Mathematical Induction

The Principle of Mathematical Induction:
Let $S(n)$ be a statement involving the integer n. Suppose that for some fixed integer n_0,

(1) $S(n_0)$ is true (that is, the statement is true if $n = n_0$) AND

(2) whenever k is an integer such that $k \geq n_0$, and $S(k)$ is true, then $S(k+1)$ is true.

Then $S(n)$ is true for all integers $n \geq n_0$.

The Strong Principle of Mathematical Induction:
Let $S(n)$ be a statement involving the integer n. Suppose that for some fixed integer n_0,

(1) $S(n_0)$ is true (that is, the statement is true if $n = n_0$) AND

(2) whenever k is an integer such that $k \geq n_0$, and $S(n_0), S(n_0+1), ..., S(k)$ are all true, then $S(k+1)$ is true.

Then $S(n)$ is true for all integers $n \geq n_0$.

In Strong induction, we assume all cases 1 through k are true (rather than just case k). Strong induction is needed when dealing with recursions:

Examples :

1. Recall in section 9.2, we used the method of iteration to compute $1 + 2 + 3 + ... + n = \dfrac{n(n+1)}{2}$. Use the principle of mathematical induction to prove this equality.

2. Prove for all positive integers n, $1^2 + 3^2 + ... + (2n-1)^2 = \dfrac{n(2n-1)(2n+1)}{3}$ using math induction.

3. Prove $n! > 3^n$ for every integer $n \geq 7$ using math induction.

4. Recall in sections 9.1 and 9.2, we saw the recurrence relation $p_n = p_{n-1} + 2$ for $n \geq 1$ with initial condition $p_0 = 92$ could have its nth term characterized by $p(n) = 2n + 92$. Prove this characterization using mathematical induction.

5. Recall in sections 9.1 and 9.2, we saw the recurrence relation $t_n = 2t_{n-1}$ for $n \geq 2$ with initial condition $t_1 = 3$ could have its nth term characterized by $t(n) = 3(2)^{n-1}$. Prove this characterization using mathematical induction.

6. Recall in section 9.2, we saw the recurrence relation $s_n = 2s_{n-1} - 3$ for $n \geq 1$ with initial condition $s_0 = 7$ could have its nth term characterized by $s(n) = 4(2)^n + 3$. Prove this characterization using mathematical induction.