1 Distance

How should we define distance in a graph?

Look at the number of edges (or weights in weighted graphs) between vertices to decide distance.

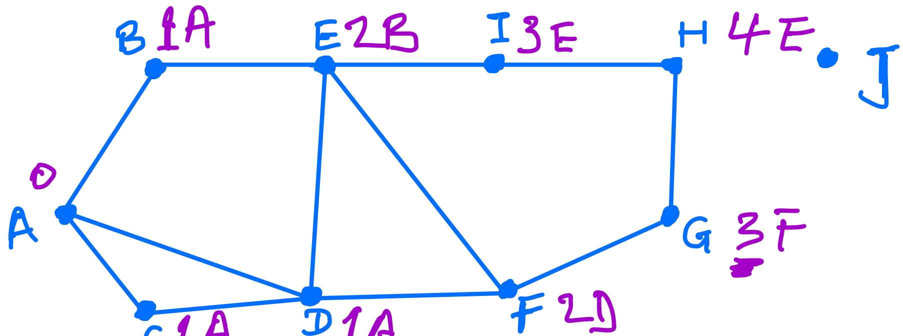
Definition:

> The smallest possible number of edges in a path between two vertices S and T is the distance between vertices S and T.

How can we find the shortest path between vertices?

Search via adjacency (Breadth-first search alg. (p. 183))
- Find all vertices adjacent to the original vertex (original labeled 0, ones adjacent labeled 1). Find all vertices adjacent to the vertices labeled 1 and label them 2 (do not label something already labeled), etc. until all vertices are labeled

Example: Using the graph below, find the distance from A to G. What is a shortest path from A to G? Is the graph connected? Why or why not?



distance from A to G is 3

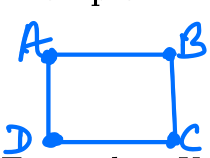
A, D, F, G

— No way to reach J from A.
— No path / no path length.

Example: If there is a path from one vertex to all others in the graph, is that sufficient to say the graph is connected? Why or why not?

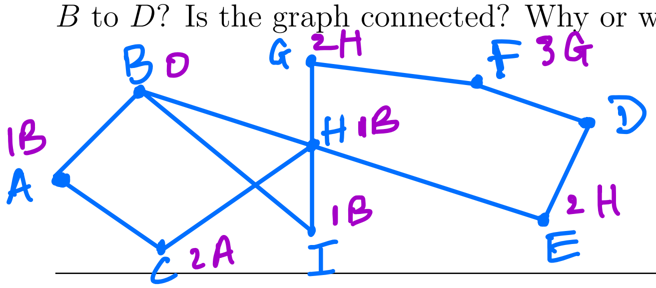
Yes! — Definition of connectedness of multigraph

Example: Must the shortest path between two vertices in a graph be unique? Why or why not?



ABC and ADC both have length 2.

Example: Using the graph below, find the distance from B to D. What is a shortest path from B to D? Is the graph connected? Why or why not?



B, H, E, D

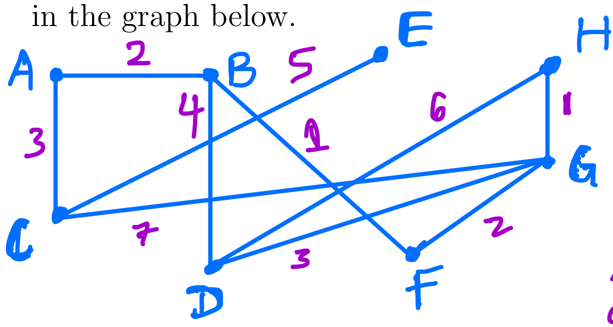
- Can reach all vertices from B / have path or length for all vertices.

2 Weighted Graphs

Definitions:

A weighted graph is a graph in which a number called the weight is assigned to each edge. The weight of a path is the sum of the weights of the edges in the path. A path of smallest weight is called a shortest path between those two vertices and the weight of that path is called the distance between them.

Example: Determine the weight of the path $\overline{A, B, D, G}$ (2 4 3) and the weight of the path $\overline{C, G, F, B, D, H}$ (7 2 1 4 6) in the graph below.



weight of path ABDG = 9
weight of path CGFBDH = 20

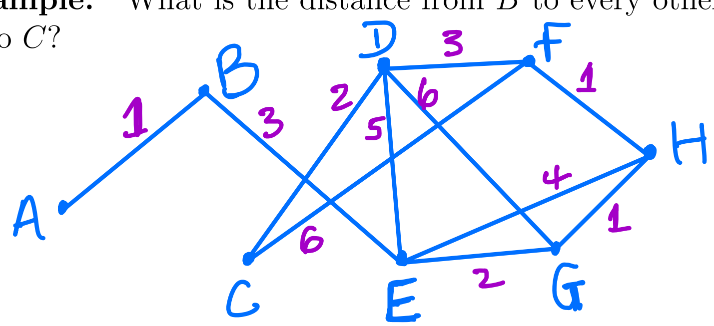
Note: in a weighted graph, the weights are isomorphism invariant.

