) + \xi_{ft})}{\sum_{g \in \mathcal{F}_t} \exp(\gamma \ln(W_{ft}) + \xi_{ft})} \overline{L} \tag{4}$$

We denote the inverse residual supply elasticities of labor and mateirals as $\psi^l_{ft} - 1$ and $\psi^m_{ft} - 1$, such that:

$$\psi^l_{ft} \equiv \frac{\partial W^l_{ft}}{\partial L_{ft}} \frac{L_{ft}}{W^l_{ft}} + 1 \qquad \psi^m_{ft} \equiv \frac{\partial W^m_{ft}}{\partial M_{ft}} \frac{M_{ft}}{W^m_{ft}} + 1 \tag{5}$$

Given the logit labor supply structure above, the residual inverse labor supply elasticity faced by firm f, $(\psi^l_{ft} - 1)$ is given by:

$$\psi^l_{ft} - 1 = \frac{1}{\gamma(1 - s_{ft})} \tag{6}$$

2.2 Behavioral Assumptions

Producers choose inputs in every period to minimize current variable costs. We denote marginal costs as λ_{ft}, such that the cost minimization problem is given by Equation (7):

$$\min_{W^l_{ft}, M_{ft}} \Big[W^m_{ft} M_{ft} + W^l_{ft} L_{ft} - \lambda_{ft} \big(Q_{ft} - G(.) \big) \Big] \tag{7}$$

As shown in De Loecker et al. (2016), the markup of the final goods price P_{ft} over marginal costs, $\mu^p_{ft} \equiv (P_{ft} - \lambda_{ft})/\lambda_{ft}$, is equal to Equation (8):

$$\mu^p_{ft} = \frac{\beta^j_{ft}}{\alpha^j_{ft} \psi^j_{ft}} - 1 \quad \forall j = l, m \tag{8}$$

where α^j_{ft} denotes the expenditure on input j as a share of gross revenues of firm f in year t, such that $\alpha^l_{ft} \equiv W^l_{ft} L_{ft} / P_{ft} Q_{ft}$ and $\alpha^m_{ft} \equiv W^m_{ft} L_{ft} / P_{ft} Q_{ft}$. Following Morlacco (2017), Brooks et al. (2021), and Yeh et al. (2022), the inverse supply elasticity of labor can be expressed relatively to the inverse supply elasticity of materials by weighting the ratio of input expenditures by the respective output elasticities of both inputs:

$$\psi^l_{ft} = \frac{\beta^l}{\beta^m}\frac{\alpha^m_{ft}}{\alpha^l_{ft}}\psi^m_{ft} \tag{9}$$

The wage markdown $\mu^w_{ft} \equiv (MRPL_{ft} - W_{ft})/MRPL_{ft}$ can be expressed in function of this inverse labor supply elasticity:

$$\mu^w_{ft} = \frac{\psi^l_{ft} - 1}{\psi^l_{ft}} \tag{10}$$

The more inelastic the labor supply curve, the greater a firm's ability to exercise monopsony power and suppress wages.

***Assumption* 5** *Residual intermediate input supply is perfectly price elastic:* $\psi^m_{ft} = 1$.

Assumption 5 implies that intermediate input prices are exogeneous to individual firms. As can be seen in Equation (8), this assumption allows to point-identify the wage markdown, rather than just the relative wage markdown compared to the material price markdown.

Solving the cost minimization problem, Equation (7), delivers the following labor demand function in the Cobb-Douglas case, denoting factor prices as W^m_{ft}, W^l_{ft}:

$$L_{ft} = \Big[\frac{\beta^l}{W^l_{ft}\psi^l_{ft}}\big(\frac{\beta^m\Omega_{ft}}{W^m_{ft}}\big)^{\frac{\beta^m}{1-\beta^m}}\Omega_{ft}\Big]^{\frac{1-\beta^m}{1-\beta^l-\beta^m}} \tag{11}$$

Optimal intermediate input demand is equal to:

$$M_{ft} = \Big(\frac{\beta^m L^{\beta^l}_{ft}\Omega_{ft}}{W^m_{ft}}\Big)^{\frac{1}{1-\beta^m}}$$

