cheese

- 88 stores
- weekly: sales volume (Q), price (P), & whether product was advertised (1 or 0) (α/β)

Estimate, for each store, the effect of displaying ads on the demand curve:

$$\log(Q) = \log(\alpha) + \beta \log(P),$$ ← log-log scale

$\beta \equiv$ price elasticity of demand

- consider $\mathbb{1}$ in α & β
- consider $|Q|$ somehow
- does β vary store-by-store? Is it worth estimating separate β for each store?
- do α & β look similar store-by-store? (hist of α & β post fit)
- Do we see any fitting problems?

model store "j"

$$\log(Q_t) \sim N(\log(\alpha_t) + \beta_t \log(P_t), \sigma^2)$$

conjugate $\sigma^2 \sim IG(100, 0.01)$ weekly homogenous error term

$$\log(\alpha_t) = a + b\,\mathbb{1}(x_t = 1)$$

- a is intercept for $\log(Q_t)$
- b is incremental effect on $\log(Q_t)$ when advertising w/o change in price (?)

not conjugate $a \sim \text{Gamma}(\alpha_a, \beta_a)$ (should be non-negative) (2nd bullet point)

conjugate $b \sim N(0, \sigma_b^2)$

$$\beta_t = c + d\,\mathbb{1}(x_t = 1)$$

conjugate $c \sim N(0, \sigma_c^2)$

conjugate $d \sim N(0, \sigma_d^2)$

Interpretation

$$\log(Q_t) \sim N\left(a + b\,\mathbb{1}(x_t = 1) + (c + d\,\mathbb{1}(x_t = 1))\log(P_t), \sigma^2\right)$$

- a is global intercept
- b is the shift in demand curve when advertising
- c is PED w/o advertising
- d is the marginal increase/decrease in PED when we advertise
- σ^2 is global homogenous error

global ≡ week-by-week per store

Full-conditionals

$$[\sigma^2 | \cdot] \propto \prod_{t=1}^{T} [\log(Q_t) | \sigma^2, \dots][\sigma^2]$$

$$\propto \prod_{t=1}^{T} N(\log(\alpha_t) + \beta_t \log(P_t), \sigma^2)\, (\sigma^2)^{-a-1} \exp(-\tfrac{b}{\sigma^2})$$

$$\propto (\sigma^2)^{-(T/2+a)-1} \exp\left\{-\frac{1}{\sigma^2}\left(b + \frac{\sum_{t=1}^{T}\left[(\log(Q_t) - \log(\alpha_t) - \beta_t \log(P_t))\right]^2}{2}\right)\right\}$$

$$\Rightarrow [\sigma^2|\cdot] \equiv IG\left(a + \frac{T}{2},\ b + \frac{1}{2}\sum_{t=1}^{T}\left[\log(Q_t) - \log(\alpha_t) - \beta_t \log(P_t)\right]\right)$$

$$[b|\cdot] \propto N\left(\log(Q_t) \,\middle|\, a + b\,\mathbb{1}(x_t=1) + (c + d\,\mathbb{1}(x_t=1))\log(P_t), \sigma^2\right)$$

$$\times \text{Normal}(b \,|\, 0, \sigma_b^2)$$

$$\propto \exp\left\{-\frac{1}{2\sigma^2}\left[\log Q_t - a - b\,\mathbb{1}(x_t=1) - c\log P_t - d\,\mathbb{1}(x_t=1)\log P_t\right]^2\right\}$$

$$\times \exp\left\{-\frac{1}{2\sigma_b^2} b^2\right\}$$

$$\propto \exp\left\{-\frac{1}{2}\left[\frac{1}{\sigma^2}\left(-2\left(\log Q_t\,\mathbb{1}(x_t=1) - a\,\mathbb{1}(x_t=1)\right) - c\log P_t\,\mathbb{1}(x_t=1) - d\,\mathbb{1}(x_t=1)\log P_t\right)b\right.\right.$$

$$+ b^2\,\mathbb{1}(x_t=1)\Big)$$

$$\left.\left. + \frac{1}{\sigma_b^2} b^2\right)\right]$$

$$\propto \exp\left\{-\frac{1}{2}\left[-2\left(\underbrace{\frac{\sum(\log Q_t\,\mathbb{1}(x_t) - a\,\mathbb{1}(x_t) - (c+d)\log P_t\,\mathbb{1}(x_t))}{\sigma^2}}_{b^T}\right)b + b^2\left(\underbrace{\frac{\sum \mathbb{1}(x_t)}{\sigma^2} + \frac{1}{\sigma_b^2}}_{A}\right)\right]\right\}$$

$$\Rightarrow [b|\cdot] \equiv N(A^{-1}b, A^{-1})$$

$$[c|\cdot] \propto \exp\left\{-\frac{1}{2\sigma^2}\left[\log Q_t - a - b\mathbb{1}(x_t=1) - c\log P_t - d\,\mathbb{1}(x_t=1)\log P_t\right]^2\right\}$$

$$\times \exp\left\{-\frac{1}{2\sigma_c^2}c^2\right\}$$

$$\propto \exp\left\{-\frac{1}{2}\left[-2\Bigg(\underbrace{\frac{\sum_t\left(\log Q_t\log P_t - a\log P_t - (b+d)\log P_t\,\mathbb{1}(x_t)\right)}{\sigma^2}}_{b^*}\Bigg)c + c^2\Bigg(\underbrace{\frac{\sum_t(\log P_t)^2}{\sigma^2} + \frac{1}{\sigma_c^2}}_{A^{-1}}\Bigg)\right]\right\}$$

$$\Rightarrow [c|\cdot] \equiv N(A^{-1}b, A^{-1})$$

Likewise, $[d|\cdot] \equiv N(A^{-1}b, A^{-1})$, where

$$A^{-1} = \frac{\sum_t \mathbb{1}(x_t=1)(\log P_t)^2}{\sigma^2} + \frac{1}{\sigma^2_d}$$

$$b^* = \frac{\sum\left(\log Q_t - a - b - c\log P_t\right)\log P_t\,\mathbb{1}(x_t=1)}{\sigma^2}$$

Non-conjugate update: a

I know this may not be best, but let's just say the proposal for M-H update is the prior, so our M-H ratio simplifies to

$$mh_a = \frac{[\log Q_t \mid a^{(*)}, \dots]}{[\log Q_t \mid a^{(h-1)}, \dots]}$$