1 Permutations

Permutations help us answer the question "How many different ordered arrangements of r objects can be formed from a set of n distinct objects?" In this section, we will answer this question in the no repetition context (and leave repetition to 8.4).

Definition:

An ordered arrangement of n distinct objects is called a ____________ of the objects. If $r \leq n$, the arrangement using r of the n distinct objects is called an _______________.

Example: For $n = 4$ and the digits 1,2,3,4, what is an example of a permutation? What is an example of a 3-permutation?

Theorem 1.2:

The number of ways an ordered list of r objects can be chosen without repetition from n objects is $P(n,r) = n(n-1)\cdots(n-r+1) = \dfrac{n!}{(n-r)!}$.

Why should this make sense?

Example: Find $P(5,2)$ and $P(13,3)$.

Example: Find $P(n,n)$ and $P(n,0)$.

Example: How many different three digit numbers can be formed using the digits 1,2,3,4,5 without repetition?

Example: How many different orders can 6 people be seated in 6 chairs?

Example: How many different arrangements are there of the letters in the word "Dekalb"?

2 Combinations

Combinations help us answer the question "How many different (unordered) collections of r objects can be made from set of of n distinct objects?" In this section, we will answer this question in the no repetition context (and leave repetition to 8.4).

Definition:

If $r \leq n$, then an unordered collection of r objects chosen from a set of n distinct objects is called an _______________.

Example: For $n = 4$ and the digits 1,2,3,4 what is an example of a 2-combination?

Theorem 2.10:

Let S be a set containing n elements, where n is a nonnegative integer. If r is an integer such that $0 \leq r \leq n$, then the number of subsets of S containing exactly r elements is $C(n, r) = \dfrac{n!}{r!(n-r)!}$.

Why should this make sense?

Example: Find $C(5, 2)$ and $C(13, 3)$.

Example: Find $C(n, n)$ and $C(n, 0)$.

Example: How many different 3-member committees can be formed from a delegation of 7 members?

Example: How many different 4-member committees can be formed from a delegation of 7 members?

Example: How many 3-letter collections can be formed from the letters in the word "Dekalb"?

3 Combining Skills

Example: Eight people are running for three at-large seats on a school board. In how many different ways can the three seats be filled?

Example: Eight people are running for all three positions of President, Vice-President, and Treasurer seats on a school board. In how many different ways can the three seats be filled?

Example: Four plenary speakers are scheduled to address a conference. In how many different orders can they appear?

Example: How many different 16-bit strings contain exactly three 1s?

Example: In how many different ways can 9 students be paired with 9 of 14 companies offering internships?

Example: How many different committees consisting of 2 representatives from math and 3 representatives from computer science can be formed from among 7 representatives from math and 9 representatives from computer science?