Recall that $[n]+[y]=[n+y]$ and $[n][y]=[ny]$.

Example: For each of the following, use a representative r such that $0 \leq r < m$ to characterize the result in $\mathbb{Z}_m$.

$[13]+[8]=[13+8]=[21]=[5]$

(a) Find $[9]+[7]$ in $\mathbb{Z}_{12}$.

$[9]+[7]=[9+7]=[16]=[4]$

(b) Find $[13]+[8]$ in $\mathbb{Z}_8$.

$[5]+[0]=[5+0]=[5]$

(c) Find $[111]+[57]$ in $\mathbb{Z}_{112}$.

$[-1]+[57]=[-1+57]=[56]$

$[111]+[57]=[111+57]$

(d) Find $[999]+[402]$ in $\mathbb{Z}_{60}$.

$[39]+[42]=[39+42]=[81]=[21]$

(e) Find $[9][7]$ in $\mathbb{Z}_{12}$.

$[9\cdot 7]=[63]=[3]$

(f) Find $[13][8]$ in $\mathbb{Z}_8$.

$[5][0]=[5\cdot 0]=[0]$

(g) Find $[111][57]$ in $\mathbb{Z}_{112}$.

$[-1][57]=[-1\cdot 57]=[-57]=[-57+112]=[\ ]$

(h) Find $[999][402]$ in $\mathbb{Z}_{60}$.

$[39][42]=[39\cdot 42]$

(i) Find $[5]^{20}$ in $\mathbb{Z}_4$.

$[1]^{20}=[1^{20}]=[1]$

(j) Find $[12]^5$ in $\mathbb{Z}_{13}$.

$[-1]^5=[-1\cdot -1\cdot -1\cdot -1\cdot -1]=[-1]$

(k) Find $[26]^{59}$ in $\mathbb{Z}_{13}$.

$[0]^{59}=[0]$

(l) Find $[23]^{18}$ in $\mathbb{Z}_{25}$.

$[-2]^{18}=[2^{18}]=[262144]=[19]$

Example: Let A denote the equivalence class containing 5 in $\mathbb{Z}_8$ and B denote the congruence class (equivalence class) containing 5 in $\mathbb{Z}_{12}$. Is $A=B$? Why or why not?

$A=[5]_8=\{\ldots,-3,5,13,21,\ldots\}$

$B=[5]_{12}=\{\ldots,-7,5,17,29,\ldots\}$