What if we want to use a minimal number of edges to be able to reach everywhere?

Definition:

Any graph that is connected and has no cycles is called a _____.

Example: Create 2 trees and 2 graphs that are not trees.

Theorem 5.1:

Let U and V be vertices in a tree. Then there is exactly one simple path from U to V.

Why should this make sense?

Theorem 5.2:

In a tree $\mathcal{T}$ with more than one vertex, there are at least two vertices of degree 1.

Why should this make sense?

Theorem 5.3:

A tree with n vertices has exactly $n-1$ edges.

Example: If a graph has $n-1$ edges and n vertices, must it be a tree? Why or why not?

Theorem 5.4:

(a) When an edge is removed from a tree (leaving all the vertices), the resulting graph is not connected and hence is not a tree.

(b) When an edge is added to a tree (without adding additional vertices), the resulting graph has a cycle and hence is not a tree.

Why should this make sense?

Theorem 5.5:

The following statements are equivalent for a graph $\mathcal{T}$:

(a) $\mathcal{T}$ is a tree.

(b) $\mathcal{T}$ is connected, and the number of vertices is one more than the number of edges.

(c) $\mathcal{T}$ has no cycles, and the number of vertices is one more than the number of edges.

(d) There is exactly one simple path between each pair of vertices in $\mathcal{T}$.

(e) $\mathcal{T}$ is connected, and the removal of any edge of $\mathcal{T}$ results in a graph that is not connected.

(f) $\mathcal{T}$ has no cycles, and the addition of any edge between two nonadjacent vertices results in a graph with a cycle.

How should I read this theorem?

Example: Hydrogen has one free electron (so it can form 1 bond) and carbon has 4 free electrons (so it can form 4 bonds). Propane is a saturated hydrocarbon (meaning it has single bonds between atoms and has the maximal number of hydrogens for each carbon atom) with 3 carbon atoms and 8 hydrogen atoms (C_3H_8). Draw a tree representing the chemical structure of propane.