2.3 Identification and Estimation

Due to Assumption 4 (input variability) and the AR(1) law of motion for productivity, Equation (2), the production function can be estimated using a dynamic panel approach. Taking ρ-differences, as in Blundell and Bond (2000), the productivity shock can be written as:

$$e_{ft} = q_{ft} - \rho q_{ft-1} - \beta^l(l_{ft} - \rho l_{ft-1}) - \beta^m(m_{ft} - \rho m_{ft-1})$$

Similarly to Ackerberg, Caves, and Frazer (2015), assuming that labor and materials are both variable inputs, the following moment conditions are formed for lags $r = 1$ up to $r = T - 1$, with the panel length being denoted as T. As in Ackerberg et al. (2015), the identifying assumption is that the variable inputs (in our case, materials and labor) are chosen after the firm observes the productivity shock e_{ft}.

$$\mathbb{E}\Big[e_{ft}(\rho, \beta^l, \beta^m)| \begin{pmatrix} L_{ft-r} \\ M_{ft-r} \end{pmatrix}\Big]_{r=1}^{T-1} = 0 \tag{12}$$

We estimate the production function coefficients (β^l, β^m) using these moment conditions, for two time lags. We use the resulting estimates $(\hat{\beta}^l, \hat{\beta}^m)$ to estimate the wage markdown ψ^l_{ft} using Equation (9). Hence, the inverse residual labor supply elasticity is estimated from the production function alone, without requiring to estimate the labor supply parameters γ and ξ_{ft}.[5]

2.4 Monte Carlo Simulation

Parametrization

We simulate a dataset of 50 independent labor markets that each contain 5 firms, which are observed during 10 years. Hence, the simulated dataset contains 250 firms that are observed during 10 times each $(N = 2500)$. We parametrize the true output elasticities of labor and materials at $\beta^l = 0.5$ and $\beta^m = 0.3$. We let intermediate input prices W^m_{ft} in the first year be distributed as a normal distribution $W^m_{f1} \sim \mathcal{N}(5, 0.05)$ and let it evolve by firm-level shocks that are $\mathcal{N}(0, 0.01)$ distributed. Similarly, we let the initial log productivity distribution be normally distributed $\omega_{f1} \sim \mathcal{N}(1, 0.01)$ and let the productivity shocks be $\mathcal{N}(0, 0.01)$ distributed. The serial correlation in productivity is set at $\rho = 0.6$. The resulting distribution of log productivity has a mean of 1/4 and a standard deviation of 1/3. We normalize the total labor market size to one.

Solving for Equilibrium

We conduct a Monte Carlo simulation with 200 independent draws. For each iteration, we numerically solve the model by finding the equilibrium wages and market shares of all firms such that labor demand (11) equals labor supply (4) at each firm in each year, and that the markets clear on aggregate.

[5]Of course, we specified the logit labor supply side to simulate the data, but the labor supply curve does not need to be estimated when estimating Equation 9.

We estimate the production function parameters ρ, β^l, and β^m on the resulting dataset using the moment conditions from Equation (12), and then plug these estimates into Equation (9) to estimate the inverse residual labor supply elasticities ψ^l_{ft} at all firms in every year.

Results

The distribution of the resulting estimates are visualized in the solid blue lines in Panel (a) of Figure 1. We find that the Hicks-neutral production function estimator yields consistent and precise estimates of the output elasticities of labor and materials. As summarized in Panel (a) of Table 1, the output elasticities of labor and materials are estimated at their true values of 0.5 and 0.3, with the standard deviation of these estimates across bootstrap iterations being very small, at 0.003 for labor and below 0.001 for materials. Hence, the production function is identified even if the labor market is imperfectly competitive. Moreover, we find that the production function estimator delivers a consistent estimate of the wage markdown ψ^l_{ft}, which is on average 1.614 and estimated at 1.615, with a standard deviation across draws of merely 0.009.

It is worth noting that the assumptions imposed on the labor supply model and on conduct were only necessary to simulate the dataset, but were not used for production function estimation: we estimated markdowns correctly using the production function while remaining agnostic about the model of competition on the labor market, the functional form for labor utility, and the distribution of wage markdowns.

