ExNEWTON_1_HuangSo

Reproduce the explicit 2×2 examples (Examples 2.5–2.7) from:

L. Huang and W. So, "On left eigenvalues of a quaternionic matrix," Linear Algebra and its Applications 323 (2001) 105–116. DOI: 10.1016/S0024-3795(00)00246-9

The shipped `script examples/ExNEWTON_1_HuangSo.m` builds the matrices using MATLAB built-in `quaternion` class and runs `checkNEWTON` plus the sphere utilities for Example 2.7.

Requirements

```
hasQuat = true;
try
    quaternion(0,0,0,0);
catch
    hasQuat = false;
end
if ~hasQuat
    disp('This toolbox requires MATLAB''s built-in quaternion class
(quaternion(w,x,y,z)).');
    disp('Examples in this page are skipped.');
    return;
end

if exist('leigqNEWTON','file') ~= 2
    thisFile = mfilename('fullpath');
    if ~isempty(thisFile)
        rootGuess = fileparts(fileparts(fileparts(thisFile))); % .../docs/source ->
toolbox root
        if exist(fullfile(rootGuess,'leigqNEWTON.m'),'file')
            addpath(rootGuess);
        end
    end
end

if exist('leigqNEWTON','file') ~= 2
    error('leigqNEWTON not found on the MATLAB path. Add the toolbox root folder.');
end
```

