Design of a pelletizing bath strain extruder for polypropylene

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ABSTRACT

Pelletizing is an important step in the processing of polymers. This paper presents the preliminary design of a strain extruder servicing polypropylene (PP). Heat transfer has been modelled using the thermal energy equation in terms of transport properties for cylindrical coordinates. The influence of the variation in coolant temperature and various modelling assumptions on bath length has been investigated. The optimum bath length has been determined.

CONTENTS

NOMENCLATURE

h_{for}	Forced convection heat transfer coefficient	W m^{-2} ℃$^{-1}$	Pr_f	Prandtl number	-
h_{mix}	Mixed convection heat transfer coefficient	W m^{-2} ℃$^{-1}$	r	Polymer radius from centerline	m
h_{nat}	Natural convection heat transfer coefficient	W m^{-2} ℃$^{-1}$	R	Polymer radius	m
C_p	Polymer heat capacity	J kg^{-1} ℃$^{-1}$	t	Time	s
C_{pf}	Coolant heat capacity	J kg^{-1} ℃$^{-1}$	T_s	Initial polymer temperature	℃
$\dot{S}$	Heat generated or evolved	W	T_w	Coolant temperature	℃
k_f	Coolant thermal conductivity	W m^{-1} ℃$^{-1}$	u	Extrusion velocity	m s^{-1}
μ_f	Coolant viscosity	Pa s	x	Initial guess of bath length	m
ρ_f	Coolant density	kg m^{-3}	Re_f	Reynolds number	-
D_s	Polymer diameter	m	T	Polymer temperature	℃$^{-1}$
Gr_f	Grashof number	-	k	Polymer thermal conductivity	W m^{-1} ℃$^{-1}$
L	Bath length	m	β	Coolant coefficient of volumetric expansion	℃$^{-1}$

Introduction

Pelletizing is one of the final stages in the processing of polymers for manufacturing [1]. Strand pelletizing baths operate by extruding cylindrical strands of polymers into some liquid media. Strands are completely submerged within the coolant during extrusion. The media that strands are submerged in acts as a coolant in which the polymer's temperature can be reduced to some desired temperature. The desired temperature of the polymer is a function of both the bath length and the extrusion velocity.

The aim of the present work is to design a pelletizing bath for the extrusion of PP. The design requires a bath that can service the extrusion of 10 strands of PP each with a diameter of 3.175×10^{-3}m. The polymer is to be extruded at a nominal temperature of 205℃, and is desired to be cut from the bed at a centerline temperature of 75℃. The extrusion rate is fixed at 3 m min^{-1}. Cooling water is available from the municipality; however, the temperature of the coolant various throughout the year.

Modelling

The thermal energy equation in terms of transport properties is presented in Equation 1 [2].

$$\rho C_p\left(\frac{\delta T}{\delta t}\right) = \nabla \cdot k\nabla T + \dot{S} \tag{1}$$

There is no energy generated through vicious dissipation; hence, the term $\dot{S}$ is null. PP strands are assumed to be extruded as perfect cylinders. For simplicity, it is assumed that heat is transferred strictly in the *r* direction. Figure 1 provides an illustration of the heat transfer. The equation reduces to

$$\rho C_p\left(\frac{\delta T}{\delta t}\right) = k\left[\frac{1}{r}\frac{\delta}{\delta r}\left(r\frac{\delta T}{\delta r}\right)\right] \tag{2}$$

which is the partial differential equation (PDE) representing the heat transfer occurring in the *r* direction along a cylinder. Two scenarios have been proposed. The first of which, is the case of no thermal resistances; hence, the surface temperature at the polymer-coolant interface is the temperature of the bulk coolant.

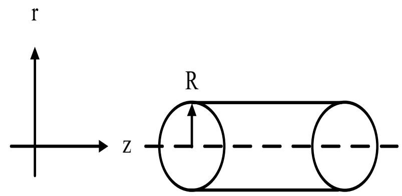

Figure 1
One dimensional heat transfer in a cylindrical PP strand. Heat transfer occurs strictly along the *r* direction.