3 Introducing Unobserved Technological Heterogeneity

3.1 Extended Model

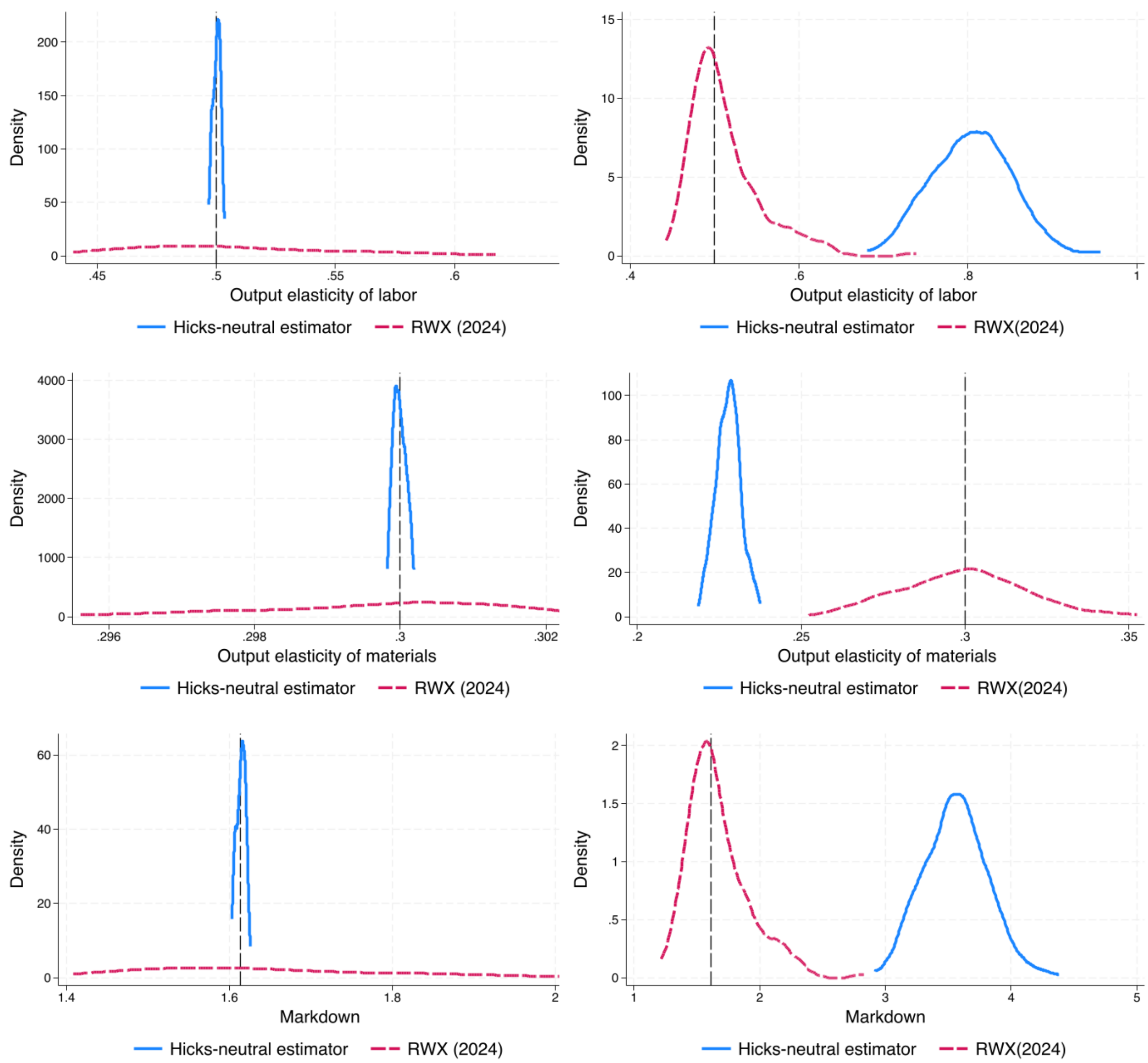
We now revisit the identification approach outlined in Section 2 by relaxing Assumption 1, which imposed Hicks-neutrality. Hicks-neutrality is assumed in most of the production function-based markdown estimators, including Morlacco (2017); Brooks et al. (2021); Yeh et al. (2022); Mertens (2019); Rubens (2023); Delabastita and Rubens (in press); Mertens and Schoefer (2024).

