

# leigqNEWTON\_quickstart

A short page of minimal example calls (Live Script friendly). You can copy/paste one-liners.

## Setup (requirements + path)

```
hasQuat = true;
try
    quaternion(0,0,0,0);
catch
    hasQuat = false;
end
if ~hasQuat
    disp('This toolbox requires MATLAB''s built-in quaternion class
(quaternion(w,x,y,z)).');
    disp('Examples in this page are skipped.');
    return;
end

if exist('leigqNewton','file') ~= 2
    thisFile = mfilename('fullpath');
    if ~isempty(thisFile)
        rootGuess = fileparts(fileparts(fileparts(thisFile))); % .../docs/source ->
toolbox root
        if exist(fullfile(rootGuess,'leigqNewton.m'),'file')
            addpath(rootGuess);
        end
    end
end

if exist('leigqNewton','file') ~= 2
    error('leigqNewton not found on the MATLAB path. Add the toolbox root folder.');
end
```

## Create a small demo matrix (Pan–Ng 4×4 circulant)

```
q = @(w,x,y,z) quaternion(w,x,y,z);
a = q(-2, 1, 1, 4);
b = q( 2, 4, 1, 1);
c = q( 1, 3, 2, 2);
d = q(-1, 2, 2, 3);
A = [a b c d;
      d a b c;
      c d a b;
      b c d a];
```

## Solve (candidates)

Request more hits than n to reduce the chance of missing distinct eigenvalues.

```
[lambda, V, res, info, lambdaU, VU, resU] = leigqNEWTON(A, 'K', 2*size(A,1),
'Seed',1, 'SolveProfile','reliable');
median(resU), max(resU)
```

```
ans =
8.4522e-17
ans =
8.0413e-14
```

