What is another way of creating spanning trees besides guess-and-check and breadth-first search?

Depth-first search — continue a path out as far as possible (numbering as you go) then back up to the last place you had a choice and go out as far as possible; repeat until all vertices have numbers and predecessors.

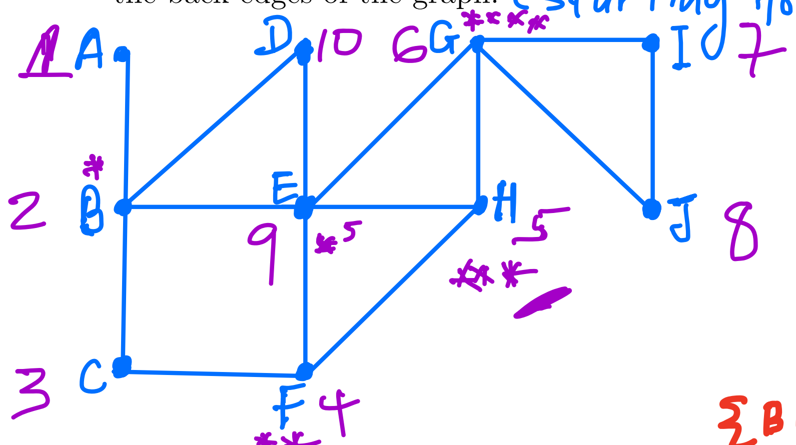
Theorem 5.7:

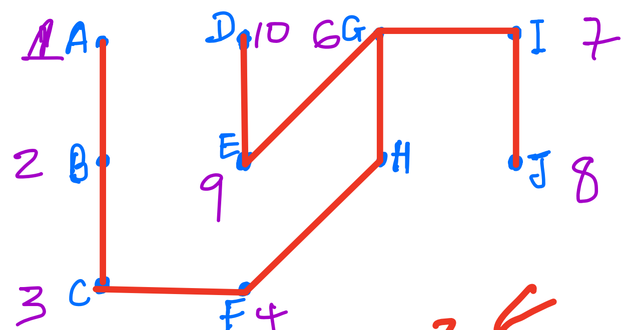
> Let the depth-first search algorithm be applied to a graph $\mathcal{G}$.
>
> (a)The edges and vertices selected form a tree.
>
> (b) If $\mathcal{G}$ is connected, this tree is a spanning tree.

Definitions:

> The tree created using the depth-first search algorithm is called a depth-first search tree.
>
> The edges are called tree edges and the other edges are called back edges. The labeling of the vertices is called a depth-first search numbering.

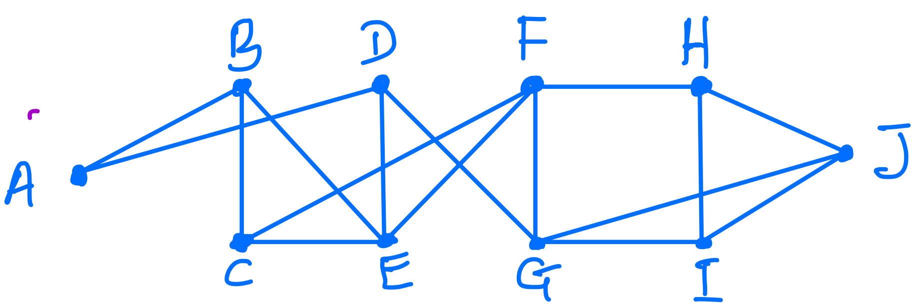
Example: Consider the graph below. Apply the depth-first search algorithm to obtain a depth-first search numbering of the vertices and use that numbering to create a spanning tree. Also list the back edges of the graph. (starting from A)



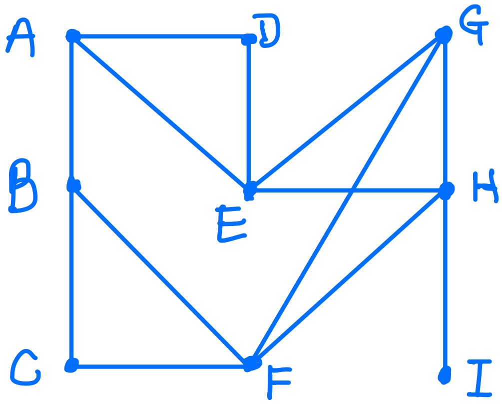


{BD, BE, EF, EH, GJ}

Example: Consider the graph below. Apply the depth-first search algorithm to obtain a depth-first search numbering of the vertices and use that numbering to create a spanning tree. Also list the back edges of the graph.



Example: Consider the graph below. Apply the (a) breadth-first search algorithm and (b) depth-first search algorithm to create a spanning tree.



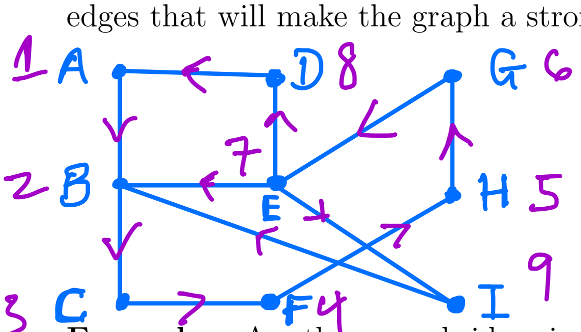
Definition:

The absence of an edge, called a bridge, whose removal disconnects the graph is necessary and sufficient to guarantee that there is a way to assign directions to edges so as to produce a strongly connected directed graph.

