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Self-organized criticality and earthquake predictability

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ABSTRACT

We analyse a seismic catalogue of South California to investigate the possibility of earthquake prediction using the hypothesis that the seismic events are self-organized critical phenomena. The relation found previously is valid only in a mean field approximation, but cannot be used for earthquake prediction because the time clustering of seismic events makes the definition of a standard deviation of waiting times of earthquakes impossible.

In a recent series of papers Bak et al. (1987, 1988) suggested that many natural phenomena evolve spontaneously towards a stationary self-organized critical state. Independently Bak and Tang (1989), Sornette and Sornette (1989) and Ito and Matsuzaki (1990) showed that earthquakes can be interpreted as a self-organized critical (SOC) phenomenon. Up to now a great deal of effort has been devoted to improving the original idea or interpreting other preceding models in terms of SOC (e.g. Brown et al., 1991; Shaw et al., 1992).

Sornette and Sornette (1989) showed that, if the SOC hypothesis is assumed for earthquakes, then many observations of occurrences and magnitudes of seismic events can be rationalized. In particular, they analysed the return times of earthquakes supposing that the average energy flux $J(t)$

$$J(t) \approx [n(t)/t]\langle E\rangle(t) \tag{1}$$

is constant. In Eq. (1) $n(t)$ is the number of earthquakes that have occurred at time t and $\langle E\rangle(t)$ is defined by

$$\langle E\rangle(t) = \int_1^{E_{\max}(t)} EN(E)\,dE \tag{2}$$

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where E is the energy, $N(E)$ is the energy distribution (generally in seismology we use the magnitude distribution well known as the Gutenberg–Richter relation) and $E_{\max}(t)$ is the energy of the strongest earthquake at time t. It should be noted that in the supposition of $J(t) = \text{const.}$ both $n(t)/t$ and $\langle E\rangle(t)$ are supposed constants.

The approximation of the $J(t)$ constant is of course a mean field approximation which neglects the fluctuations. Using such an hypothesis Sornette and Sornette found the following relation for 'waiting times' at a given magnitude

$$t = t_0 \exp(cM) \tag{3}$$

where M is the magnitude and c is the slope of log energy vs. magnitude relation ($c \approx 1$ for $M < 7$ and $c \approx 1.5$ for $M > 7$). Note that this relation is derived from the condition that $E_{\max}$ is proportional to the waiting time.

This paper describes the experimental study of the limits of applicability of Eq. (3). With this aim we used the seismic catalogue of South California (1980–1990) using magnitudes greater than 2.5.

The first step was the evaluation of $n(t)$ to be sure of the stationarity of this quantity. Figure 1 shows that the stationary hypothesis is valid, in

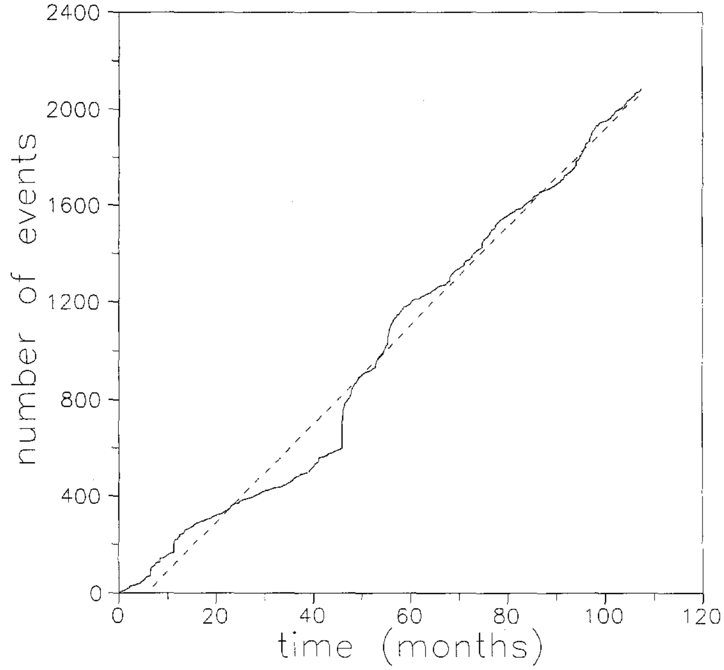

Fig. 1. Number of earthquakes vs. time.