The second is the case of thermal resistance between the polymer-coolant interface.

In the first case, it is assumed that the coolant is in perfect contact with the polymer. It is also assumed that the bulk coolant temperature is maintained at the temperature it is provided at from the municipality. The boundary and initial conditions the PDE given in Equation 2 can be expressed as

B.C.1:	at $r = R$,	$T(r,t) = T_w$	(3)
B.C.2:	at $r = 0$,	$\frac{\delta T}{\delta r} = 0$	(4)
I.C.:	at $t = 0$,	$T(r,0) = T_s$	(5)

Case 2 has similar boundary conditions and assumptions; however, there is a heat flux at $r = R$ as a result of thermal resistances between the coolant and polymer. In terms of boundary and initial conditions for Equation 2, this can be expressed as

B.C.1:	at $r = R$,	$T(r,t) = -k\frac{\delta T}{\delta r}(R,t)$	(6)
B.C.2:	at $r = 0$,	$\frac{\delta T}{\delta r} = 0$	(7)
I.C.:	at $t = 0$,	$T(r,0) = T_s$	(8)

The term $T(r,t) = -k\frac{\delta T}{\delta r}(R,t)$ in Equation 6 can also be expressed as in Equation 9

$$T(r,t) = -k\frac{\delta T}{\delta r}(R,t) = h_{mix}(T(R,t) - T_w) \tag{9}$$

h_{mix} can be evaluated through an analysis of the natural and forced convection heat transfer coefficients. The correlation for natural convection is given by Baird and Collias [2], as expressed in Equation 10

$$\frac{h_{nat}D_s}{k_f} = 0.518\left(Gr_f Pr_f\right)^{0.25} \tag{10}$$

where

$$Gr_f = \frac{D_s^3 \beta \rho_f \Delta T}{\mu_f^2} \tag{11}$$

and

$$\mathrm{Pr}_f = \frac{C_{pf}\mu_f}{k_f} \tag{12}$$

Fluid properties have been evaluated at the mean temperature at the coolant-polymer interface using coolant fluid data adapted from Bergman [3] (see Appendix 2; all units are SI). Fluid properties for PP have been adapted from Baird and Collias [2].

The forced convection correlation is provided by Baird and Collias [2] in a simplification that assumed the geometry of a smooth flat plate. This is expressed in Equation 13.

$$\frac{h_{for}x}{k_f} = 2 \times 0.332 Pr_f^{\frac{1}{3}} Re_f^{\frac{1}{2}} \tag{13}$$

where

$$Re_f = \frac{u\rho_f}{\mu_f} \tag{14}$$

The mixed heat transfer coefficient can be evaluated as suggested by Cengel [3], as expressed in Equation 15.

$$h_{mix} + \left(h_{for}^{3.2} + h_{nat}^{3.2}\right)^{\frac{1}{3.2}} \tag{15}$$

Methodology

The PDE expressed in Equation 2 has been evaluated using the foregoing boundary and initial conditions for both the case of no thermal resistance and the case of thermal resistance using the PDEPE function in MATLAB [4]. All calculations have been implemented in MATLAB for computing convenience. The script used has been appended to this document.

The time required to cool a strand to 75℃ was used to evaluate the bath length required. Equation 16 represents the relationship between extrusion velocity and bath length

$$L = ut \tag{15}$$

The required bath length was evaluating using cooling water available from the municipality at temperature of 6℃, 12℃, 16℃ and 26℃ for both the case of no thermal resistance and thermal resistance at the polymer-coolant interface. Values for h_{mix} were evaluated at the start and end of the fluid bath (i.e., at T_s = 205℃ and T_s = 75℃), and an atheromatic mean was used as a parameter for the boundary condition listed in Equation 6. All fluid properties have been appended to this paper.

