Mathematical Induction

The Principle of Mathematical Induction:

Let S(n) be a statement involving the integer n. Suppose that for some fixed integer n_0 ,

- (1) $S(n_0)$ is true (that is, the statement is true if $n = n_0$) AND
- (2) whenever k is an integer such that $k \geq n_0$, and S(k) is true, then S(k+1) is true.

Then S(n) is true for all integers $n \geq n_0$.

The Strong Principle of Mathematical Induction:

Let S(n) be a statement involving the integer n. Suppose that for some fixed integer n_0 ,

- (1) $S(n_0)$ is true (that is, the statement is true if $n = n_0$) AND
- (2) whenever k is an integer such that $k \geq n_0$, and $S(n_0), S(n_0 + 1), ..., S(k)$ are all true, then S(k+1) is true.

Then S(n) is true for all integers $n \geq n_0$.

In strong induction, we assume all cases 1 through k are true (rather than just case k). Strong induction is needed when dealing with recursions:

Examples:

1. Recall in section 9.2, we used the method of iteration to compute 1+2+3+...+n= $\frac{n(n+1)}{2}$. Use the principle of mathematical induction to prove this equality. 1+2+3+...+1

Assume fluct S(K) is frue 1+2+3+...+K= K(K+1)

Our fost: Show that 5(k+1) is true 1+2+3+ · · · + k + k+1 = (k+1)

1+2+3+...+k,+k+1 = K(K+1)

K(x+1) + 2(x+1) Conclusion that SCM is true for all 2. Prove for all positive integers n, $1^2 + 3^2 + \dots + (2n-1)^2 = \frac{(\cancel{\textbf{K+1}}) (\cancel{\textbf{K+1}})}{n(2n-1)(2n+1)}$ using $S(n) = 1^2 + 3^2 + \cdots + (2n-1)^2 = n(2n-1)(2n+1)$ Base Case: n=1 $1(2-1)(2+1) = 1\cdot 1\cdot 3 - 1$ some that s(x) is true =) $|^2 + 3^2 + \cdots + (2k-1)^2 = | k(2k-1)(2k+1)$ Our task! Show that 12+22+···+(2K+1)2=[(K+1)(2K+3) 1 + 32 + ... + (2K-1) + (2K+1) = K(2K-1)(2K+1) + (2K+1) K(2F-1)(2K+1) + (2K+1) K(2K-1)(2K+1) + 3(2K+1) = (2K+1) (K(2F-1) + 3 (2K+1)) 242-K-194 +3

$$2k^{2}+5k+3=2k^{2}+2k+3k+6$$

= $2k(k+2)=(k+2)(2k+3)$

3. Prove $n! > 3^n$ for every integer $n \ge 7$ using math induction.

Parke Case;
$$SC7$$
) $7 = 7.6-5.4.3.2.1$
 $K+17K$

Askume $K = 7.6$
 $S(K)$ is true

Sum that $S(K+1)$ is true

Our factor $S(K+1)$ is $S(K$

Base Case:
$$n=0$$
 $p(0) = 2.0+92 = 92 = P_0$

Assume $P_0 = 92$, $P_1 = P_{2-1} + 2$ for all case

A through $P_1 = 212 + 92$

Dur $P_2 = 212 + 92$

$$P_{k+1} = 2(k+1) + 92 = 2k + 94$$

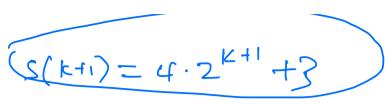
$$P_{k+1} = P_{k} + 2 = (2k + 92) + 2 = [2(k+1) + 92]$$

5. Recall in sections 9.1 and 9.2, we saw the recurrence relation $t_n = 2t_{n-1}$ for $n \ge 2$ with initial condition $t_1 = 3$ could have its *n*th term characterized by $t(n) = 3(2)^{n-1}$. Prove this characterization using mathematical induction.

Bese case: $t(1) = 3 \cdot 2^{-1} = 3 = \ell_1$ Assume that $t_1 = 3$, $t_k = 2t_{k-1}$ for all cases 1 through k, $t_k = 3 \cdot 2^{k-1}$ Our task: $t_{k+1} = 3 \cdot 2^k$ $t_{k+1} = 2t_k = 2 \cdot (3 \cdot 2^{k-1}) = 3 \cdot 2^{k-1+1}$ $= 3 \cdot 2^k$

6. Recall in section 9.2, we saw the recurrence relation $s_n = 2s_{n-1} - 3$ for $n \ge 1$ with initial condition $s_0 = 7$ could have its *n*th term characterized by $s(n) = 4(2)^n + 3$. Prove this characterization using mathematical induction.

Base case: N = 0 $3(0) = 4 \cdot 2^{0} + 3 = 7 = 5$ Assume that 50 = 7,8 $5_{k} = 2,5_{k-1} - 3$ for all cases 4 through k; $5_{k} = 42^{k} + 3$. Our task: 5(k+1) is force $5_{k+1} = 2 \cdot 5_{k} - 3^{4} = 2 \cdot (4 \cdot 2^{k} + 3) - 3$ $= 4 \cdot 2^{k+1} + 6 - 3 = 4 \cdot 2^{k+1}$



Step 1: Me check the base true. Step 2: We assume that the for some KZ No ((16) 15 Step 3: Our tousk of showing that S(K+1) is 67 Mg true. Strong PMI Step 1: Base case step 3

 $P(x) = x^{2} + 5x + 6$ d(x) = x + 1 $P(x) = (x + 1) \cdot (x + 4) + 2$ d(x)

2+5x+6 22+x 0+4x+6 LONG DIVISION