## Refine + certify (recommended)

```
[lambdaR, VR, cert] = leigqNEWTON_refine_batch(A, lambdaU, 'DoPolish',true,
'TargetResMin',1e-14);
```

```
==== leigqNEWTON_refine_batch: start (n=4, K=4, mode=auto, target=1.0e-14) ===
```

```
[k=1/4] start r_min=2.978e-16
[k=1] DONE: r_min 2.978e-16 -> 2.978e-16 (dt=0.00s)
```

```
[k=2/4] start r_min=1.405e-15
[k=2] DONE: r_min 1.405e-15 -> 1.405e-15 (dt=0.00s)
```

```
[k=3/4] start r_min=5.199e-16
[k=3] DONE: r_min 5.199e-16 -> 5.199e-16 (dt=0.00s)
```

```
[k=4/4] start r_min=3.767e-12
[k=4/4] rand probe: radius=1, NRand=2000
t= 200/ 2000 best=3.767e-12 dt=0.07s
t= 400/ 2000 best=3.767e-12 dt=0.15s
t= 600/ 2000 best=3.767e-12 dt=0.21s
t= 800/ 2000 best=3.767e-12 dt=0.28s
t= 1000/ 2000 best=3.767e-12 dt=0.35s
t= 1200/ 2000 best=3.767e-12 dt=0.42s
t= 1400/ 2000 best=3.767e-12 dt=0.49s
t= 1600/ 2000 best=3.767e-12 dt=0.56s
t= 1800/ 2000 best=3.767e-12 dt=0.62s
t= 2000/ 2000 best=3.767e-12 dt=0.69s
done radius=1 -> best=3.767e-12 (dt=0.69s)
```

```
[k=4/4] rand probe: radius=0.3, NRand=2000
t= 200/ 2000 best=3.767e-12 dt=0.06s
t= 400/ 2000 best=3.767e-12 dt=0.13s
t= 600/ 2000 best=3.767e-12 dt=0.20s
t= 800/ 2000 best=3.767e-12 dt=0.26s
t= 1000/ 2000 best=3.767e-12 dt=0.33s
t= 1200/ 2000 best=3.767e-12 dt=0.41s
t= 1400/ 2000 best=3.767e-12 dt=0.47s
t= 1600/ 2000 best=3.767e-12 dt=0.55s
t= 1800/ 2000 best=3.767e-12 dt=0.62s
t= 2000/ 2000 best=3.767e-12 dt=0.71s
done radius=0.3 -> best=3.767e-12 (dt=0.71s)
```

```
[k=4/4] rand probe: radius=0.1, NRand=2000
t= 200/ 2000 best=3.767e-12 dt=0.07s
t= 400/ 2000 best=3.767e-12 dt=0.15s
t= 600/ 2000 best=3.767e-12 dt=0.22s
t= 800/ 2000 best=3.767e-12 dt=0.30s
t= 1000/ 2000 best=3.767e-12 dt=0.37s
t= 1200/ 2000 best=3.767e-12 dt=0.45s
t= 1400/ 2000 best=3.767e-12 dt=0.52s
t= 1600/ 2000 best=3.767e-12 dt=0.60s
t= 1800/ 2000 best=3.767e-12 dt=0.66s
t= 2000/ 2000 best=3.767e-12 dt=0.74s
```

```

done radius=0.1 -> best=3.767e-12 (dt=0.74s)
[k=4/4] rand probe: radius=0.03, NRand=2000
t= 200/ 2000 best=3.767e-12 dt=0.07s
t= 400/ 2000 best=3.767e-12 dt=0.15s
t= 600/ 2000 best=3.767e-12 dt=0.21s
t= 800/ 2000 best=3.767e-12 dt=0.28s
t= 1000/ 2000 best=3.767e-12 dt=0.35s
t= 1200/ 2000 best=3.767e-12 dt=0.42s
t= 1400/ 2000 best=3.767e-12 dt=0.49s
t= 1600/ 2000 best=3.767e-12 dt=0.55s
t= 1800/ 2000 best=3.767e-12 dt=0.62s
t= 2000/ 2000 best=3.767e-12 dt=0.69s
done radius=0.03 -> best=3.767e-12 (dt=0.69s)
[k=4/4] rand probe: radius=0.01, NRand=2000
t= 200/ 2000 best=3.767e-12 dt=0.06s
t= 400/ 2000 best=3.767e-12 dt=0.13s
t= 600/ 2000 best=3.767e-12 dt=0.20s
t= 800/ 2000 best=3.767e-12 dt=0.27s
t= 1000/ 2000 best=3.767e-12 dt=0.33s
t= 1200/ 2000 best=3.767e-12 dt=0.40s
t= 1400/ 2000 best=3.767e-12 dt=0.47s
t= 1600/ 2000 best=3.767e-12 dt=0.54s
t= 1800/ 2000 best=3.767e-12 dt=0.60s
t= 2000/ 2000 best=3.767e-12 dt=0.68s
done radius=0.01 -> best=3.767e-12 (dt=0.68s)
[k=4/4] rand probe: radius=0.003, NRand=2000
t= 200/ 2000 best=3.767e-12 dt=0.07s
t= 400/ 2000 best=3.767e-12 dt=0.14s
t= 600/ 2000 best=3.767e-12 dt=0.21s
t= 800/ 2000 best=3.767e-12 dt=0.28s
t= 1000/ 2000 best=3.767e-12 dt=0.35s
t= 1200/ 2000 best=3.767e-12 dt=0.42s
t= 1400/ 2000 best=3.767e-12 dt=0.48s
t= 1600/ 2000 best=3.767e-12 dt=0.55s
t= 1800/ 2000 best=3.767e-12 dt=0.62s
t= 2000/ 2000 best=3.767e-12 dt=0.70s
done radius=0.003 -> best=3.767e-12 (dt=0.70s)
[k=4/4] rand probe: radius=0.001, NRand=2000
t= 200/ 2000 best=3.767e-12 dt=0.06s
t= 400/ 2000 best=3.767e-12 dt=0.14s
t= 600/ 2000 best=3.767e-12 dt=0.21s
t= 800/ 2000 best=3.767e-12 dt=0.28s
t= 1000/ 2000 best=3.767e-12 dt=0.35s
t= 1200/ 2000 best=3.767e-12 dt=0.42s
t= 1400/ 2000 best=3.767e-12 dt=0.49s
t= 1600/ 2000 best=3.767e-12 dt=0.56s
t= 1800/ 2000 best=3.767e-12 dt=0.63s
t= 2000/ 2000 best=3.767e-12 dt=0.70s
done radius=0.001 -> best=3.767e-12 (dt=0.70s)
[k=4/4] rand probe: radius=0.0003, NRand=2000
t= 200/ 2000 best=3.767e-12 dt=0.06s
t= 400/ 2000 best=3.767e-12 dt=0.13s
t= 600/ 2000 best=3.767e-12 dt=0.20s
t= 800/ 2000 best=3.767e-12 dt=0.27s
t= 1000/ 2000 best=3.767e-12 dt=0.34s
t= 1200/ 2000 best=3.767e-12 dt=0.41s
t= 1400/ 2000 best=3.767e-12 dt=0.48s
t= 1600/ 2000 best=3.767e-12 dt=0.55s
t= 1800/ 2000 best=3.767e-12 dt=0.62s
t= 2000/ 2000 best=3.767e-12 dt=0.69s
done radius=0.0003 -> best=3.767e-12 (dt=0.69s)
[k=4/4] rand probe: radius=0.0001, NRand=2000
t= 200/ 2000 best=3.767e-12 dt=0.07s
t= 400/ 2000 best=3.767e-12 dt=0.14s