Results

Figure 2 illustrates the solution to the PDE expressed in Equation 2 for the condition of thermal resistance at the polymer-coolant interface. Similar graphs were developed for the case of no thermal resistance at the polymer-coolant interface using varying coolant water temperature. Table 1 summarises the results of this investigation. Figure 3 provides an illustration of the results summarised in Table 1 for visual comparison of the varying bath lengths.

Table 1
Summary of results.

Parameter	T_w	T	L
Thermal Resistance			
	6	75.03	0.58
	12	75.00	0.60
	16	74.99	0.62
	24	75.07	0.66
No Thermal Resistance			
	6	75.01	0.56
	12	75.02	0.58
	16	75.02	0.60
	24	74.99	0.64

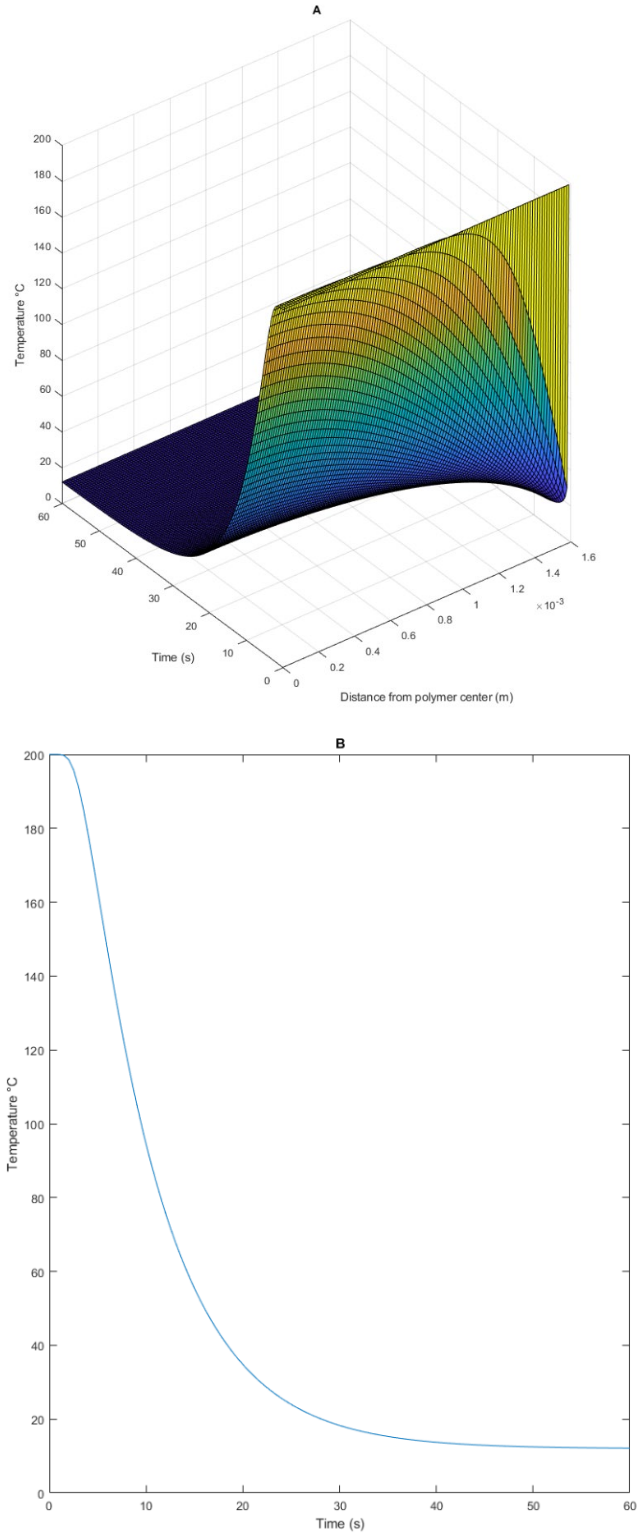

Figure 2
Results from the appended script (Appendix 1) for T_w = 16 ℃ with thermal resistance modeling. Figure A shows a surface plot for the 3-dimensional solution of the PDE. Figure B shows centerline-temperature as a function of time. Note that at T = 75 ℃ t = 12s; hence, the corresponding bath length is 0.56 given the extrusion rate of 0.05 m s^{-1}.

