# Zoo bundle overview and quick-start runnable walkthrough.

(`LAA_Zoo_Bundle_Overview.m`)

**This script is part of the "Zoo" demo bundle accompanying the paper on Newton-based computation of left eigenvalues of quaternion matrices. The bundle enables running each of the Zoo examples.**

**Where are these examples in the paper?**

- Supplement B, Section 2: more than n distinct isolated eigenvalues (Examples 1 and 2)
- Supplement B, Section 2: less than n distinct isolated eigenvalues (Example 3)
- Supplement B, Section 4: spherical components (Examples 5, 6, ...)

**What do the demo scripts do?**

*Computation scripts* (`LAA_Zoo_Ex_*_computation.m`) typically:

1. define the matrix A from the paper,
2. run a multi-start Newton solver to collect candidate left eigenvalues,
3. filter duplicates (distinctness tolerance) and report K distinct candidates,
4. (optional) refine/polish candidates and report improved certificates,
5. for spherical cases: detect sphere models and report centers/radii/inliers,
6. print a short "workspace guide" listing key output variables.

*Evidence scripts* (`LAA_Zoo_Ex_*_evidence.m`) are complementary to the randomly generated exmaples:

1. they run a larger/batched multi-start search to see whether NEW distinct isolated eigenvalues appear at the stated tolerances,
2. and/or they probe off-axis points on a hypothesized eigen-sphere to confirm that the reported eigenvalues are isolated (not spherical).

**Requirements**

- The **leigqNEWTON toolbox** must be on the MATLAB path.
- Quaternion support as used by your toolbox (typically MATLAB's quaternion).

**Author/citation**

This demo bundle accompanies the leigqNEWTON reference implementation and the associated manuscript (see the paper's "Data and code availability").

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--------------------------------------------------------------------------

## Setup: paths and reproducibility

```
thisFile = mfilename('fullpath');
thisDir  = fileparts(thisFile);
addpath(thisDir);
```

```
rng(24680,'twister');

if exist('leigqNEWTON','file') ~= 2
    error('Cannot find leigqNEWTON on the MATLAB path.\n');
end
```

## Bundle contents (Zoo scripts)

*Computations (Supplement B examples):*

- LAA_Zoo_Ex_2_1_computation : 3x3, five distinct isolated eigenvalues
- LAA_Zoo_Ex_2_2_computation : (as in Supplement B)
- LAA_Zoo_Ex_3_3_computation : (as in Supplement B)
- LAA_Zoo_Ex_4_2_computation : 4x4, spherical eigenvalues
- LAA_Zoo_Ex_4_5_computation : 4x4, one sphere + isolated points
- LAA_Zoo_Ex_4_6_computation : 6x6, two spherical components

*Evidence (Supplement B):*

- LAA_Zoo_Ex_2_1_evidence : evidence for "no extra isolated eigs" + no sphere -
- LAA_Zoo_Ex_2_2_evidence : analogous evidence run for Example 2-2

## What the scripts print:

*Distinct eigenvalue cases* (Supplement B, Section 2 and 3): - distinct eigenvalues as returned by solver (before refinement) with certificates - distinct eigenvalues after refine with certificates - tips, where to find results in the workspace

*Spherical cases (*Supplement B, Section 4)

- Stage 1 (sampling + detection): - collect K distinct candidates, - cluster and fit sphere models in R^4, - report centers/radii/inliers and isolated/outlier candidates.
- Stage 2 (refinement + optional sphere re-fit): - refine to improve eigenvalue certificates, - optionally re-fit spheres using refined candidates (diagnostic consistency check). - tips, where to find results in the workspace