```

```

t= 600/ 2000 best=3.767e-12 dt=0.21s
t= 800/ 2000 best=3.767e-12 dt=0.28s
t= 1000/ 2000 best=3.767e-12 dt=0.35s
t= 1200/ 2000 best=3.767e-12 dt=0.42s
t= 1400/ 2000 best=3.767e-12 dt=0.49s
t= 1600/ 2000 best=3.767e-12 dt=0.56s
t= 1800/ 2000 best=3.767e-12 dt=0.63s
t= 2000/ 2000 best=3.767e-12 dt=0.71s
done radius=0.0001 -> best=3.767e-12 (dt=0.71s)
[k=4] fminsearch starting from r_min=3.767e-12 ...

```

Iteration	Func-count	f(x)	Procedure
0	1	3.76692e-12	
1	5	3.76692e-12	initial simplex
2	7	3.76692e-12	contract outside
3	9	3.76692e-12	contract inside
4	10	3.76692e-12	reflect
5	11	3.76692e-12	reflect
6	12	3.76692e-12	reflect
7	14	3.76692e-12	contract inside
8	16	3.76692e-12	contract inside
9	18	3.76692e-12	contract outside
10	20	3.76692e-12	contract inside
11	22	3.76692e-12	contract outside
12	24	3.76692e-12	contract outside
13	26	3.76692e-12	contract inside
14	27	3.76692e-12	reflect
15	29	3.76692e-12	contract inside
16	31	3.76692e-12	contract outside
17	33	3.76692e-12	contract outside
18	35	3.76692e-12	contract inside
19	37	3.76692e-12	contract inside
20	39	3.76692e-12	contract inside
21	41	3.76692e-12	contract inside
22	43	3.76692e-12	contract inside
23	45	3.76692e-12	contract inside
24	47	3.76692e-12	contract inside
25	49	3.76692e-12	contract inside
26	51	3.76692e-12	contract outside
27	52	3.76692e-12	reflect
28	54	3.76692e-12	contract inside
29	56	3.76692e-12	contract inside
30	58	3.76692e-12	contract inside
31	60	3.76692e-12	contract inside
32	62	3.76692e-12	contract inside
33	64	3.76692e-12	contract inside
34	66	3.76692e-12	contract outside
35	68	3.76692e-12	contract inside
36	70	3.76692e-12	contract inside
37	72	3.76692e-12	contract outside
38	74	3.76692e-12	contract inside
39	76	3.76692e-12	contract inside
40	77	3.76692e-12	reflect
41	79	3.76692e-12	contract inside
42	81	3.76692e-12	contract inside
43	83	3.76692e-12	contract inside
44	85	3.76692e-12	contract inside
45	87	3.76692e-12	contract inside
46	89	3.76692e-12	contract inside
47	91	3.76692e-12	contract outside
48	93	3.76692e-12	contract inside
49	95	3.76692e-12	contract outside
50	96	3.76692e-12	reflect
51	98	3.76692e-12	contract inside