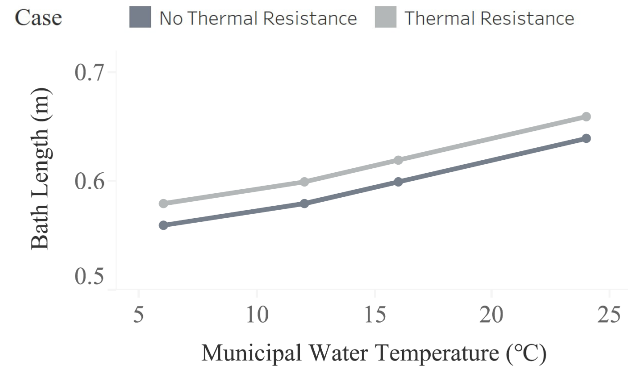

Figure 3
Bath length as a function of temperature.

Discussion

Baird and Collias [2] provides a sample solution similar to the design problem outlined in the present work; albeit, the solution provided only covers the evaluation of the heat transfer coefficient for the thermal resistance case (see page 132 of Baird and Collias). The heat transfer coefficient calculated in the present works deviates from the solution in the literature. For example, the product of the Grashof and Prandtl number was evaluated in this work as 2.75×10^6 whereas the solution from the literature was 7.40×10^5. Additionally, the solution in the present work evaluated the mixed heat transfer coefficient, whereas the solution from the literature neglected forced convection coefficient due to its small relative magnitude compared to the natural convection coefficient. It is difficult to audit the solution provided within the literature as there is ambiguity as to how fluid properties have been evaluated.

It was expected that the case of thermal resistance would result in a longer bath. The modelling of this case imposed a layer of "hot coolant" that is at the polymer-coolant interface. This causes a layer of resistance such that the bulk liquid must first cool the hot coolant layer before the polymer can be cooled. This phenomenon is illustrated in Figure 4. In contrast, the case with no thermal resistance was modelled such that there was no hot coolant layer at the surface of the polymer. This implies that, as expressed in Equation 3, the temperature at the polymer-coolant interface is exactly that of the bulk liquid. Results were in alignment with theoretical expectations. The bath length was, on average for all coolant temperatures, 3.4% greater than the case with no thermal resistance.

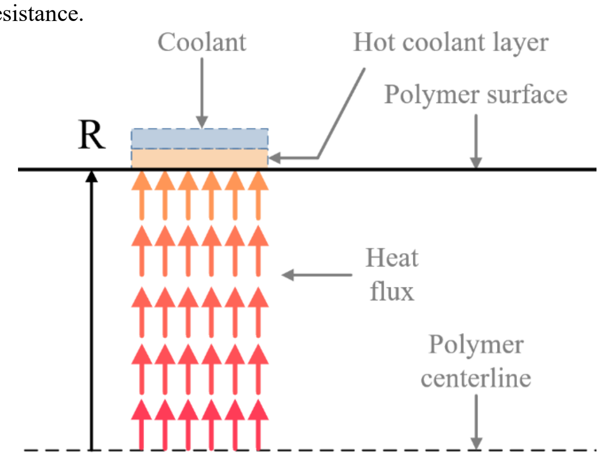

Figure 4
Illustration of convective heat transfer with resistance.

The variation of coolant water from the municipality had an impact on bath length. For both cases, bath length varied by, on average, 5.6% through each step change of feed water temperature. Results align with theoretical expectations, as the rate of heat transfer is directly proportional to the difference in temperatures between the hot and cold temperature revivor.

