1 What is a set?

Definition:

A Set is a collection of elements such that, given any element, we can tell whether that element is in the set or not.

Notation: If the element x is in set S, we write $x \in S$ (in) and if not, we write $x \notin S$ (not in).

Example: Based on this definition, does the order of elements matter in a set? Why or why not?

No- in the set or not in the set is all that is required from our definition

Example: Does the number of times an element is listed matter? Why or why not?

No- same rationale- whether in or not is what matters

Example: Let C be the set of all cities in Illinois. What are some elements in set C? What are some non-elements?

Dekalb, Chicago, Rockford

Iowa City, New York, Michigan, Pacific Ocean

Example: Let U be the set of all integers from 12 to 17. List all elements in set U.

$U = \{12, 13, 14, 15, 16, 17\}$

Definition:

Let A and B be sets. Then A is a Subset of B (notation: $A \subseteq B$ OR $B \supseteq A$) if every element of A is also in B.

Example: What are examples of subsets of C and U above?

one subset of C is our list above: {Dekalb, Chicago, Rockford}

subsets of U: $\{12, 14, 16\}$ OR $\{13, 15, 17\}$ OR $\{12, 13, 14, 15, 16, 17\}$

Notation:

If A is a finite set, we will denote the number of elements in A by $|A|$.

Definition:

The emptyset is the set that has no elements (notation: $\emptyset$).

Definition:

We say two sets are equal if every element in the first is also in the second and, conversely, every element in the second is also in the first. Thus, $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

If $A=B$, then $A \subseteq B$ and $B \subseteq A$
If $A \subseteq B$ and $B \subseteq A$, then $A=B$.
translates to

Example: Provide two examples of equal sets. (Be creative: what must remain the same and what can change?) Kids of same parents,

order, number of times appearing $\{1,2,3\} = \{3,3,2,1,1\}$

$\{\text{multiples of } 2\} = \{\text{even numbers}\}$ phrasing different

2 Set Operations

Definition:

The union of sets A and B (notation $A \cup B$) is the set consisting of all elements in A or B, meaning one of these scenarios is true:

(1) $x \in A$ and $x \notin B$

(2) $x \notin A$ and $x \in B$

(3) $x \in A$ and $x \in B$.

Definition:

The intersection of sets A and B (notation $A \cap B$) is the set consisting of all elements in A and B, meaning $x \in A$ and $x \in B$.

Definition:

If the intersection of two sets is the empty set, then the sets are said to be disjoint.

Example: Suppose $A = \{1, 3, 5, 7, 9\}$ (odds), $B = \{2, 3, 5, 7\}$ (primes), and $C = \{2, 4, 6, 8\}$ (evens). 1-9

(a) Determine $A \cup B$. $= \{1,2,3,5,7,9\}$ $A \cap B = \{3,5,7\}$

(b) Determine $B \cap C$. $= \{2\}$ $B \cup C = \{2,3,4,5,6,7,8\}$

(c) Which, if any, sets are disjoint?

A and C

Bonus question: what is $|A \cup B|$? $=6$

(6 elements in

$\{1,1,1\}$ is a subset of $\{1,2,3\}$ because $\{1,1,1\} = \{1\}$

Definition:

The difference of sets A and B (notation: $A - B$) is the set consisting of the elements in A that are not in B.

Example: Using $A = \{1,3,5,7,9\}$ and $B = \{2,3,5,7\}$, find $A-B$ and $B-A$. Will $A-B = B-A$ in general?

$A - B = \{1, 9\}$

$B - A = \{2\}$

No

Definition:

A set consisting of all of the elements of interest in a particular situation is called a universal set.

Definition:

Given a universal set U and a subset A of U, the set $U - A$ is called the complement of A (notation: $\overline{A}$). (together A and $\overline{A}$ "complete" U: $A \cup \overline{A} = U$)