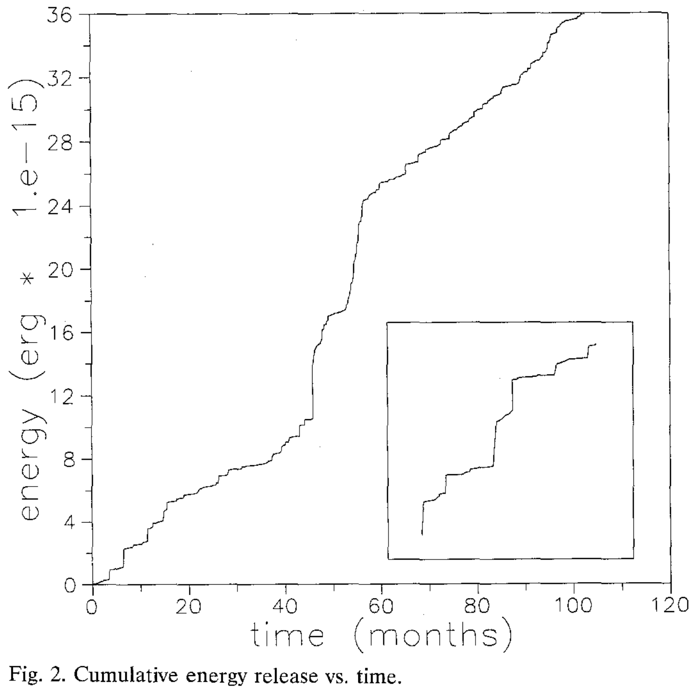

Fig. 2. Cumulative energy release vs. time.

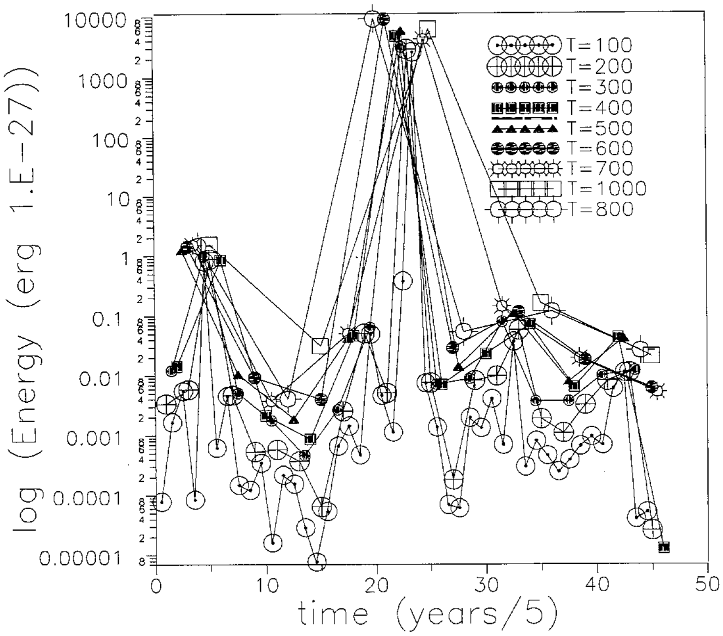

Fig. 3. Average energy in a given time interval vs. time for different time intervals.

fact $n(t)$ is proportional to t or in other words $n(t)/t$ is constant (the number of used events in the figure is about 2000). On the contrary the cumulative energy of earthquakes vs. time exhibits fluctuations due to the occurrence of strong earthquakes. In fact, in Fig. 2 we can almost

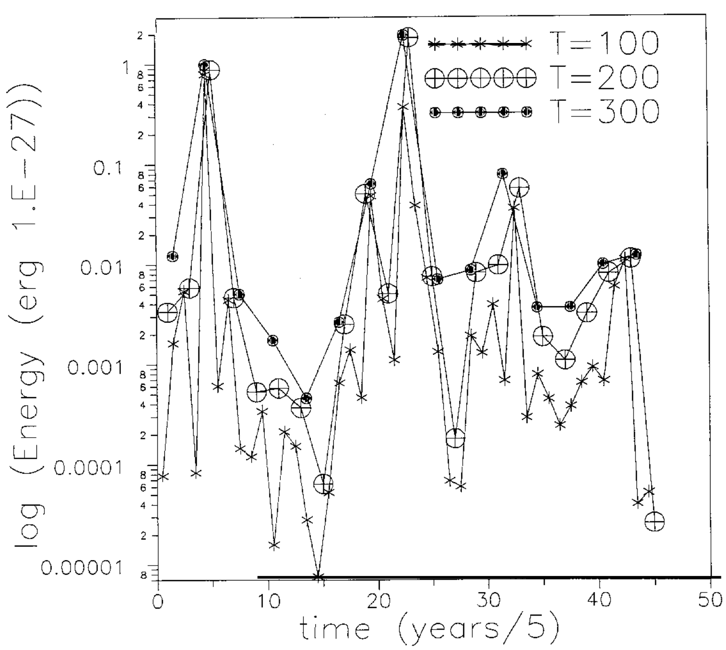

Fig. 4. The same as in Fig. 3, but for magnitudes of less than 4.5.