52	100	3.76692e-12	contract inside
53	102	3.76692e-12	contract inside
54	104	3.76692e-12	contract outside
55	106	3.76692e-12	contract inside
56	108	3.76692e-12	contract inside
57	109	3.76692e-12	reflect
58	111	3.76692e-12	contract inside
59	113	3.76692e-12	contract inside
60	114	3.76692e-12	reflect
61	116	3.76692e-12	contract inside
62	118	3.76692e-12	contract outside
63	120	3.76692e-12	contract outside
64	122	3.76692e-12	contract inside
65	124	3.76692e-12	contract inside
66	126	3.76692e-12	contract inside
67	128	3.76692e-12	contract outside
68	130	3.76692e-12	contract inside
69	132	3.76692e-12	contract inside
70	134	3.76692e-12	contract inside
71	136	3.76692e-12	contract outside
72	138	3.76692e-12	contract inside
73	140	3.76692e-12	contract inside
74	142	3.76692e-12	contract inside
75	144	3.76692e-12	contract outside
76	146	3.76692e-12	contract inside
77	148	3.76692e-12	contract inside
78	150	3.76692e-12	contract inside
79	152	3.76692e-12	contract outside
80	154	3.76692e-12	contract inside
81	156	3.76692e-12	contract inside
82	158	3.76692e-12	contract inside
83	160	3.76692e-12	contract outside
84	162	3.76692e-12	contract outside
85	164	3.76692e-12	contract inside
86	166	3.76692e-12	contract inside
87	168	3.76692e-12	contract inside
88	170	3.76692e-12	contract inside
89	172	3.76692e-12	contract inside
90	174	3.76692e-12	contract inside
91	176	3.76692e-12	contract inside
92	178	3.76692e-12	contract inside
93	180	3.76692e-12	contract inside
94	182	3.76692e-12	contract inside
95	184	3.76692e-12	contract inside
96	186	3.76692e-12	contract inside
97	188	3.76692e-12	contract outside
98	189	3.76692e-12	reflect
99	191	3.76692e-12	contract inside
100	193	3.76692e-12	contract inside
101	199	3.76692e-12	shrink
102	201	3.76692e-12	contract outside
103	203	3.76692e-12	contract outside
104	205	3.76692e-12	contract inside
105	207	3.76692e-12	contract inside
106	209	3.76692e-12	contract inside
107	211	3.76692e-12	contract outside
108	212	3.76692e-12	reflect
109	214	3.76692e-12	contract inside
110	216	3.76692e-12	contract outside
111	218	3.76692e-12	contract inside
112	220	3.76692e-12	contract outside
113	222	3.76692e-12	contract inside
114	224	3.76692e-12	contract outside
115	225	3.76692e-12	reflect

