ExNEWTON_2_MVPS

Reproduce the explicit 3×3 examples (topological cases) from:

E. Macías-Virgós and M. J. Pereira-Sáez. Cayley–Hamilton theorem for left eigenvalues of 3 × 3 quaternionic matrices. Special Matrices, 2(1):11–18, 2014. DOI: 10.2478/SPMA-2014-0002.

Reproduce explicit 3×3 examples from paper by Macías-Virgos & Pereira-Sáez (MVPS) using the shipped script `examples/ExNEWTON_2_MVPS.m`.

Requirements

```
hasQuat = true;
try
    quaternion(0,0,0,0);
catch
    hasQuat = false;
end
if ~hasQuat
    disp('This toolbox requires MATLAB''s built-in quaternion class
(quaternion(w,x,y,z)).');
    disp('Examples in this page are skipped.');
    return;
end

if exist('leigqNEWTON','file') ~= 2
    thisFile = mfilename('fullpath');
    if ~isempty(thisFile)
        rootGuess = fileparts(fileparts(fileparts(thisFile))); % .../docs/source ->
toolbox root
        if exist(fullfile(rootGuess,'leigqNEWTON.m'),'file')
            addpath(rootGuess);
        end
    end
end

if exist('leigqNEWTON','file') ~= 2
    error('leigqNEWTON not found on the MATLAB path. Add the toolbox root folder.');
end
```