Instead of the Cobb-Douglas production function with constant output elasticities from Equation (1b), we allow for unobserved random coefficients β^l_{ft}, β^m_{ft} , as shown in Equation (13).

$$q_{ft} = \beta^l_{ft} l_{ft} + \beta^m_{ft} m_{ft} + \omega_{ft} \tag{13}$$

Figure 1: Monte-Carlo Simulations

(a) Hicks-neutral DGP: **(b) Factor-Biased DGP:**



We let the output elasticities of labor and materials be distributed around the same values β^l and β^m as before, with idiosyncratic error terms ϵ^l_{ft} and ϵ^m_{ft}:

$$\begin{cases} \beta^l_{ft} &= \beta^l + \epsilon^l_{ft} \\ \beta^m_{ft} &= \beta^m + \epsilon^m_{ft} \end{cases}$$

Equation (13) is a simple way of allowing for unobserved heterogeneity while maintaining the analytical simplicity of the Cobb-Douglas production function. We refer to Rubens et al. (2024) for estimation of a Constant Elasticity of Substitution production function under imperfect labor market competition, and for an empirical application in the context of the Chinese nonferrous metals industry.

3.2 Identification Challenge

We rewrite the markdown estimator from Equation (9) while relaxing the homogeneous output elasticities assumption. Equation (14) makes clear that to identify the wage markdown, it is crucial to fully estimate the random coefficients β^m_{ft} and β^l_{ft}.

$$\psi^l_{ft} = \frac{\beta^l_{ft}}{\beta^m_{ft}} \frac{\alpha^m_{ft}}{\alpha^l_{ft}} \psi^m_{ft} \tag{14}$$

Although there is a literature on estimating production functions with non-scalar unobservables, such as Doraszelski and Jaumandreu (2018) and Demirer (2019), these estimators rely on the assumption of perfect factor market competition, which imposes $\psi^l_{ft} = \psi^m_{ft} = 1$. In contrast, Rubens et al. (2024) develops an estimator that allows for both non-scalar unobservables in production and imperfect factor market competition. We lay out this estimation procedure below in the context of our simple production model.

3.3 Estimation

Rubens et al. (2024) relies on jointly estimating the labor supply curve and the production function. Using the discrete choice labor supply model imposed above, the labor supply equation to be estimated is given by:

$$s_{ft} - s^0_{it} = \gamma \ln(W_{ft}) + \xi_{ft} \tag{15}$$

Under the assumption of Nash-Bertrand conduct, the markdown ψ^l_{ft} can be recovered as a function of the estimated parameter $(\hat{\gamma})$ and the observed labor market share s_{ft}:

$$\hat{\psi}^l_{ft} = 1 + \frac{1}{\hat{\gamma}(1 - s_{ft})} \tag{16}$$

Then, from the first order conditions, one can express the output elasticity of labor as a function of the estimated wage markdown $\hat{\psi}^l_{ft}$, the observed revenue shares α^l_{ft} and α^m_{ft}, and the yet-to-be-estimated materials coefficient β^m.

$$\hat{\beta}^l_{ft} = \frac{\hat{\psi}^l_{ft}\alpha^l_{ft}\beta^m}{\alpha^m_{ft}} \tag{17}$$

Substituting this output elasticity of labor into the production function results in Equation (18), in which the term $a_{ft} \equiv \frac{\hat{\psi}^l_{ft}\alpha^l_{ft}l_{ft}}{\alpha^m_{ft}} + m_{ft}$ is composed solely of observed and estimated terms. Hence, the error term in the production function is again reduced to a scalar unobservable ω_{ft}.

$$q_{ft} = \beta^m \underbrace{[\frac{\hat{\psi}^l_{ft}\alpha^l_{ft}l_{ft}}{\alpha^m_{ft}} + m_{ft}]}_{a_{ft}} + \omega_{ft} \tag{18}$$

Again using the equation of motion for productivity, we isolate the productivity shock e_{ft} as:

$$e_{ft} = q_{ft} - \rho q_{ft-1} - \beta^m(a_{ft} - \rho a_{ft-1})$$

The moment conditions to estimate the parameters (β^m, ρ) are given by:

$$\mathbb{E}\Big[e_{ft}(\rho, \beta^m)| \begin{pmatrix} L_{ft-r} \\ M_{ft-r} \end{pmatrix} \Big]_{r=1}^{T-1} = 0 \tag{19}$$

We again estimate the production function parameters taking up to two lags. Using the estimated materials coefficient $\hat{\beta}^m$, the full distribution of the output elasticities of labor β^l_{ft} s can be recovered using Equation (17), which is now a function of data and estimated parameters.