116	227	3.76692e-12	contract inside
117	229	3.76692e-12	contract inside
118	230	3.76692e-12	reflect
119	232	3.76692e-12	contract inside
120	234	3.76692e-12	contract outside
121	236	2.52777e-12	contract inside
122	238	2.52777e-12	contract inside
123	240	2.52777e-12	contract outside
124	242	2.39765e-12	contract inside
125	244	1.33557e-12	contract inside
126	246	1.33557e-12	contract outside
127	248	1.1612e-12	contract inside
128	250	1.09099e-12	contract inside
129	252	6.92566e-13	contract outside
130	254	6.49651e-13	contract inside
131	256	3.64665e-13	contract inside
132	257	3.64665e-13	reflect
133	259	3.64665e-13	contract inside
134	261	1.77102e-13	contract inside
135	263	1.77102e-13	contract inside
136	264	1.77102e-13	reflect
137	266	1.77102e-13	contract inside
138	268	1.72467e-13	contract inside
139	270	1.72467e-13	contract inside
140	272	1.5983e-13	contract inside
141	274	6.54049e-14	contract inside
142	276	6.54049e-14	contract inside
143	278	6.54049e-14	contract inside
144	280	6.54049e-14	contract inside
145	282	6.54049e-14	contract inside
146	283	6.54049e-14	reflect
147	285	2.64575e-14	contract inside
148	287	2.64575e-14	contract inside
149	289	2.64575e-14	contract outside
150	290	2.64575e-14	reflect
151	292	2.32462e-14	contract inside
152	294	2.10865e-14	contract inside
153	296	1.44621e-14	contract inside
154	298	1.44621e-14	contract inside
155	300	1.15599e-14	contract inside
156	302	7.22256e-15	contract inside
157	304	7.22256e-15	contract outside
158	306	4.37682e-15	contract inside
159	308	4.37682e-15	contract outside
160	310	3.89828e-15	contract inside
161	312	2.60486e-15	contract inside
162	314	2.10951e-15	contract inside
163	315	2.10951e-15	reflect
164	317	5.65456e-16	contract inside
165	319	5.65456e-16	contract inside
166	320	5.65456e-16	reflect
167	322	2.61128e-16	contract inside
168	324	2.61128e-16	contract inside
169	326	2.61128e-16	contract inside
170	327	2.61128e-16	reflect
171	329	1.40004e-16	contract inside
172	331	1.40004e-16	contract inside
173	332	1.40004e-16	reflect
174	334	9.9982e-17	contract inside
175	340	9.4032e-17	shrink
176	342	9.4032e-17	contract inside
177	344	9.4032e-17	contract outside
178	350	9.4032e-17	shrink
179	352	9.28756e-17	contract inside

180	353	9.28756e-17	reflect
181	355	9.28756e-17	contract inside
182	357	9.28756e-17	contract inside
183	359	7.87333e-17	reflect
184	365	4.34872e-17	shrink
185	366	4.34872e-17	reflect
186	372	9.24455e-18	shrink
187	373	9.24455e-18	reflect
188	374	9.24455e-18	reflect
189	375	9.24455e-18	reflect
190	376	9.24455e-18	reflect
191	382	9.24455e-18	shrink
192	383	9.24455e-18	reflect
193	384	9.24455e-18	reflect
194	385	9.24455e-18	reflect
195	391	9.24455e-18	shrink
196	393	9.24455e-18	contract outside
197	394	9.24455e-18	reflect
198	395	9.24455e-18	reflect
199	397	9.24455e-18	contract inside
200	399	9.24455e-18	contract inside
201	400	9.24455e-18	reflect
202	401	9.24455e-18	reflect
203	402	9.24455e-18	reflect
204	404	9.24455e-18	contract outside
205	410	9.24455e-18	shrink
206	411	9.24455e-18	reflect
207	413	9.24455e-18	contract inside
208	419	9.24455e-18	shrink
209	420	9.24455e-18	reflect
210	421	9.24455e-18	reflect
211	427	9.24455e-18	shrink
212	428	9.24455e-18	reflect
213	430	9.24455e-18	contract inside
214	436	9.24455e-18	shrink
215	438	9.24455e-18	contract inside
216	444	9.24455e-18	shrink
217	450	9.24455e-18	shrink
218	451	9.24455e-18	reflect
219	457	9.24455e-18	shrink
220	459	9.24455e-18	contract outside
221	460	9.24455e-18	reflect
222	466	9.24455e-18	shrink
223	467	9.24455e-18	reflect
224	468	9.24455e-18	reflect
225	474	9.24455e-18	shrink
226	475	9.24455e-18	reflect
227	476	9.24455e-18	reflect
228	477	9.24455e-18	reflect
229	478	9.24455e-18	reflect
230	484	9.24455e-18	shrink
231	485	9.24455e-18	reflect
232	487	4.54138e-18	reflect
233	488	4.54138e-18	reflect
234	494	4.54138e-18	shrink
235	495	4.54138e-18	reflect
236	501	4.54138e-18	shrink
237	503	4.54138e-18	contract outside
238	505	4.54138e-18	contract inside
239	507	4.54138e-18	contract inside
240	509	4.54138e-18	contract inside
241	515	4.54138e-18	shrink
242	517	4.54138e-18	contract outside
243	519	4.54138e-18	contract inside

```

244      521      4.54138e-18      contract inside

Optimization terminated:
the current x satisfies the termination criteria using OPTIONS.TolX of 1.000000e-16
and F(X) satisfies the convergence criteria using OPTIONS.TolFun of 1.000000e-16

[k=4] after fminsearch: r_min=4.541e-18 (exitflag=1)
[k=4] DONE: r_min 3.767e-12 -> 4.541e-18 (dt=6.68s)

== leigqNEWTON_refine_batch: summary ==
k=1 r_min=6.736e-18 respair=3.326e-15 resInf(polish)=8.204e-17
k=2 r_min=1.188e-17 respair=2.794e-15 resInf(polish)=5.586e-16
k=3 r_min=7.289e-17 respair=3.001e-15 resInf(polish)=6.668e-16
k=4 r_min=7.960e-16 respair=2.895e-15 resInf(polish)=9.366e-16
min pairwise distance = 9.272e-01 between (3,2)