## Quick-start runnable walkthrough: Example 2-1 (fastest)

```
fprintf('\n=== Quick-start: running LAA_Zoo_Ex_2_1_computation ===\n');
```

```
=== Quick-start: running LAA_Zoo_Ex_2_1_computation ===
```

```
LAA_Zoo_Ex_2_1_computation;
```

```
=== LAA demo: Zoo Example 2--1 (3x3, K=5 distinct isolated left eigenvalues) ===
UniqueTol=1.0e-08 | TolNewton=1.0e-12 | TargetResMin=1.0e-16
Runs=50 | Restarts/run=500 | Krequest=3 | Seed0=24680

Collected accepted hits: 150
Distinct eigenvalues (tol = 1.0e-08): 5
```

```
--- DISTINCT eigenvalues as returned by solver (before refinement) ---
 1) -22.877487 +15.850469i -17.069787j -11.791606k   | r_min = 2.955e-13
 2) 11.833188 +9.698189i -13.382634j -19.325731k   | r_min = 4.020e-13
 3) 13.399540 +15.934883i -12.000914j -0.414566k   | r_min = 6.436e-13
 4) 14.897483 +16.835221i -11.965564j -2.863713k   | r_min = 1.336e-15
 5) 21.109974 +21.579378i +5.435201j -2.138868k   | r_min = 8.827e-14

Refinement will now run (refine_batch).
Progress:
Refinement finished (refine_batch), elapsed 24.15s.

--- DISTINCT eigenvalues after refine (refine_batch) ---
 1) -22.877487 +15.850469i -17.069787j -11.791606k   | r_min = 1.618e-15
 2) 11.833188 +9.698189i -13.382634j -19.325731k   | r_min = 8.922e-16
 3) 13.399540 +15.934883i -12.000914j -0.414566k   | r_min = 3.032e-15
 4) 14.897483 +16.835221i -11.965564j -2.863713k   | r_min = 4.734e-15
 5) 21.109974 +21.579378i +5.435201j -2.138868k   | r_min = 1.014e-15

Certificate range r_min: [8.922e-16, 4.734e-15]
Minimum pairwise separation (R^4 distance): 3.00899

Where to find results in the workspace:
  A              : 3x3 quaternion matrix
  lamAll         : raw accepted hits (quaternion, 150-by-1)
  lamDistinct    : distinct eigenvalues BEFORE refinement (quaternion, 5-by-1)
  resMinDistinct : certificates for lamDistinct (double, 5-by-1)
  lamRef         : refined eigenvalues (quaternion, 5-by-1)
  resMinRef      : certificates for lamRef (double, 5-by-1)
  refineNote     : which refiner was used
=== End of demo ===
```

```
% The computation script should print a short "workspace guide" at the end.
% Below we additionally *show* a few key variables if they exist.

varsWanted = {'A','lamDistinct','resMinDistinct','lamRef','resMinRef'};
for ii = 1:numel(varsWanted)
    if evalin('base', sprintf("exist('%s','var')", varsWanted{ii}))
        fprintf('  (workspace) %s is available.\n', varsWanted{ii});
    end
end
```

```
  (workspace) A is available.
  (workspace) lamDistinct is available.
  (workspace) resMinDistinct is available.
  (workspace) lamRef is available.
  (workspace) resMinRef is available.
```

```
if evalin('base',"exist('A','var')")
    A = evalin('base','A');
    fprintf('\nA (Example 2-1 matrix):\n');
    disp(A);
end
```

```
A (Example 2-1 matrix):
     -7 +  6i -  6j -  7k      3 -  7i + 11j -  2k     11 -  9i +  0j +  0k
      6 +  1i -  5j +  3k      9 +  7i -  6j - 10k     -5 + 15i + 14j -  1k
     16 +  6i + 14j + 11k     20 -  3i +  4j +  6k     -5 + 19i +  1j -  3k
```

```
if evalin('base',"exist('lamDistinct','var') && exist('resMinDistinct','var')")
    lamDistinct = evalin('base','lamDistinct');
    resMinDistinct = evalin('base','resMinDistinct');
    fprintf('\nDistinct candidates (pre-polish):\n');
    disp(table(lamDistinct(:), resMinDistinct(:), 'VariableNames',
{'lamDistinct','resMinDistinct'}));
end
```

```
Distinct candidates (pre-polish):
     lamDistinct      resMinDistinct
    ______________    ______________

    1×1 quaternion      2.9548e-13
    1×1 quaternion      4.0201e-13
    1×1 quaternion      6.4356e-13
    1×1 quaternion      1.3357e-15
    1×1 quaternion      8.8275e-14
```

```
if evalin('base',"exist('lamRef','var') && exist('resMinRef','var')")
    lamRef = evalin('base','lamRef');
    resMinRef = evalin('base','resMinRef');
    fprintf('\nRefined/polished candidates:\n');
    disp(table(lamRef(:), resMinRef(:), 'VariableNames',
{'lambdaRef','resMinRef'}));
end
```

```
Refined/polished candidates:
      lambdaRef       resMinRef
    ______________    __________

    1×1 quaternion    1.6177e-15
    1×1 quaternion    8.9222e-16
    1×1 quaternion    3.0316e-15
    1×1 quaternion    4.7338e-15
    1×1 quaternion    1.0144e-15
```

```
fprintf('\n=== Quick-start: running LAA_Zoo_Ex_2_1_evidence ===\n');
```

```
=== Quick-start: running LAA_Zoo_Ex_2_1_evidence ===
```

```
LAA_Zoo_Ex_2_1_evidence;
```

```
=== LAA evidence: Zoo Example 2–1 (3x3, K=5) ===
UniqueTol=1.0e-08 | TolNewton=1.0e-12 | TargetResMin=1.0e-16
Sampling: batchRuns=25 | restarts/run=500 | maxRuns=400 | patience=8 batches
Sphere-probe: 300 directions per candidate (seed=24680)

[runs=  25] DISTINCT increased: 0 -> 0
[runs=  50] DISTINCT stays at 0   (no-new batches: 2/8)
[runs=  75] DISTINCT stays at 0   (no-new batches: 3/8)
[runs= 100] DISTINCT stays at 0   (no-new batches: 4/8)
[runs= 125] DISTINCT stays at 0   (no-new batches: 5/8)
[runs= 150] DISTINCT stays at 0   (no-new batches: 6/8)
[runs= 175] DISTINCT stays at 0   (no-new batches: 7/8)
[runs= 200] DISTINCT stays at 0   (no-new batches: 8/8)

Stage 1 done. Total runs=200, raw hits=0, distinct=0
```

```
Distinct eigenvalues found after refine (empty):
Min pairwise ||lambda_i - lambda_j||_{R^4}: Inf

Discovery log (runs vs Kdistinct):
    Runs    Kdistinct
      25          0
      50          0
      75          0
     100          0
     125          0
     150          0
     175          0
     200          0

=== Stage 2: no-sphere diagnostics ===
Invariant groups by (Re, ||Im||) within tol=1.0e-08:
No group contains 2+ points with matching (Re, ||Im||) invariants.

Sphere-probe test (r_min over random directions on candidate spheres):

Interpretation:
  - If an eigen-sphere existed and were reachable, many off-axis probes would yield r_min ~ 0.
  - For isolated eigenvalues, off-axis r_min should stay bounded away from 0.

=== End of evidence script ===
```