

Recall that $[x] + [y] = [x + y]$ and $[x][y] = [xy]$.

Example: For each of the following, use a representative r such that $0 \leq r < m$ to characterize the result in \mathbb{Z}_m .

$$[13] + [8] = [13 + 8] = [21] = [5]$$

$$\begin{array}{r} 16 \\ 60 \overline{) 999} \\ \underline{960} \\ 39 \end{array}$$

(a) Find $[9] + [7]$ in \mathbb{Z}_{12} .

$$[9] + [7] = [9 + 7] = [16] = [4]$$

(b) Find $[13] + [8]$ in \mathbb{Z}_8 .

$$[5] + [0] = [5 + 0] = [5]$$

(c) Find $[111] + [57]$ in \mathbb{Z}_{112} .

$$[-1] + [57] = [-1 + 57] = [56] \quad \begin{array}{l} [11] + [57] \\ = [111 + 57] \end{array}$$

$$\begin{array}{r} 6 \\ 60 \overline{) 402} \\ \underline{360} \\ 42 \end{array}$$

(d) Find $[999] + [402]$ in \mathbb{Z}_{60} .

$$[39] + [42] = [39 + 42] = [81] = [21]$$

(e) Find $[9][7]$ in \mathbb{Z}_{12} .

$$[9 \cdot 7] = [63] = [3]$$

$$\begin{array}{r} 6 \\ 60 \overline{) 378} \\ \underline{360} \\ 18 \end{array}$$

(f) Find $[13][8]$ in \mathbb{Z}_8 .

$$[5][0] = [5 \cdot 0] = [0]$$

(g) Find $[111][57]$ in \mathbb{Z}_{112} .

$$[-1][57] = [-1 \cdot 57] = [-57] = [-57 + 112] = [55]$$

(h) Find $[999][402]$ in \mathbb{Z}_{60} .

$$[39][42] = [39 \cdot 42]$$

$$60 \overline{) 39 \cdot 42}$$

$$2^{18} = 262144$$

(i) Find $[5]^{20}$ in \mathbb{Z}_4 .

$$[1]^{20} = [1^{20}] = [1]$$

(j) Find $[12]^5$ in \mathbb{Z}_{13} .

$$[-1]^5 = [-1 \cdot -1 \cdot -1 \cdot -1 \cdot -1] = [-1]$$

$$\begin{array}{r} 10485 \\ 25 \overline{) 262144} \\ \underline{262125} \\ 19 \end{array}$$

(k) Find $[26]^{59}$ in \mathbb{Z}_{13} .

$$[0]^{59} = [0]$$

(l) Find $[23]^{18}$ in \mathbb{Z}_{25} .

$$[-2]^{18} = [2^{18}] = [262144] = [19]$$

Example: Let A denote the equivalence class containing 5 in \mathbb{Z}_8 and B denote the congruence class (equivalence class) containing 5 in \mathbb{Z}_{12} . Is $A = B$? Why or why not?

$$A = [5]_8 = \{\dots, -3, 5, 13, 21, \dots\}$$

$$B = [5]_{12} = \{\dots, -7, 5, 17, 29, \dots\}$$