Overview

This project contains MATLAB files created to visualize and verify quaternion calculations, specifically those employed in Madgwick 2010 [1]. This document was made to show which code matches with which equations in Madgwick 2010 [1] and to discuss some results. To quickly view a rotation of a vector by a quaternion, run **driver.m**.

Selected Code

$$\begin{aligned}\boldsymbol{a} \otimes \boldsymbol{b} &= \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix} \otimes \begin{bmatrix} b_1 & b_2 & b_3 & b_4 \end{bmatrix} \\ &= \begin{bmatrix} a_1 b_1 - a_2 b_2 - a_3 b_3 - a_4 b_4 \\ a_1 b_2 + a_2 b_1 + a_3 b_4 - a_4 b_3 \\ a_1 b_3 - a_2 b_4 + a_3 b_1 + a_4 b_2 \\ a_1 b_4 + a_2 b_3 - a_3 b_2 + a_4 b_1 \end{bmatrix}^T \end{aligned} \qquad \text{(4, Madgwick)}$$

quatMult.m

```
28  a1 = q_a(1); b1= q_b(1);
29  a2 = q_a(2); b2= q_b(2);
30  a3 = q_a(3); b3= q_b(3);
31  a4 = q_a(4); b4= q_b(4);
32
33  q_out = [a1*b1 - a2*b2 - a3*b3 - a4*b4...
34           a1*b2 + a2*b1 + a3*b4 - a4*b3...
35           a1*b3 - a2*b4 + a3*b1 + a4*b2...
36           a1*b4 + a2*b3 - a3*b2 + a4*b1];
```

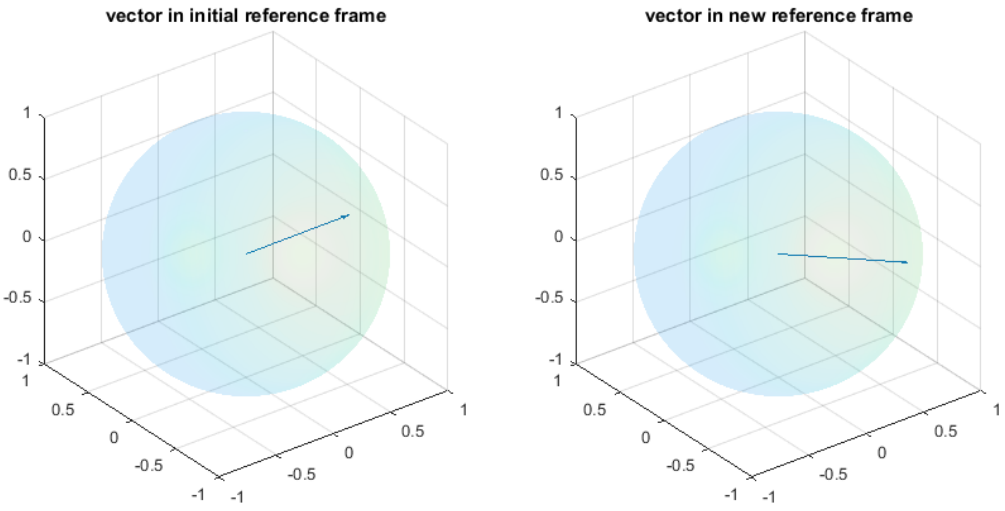
$${}^{B}\boldsymbol{v} = {}^{A}_{B}\hat{\boldsymbol{q}} \otimes {}^{A}\boldsymbol{v} \otimes {}^{A}_{B}\hat{\boldsymbol{q}}^{*} \qquad \text{(5, Madgwick)}$$

quatRotateDup.m

```
 9  qconj = [q(1) -q(2) -q(3) -q(4)];
10
11  first = quatMult(q,[0 vect_a]);
12  vect_b = quatMult(first, qconj);
13  vect_b = vect_b(2:4);
```

Example

The below figure demonstrates the result of **driver.m** for a $\pi/4$ angle rotation about the z axis. The initial vector [1 0 0], a unit vector in the x axis, was hardcoded in **quatTest.m**.



Directory Tree

```
./
├── README.md
├── driver.m
├── quatRotateDup.m
├── quatMult.m
└── quatTest.m
```

README.md

```
# Quaternion Testing
# Michael Sikora
# 2018.1.14

for quick visualization run driver.m and change the ax (axis of rotation) and angle given in quatTest call.

quatRotateDup.m is a function that rotates a vector given a quaternion representing a 3D rotation.
    It was intended as a duplicate of the quatrotate function available in the Aerospace Toolbox.

quatMult.m is the quaternion multiply operation.

quatTest.m was originally not a function, just a test script for rotating a vector in 3D using the
    quaternion calculation. The function was added to quickly see the effect of various angles and axiis.

THEORY

A rotation in three dimensional space can be mathematically represented using quaternion algebra. Quaternion
algebra is often preferred for heavy calculations, because only 4 rational numbers are needed. A quaternion
represents a change in coordinate reference frame.
```

References

[1] S. O. Madgwick, "An efficient orientation filter for inertial and inertial/magnetic sensor arrays," April 2010. [Online]. Available: http://x-io.co.uk/res/doc/madgwick_internal_report.pdf