1 What is a set?

Definition:

A ______ is a collection of elements such that, given any element, we can tell whether that element is in the set or not.

Notation: If the element x is in set S, we write $x \in S$ and if not, we write $x \notin S$.

Example: Based on this definition, does the order of elements matter in a set? Why or why not?

Example: Does the number of times an element is listed matter? Why or why not?

Example: Let C be the set of all cities in Illinois. What are some elements in set C? What are some non-elements?

Example: Let U be the set of all integers from 12 to 17. List all elements in set U.

Definition:

Let A and B be sets. Then A is a __________ of B (notation: $A \subseteq B$ OR $B \supseteq A$) if every element of A is also in B.

Example: What are examples of subsets of C and U above?

Notation:

If A is a finite set, we will denote the number of elements in A by $|A|$.

Definition:

The __________ is the set that has no elements (notation: $\emptyset$).

Definition:

We say two sets are ____________ if every element in the first is also in the second and, conversely, every element in the second is also in the first. Thus, $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.

Example: Provide two examples of equal sets. (Be creative: what must remain the same and what can change?)

2 Set Operations

Definition:

The ____________ of sets A and B (notation $A \cup B$) is the set consisting of all elements in A or B, meaning one of these scenarios is true:

(1) $x \in A$ and $x \notin B$

(2) $x \notin A$ and $x \in B$

(3) $x \in A$ and $x \in B$.

Definition:

The ____________ of sets A and B (notation $A \cap B$) is the set consisting of all elements in A and B, meaning $x \in A$ and $x \in B$.

Definition:

If the intersection of two sets is the empty set, then the sets are said to be ____________.

Example: Suppose $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 5, 7\}$, and $C = \{2, 4, 6, 8\}$.

(a) Determine $A \cup B$.

(b) Determine $B \cap C$.

(c) Which, if any, sets are disjoint?

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Definition:

The ____________________ of sets A and B (notation: $A - B$) is the set consisting of the elements in A that are not in B.

Example: Using $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 3, 5, 7\}$, find $A - B$ and $B - A$. Will $A - B = B - A$ in general?

Definition:

A set consisting of all of the elements of interest in a particular situation is called a ____________________.

Definition:

Given a universal set U and a subset A of U, the set $U - A$ is called the ______________ of A (notation: $\overline{A}$).

Example: Suppose $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is the universal set and $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 5, 7\}$, and $C = \{2, 4, 6, 8\}$. What are $\overline{A}$, $\overline{B}$, and $\overline{C}$?

Definition:

A ____________________ is a visual representation of the relationships among sets in which the universal set is represented by a rectangular region and subsets of the universal set are represented by circular disks drawn within the rectangular region. Sets not known to be disjoint should be represented by overlapping circles.

Example: Draw Venn diagrams to represent $A \cup B$, $A \cap B$, $A - B$, $\overline{A}$, $A \cup \overline{B}$, and $(\overline{A \cup \overline{B}})$.

Example: Suppose $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is the universal set and $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 3, 5, 7\}$, and $C = \{2, 4, 6, 8\}$. Draw a Venn diagram that places each number appropriately.

Theorem 2.1:

Let U be a universal set. For any subsets A, B, and C of U, the following are true:

(a) $A \cup B = B \cup A$ and $A \cap B = B \cap A$

(b) $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$

(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(d) $\overline{\overline{A}} = A$

(e) $A \cup \overline{A} = U$

(f) $A \cap \overline{A} = \emptyset$

(g) $A \subseteq A \cup B$ and $B \subseteq A \cup B$

(h) $A \cap B \subseteq A$ and $A \cap B \subseteq B$

(i) $A - B = A \cap \overline{B}$

Theorem 2.2, De Morgan's Laws:

For any subsets A and B of a universal set U, the following are true:

(a) $(\overline{A \cup B}) = \overline{A} \cap \overline{B}$

(b) $(\overline{A \cap B}) = \overline{A} \cup \overline{B}$

Example: Use Theorems 2.1 and/or 2.2 to simplify $\overline{A} \cap (A \cup B)$.

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Definition:

An ____________________ of elements (notation: (a, b)) lists two elements and attends to the order of entries. Thus $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$.

Definition:

The __________________________ of A and B is the set consisting of all ordered pairs (a, b) where $a \in A$ and $b \in B$ (notation: $A \times B$).

Example: Let $C = \{1, 5\}$ and $D = \{2, 4, 5\}$. Find $C \times D$ and $D \times C$.

Example: In general, will $A \times B = B \times A$? Why or why not?