```

```
median(cert.resMin), max(cert.resMin)
```

```

ans =
4.2385e-17
ans =
7.9596e-16

```

## Certificates only (eigenvalue-only vs eigenpair)

```

rMin = leigqNEWTON_cert_resMin(A, lambdaR(1));
rPair = leigqNEWTON_cert_resPair(A, lambdaR(1), VR(:,1));
[rMin rPair]

```

```

ans = 1×2
10-14 ×
    0.0007    0.3326

```

## One-call report

```
out = checkNEWTON(A, lambdaR, VR, 'SphereCheck', 'off');
```

```

== checkNEWTON ===
Matrix: 4x4 quaternion
Lambdas: 4 (provided). Vectors: 4 columns (provided).
Certificate/residual summary (median | max):
resMin abs: 4.24e-17 | 7.96e-16    resMin rel: 1.89e-18 | 3.47e-17
resPair abs: 3.07e-15 | 3.21e-15    resPair rel: 1.36e-16 | 1.50e-16

Per-eigenvalue residuals:
# lambda (cleaned/rounded)          resMin(abs/rel)          resPair(abs/rel)
1 -2 -2i +0j +2k                  6.74e-18/ 3.15e-19  3.19e-15/ 1.50e-16
2 -1.609 -1.752i +2.986j +2.341k  1.19e-17/ 5.31e-19  2.71e-15/ 1.21e-16
3 -1.339 -1.5i +2.699j +3.142k   7.29e-17/ 3.24e-18  3.21e-15/ 1.43e-16
4 -3.688 +1.143i +1.652j +2.785k  7.96e-16/ 3.47e-17  2.95e-15/ 1.29e-16

```

```

Interestingness:
Standard: Kdistinct = n = 4.
Sphere check: not run. Use checkNEWTON(...,'SphereCheck','on') to force.

```

```

Outputs:
out       : main struct (out.lambda, out.v, out.resMinAbs, out.resPairAbs, ...).
cases{1}  : simple case struct with fields A, lamAll, lamSamples, sph, info (sphere tooling style).
S         : summary struct (n, Ktot, Kdistinct, spheresFound, ...).

```

## **See also**

[doc\\_GettingStarted](#), [doc\\_leigqNEWTON](#), [doc\\_RefinementAndCertificates](#)