A —1— B
A —2— B
not isomorphic

How can we find the shortest path between two vertices, when the weights are important?

Dijkstra's Algorithm (P.185) — Summary: start with the initial vertex of path, find the weight of each adjacent vertex, then repeat with adjacent vertex of least weight, assign the lesser of old weight and new (cummulative) weight from that vertex to what it is adjacent to and proceed until all vertices are labeled. Call nonadjacent vertices weight ∞ (will be replaced later if connected). Keep track of finalized vertices.

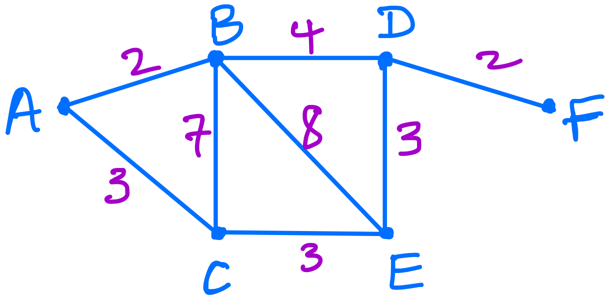
Example: What is the distance from B to every other vertex? What is the shortest path from B to C?



100%

PLEASE CHECK OUT REVIEW PROBLEMS ON THE BB!

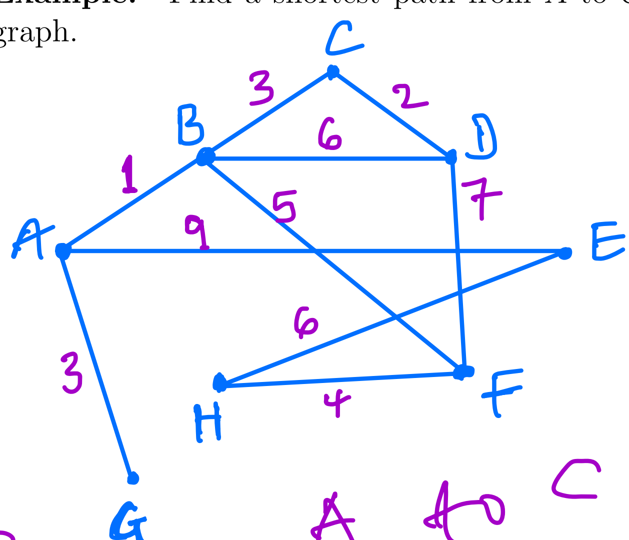
Example: What is the distance from B to every other vertex? What is the shortest path from B to C?



Example: Find a shortest path from A to G that goes through the vertex C in the weighted graph.

SD PV

A
B
C
D
E
F
G
H



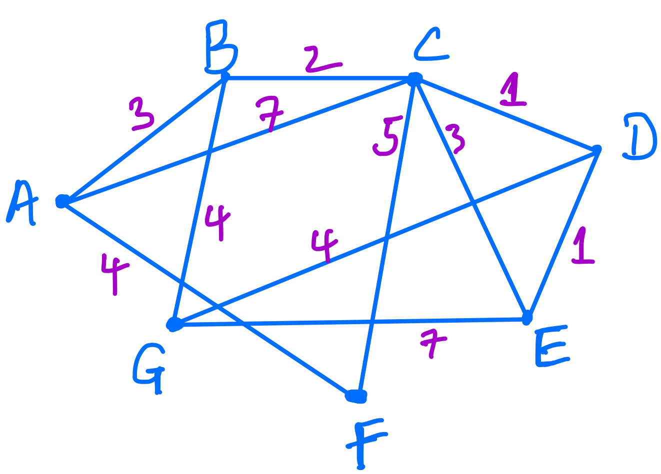
C to G

A to C

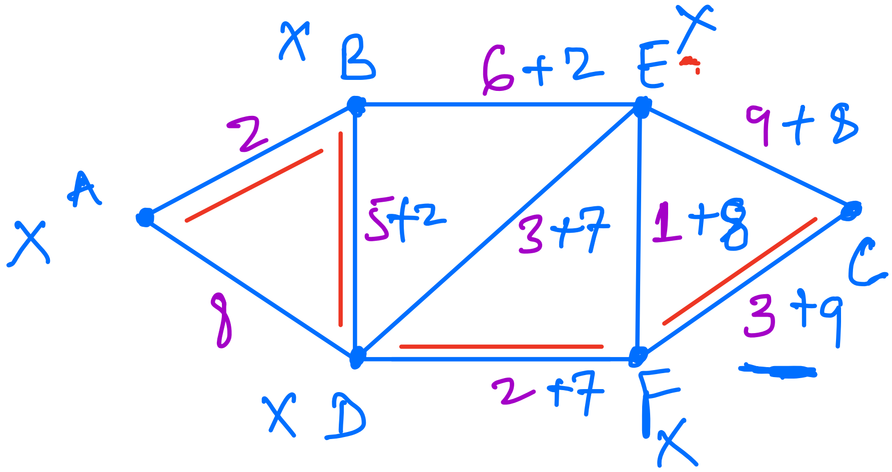
Example: Find a shortest path from A to G that goes through the vertex C in the weighted graph.

