More problems on relations

Definition :

A **relation from the set** A **to** B is any subset of the Cartesian product $A \times B$.

- If R is a relation from set A to set B and (x, y) is an element of R, we say x **is related to** y **by** R (notation: $x\ R\ y$).
- A relation from a set S to itself is called a **relation on** S.

Definition :

A relation R on a set S may have any of the following special properties:

1. If for each $x \in S$, $x\ R\ x$ is true, then R is called **reflexive**.
2. If $x\ R\ y$ is true implies $y\ R\ x$ is true, then R is called **symmetric**.
3. If $y\ R\ x$ and $x\ R\ y$ are true implies $x = y$, then R is called **antisymmetric.**
4. If $x\ R\ y$ and $y\ R\ z$ are both true implies $x\ R\ z$ is true, then R is called **transitive**.

Examples :

1. Suppose $R = \{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}$ is a relation on $S = \{1,2,3\}$. Does this relation satisfy the reflexive, symmetric, antisymmetric, and transitive properties? Why or why not?

2. Suppose $R = \{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}$ is a relation on $S = \{1,2,3,4\}$. Does this relation satisfy the reflexive, symmetric, antisymmetric, and transitive properties? Why or why not?

3. Suppose $R = \{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3),(3,1)\}$ is a relation on $S = \{1,2,3\}$. Does this relation satisfy the reflexive, symmetric, antisymmetric, and transitive properties? Why or why not?

4. Let S be the set of nonempty subsets of $\{1,2,3\}$. Define a relation R on S by $A\ R\ B$ if $A \subseteq B$. Does this relation satisfy the reflexive, symmetric, antisymmetric, and transitive properties?

5. Let S be the set of nonempty subsets of $\{1, 2, 3\}$. Define a relation R on S by $A \ R \ B$ if $B \subseteq A$. Does this relation satisfy the reflexive, symmetric, antisymmetric, and transitive properties?