Run the shipped script

```
root = fileparts(which('leigqNEWTON'));
ex   = fullfile(root,'examples','ExNEWTON_2_MVPS.m');
if exist(ex,'file')
    run(ex);
else
    error('Example not found: %s', ex);
end
```

```
MVPS topological examples (leigqNEWTON)
=============================================================
A19 =
     0 + 1i + 0j + 0k     0 + 0i + 0j + 0k     0 + 0i + 0j + 0k
     0 + 0i + 0j + 1k     0 + 0i + 1j + 0k     0 + 0i + 0j + 0k
     0 - 3i + 0j + 0k     0 + 0i + 0j + 2k     0 + 0i + 0j + 1k
A38 =
     0 + 0i + 0j + 0k     0 + 1i + 0j + 0k     1 + 0i + 0j + 0k
     0 + 3i + 0j - 1k     0 + 0i + 0j + 0k     1 + 0i + 0j + 0k
     0 + 0i + 0j + 1k    -1 + 0i + 1j + 1k     0 + 0i + 0j + 0k
B51 =
     0 + 1i + 0j + 0k     0 + 1i + 0j + 0k     1 + 0i + 0j + 0k
     0 + 3i + 0j - 1k     0 + 1i + 0j + 0k     1 + 0i + 0j + 0k
     0 + 0i + 0j + 1k    -1 + 0i + 1j + 1k     0 + 1i + 0j + 0k
Binv51 =
       0 + 0.4i +   0j - 0.2k       0 - 0.4i +   0j + 0.2k       0 +   0i +   0j +   0k
    -0.1 - 0.3i + 0.8j - 0.6k     0.1 + 0.3i - 0.3j + 0.1k       0 +   0i - 0.5j - 0.5k
     1.1 + 0.1i - 0.8j - 0.8k    -0.1 - 0.1i + 0.3j + 0.3k       0 +   0i - 0.5j + 0.5k
A52 =
     0 + 0i + 1j + 0k     1 + 0i + 0j + 0k     0 + 0i + 0j + 0k
     0 + 2i + 0j + 0k     0 + 0i + 0j - 1k     1 + 0i + 0j + 0k
     2 - 1i - 2j + 0k    -1 + 0i - 1j + 1k     0 - 1i + 0j - 1k
A54 =
       0 +   0i +   0j +   0k       0 +   0i -   1j +   0k       0 +   1i +   0j +   0k
      -1 +   0i +   1j +   0k       0 +   0i +   1j +   0k       0 +   0i +   0j +   1k
     NaN + NaNi + NaNj + NaNk     NaN + NaNi + NaNj + NaNk     NaN + NaNi + NaNj + NaNk
A55 =
     0 + 0i + 0j + 1k     0 + 0i + 0j + 0k     0 + 0i + 0j + 0k
     0 + 3i - 1j + 0k     0 - 1i + 0j + 0k     0 + 1i + 0j + 0k
     1 + 0i + 0j - 2k     0 + 0i + 1j + 0k     0 + 0i - 1j + 0k
A56 =
     0 - 1i - 1j + 0k     0 + 0i + 0j + 0k     0 + 0i + 0j + 0k
     0 + 0i + 0j + 1k     0 - 1i + 0j + 0k     0 + 1i + 0j + 0k
     1 - 1i + 0j + 0k     0 + 0i + 1j + 0k     0 + 0i - 1j + 0k

Inverse check for Example 51: max|B51*Binv51 - I| = 8.327e-17

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Example 19
=== checkNEWTON ===
Matrix: 3x3 quaternion
Lambdas: 3 (computed).  Vectors: 3 columns (computed).
Certificate/residual summary (median | max):
  resMin abs: 1.37e-18 | 9.18e-18    resMin rel: 2.68e-19 | 1.79e-18
  resPair abs:2.29e-16 | 3.48e-16    resPair rel:4.48e-17 | 6.80e-17

Per-eigenvalue residuals:
  #  lambda (cleaned/rounded)                          resMin(abs/rel)        resPair(abs/rel)
   1   0 +1i +0j +0k                                     9.18e-18/ 1.79e-18    3.48e-16/ 6.80e-17
   2   0 +0i +1j +0k                                     1.37e-18/ 2.68e-19    2.29e-16/ 4.48e-17
   3   0 +0i +0j +1k                                     0.00e+00/ 0.00e+00    0.00e+00/ 0.00e+00

Interestingness:
  Standard: Kdistinct = n = 3.
  Sphere check: not run. Use checkNEWTON(...,'SphereCheck','on') to force.

Outputs:
  out           : main struct (out.lambda, out.v, out.resMinAbs, out.resPairAbs, ...).
  cases{1}      : simple case struct with fields A, lamAll, lamSamples, sph, info (sphere tooling style).
  S             : summary struct (n, Ktot, Kdistinct, spheresFound, ...).

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Example 38
=== checkNEWTON ===
Matrix: 3x3 quaternion
Lambdas: 3 (computed).  Vectors: 3 columns (computed).
Certificate/residual summary (median | max):
  resMin abs: 1.03e-15 | 1.38e-13    resMin rel: 1.76e-16 | 2.53e-14
  resPair abs:1.33e-15 | 1.39e-13    resPair rel:2.28e-16 | 2.53e-14

Per-eigenvalue residuals:
  #  lambda (cleaned/rounded)                          resMin(abs/rel)        resPair(abs/rel)
   1   0.02387 -1.716i +0.2257j -0.04859k                3.63e-17/ 6.20e-18    6.04e-16/ 1.03e-16
   2   0.0983 -0.07397i +1.137j -0.7198k                 1.38e-13/ 2.53e-14    1.39e-13/ 2.53e-14
   3   0.02387 -1.716i +0.2257j -0.04859k                1.03e-15/ 1.76e-16    1.33e-15/ 2.28e-16

Interestingness:
  Nonstandard: Kdistinct = 2 < n = 3. Consider running longer: leigqNEWTON(A,'SolveProfile','reliable').
  Sphere check: not run. Use checkNEWTON(...,'SphereCheck','on') to force.

Outputs:
  out           : main struct (out.lambda, out.v, out.resMinAbs, out.resPairAbs, ...).
  cases{1}      : simple case struct with fields A, lamAll, lamSamples, sph, info (sphere tooling style).
  S             : summary struct (n, Ktot, Kdistinct, spheresFound, ...).

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Example 51 (B)
=== checkNEWTON ===
Matrix: 3x3 quaternion
Lambdas: 3 (computed).  Vectors: 3 columns (computed).
Certificate/residual summary (median | max):
  resMin abs: 8.79e-16 | 1.39e-13    resMin rel: 1.68e-16 | 2.27e-14
  resPair abs:1.55e-15 | 1.39e-13    resPair rel:2.96e-16 | 2.27e-14

Per-eigenvalue residuals:
  #  lambda (cleaned/rounded)                          resMin(abs/rel)        resPair(abs/rel)
   1   0.02387 -0.7155i +0.2257j -0.04859k               5.35e-17/ 1.02e-17    5.29e-16/ 1.01e-16
   2   0.0983 +0.926i +1.137j -0.7198k                   1.39e-13/ 2.27e-14    1.39e-13/ 2.27e-14
   3   0.02387 -0.7155i +0.2257j -0.04859k               8.79e-16/ 1.68e-16    1.55e-15/ 2.96e-16

Interestingness:
  Nonstandard: Kdistinct = 2 < n = 3. Consider running longer: leigqNEWTON(A,'SolveProfile','reliable').
  Sphere check: not run. Use checkNEWTON(...,'SphereCheck','on') to force.

Outputs:
  out           : main struct (out.lambda, out.v, out.resMinAbs, out.resPairAbs, ...).
  cases{1}      : simple case struct with fields A, lamAll, lamSamples, sph, info (sphere tooling style).
  S             : summary struct (n, Ktot, Kdistinct, spheresFound, ...).

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Example 52
=== checkNEWTON ===
Matrix: 3x3 quaternion
Lambdas: 3 (computed).  Vectors: 3 columns (computed).
Certificate/residual summary (median | max):
  resMin abs: 2.20e-13 | 2.99e-11    resMin rel: 4.06e-14 | 6.37e-12
  resPair abs:2.21e-13 | 2.99e-11    resPair rel:4.07e-14 | 6.37e-12

Per-eigenvalue residuals:
  #  lambda (cleaned/rounded)                          resMin(abs/rel)        resPair(abs/rel)
   1  -0.1941 +0.1022i -0.01561j -0.5378k                3.98e-17/ 7.54e-18    1.12e-15/ 2.13e-16
   2   0 +0i +0j +6.501e-06k                             2.99e-11/ 6.37e-12    2.99e-11/ 6.37e-12
   3   0.2796 +0.3996i +0.3945j +0.3843k                 2.20e-13/ 4.06e-14    2.21e-13/ 4.07e-14

Interestingness:
  Standard: Kdistinct = n = 3.
  Sphere check: not run. Use checkNEWTON(...,'SphereCheck','on') to force.
```

```
Outputs:
  out           : main struct (out.lambda, out.v, out.resMinAbs, out.resPairAbs, ...).
  cases{1}      : simple case struct with fields A, lamAll, lamSamples, sph, info (sphere tooling style).
  S             : summary struct (n, Ktot, Kdistinct, spheresFound, ...).

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Example 54
Skipped: matrix contains NaN placeholders (fill p,q,r first).

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Example 55
=== checkNEWTON ===
Matrix: 3x3 quaternion
Lambdas: 3 (computed).  Vectors: 3 columns (computed).
Certificate/residual summary (median | max):
  resMin abs: 8.90e-12 | 2.23e-11    resMin rel: 1.51e-12 | 4.08e-12
  resPair abs:8.90e-12 | 2.23e-11    resPair rel:1.51e-12 | 4.08e-12

Per-eigenvalue residuals:
  #  lambda (cleaned/rounded)                          resMin(abs/rel)        resPair(abs/rel)
   1   0 +0i +0j +0k                                     6.12e-18/ 1.37e-18    3.65e-16/ 8.16e-17
   2  -1.091e-11 -1i -1j -8.211e-12k                     8.90e-12/ 1.51e-12    8.90e-12/ 1.51e-12
   3  -1.401e-11 +3.79e-11i +4.676e-11j +1k              2.23e-11/ 4.08e-12    2.23e-11/ 4.08e-12

Interestingness:
  Standard: Kdistinct = n = 3.
  Sphere check: not run. Use checkNEWTON(...,'SphereCheck','on') to force.

Outputs:
  out           : main struct (out.lambda, out.v, out.resMinAbs, out.resPairAbs, ...).
  cases{1}      : simple case struct with fields A, lamAll, lamSamples, sph, info (sphere tooling style).
  S             : summary struct (n, Ktot, Kdistinct, spheresFound, ...).

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Example 56
=== checkNEWTON ===
Matrix: 3x3 quaternion
Lambdas: 3 (computed).  Vectors: 3 columns (computed).
Certificate/residual summary (median | max):
  resMin abs: 2.99e-11 | 3.75e-11    resMin rel: 6.78e-12 | 8.49e-12
  resPair abs:2.99e-11 | 3.75e-11    resPair rel:6.78e-12 | 8.49e-12

Per-eigenvalue residuals:
  #  lambda (cleaned/rounded)                          resMin(abs/rel)        resPair(abs/rel)
   1   0 +0i +0j +0k                                     4.40e-18/ 1.47e-18    5.00e-16/ 1.67e-16
   2   4.24e-06 -1i -1j -3.338e-06k                      2.99e-11/ 6.78e-12    2.99e-11/ 6.78e-12
   3  -7.412e-06 -1i -1j -6.125e-07k                     3.75e-11/ 8.49e-12    3.75e-11/ 8.49e-12

Interestingness:
  Standard: Kdistinct = n = 3.
  Sphere check: not run. Use checkNEWTON(...,'SphereCheck','on') to force.

Outputs:
  out           : main struct (out.lambda, out.v, out.resMinAbs, out.resPairAbs, ...).
  cases{1}      : simple case struct with fields A, lamAll, lamSamples, sph, info (sphere tooling style).
  S             : summary struct (n, Ktot, Kdistinct, spheresFound, ...).

Done.
```

What to look for

- The script prints the matrices (cleaned) and runs `checkNEWTON` on each case.
- Some cases are non-generic (e.g., more than n distinct left eigenvalues).
- If you want to increase solver budgets, edit the script and change `SolveProfile` (e.g., `'default'` → `'reliable'`) or pass larger trial counts.

See also

`checkNEWTON`, `doc_RefinementAndCertificates`, `leigqNEWTON_refine_batch`