The minimum bath length of 0.56 m was determined assuming no thermal resistance and a coolant temperature of 6℃. The accepted design, however, is a bath with length of 0.61 m. This length is the average bath length for the thermal resistance case for the varying coolant water temperatures. Figure 5 illustrates the preliminary design of the bath.

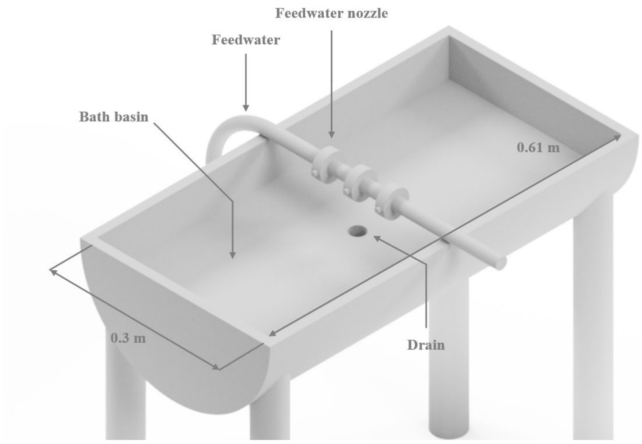

Figure 5
Preliminary CAD of the proposed design.

Conclusion

The preliminary design of a strand pelletizing bath was conducted. Note that, for an alternative approach to this design, the use of Heisler charts would provide a similar solution to that of the present works. Although a minimum bath length of 0.56m was obtained, the preliminary bath design is of length 0.61m; this is because this length more accurately represents the transport phenomena occurring at the polymer-coolant interface. Additionally, this proposed bath length was derived using the mean municipal water temperature; hence, it is the most accurate solution for year-round purposes. It is recommended to conduct the detailed mechanical design of this bath following these preliminary calculations. Simulation of the process using CFD could be used to validate results for approval to manufacture this bath.

References

[1] C. G. Ggos and Z. Tadmor, Principles of Polymer Processing, John Wiley & Sons, 1979.

[2] G. D. Baird and D. I. Collias, Polymer Processing: principles and Design, Wiley, 2014.

[3] T. L. Bergman, A. S. Lavine, F. P. Incropera and D. P. Dewitt, Introduction to Heat Transfer (sixth edition), John Wiley & Sons, 2011.

[4] Y. Cengel, Heat Transfer: A practical approach, 2003.

[5] MATLAB, "pdepe," 2020. [Online]. Available: https://www.mathworks.com/help/matlab/ref/pdepe.html. [Accessed 25 March 2020].

