Relations

Definition :

A **relation from the set** A **to** B is any subset of the Cartesian product $A \times B$.

- If R is a relation from set A to set B and (x, y) is an element of R, we say x **is related to** y **by** R (notation: $x\ R\ y$).
- A relation from a set S to itself is called a **relation on** S.

Definition :

A relation R on a set S may have any of the following special properties:

1. If for each $x \in S$, $x\ R\ x$ is true, then R is called **reflexive**.
2. If $x\ R\ y$ is true implies $y\ R\ x$ is true, then R is called **symmetric**.
3. If $y\ R\ x$ and $x\ R\ y$ are true implies $x = y$, then R is called **antisymmetric.**
4. If $x\ R\ y$ and $y\ R\ z$ are both true implies $x\ R\ z$ is true, then R is called **transitive**.

Examples :

1. Suppose $R = \{(5,5), (5,4), (5,3), (4,4), (3,4), (3,3)\}$ and $S = \{3,4,5\}$. Determine which of the reflexive, symmetric, antisymmetric, and transitive properties are satisfied by relation R on set S and justify your conclusions.

2. Suppose $R = \{(1,1),(1,3),(2,2),(2,3),(3,1),(3,2),(3,3)\}$ is a relation on $S = \{1,2,3\}$. Does this relation satisfy the reflexive, symmetric, antisymmetric, and transitive properties? Why or why not?

3. Suppose $R = \{(2,2),(2,3),(3,2),(3,3,(4,1),(1,4),(1,1),(4,4),(5,5)\}$ is a relation on $S = \{1,2,3,4,5\}$. Does this relation satisfy the reflexive, symmetric, antisymmetric, and transitive properties? Why or why not?

4. Suppose $R = \{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}$ is a relation on $S = \{1,2,3\}$. Does this relation satisfy the reflexive, symmetric, antisymmetric, and transitive properties? Why or why not?

5. Suppose $R = \{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}$ is a relation on $S = \{1,2,3,4\}$. Does this relation satisfy the reflexive, symmetric, antisymmetric, and transitive properties? Why or why not?

6. Suppose $R = \{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3),(3,1)\}$ is a relation on $S = \{1,2,3\}$. Does this relation satisfy the reflexive, symmetric, antisymmetric, and transitive properties? Why or why not?

7. Let $S = \{1,2,3,4\}$. Define a relation R on S by letting $x \; R \; y$ mean $x \geq y$. Does this relation satisfy the reflexive, symmetric, antisymmetric, and transitive properties?

8. Let $S = \{1, 2, 3, 4\}$. Define a relation R on S by letting $x \ R \ y$ mean $x = y$. Does this relation satisfy the reflexive, symmetric, antisymmetric, and transitive properties?