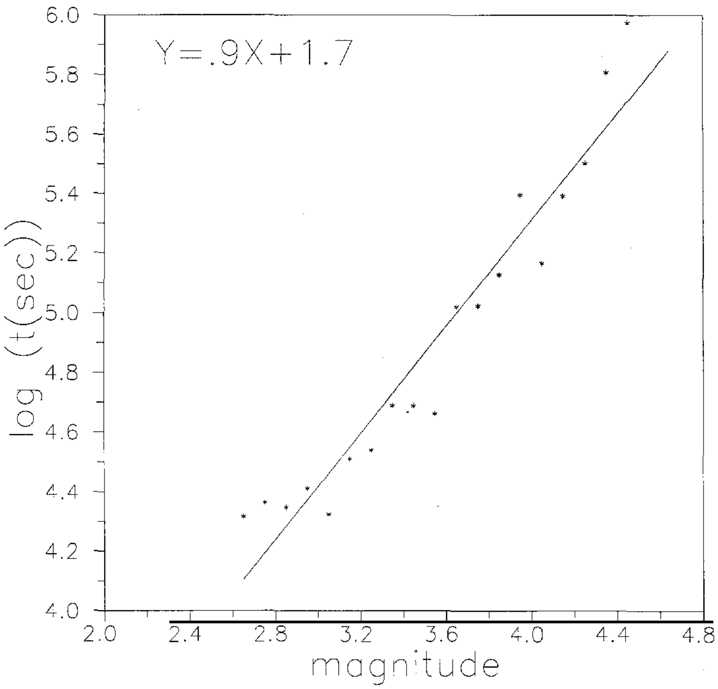

Fig. 5. Fit of Eq. (3) linearized.

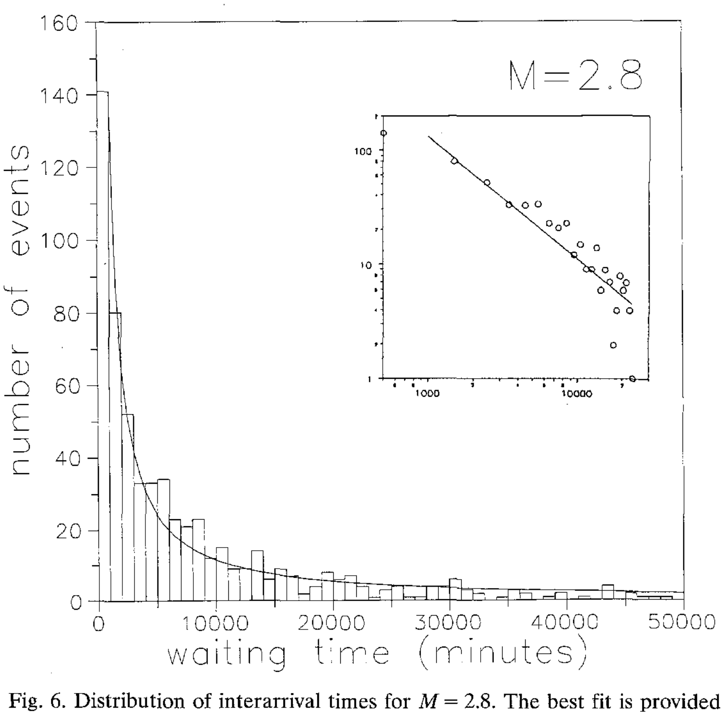

Fig. 6. Distribution of interarrival times for $M = 2.8$. The best fit is provided by the equation $Y = 2.2 \times 10^5 X^{-1.07}$

observe two discontinuities which shift up the line of an apparently constant energy flux.

In Fig. 3 $\langle E \rangle(t)$ calculated by Eq. (2) shows great fluctuations, in fact this curve represents the derivative of the one shown in Fig. 2, and, as is well known, the derivative enhances the fluctuations. At the moment that $\langle E \rangle(t)$ was evaluated for a given time interval, we evaluate this quantity for different time intervals (the units for our time interval are days) to be sure that fluctuations cannot be smoothed by 'more appropriate' choices of the time interval. As can be seen in Fig. 2 the feature of $\langle E \rangle(t)$ is quite independent of the time interval.

The observed fluctuations are scale invariant: if we introduce a magnitude cutoff, new fluctuations will appear (Fig. 4). In fact the inset of Fig. 2 shows the fluctuating amplified signal between 10 and 20 months.