Appendix 1: MATLAB script

```
% Declarations
  global Tw;
  global kpolymer;
  global cppolymer;
  global rhopolymer;
  global hmixavg;
  kpolymer = 0.142;       % Polymer heat cond (W/mK)
  cppolymer = 2.80*1000;  % Polymer heat cap (J/kgK)
  rhopolymer = 867;       % Polymer denisty (kg/m^3)
  Ts = 205;               % Strand Temperature (degC)
  Ds = 3.175*10^-3;       % Strand diameter (m)
  mw = 3/60;              % Bath water rate (m/s)
  Tw = 12;                % Bath temp (degC)
  T2 = 75;                % Final strand temp temperature (degC)
  rhof = 988;             % Density of water (kg/m^3)
  g = 9.81;               % Aceleration due to gravity (m/s^2)
  L = 1;                  % Arbitrary bath length (m)

% Assumptions
  % (1) Strand can be modelled as a horizontal cylinder
  % (2) The bath is infinite relative to the cylinder

% Solution
  Tf1 = (Ts + Tw)/2;
  Tf2 = (T2 + Tw)/2;
  % Fluid Data
    filename = 'fluiddat.txt';
    data = readtable(filename);
    tempdat = table2array(data(:,1))' - 273.15;
    cpdat = table2array(data(:,6))';
    mudat = table2array(data(:,8))'*10^-6;
    kdat = table2array(data(:,10))'*10^-3;
    prdat = table2array(data(:,12))';
    betadat = table2array(data(:,15))'*10^-6;
        % Start of Bath
          cpactual = interp1(tempdat,cpdat,Tf1);
          muactual = interp1(tempdat,mudat,Tf1);
          kactual = interp1(tempdat,kdat,Tf1);
          betaactual = interp1(tempdat,betadat,Tf1);
          Prf = interp1(tempdat,prdat,Tf1);
          Grf = (g*betaactual*((Ts-Tw)
+273.15)*Ds^3*rhof^2)/(muactual)^2;
      % Natural Convection
        % Nussult Correlation
          if Grf*Prf > 10^4;
            fprintf('Nusselt correlation is valid')
            Num = 0.518*(Grf*Prf)^0.25; % Nusselt correlation
          else
            fprintf('Nusselt correlation is NOT valid')
          end
        % Heat Transfer Coefficient
          hnat = Num*kactual/Ds;
      % Forced Convection
        % Reynolds Number
          Ref = (mw*rhof*L)/muactual;
        % Heat Transfer Coefficient
          hfor = 2*(kactual/L)*(0.332*(Prf^1/3)*(Ref^0.5));
      % Mixed Convection
        hmix = (hnat^3.2 + hfor^3.2)^(0.3125);
  % End of Bath
          cpactual2 = interp1(tempdat,cpdat,Tf2);
          muactual2 = interp1(tempdat,mudat,Tf2);
          kactual2 = interp1(tempdat,kdat,Tf2);
          betaactual2 = interp1(tempdat,betadat,Tf2);
          Prf2 = interp1(tempdat,prdat,Tf2);
          Grf2 = (g*betaactual2*((T2-Tw) +
273.15)*Ds^3*rhof^2)/(muactual2)^2;
      % Natural Convection
        % Nussult Correlation
          if Grf2*Prf2 > 10^4;
            fprintf('Nusselt correlation is valid')
            Num2 = 0.518*(Grf2*Prf2)^0.25; % Nusselt
correlation
          else
            fprintf('Nusselt correlation is NOT valid')
          end
        % Heat Transfer Coefficient
          hnat2 = Num*kactual/Ds;
      % Forced Convection
        % Reynolds Number
          Ref2 = (mw*rhof*L)/muactual2;
        % Heat Transfer Coefficient
          hfor2 =
2*(kactual2/L)*(0.332*(Prf2^1/3)*(Ref2^0.5));
      % Mixed Convection
        hmix2 = (hnat2^3.2 + hfor2^3.2)^(0.3125);
  % Avg HTF
  hmixavg = (hmix + hmix2)/2;
% PDE
        function [c,f,s] = pdefun (x,t,u,DuDx)
        global kpolymer
        global rhopolymer
        global cppolymer
        c = 1/(kpolymer/(rhopolymer*cppolymer));
        f = DuDx;
        s = 0;
        end
        function u0 = pdeic(x)
          u0 = 200;
        end
        function [pl,ql,pr,qr] = pdebc(xl,ul,xr,ur,t)
        global hmixavg;
        global kpolymer;
        global Tw;
          pl = 0;
          ql = 1;
          pr = (hmixavg/kpolymer)*(ur-Tw);
          qr = 1;
        end
  x = linspace(0,Ds/2,120);
  t = linspace(0,60,120);
  sol = pdepe(1,@pdefun,@pdeic,@pdebc,x,t);
  u = sol(:,:,1);
 % plot
  subplot(1,2,1);
    surf(x,t,u);
    xlabel('Distance from polymer center (m)');
    ylabel('Time (s)');
    zlabel(['Temperature ' char(176) 'C']);
    title ('A')
  subplot(1,2,2)
    plot(t,sol(:,1));
    xlabel('Time (s)');
    ylabel(['Temperature ' char(176) 'C']);
    title('B')
```

Appendix 2: Thermal properties of water (fluiddat.txt)