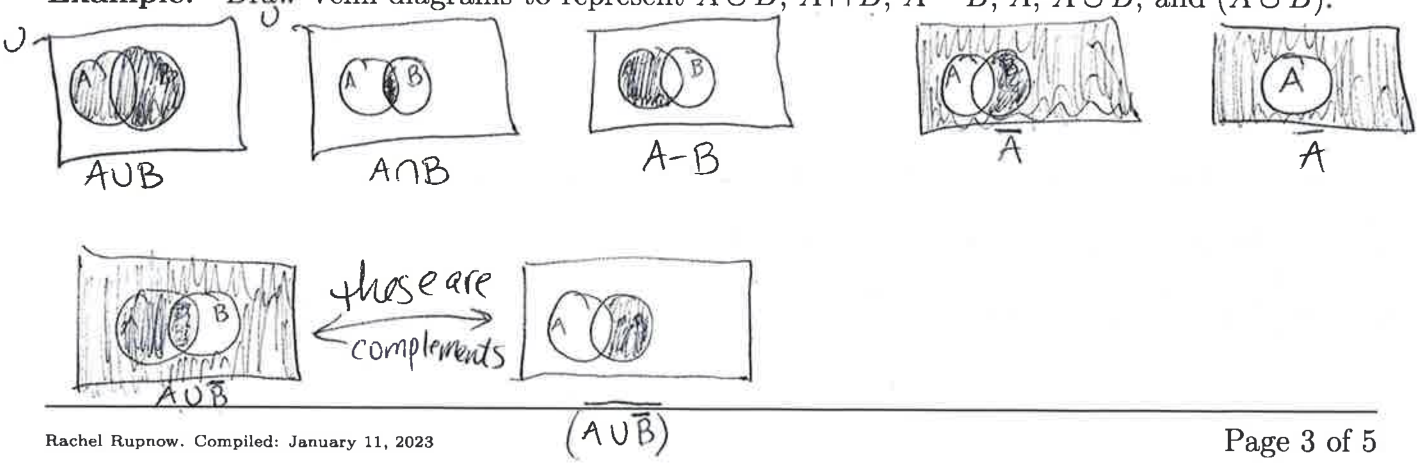
Example: Suppose $U = \{1,2,3,4,5,6,7,8,9\}$ is the universal set and $A = \{1,3,5,7,9\}$, $B = \{2,3,5,7\}$, and $C = \{2,4,6,8\}$. What are $\overline{A}$, $\overline{B}$, and $\overline{C}$?

$\overline{A} = \{2,4,6,8\}$; $\overline{B} = \{1,4,6,8,9\}$; $\overline{C} = \{1,3,5,7,9\} (= A)$

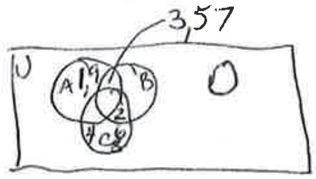
Definition:

A Venn diagram is a visual representation of the relationships among sets in which the universal set is represented by a rectangular region and subsets of the universal set are represented by circular disks drawn within the rectangular region. Sets not known to be disjoint should be represented by overlapping circles.

Example: Draw Venn diagrams to represent $A \cup B$, $A \cap B$, $A - B$, $\overline{A}$, $A \cup \overline{B}$, and $\overline{(A \cup \overline{B})}$.



Example: Suppose $U = \{0,1,2,3,4,5,6,7,8,9\}$ is the universal set and $A = \{1,3,5,7,9\}$, $B = \{2,3,5,7\}$, and $C = \{2,4,6,8\}$. Draw a Venn diagram that places each number appropriately.



Theorem 2.1:

Let U be a universal set. For any subsets A, B, and C of U, the following are true:

(a) $A \cup B = B \cup A$ and $A \cap B = B \cap A$ (commutative property for union or intersection)

(b) $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$ (associative property for union or intersection)

(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (distributive property)

(d) $\overline{\overline{A}} = A$

(e) $A \cup \overline{A} = U$

(f) $A \cap \overline{A} = \emptyset$

(g) $A \subseteq A \cup B$ and $B \subseteq A \cup B$

(h) $A \cap B \subseteq A$ and $A \cap B \subseteq B$

(i) $A - B = A \cap \overline{B}$

Theorem 2.2, De Morgan's Laws:

For any subsets A and B of a universal set U, the following are true:

(a) $(\overline{A \cup B}) = \overline{A} \cap \overline{B}$

(b) $(\overline{A \cap B}) = \overline{A} \cup \overline{B}$

Example: Use Theorems 2.1 and/or 2.2 to simplify $\overline{A} \cap (A \cup B)$.

Using Thm 2.1c: $(\overline{A} \cap A) \cup (\overline{A} \cap B)$

Using Thm 2.1f, we know $\overline{A} \cap A = \emptyset$ so $(\overline{A} \cap A) \cup (\overline{A} \cap B) = \boxed{\overline{A} \cap B}$

Definition:

An ordered pair of elements (notation: (a, b)) lists two elements and attends to the order of entries. Thus $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$.

Definition:

The Cartesian product of A and B is the set consisting of all ordered pairs (a, b) where $a \in A$ and $b \in B$ (notation: $A \times B$).

Example: Let $C = \{1, 5\}$ and $D = \{2, 4, 5\}$. Find $C \times D$ and $D \times C$.

$C \times D = \{(1,2), (1,4), (1,5), (5,2), (5,4), (5,5)\}$

$D \times C = \{(2,1), (2,5), (4,1), (4,5), (5,1), (5,5)\}$

Example: In general, will $A \times B = B \times A$? Why or why not?

No - $C \times D$ and $D \times C$ above are already different (have a counterexample)