3.4 Monte Carlo Simulations

Parametrization and Estimation

We keep the same parametrization of the Monte Carlo simulation in Section 2, with the only difference that we now allow for unobserved heterogeneity in the output elasticity of labor. We parametrize this unobserved heterogeneity as $\beta^l_{ft} \sim \mathcal{U}[\frac{1}{3}, \frac{2}{3}]$. We solve for labor market equilibrium using the same procedure that was outlined in Section 2.4.

We estimate the production function twice. First we "naively" estimate the production function assuming the DGP is Hicks-neutral, using the moment conditions in Equation (12), and estimate the markdown using the cost-side markdown estimator from Equation (9).

Second, we estimate the production function using the estimation procedure from Rubens et al. (2024) that was outlined above. We start by estimating Equation (15). Given the latent firm amenities ξ_{ft}, we need to find an instrument for wages that is excluded from the error term ξ_{ft}. We assume that a labor demand shifter z is available, which we construct as a variable that is correlated with productivity but uncorrelated to the amenity firm ξ_{ft}. We parametrize this labor demand shifter as the sum of TFP and an error term u_{ft}, which is normally distributed with a zero mean and standard deviation of 0.01.

$$z_{ft} = \frac{\omega_{ft}}{2} + u_{ft}$$

With this labor demand shifter at hand, we estimate the labor supply curve (15) using 2SLS. Using the estimated parameter $\hat{\gamma}$ and the observed labor market share s_{ft}, we compute wage markdowns following Equation (16). Finally, we substitute the markdown estimate ψ^l_{ft} into Equation (18) and form the moment conditions (19) to estimate the production function parameters β^m and ρ. The full distribution of the output elasticities α^l_{ft} and α^m_{ft} can then be recovered using Equation (18)

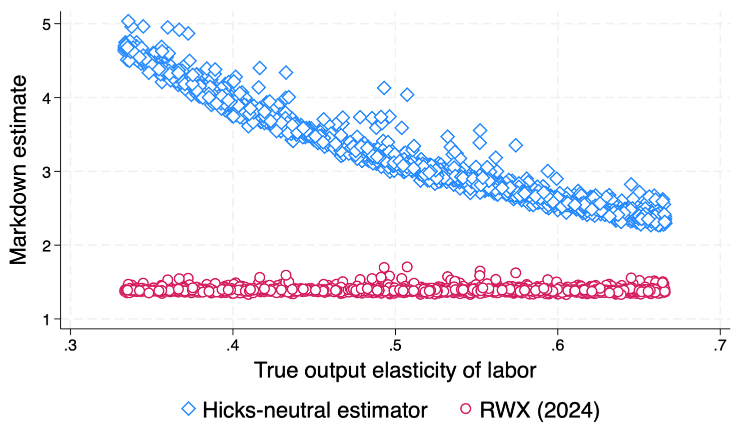
Results under the Factor-Biased Data Generating Process

We visualize the production function estimates for the DGP with random coefficients in the production function in panel (b) of Figure 1. The solid blue lines in Figure 1 report the estimates using the Hicks-neutral production function estimator that assumes homogeneous output elasticities. It is clear that the markdown estimator that relies on Hicks neutrality does a poor job a estimating the production function coefficients: the labor coefficient is estimated at 0.8, which is 60% above tis true value, whereas the materials coefficient is estimated at

0.23, which is 25% below the true value. As a result, the Hicks-neutral model estimates the inverse labor supply elasticity at 3.559 on average, which is three times higher than the true average value of 1.613. This leads the econometrician to believe that wages are marked down by 72% below the marginal revenue product of labor,[6] whereas wages are in reality marked down by 38%.