We tried to obtain a relation similar to Eq. (3) with the aim of investigating the agreement of Eq. (3) with experimental data. So we divided the catalogue into subcatalogues for magnitude classes ($M = 0.1$) and averaged the time intervals between earthquakes for any magnitude class (the number of earthquakes in each magnitude class ranges from about 200 for the lowest class to some units for magnitude 4.5). The fit shown in Fig. 5 is in a very good agreement with Eq. (3) producing a c value equal to 0.9 (in our catalogue events with $M > 7$ do not appear). This means that, as suggested by Sornette and Sornette (1989), although $J(t)$ strongly fluctuates, Eq. (3) remains valid in a mean field approximation (average waiting times are used to obtain an experimental fit of Eq. (3)).

Figure 6 shows the distribution of waiting times for $M = 2.8$: as can be seen the prediction of Sornette and Sornette of a power-law distribution $1/t$ is in good agreement with experimental data. This result implies that earthquakes are correlated in time: as is well known, earthquakes exhibit a strong clustering generally interpreted in terms of a Poisson Generalized model (Shlien

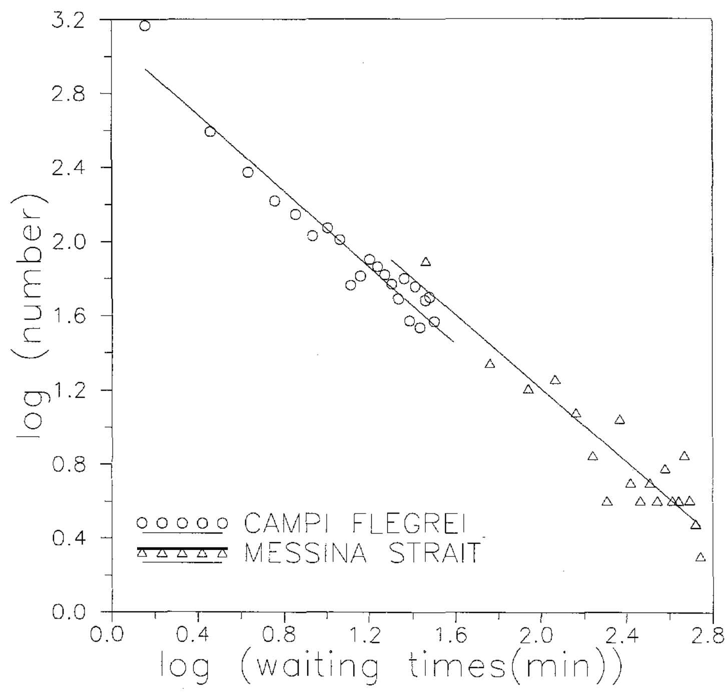

Fig. 7. Log–log plot of waiting times distribution for the Messina Strait area and the Campi Flegrei caldera. The exponents are, respectively, 0.99 ± 0.09 and 1.03 ± 0.06.

and Toksoz, 1970; Udias and Rice, 1975) or Self-exciting Intensity model (Ogata et al., 1982; De Natale and Godano, 1993) which implies a correlation between earthquakes and a power-law distribution of waiting times. Such a power law should be a general feature of earthquake catalogues; in fact Fig. 7 shows the same distribution for two other different ones (the Messina Strait area and Campi Flegrei caldera — Italy). Note that this result is linked, by the approach of Sornette and Sornette (1989), to the b value of the Gutenberg and Richter distribution which is very close to the generally observed value of 1 for the three catalogues examined.

The $1/t$ distribution of waiting times, is consistent with a $1/f$ noise system as predicted by SOC models. The power spectrum

$$S(f) = \int \langle E(t_0 + t) E(t_0) \rangle \exp(2\pi i f t) dt$$

where the average $\langle * \rangle$ is taken over all t_0 and the energy calculated for each week is shown in Fig. 8. As can be seen, the predicted $1/f$ feature is not observed. On the contrary a very low frequency dependence marks a behaviour not far from that of white noise. This means that the energy flux fluctuations are so strong as to destroy the correlation of the earthquake number. In other terms, the external field fluctuations (the external flow of sand) cause energy release variations that can not be interpreted in terms of $1/f$ noise, while earthquakes occurring remain correlated with a $1/t$ distribution of waiting times.

We can conclude that Eq. (3) and a power-law distribution of waiting times are in good agreement with experimental data but the $1/f$ noise prediction is not fulfilled probably due to fluctuations of the external field.