Run the shipped script

```
root = fileparts(which('leigqNEWTON'));
ex   = fullfile(root,'examples','ExNEWTON_1_HuangSo.m');
if exist(ex,'file')
    run(ex);
else
    error('Example not found: %s', ex);
end
```

```
Huang--So (LAA 323, 2001) explicit 2x2 examples
DOI: 10.1016/S0024-3795(00)00246-9
============================================================

EXAMPLE 2.5
A25 =
     0 + 0i + 0j + 0k     1 + 1i + 0j + 0k
     1 - 1i + 0j + 0k     0 + 0i + 0j + 0k
Expected left eigenvalues (paper) =
     1.4142 +      0i +      0j +      0k
    -1.4142 +      0i +      0j +      0k
=== checkNEWTON ===
Matrix: 2x2 quaternion
Lambdas: 2 (computed).  Vectors: 2 columns (computed).
Certificate/residual summary (median | max):
  resMin abs: 9.70e-13 | 1.94e-12    resMin rel: 2.84e-13 | 5.68e-13
  resPair abs:9.71e-13 | 1.94e-12    resPair rel:2.84e-13 | 5.68e-13

Per-eigenvalue residuals:
  #  lambda (cleaned/rounded)                          resMin(abs/rel)        resPair(abs/rel)
   1   1.414 +0i +2.192e-12j +1.168e-12k                1.94e-12/ 5.68e-13    1.94e-12/ 5.68e-13
   2  -1.414 +0i +0j +0k                                1.47e-16/ 4.31e-17    4.56e-16/ 1.34e-16

Interestingness:
  Standard: Kdistinct = n = 2.
  Sphere check: not run. Use checkNEWTON(...,'SphereCheck','on') to force.

Outputs:
  out           : main struct (out.lambda, out.v, out.resMinAbs, out.resPairAbs, ...).
  cases{1}      : simple case struct with fields A, lamAll, lamSamples, sph, info (sphere tooling style).
  S             : summary struct (n, Ktot, Kdistinct, spheresFound, ...).
  Nearest distances to the reference set (in 4D):
    lambda[1] ~ ref[1], dist = 2.603e-12, lambda =          1.4142 +          0i + 2.1921e-12j +  1.168e-12k
    lambda[2] ~ ref[2], dist = 0.000e+00, lambda =     -1.4142 +      0i +      0j +      0k

EXAMPLE 2.6
A26 =
     0 + 0i + 0j + 0k     0 + 1i + 0j + 0k
     0 + 0i + 1j + 0k     1 + 0i + 0j + 0k
Expected left eigenvalues (paper) =
     0.5 + 0.5i + 0.5j - 0.5k
     0.5 - 0.5i - 0.5j - 0.5k
=== checkNEWTON ===
Matrix: 2x2 quaternion
Lambdas: 2 (computed).  Vectors: 2 columns (computed).
Certificate/residual summary (median | max):
  resMin abs: 1.34e-11 | 2.68e-11    resMin rel: 4.91e-12 | 9.81e-12
  resPair abs:1.34e-11 | 2.68e-11    resPair rel:4.91e-12 | 9.81e-12

Per-eigenvalue residuals:
  #  lambda (cleaned/rounded)                          resMin(abs/rel)        resPair(abs/rel)
   1   0.5 +0.5i +0.5j -0.5k                            2.68e-11/ 9.81e-12    2.68e-11/ 9.81e-12
   2   0.5 -0.5i -0.5j -0.5k                            6.31e-17/ 2.31e-17    3.16e-16/ 1.15e-16

Interestingness:
  Standard: Kdistinct = n = 2.
  Sphere check: not run. Use checkNEWTON(...,'SphereCheck','on') to force.

Outputs:
  out           : main struct (out.lambda, out.v, out.resMinAbs, out.resPairAbs, ...).
  cases{1}      : simple case struct with fields A, lamAll, lamSamples, sph, info (sphere tooling style).
  S             : summary struct (n, Ktot, Kdistinct, spheresFound, ...).
  Nearest distances to the reference set (in 4D):
    lambda[1] ~ ref[1], dist = 3.721e-11, lambda =      0.5 + 0.5i + 0.5j - 0.5k
```

```
    lambda[2] ~ ref[2], dist = 3.331e-16, lambda =      0.5 - 0.5i - 0.5j - 0.5k

EXAMPLE 2.7
A27 =
     2 + 0i + 0j + 0k     0 + 1i + 0j + 0k
     0 - 1i + 0j + 0k     2 + 0i + 0j + 0k
Paper claim: sigma_l(A27) is infinite (a sphere family).
=== checkNEWTON ===
Matrix: 2x2 quaternion
Lambdas: 2 (computed).  Vectors: 2 columns (computed).
Certificate/residual summary (median | max):
  resMin abs: 1.11e-11 | 1.89e-11    resMin rel: 2.25e-12 | 3.84e-12
  resPair abs:1.11e-11 | 1.89e-11    resPair rel:2.25e-12 | 3.84e-12

Per-eigenvalue residuals:
  #  lambda (cleaned/rounded)                          resMin(abs/rel)        resPair(abs/rel)
   1   1.526 +1.848e-11i -0.01666j +0.8802k              1.89e-11/ 3.84e-12    1.89e-11/ 3.84e-12
   2   1.672 +3.249e-12i -0.6181j +0.7145k               3.33e-12/ 6.55e-13    3.33e-12/ 6.55e-13

Interestingness:
  Standard: Kdistinct = n = 2.
  Sphere check: not run. Use checkNEWTON(...,'SphereCheck','on') to force.

Outputs:
  out           : main struct (out.lambda, out.v, out.resMinAbs, out.resPairAbs, ...).
  cases{1}      : simple case struct with fields A, lamAll, lamSamples, sph, info (sphere tooling style).
  S             : summary struct (n, Ktot, Kdistinct, spheresFound, ...).
=== leigqNEWTON_sphere_sample: collecting samples (n=2, targetDistinct=20) ===
  run   1 | Seed=1 | addedDistinct  2 | totalDistinct   2 | counted   2
  run   2 | Seed=2 | addedDistinct  2 | totalDistinct   4 | counted   4
  run   3 | Seed=3 | addedDistinct  2 | totalDistinct   6 | counted   6
  run   4 | Seed=4 | addedDistinct  2 | totalDistinct   8 | counted   8
  run   5 | Seed=5 | addedDistinct  2 | totalDistinct  10 | counted  10
  run   6 | Seed=6 | addedDistinct  2 | totalDistinct  12 | counted  12
  run   7 | Seed=7 | addedDistinct  2 | totalDistinct  14 | counted  14
  run   8 | Seed=8 | addedDistinct  2 | totalDistinct  16 | counted  16
  run   9 | Seed=9 | addedDistinct  2 | totalDistinct  18 | counted  18
  run  10 | Seed=10 | addedDistinct  2 | totalDistinct  20 | counted  20

Please note: The refinement may take about ten minutes.

=== leigqNEWTON_sphere_refine: idx=NaN, seed=NaN, origin= ===
Ktot=20, Kdistinct=20, spheres=1
Initial residuals: lamAll  med=1.91e-15  max=1.89e-11 | lamSamples med=1.91e-15  max=1.89e-11
```

What to look for

- Example 2.5 and 2.6: two isolated left eigenvalues, agreement with the paper.
- Example 2.7: an infinite left spectrum (sphere family). The script runs `leigqNEWTON_sphere_sample` and then `leigqNEWTON_sphere_validate` (which calls `leigqNEWTON_sphere_refine` in the public release) and prints the fitted sphere model sphere model and a confidence report. Validation is a lengthy process and may take up to 10 minutes.

See also

`doc_SphereHunting, leigqNEWTON_sphere_sample, leigqNEWTON_sphere_refine, checkNEWTON`