Figure 2 shows the source of the identification problem by plotting the estimated inverse labor supply elasticity estimates against the true output elasticity of labor, β^l_{ft} across observations in a single bootstrap iteration (the first of the 200 iterations), for both estimators. In the Hicks-neutral model, the latent variation in the output elasticity of labor is interpreted as wage markdown variation: firms with high output elasticities of labor are estimated to set a low wage markdown, because their cost share of labor is higher than average. In contrast, our estimator delivers inverse labor supply elasticity estimates that are independent of the output elasticity of labor, as is true in the underlying DGP.

Figure 2: Markdowns and the Output Elasticity of Labor



The red dashed lines in panel (b) of Figure 1 plot the estimates using the method of Rubens et al. (2024) for the random coefficients DGP. The markdown is estimated with a small negative bias, which is due to the small-sample properties of the instrumental variables estimator of labor supply, but close to the true value of 1.613. Turning to the production function coefficients, we find that our estimator delivers consistent output elasticity estimates. Hence, the production function can be estimated even with random coefficients and imperfect labor market competition, but it needs to be estimated jointly with the labor supply curve.

[6] $(1 - 1/3.559)$

Results under the Hicks-Neutral Data Generating Process

How does the estimator of Rubens et al. (2024) perform if there is in reality no unobserved heterogeneity in the output elasticities? Panel (a) of Table 1 shows that the output elasticities of labor is still estimated reasonable close to the truth, at 0.516, which implies an upward bias of 3.2%, whereas the materials elasticity is estimated consistently. The standard errors on these estimates, which are 0.072 and 0.003 for labor and materials, respectively, are much higher than when using the Hicks-neutral estimator, but still relatively precise. The full distribution of the output elasticity and markdown estimates are visualized as the red lines in Panel (a) of Figure 1.

Assuming Exogenous Input Prices

Finally, we re-estimate the production function under both DGPs using the method of Rubens et al. (2024), but assume exogenous input prices. This effectively corresponds to the estimator of Doraszelski and Jaumandreu (2018). We find that imposing exogenous input prices when the true DGP is oligopsonistic and Hicks-neutral results in a serious bias in the materials coefficient, which is estimated at 0.492 whereas the true β^m is 0.3, as can be seen in the middle columns of Panel (a) in Table 1. The estimates are very similar when the the DGP is factor-biased, as shown in Panel (b) of Table 1.

4 Further Extensions

4.1 Nonsubstitutable Inputs

Rubens (2023) relaxes Assumption 2 by allowing for labor and materials to be perfect complements, while still allowing labor to be substitutable with other inputs, such as capital K. The updated production function is given by Equation (20), which is used in Rubens (2023) to study cigarettes production in China. For tobacco leaves, and for many other types of intermediate inputs, it is more reasonable to assume perfect complementarity with labor, rather than substitutability.

$$Q_{ft} = \min\{\beta^l l_{ft} \beta^k k_{ft}; \beta^m m_{ft}\}\Omega_{ft} \tag{20}$$

As shown in Rubens (2023), which introduces imperfect factor market competition to the model of De Loecker and Scott (2022) , the markup now takes on a different form, which

Table 1: Monte Carlo Simulations: Summary

(a) DGP 1: Hicks-neutral		Hicks-neutral estimator		RWX(2024) with exo. wage		RWX(2024) with endo. wage	
		Est.	S.E.	Est.	S.E.	Est.	S.E.
mean(β^l)	true = 0.5	0.500	0.003	0.508	0.000	0.516	0.072
sd(β^l)	true = 0	0.000	0.000	0.006	0.000	0.002	0.002
β^m	true = 0.3	0.300	0.000	0.492	0.000	0.299	0.003
ψ^l	true = 1.614	1.615	0.009	0.000	0.000	1.670	0.251
$corr(\beta^l, \psi^l)$		0.000	0.000	0.000	0.000	-0.008	0.997

