1 Method of Iteration

Procedure Description:

The method of iteration allows us to compute the general term of a sequence from its recurrence relation and initial conditions. This is done by computing successive terms in the sequence until a pattern is observed.

Example: Suppose the recurrence relation is $\boxed{t_n = 2t_{n-1}}$ for $n \geq 2$ with initial condition $t_1 = 3$ (from 9.1). How could we describe the nth term of this sequence?

$t_1 = 3 = 3\cdot 2^0$

$t_2 = 2\cdot 3 = 6 = 3\cdot 2 = 3\cdot 2^1$

$t_3 = 2\cdot t_2 = 2\cdot 6 = 12 = 3\cdot 4 = 3\cdot 2^2$

$t_4 = 2\cdot 12 = 24 = 3\cdot 8 = 3\cdot 2^3$

$t_5 = 2\cdot 24 = 48 = 3\cdot 16 = 3\cdot 2^4$

$t_6 = 2\cdot 48 = 96 = 3\cdot 32 = 3\cdot 2^5$

$t_n = 3\cdot 2^{n-1}; \; n \geq 1$

Example: Suppose the recurrence relation is $p_n = p_{n-1} + 2$ for $n \geq 1$ with initial condition $p_0 = 92$ (from 9.1). How could we describe the nth term of this sequence?

$p_0 = 92$

$p_1 = 94 = 92 + \boxed{2 = 2\cdot 1}$

$p_2 = 96 = 92 + \boxed{4 = 2\cdot 2}$

$p_3 = 98 = 92 + \boxed{6 = 2\cdot 3}$

$p_4 = 100 = 92 + \boxed{8 = 2\cdot 4}$

$p_5 = 102 = 92 + \boxed{10 = 2\cdot 5}$

$p_6 = 104 = 92 + 12$

$$\boxed{p_n = 92 + 2\cdot n \quad n \geq 0}$$

Example: Suppose the recurrence relation is $s_n = 2s_{n-1} - 3$ for $n \geq 1$ with initial condition $s_0 = 7$. How could we describe the nth term of this sequence?

s_0

$\frac{2\cdot(2+1)}{2} = \frac{6}{2} = 3$ $\qquad \frac{1\cdot(1+1)}{2} = 1$

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Example: Use the method of iteration to help you compute $1 + 2 + 3 + ... + n$.

$s_1 = 1$

$s_2 = 1 + 2 = 3$

$s_3 = 1 + 2 + 3 = 6$

$s_4 = 1 + 2 + 3 + 4 = 10$

$s_5 = 1 + 2 + 3 + 4 + 5 = 15$

$s_n = \frac{n(n+1)}{2}$

$\frac{3\cdot(3+1)}{2}$

$\frac{3\cdot 4}{2} = 6$

$n = 4 \quad \frac{4\cdot(4+1)}{2} = \frac{4\cdot 5}{2}$

$\frac{5\cdot 6}{2} = 15$

Example: Use the method of iteration to find a formula expressing s_n as a function of n for the given recurrence relation and initial conditions: $s_n = -s_{n-1} + 10, s_0 = -4$.

$n=0$ $s_0 = -4$

$n=1$ $s_1 = -(-4) + 10 = 14$

$n=2$ $s_2 = -14 + 10 = -4$

$n=3$ $s_3 = -(-4) + 10 = 14$

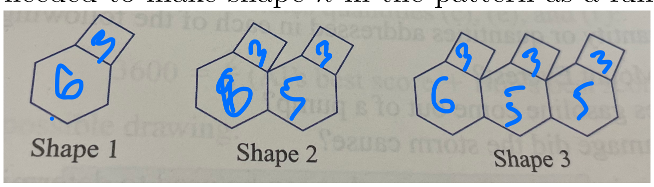
$n=4$ $s_4 = -14 + 10 = -4$

$n=5$ $s_5 = -(-4) + 10 = 14$

$n=6$ $s_6 = -14 + 10 = -4$

$$s_n = \begin{cases} -4 & \text{if } n \ge 0 \text{ is even} \\ 14 & \text{if } n \ge 0 \text{ is odd} \end{cases}$$

Example: Use the method of iteration to find a formula expressing the number of toothpicks needed to make shape n in the pattern as a function of n:



$s_1 = 9 + 8\cdot 0$

$s_n = 9 + 8(n-1)$

$s_2 = 9 + 8\cdot 1$

$n \ge 1$

$s_3 = 9 + 8 + 8$

Example: Use the method of iteration to find a formula expressing s_n as a function of n for the given recurrence relation and initial conditions: $s_n = s_{n-1} + 4(n-3), s_0 = 10$.

