1 Sequences and Recurrence Relations

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Definitions:

An (infinite) ordered list is called a sequence. Individual items in such a list are called terms of the sequence.

Relations

An equation relating a general term to terms that precede it is called a recurrence relation. The assignment of values for a set of terms in the sequence, usually the beginning terms, is called the set of initial conditions.

Note:

When describing sequences we often use notation of the form s_n. The n subscript specifies which term we are talking about whereas s_n refers to the value of the nth term.

Example: Suppose sequence s is $\{2, 4, 6, 8, 10, ...\}$. Determine s_1, s_2, and s_6.

$f : \mathbb{N} \to \mathbb{R}$

s_1 s_2 $s_6 = 12$

$f(n) = s_n$

Example: The sequence of Fibonacci numbers (Fibonacci sequence) is defined by the recurrence relation $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$ and $F_1 = F_2 = 1$. What are the first 7 terms of the Fibonacci sequence?

$F_1 = 1$

$F_2 = 1$

$F_3 = F_2 + F_1 = 2$

$F_4 = F_3 + F_2 = 3$

$F_5 = F_4 + F_3 = 5$

$F_6 = F_5 + F_4 = 8$

$F_7 = F_6 + F_5 = 13$

Example: Suppose the recurrence relation $s_n = s_{n-1} + s_{n-2}$ is maintained but with initial conditions $s_1 = 1$ and $s_2 = 2$. What are the first 7 terms of this sequence?

DONE!

$n = 0$

$n > \cancel{0}$ or $n \geq 2$

Example: Suppose the recurrence relation $s_n = s_{n-1} + s_{n-2}$ is maintained but with initial conditions $s_1 = 1$ and $s_2 = 3$. What are the first 7 terms of this sequence?

DONE!

$n > 2$ or $n \geq 2$

$n \geq 1$

Example: Suppose the recurrence relation is $t_n = 2t_{n-1}$ with initial condition $t_1 = 3$. What are the first 6 terms of this sequence?

$t_1 = 3$

$t_2 = 2t_{2-1} = 2t_1 = 6$

$t_3 = 2t_{3-1} = 2t_2 = 12$

$t_4 = 2t_{4-1} = 2t_3 = 24$

$t_5 = 2t_{5-1} = 2t_4 = 48$

$t_6 = 2t_5 = 2 \cdot 48$

Example: Suppose the recurrence relation is $t_n = 4t_{n-2} - 3t_{n-1}$ with initial conditions $t_0 = 3$ and $t_1 = 2$. What are the first 7 terms of this sequence?

$n \geq 2$

$n > 1$

$t_2 = 4t_{2-2} - 3t_{2-1} = 4t_0 - 3t_1$

$= 4(3) - 3(2)$

$t_3 = 4t_{3-2} - 3t_{3-1} = 4t_1 - 3t_2$

$= 4(2) - 3(\quad)$

Example: Annual student parking permits at NIU are \$92 this academic year Fall 2023-Spring 2024. Suppose parking permits increase \$2 per year. Write a recurrence relation and initial conditions for p_n, the parking permit cost n years after the Fall 23-Spring 24 academic year.

Example: Factorials, which are defined as $n! = n(n-1)(n-2)\cdots(2)(1)$ for n a positive integer and $0! = 1$ can be defined recursively. Write a recurrence relation and initial condition to characterize factorials on the nonnegative integers.

Example: Write a recurrence relation requiring at least three initial conditions to be given. Determine the first 7 terms for your sequence.

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