(b) DGP 2: Random coefficients		Hicks-neutral estimator		RWX(2024) with exo. wage		RWX(2024) with endo. wage	
		Est.	S.E.	Est.	S.E.	Est.	S.E.
mean(β^l)	true = 0.5	0.805	0.048	0.503	0.001	0.512	0.043
sd(β^l)	true = 0.097	0.000	0.000	0.050	0.000	0.098	0.008
β^m	true = 0.3	0.228	0.004	0.497	0.001	0.299	0.019
ψ^l	true = 1.613	3.559	0.252	0.000	0.000	1.669	0.248
$corr(\beta^l, \psi^l)$		.	.	0.000	0.000	-0.111	0.033

Notes: Monte-Carlo simulation with 200 iterations. We use the coefficients from 2SLS regressions as the initial values for GMM in the prior approach in DGP 1.

reflects that marginal costs are additive in labor and materials:

$$\mu_{ft} = (\frac{\alpha^l_{ft}}{\beta^l}\psi^l_{ft} + \alpha^m_{ft})^{-1} \tag{21}$$

In contrast to the models in Section 2 and 3, the first order conditions for labor and materials are no longer linearly independent, rather, there is a single first order condition that takes into account both input prices and input supply elasticities. The reason for this reduction in the number of first order conditions is that firms do not choose labor and materials separately, as one input choice determines the other input quantity as well. This is a problem for identification of either input price markdown, as one can no longer divide the two first order conditions by each other to express the markdown in function of output elasticities and revenue shares.

Of course, it is always possible that there is a third variable input, such as energy. If this third input is substitutable with the input over which monopsony power is exerted (so far, labor), than the markdown on that substitutable input can still be identified by solving for the energy first order condition and the markup expression (21). However, this does not apply if firms exert monopsony power on the non-substitutable input, which is materials in Equation 20. In this case, the markup expression becomes:

$$\mu_{ft} = (\frac{\alpha^l_{ft}}{\beta^l}\psi^l_{ft} + \alpha^m_{ft}\psi^m_{ft})^{-1} \tag{22}$$

Even if one can add variable inputs that substitute with labor, which results in additional first order conditions, this does not allow to write the inverse intermediate input supply elasticity ψ^m_{ft} as a function of output elasticities and data, because materials are perfect complements to any of these other variable inputs. In this case, one needs to either estimate or impose a markup, or estimate the factor supply elasticity, as discussed in Rubens (2023). This identification strategy has been implemented in various industries, including Chinese tobacco manufacturing in Rubens (2023), German car manufacturing in Hahn (2024), French dairy production in Avignon and Guigue (2022), and Chinese coal mining in Zheng (2024).

4.2 Labor Market Conduct

The labor market simulations in Sections 2 and 3 imposed that firms compete oligopsonistically following Nash-Bertrand conduct. This model nests models of monopsonistic competition, when firms become atomistic (market shares go to zero). In this subsection, we consider other types of labor market competition than oligopsonistic or monopsonistic competition.

Collusion

A first possibility is that firms collude on their input markets, coordinating their wage or employment choices rather than making these decisions independently. Delabastita and Rubens (in press) considers markdown estimation when firms potentially collude. They show that, maintaining the assumption of Hicks neutrality, the wage markdown can still be estimated using the production approach, even if firms collude on their labor markets. Next, they combine estimation of a labor supply model with the production estimates to identify conduct on the labor market, and find that their collusion estimates align with the observed introduction of a cartel in the Belgian coal mining industry.

Bargaining

In many labor market settings, firms and workers bargain over wages, rather than that firms post wages (Caldwell, Haegele, & Heining, 2025). This bargaining can either be individual or collective, through a labor union. Rubens (2024) considers cost-side markdown estimation when wages are bargained. Its empirical application focuses on Illinois coal operators, which bargain over wages with miner unions. A methodological challenge arises because to identify bargaining parameters, an estimate of the production function is needed, but bargaining parameters need to be known in order to identify the production function. Rubens (2024) addresses this problem using a fixed point estimator, in which production function estimation is nested in a loop over which bargaining parameters are guessed. As is explained in Rubens (2024) , this estimation procedure converges quickly towards a stable set of estimates of both bargaining abilities and production function coefficients.