Theorem 5.8:

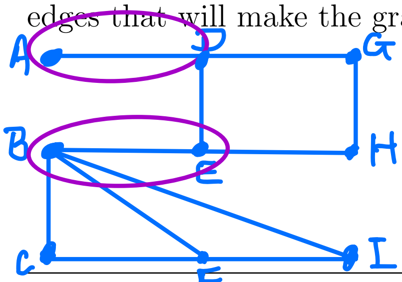
Suppose depth-first search is applied to a connected graph without a bridge. If directions are assigned to tree edges by going from the lower depth-first search number to the higher and to back edges by going from the higher number to the lower, then the resulting directed graph is strongly connected.

Example: Are there any bridges in the graph below? If so, where? If not, apply the depth-first search algorithm to obtain a depth first search numbering of the vertices to assign directions to edges that will make the graph a strongly connected directed graph.



has no bridges

Example: Are there any bridges in the graph below? If so, where? If not, apply the depth-first search algorithm to obtain a depth first search numbering of the vertices to assign directions to edges that will make the graph a strongly connected directed graph.

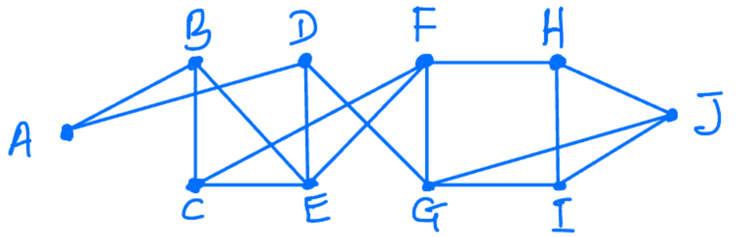


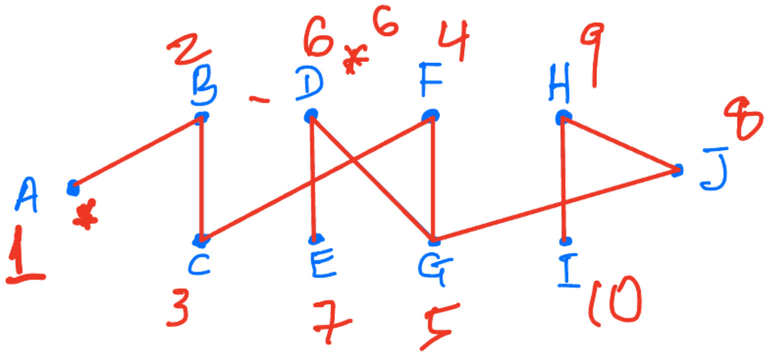
BE is a bridge between ADGHE $ BCFI

AD is a bridge between A and all other vertices.

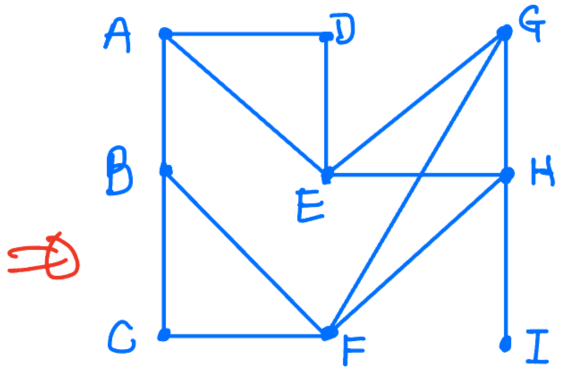
– either bridge is enough to say we cannot make a strongly connected directed graph.

 Rachel Rupnow. Compiled: November 1, 2022

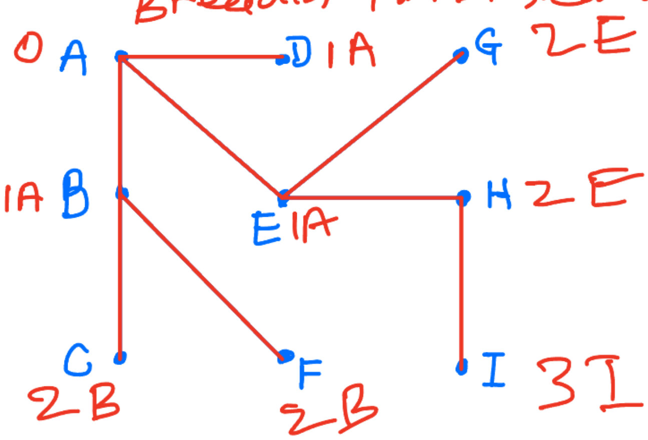




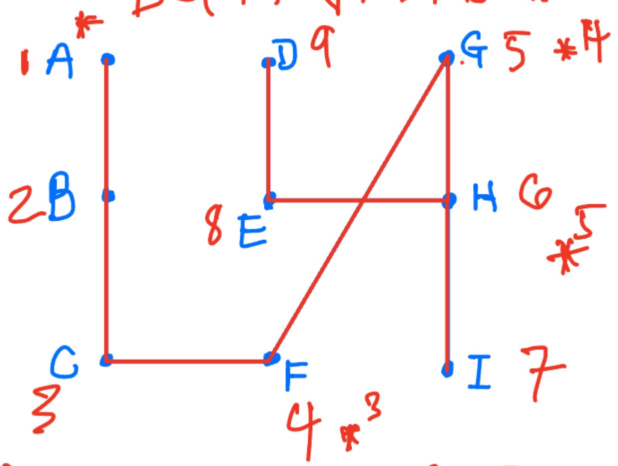
Back edges = {AD, BE, FH, IJ, GI, EF, CE}



Breadth first search



Depth first search



{AD, AE, EG, FH, BF}