T	p	vf.10^3	vg	hfg	cpf	cpg	uf.10^6	ug.10^6	kf.10^3	kg.10^3	Prf	Prg	sigmaf.10^3	betaf.10^6
273.15	0.00611	1	206.3	2502	4.217	1.854	1750	8.02	569	18.2	12.99	0.815	75.5	68.05
275	0.00697	1	181.7	2497	4.211	1.855	1652	8.09	574	18.3	12.22	0.817	75.3	32.74
280	0.0099	1	130.4	2485	4.198	1.858	1422	8.29	582	18.6	10.26	0.825	74.8	46.04
285	0.01387	1	99.4	2473	4.189	1.861	1225	8.49	590	18.9	8.81	0.833	74.3	114.1
290	0.01917	1.001	69.7	2461	4.184	1.864	1080	8.69	598	19.3	7.56	0.841	73.7	174
295	0.02617	1.002	51.94	2449	4.181	1.868	959	8.89	606	19.5	6.62	0.849	72.7	227.5
300	0.03531	1.003	39.13	2438	4.179	1.872	855	9.09	613	19.6	5.83	0.857	71.7	276.1
305	0.04712	1.005	29.74	2426	4.178	1.877	769	9.29	620	20.1	5.2	0.865	70.9	320.6
310	0.06221	1.007	22.93	2414	4.178	1.882	695	9.49	628	20.4	4.62	0.873	70	361.9
315	0.08132	1.009	17.82	2402	4.179	1.888	631	9.69	634	20.7	4.16	0.883	69.2	400.4
320	0.1053	1.011	13.98	2390	4.18	1.895	577	9.89	640	21	3.77	0.894	68.3	436.7
325	0.1351	1.013	11.06	2378	4.182	1.903	528	10.09	645	21.3	3.42	0.901	67.5	471.2
330	0.1719	1.016	8.82	2366	4.184	1.911	489	10.29	650	21.7	3.15	0.908	66.6	504
335	0.2167	1.018	7.09	2354	4.186	1.92	453	10.49	656	22	2.88	0.916	65.8	535.5
340	0.2713	1.021	5.74	2342	4.188	1.93	420	10.69	660	22.3	2.66	0.925	64.9	566
345	0.3372	1.024	4.683	2329	4.191	1.941	389	10.89	664	22.6	2.45	0.933	64.1	595.4
350	0.4163	1.027	3.846	2317	4.195	1.954	365	11.09	668	23	2.29	0.942	63.2	624.2
355	0.51	1.03	3.18	2304	4.199	1.968	343	11.29	671	23.3	2.14	0.951	62.3	652.3
360	0.6209	1.034	2.645	2291	4.203	1.983	324	11.49	674	23.7	2.02	0.96	61.4	697.9
365	0.7514	1.038	2.212	2278	4.209	1.999	306	11.69	677	24.1	1.91	0.969	60.5	707.1
370	0.904	1.041	1.861	2265	4.214	2.017	289	11.89	679	24.5	1.8	0.978	59.5	728.7
373.15	1.0133	1.044	1.679	2257	4.217	2.029	279	12.02	680	24.8	1.76	0.984	58.9	750.1
375	1.0815	1.045	1.574	2252	4.22	2.036	274	12.09	681	24.9	1.7	0.987	58.6	761
380	1.2869	1.049	1.337	2239	4.226	2.057	260	12.29	683	25.4	1.61	0.999	57.6	788
385	1.5233	1.053	1.142	2225	4.232	2.08	248	12.49	685	25.8	1.53	1.004	56.6	814
390	1.794	1.058	0.98	2212	4.239	2.104	237	12.69	686	26.3	1.47	1.013	55.6	841
400	2.455	1.067	0.731	2183	4.256	2.158	217	13.05	688	27.2	1.34	1.033	53.6	896
410	3.302	1.077	0.553	2153	4.278	2.221	200	13.42	688	28.2	1.24	1.054	51.5	952
420	4.37	1.088	0.425	2123	4.302	2.291	185	13.79	688	29.8	1.16	1.075	49.4	1010