4.3 Differentiation and Multi-Product Firms

Product and Input Differentiation

Assumption 3 imposed that both goods and inputs are homogeneous, which is clearly a strong assumption in many settings. Although vertical product differentiation can be easily allowed using a price control in the production function De Loecker et al. (2016); Rubens (2023), most goods are horizontally differentiated as well. Hahn (2024) addresses this challenge by estimating a hedonic price model for car manufacturers, which incorporates car characteristics, in addition to a production function. This model is then used to estimate markdowns and examine bargaining between car manufacturers and parts producers. A distinct challenge is raised when the inputs, rather than the products, are differentiated.

Lamadon, Mogstad, and Setzler (2022) addresses this challenge by allowing for heterogeneous worker quality using matched employer-employee data.

Multi-Product Firms

Production function estimation with multi-product firms is challenging even if input markets are perfectly competitive, because inputs are usually not disaggregated at the product level in the data. Various approaches have been developed to address this challenge (De Loecker et al., 2016; Orr, 2022; Dhyne, Petrin, Smeets, & Warzynski, 2022; Valmari, 2023) without allowing for imperfect factor market competition. Avignon and Guigue (2022) estimate factor price markdowns for French dairy industries while allowing for multi-product firms. They combine engineering data to assign input costs to the various products with production function estimation to recover markups and markdowns.

4.4 Adjustment Frictions

Assumption 5 imposed that both materials and labor are variable and static inputs. In many applications, it is reasonable that at least a subset of these inputs will be subject to adjustment frictions, such as hiring or firing costs. Although estimation of the production function is not hampered by such adjustment frictions (only the timing assumptions imposed would change), they do pose a challenge for markdown identification using the production function approach because the markup and markdown expressions 8 and 10 follow from solving a static cost minimization problem. Adjustment frictions lead to additional wedges between marginal revenue products and input prices that are unrelated to the exercise of monopsony power. One possibility to separately identify adjustment costs from monopsony distortions is to, again, jointly estimate a labor supply model and a production model, which is carried out in Chan, Mattana, Salgado, and Xu (2023) using Danish data.

4.5 No Competitive Input Market

Finally, Assumption 4 imposed that intermediate input prices are exogenous to firms. This is a commonly made assumption in the literature (Morlacco, 2017; Brooks et al., 2021; Yeh et al., 2022; Delabastita & Rubens, in press), and is needed to point-identify the markdown when only using the production function, as made clear by Equation (10). In case all input markets are imperfectly competitive, meaning that no input price is exogenous, there are two potential solutions. First, one could impose a model of imperfect competition and estimate a factor supply curve for one of the inputs, as carried out in Section 3, and still identify the markdown of the remaining inputs using the production approach. Alternatively, Treuren

(2022) proposes estimating a *revenue* production function, in contrast to the *quantity* production functions used in this article so far, to identify wage markdowns without having to assume competitive material markets. Whereas the benefit of allowing for endogenous material prices is clear, using a revenue production function comes at the cost of imposing homogeneous goods demand elasticities between firms, which restricts the set of models of imperfect competition on the product market one can allow for. As with any assumption, the tradeoff between imposing additional restrictions on product market competition while relaxing the assumptions in terms of input market competition is specific to the empirical application, and depends on the type of industry at hand.

5 Conclusions

In this article, we review 'production approaches' to estimate factor price markdowns. We discuss the commonly made assumptions in this class of estimators and test this class of estimators using Monte Carlo simulations for oligopsonistic labor markets in which firms compete in wages in a static Nash-Bertrand equilibrium. We find that when production is Hicks-neutral, existing 'cost-side' markdown estimators recover markdowns consistently. This implies that it is possible to estimate wage markdowns without having tospecify and estimate a labor supply model, and while remaining agnostic about the underlying model of labor market conduct. However, we find that allowing for unobserved technological heterogeneity in production leads to severely biased estimates of factor price markdowns using the production approaches that rely on Hicks neutrality. When implementing the estimation procedure suggested by Rubens et al. (2024), which is designed to allow for departured from Hicks-neutrality, we find that the production function coefficients and heterogeneity can be estimated consistently in the presence of imperfect labor market competition. Finally, we discuss approaches in the literature that have extended cost-side markdown estimation to relax other assumptions, such as allowing for nonsubstitutable inputs, different types of labor market conduct, and multi-product production.

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