$s_0 = 1 = 3 - 2 = 3^1 - 2$

$s_1 = 2 = 4 - 2$ ($4\cdot 3^0$)

$s_2 = 7 = 9 - 2 = 3^2 - 2$

$s_3 = 10 = 12 - 2$ ($4\cdot 3^1$)

$s_4 = 25 = 27 - 2 = 3^3 - 2$

$s_5 = 34 = 36 - 2$ ($4\cdot 9$, $4\cdot 3^2$)

$s_6 = 79 = 81 - 2 = 3^4 - 2$

$s_n = 3^{\frac{n+2}{2}} - 2$; $n \ge 0$ (n is even)

$s_n = 4\cdot 3^{\frac{n-1}{2}} - 2$ $n \ge 1$ (n is odd)

Example: Use the method of iteration to find a formula expressing s_n as a function of n for the given recurrence relation and initial conditions: $s_n = 3s_{n-2} + 4, s_0 = 1, s_1 = 2$.

$s_0 = 1$

$s_1 = 2$

$s_3 = 3\cdot s_1 + 4 = 6 + 4 = 10$

$s_2 = 3\cdot s_0 + 4 = 7$

$s_4 = 3\cdot s_2 + 4 = 25$

$s_5 = 3\cdot s_3 + 4 = 34$

$s_6 = 3\cdot s_4 + 4 = 79$

Example: Suppose the recurrence relation is $s_n = 2s_{n-1} - 3$ for $n \geq 1$ with initial condition $s_0 = 7$. How could we describe the nth term of this sequence?

$s_0 = 7$

$s_1 = 2 \cdot s_0 - 3 = 14 - 3 = 11$ $\quad +4$

$s_2 = 2 \cdot s_1 - 3 = 22 - 3 = 19$ $\quad +8 \quad 2^3$

$s_3 = 2 \cdot s_2 - 3 = 38 - 3 = 35$ $\quad +16 \quad 2^4$

$s_4 = 2 \cdot s_3 - 3 = 70 - 3 = 67$ $\quad +32 \quad 2^5$

$s_5 = 2 \cdot 67 - 3 = 134 - 3 = 131$ $\quad +64 \quad 2^6$

$$\boxed{s_n = 4 \cdot 2^n + 3} \qquad n \geq 0$$

Example: Use the method of iteration to find a formula expressing s_n as a function of n for the given recurrence relation and initial conditions: $s_n = s_{n-1} + 4(n-3)$, $s_0 = 10$.

$s_0 = 10$

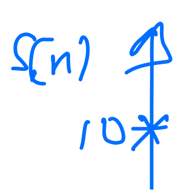
$s_1 = s_0 + 4 \cdot (1-3) = 10 - 8 = 2$

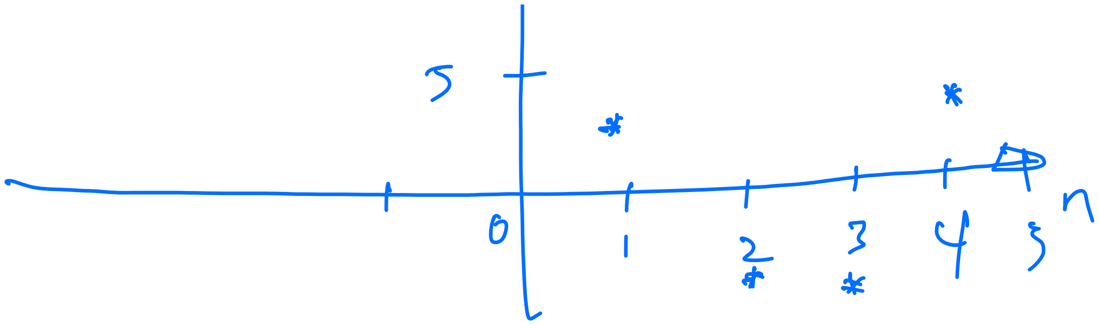
$s_2 = s_1 + 4(2-3) = 2 + 4 \cdot (-1) = -2$

$s_3 = s_2 + 4(3-3) = -2 + 0 = -2$

$s_4 = s_3 + 4(4-3) = -2 + 4 = 2$

$s_5 = s_4 + 4(5-3) = 2 + 8 = 10$





$$y = ax^2 + bx + c$$

$$\boxed{S_n = an^2 + bn + c}$$

$n = 0 \quad S_0 = 10$

$10 = a \cdot 0^2 + b \cdot 0 + c$

$\Rightarrow c = 10$

$$\boxed{S_n = 2n^2 - 10n + 10}$$

$n = 2 \quad S_2 = -2 \; ; \; n = 3 \quad S_3 = -2$

(1) $-2 = a \cdot 2^2 + b \cdot 2 + 10$

(2) $-2 = a \cdot 3^2 + b \cdot 3 + 10$

(1) $\Rightarrow -12 = 4a + 2b \Rightarrow -6 = 2a + b$

$\boxed{b = -6 - 2a}$ (3)

(2) $\Rightarrow -12 = 9 \cdot a + 3b \Rightarrow \boxed{-4 = 3a + b}$ (4)

$-4 = 3a - 6 - 2a$ in (3)

$2 = a$

$b = -6 - 2 \cdot 2$
$= -10$