We need a more detailed model including spatio-temporal fluctuations to obtain a correlation lower that the one predicted by a mean field approximation (see e.g. Sornette et al., 1990; Sornette and Virieux, 1992).

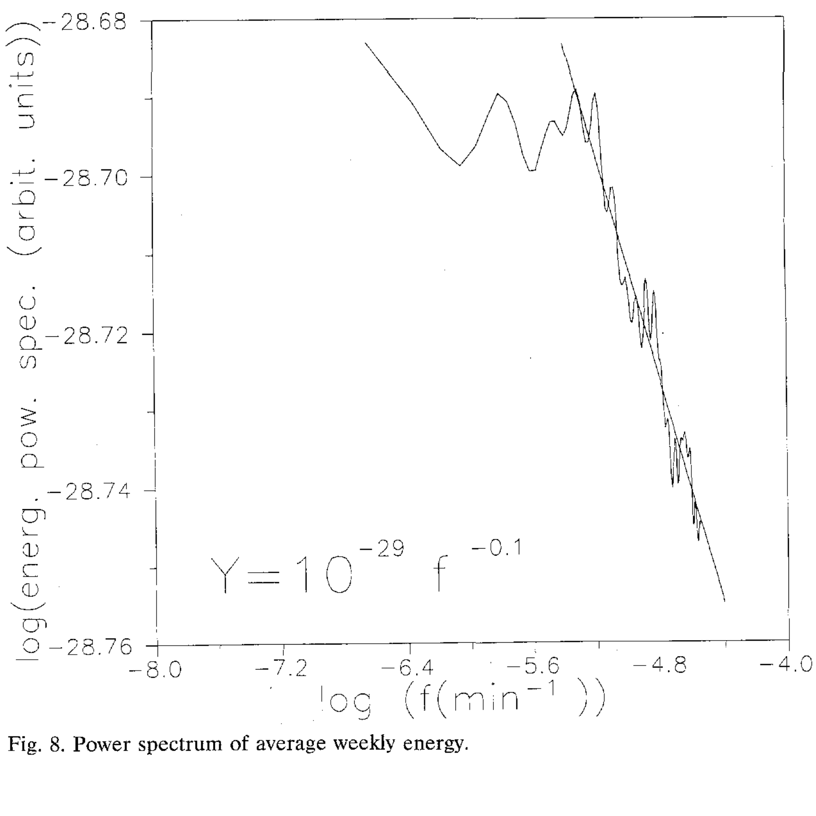

Fig. 8. Power spectrum of average weekly energy.

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References

Bak, P. and Tang, C., 1989. Earthquakes as a self-organized critical phenomenon. J. Geophys. Res., 94: 15 635–15 637.

Bak, P., Tang, C. and Wiesenfeld, K., 1987. Self organized criticality: An explanation of 1/f noise. Phys. Rev. Lett., 59: 381–384.

Bak, P., Tang, C. and Wiesenfeld, K., 1988. Self organized criticality. Phys. Rev. A, 38: 364–374.

Brown, S.R., Scholz, C.H. and Rundle, J.B., 1991. A simplified spring-block model of earthquakes. Geophys. Res. Lett., 18: 215–218.

De Natale, G. and Godano, C., 1993. Earthquakes time clustering and the fracture fractal geometry. Submitted.

Ito, K. and Matsuzaki, M., 1990. Earthquakes as self-organized critical phenomena. J. Geophys. Res., 95: 6853–6860.

Ogata, Y., Akaike, H. and Katsura, K., 1982. The application of linear intensity model to the investigation of casual relation between a point process another stochastic process. Ann. Inst. Stat. Math., 34: 373–387.

Shaw, B.E., Carlson, J.M. and Langer, J.S., 1992. Pattern of seismic activity preceding large earthquakes. J. Geophys. Res., 97: 479–488.

Shlien, A. and Toksoz, M.N., 1970. A clustering model for earthquakes occurrence. Bull. Seismol. Soc. Am., 60: 173–185.

Sornette, A. and Sornette, D., 1989. Self-organized criticality and earthquakes. Europhys. Lett., 9: 197–202.

Sornette, D. and Virieux, J., 1992. A theory linking large time tectonics and short time deformations of the lithosphere. Nature, 357: 401–403.

Sornette, D., Davy, Ph. and Sornette, A., Structuration of the lithosphere in plate tectonics as a self-organized critical phenomenon. J. Geophys. Res., 95: 17353–17361.

Udias, A. and Rice, J., 1975. Statistical analysis of microearthquake activity near San Andreas geophysical observatory, Hollister, California. Bull. Seismol. Soc. Am., 65: 809–827.