SD PV

A
B
C
D
E
F
G



Find a shortest path from A to C



vertex	SD	PV
x A	0	—
x B	2	A
C	12	F
x D	7	B
→ x E	8	B
x F	9	D

SD ≡ shortest distane

PV ≡ previous vertex

A B D F C

$2+5+2+3$

A to B = 2 ↑

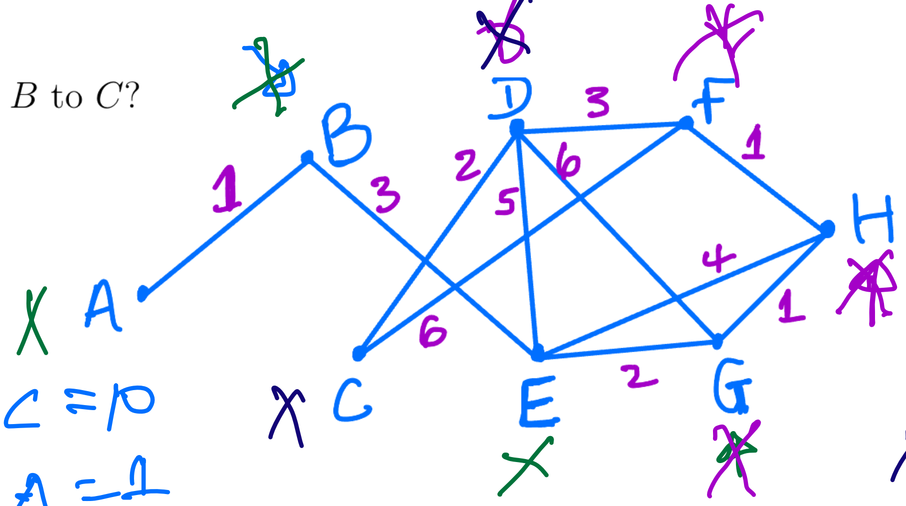
A to D = 7

A to E = 8   ABE

A to C = 12

A to F = 9

B to C?



	SD	PV
A	~~∞~~ 1	B
B	0	—
C	~~∞~~ ~~13~~ 10	~~F~~ D
D	~~∞~~ 8	E
E	~~∞~~ 3	B
F	~~∞~~ 7	H
G	~~∞~~ 5	E
H	~~∞~~ ~~7~~ 6	~~E~~ G

B to C = 10

B to A = 1

B to E = 3 B, E, D, C

B to D = 8 3 + 5 + 2 = 10

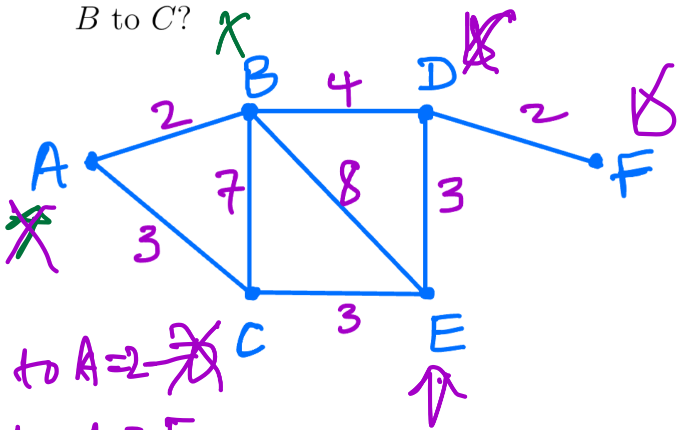
B to F = 7

B to H = 6

B E D
3+5

B E G H F
3+2+1+1

Example: What is the distance from B to every other vertex? What is the shortest path from B to C?



	SD	PV
A	~~∞~~ 2	B
B	0	—
C	~~∞~~ ~~7~~ 5	~~B~~ A
D	~~∞~~ 4	B
E	~~∞~~ ~~8~~ 7	~~B~~ D
F	~~∞~~ 6	D

B to A = 2 ~~7~~

B to C = 5

B to D = 4

B to E = 7

B to F = 6

B A C

2 + 3 = 5

B D E
4 3

B D F
4+2