Hand-in 1

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Exercise 3

a)

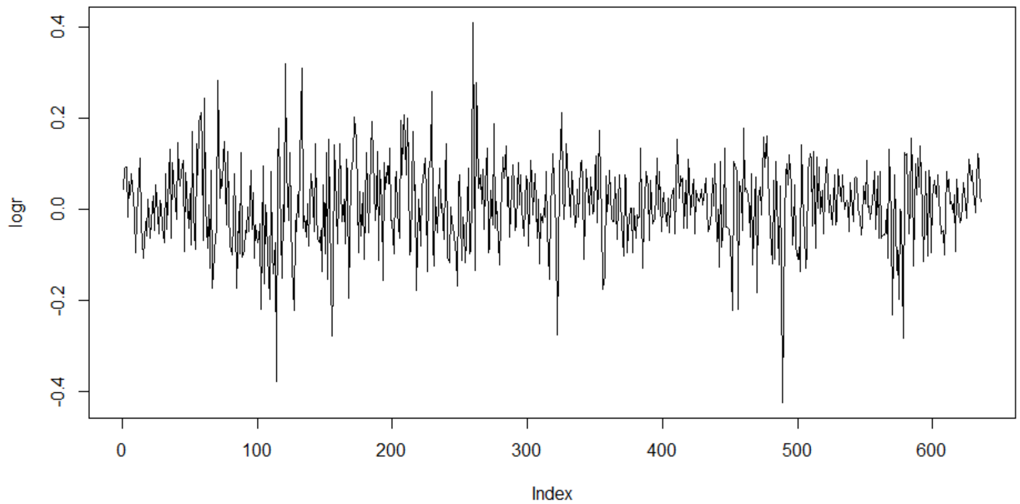

Figure 1: Plotted log returns against time.

The plotted log returns in Figure 1 shows signs of some variance clustering i.e. conditional hetroscedasticity might be present. There is no significant auto correlation at the 5% level according to the ACF plot in Figure 2.

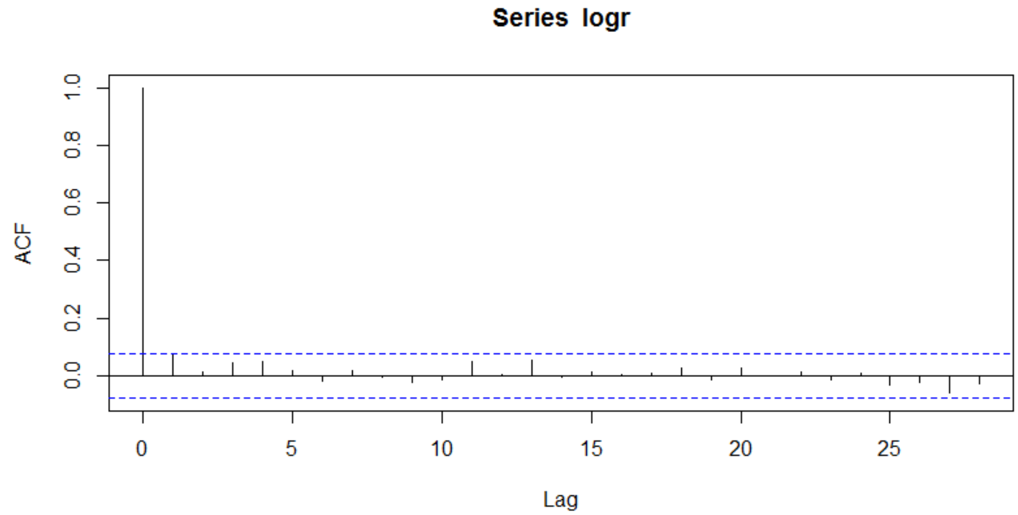

Figure 2: Plotted log returns against time.

A Ljung-Box test statistic was calculated for 7 lags and equaled 7.2948, the corresponding p-value=0.3988 means that the null hypothesis of zero autocorrelation can not be rejected.

b)

From the results in **a** it seems like the proper model for the log returns is $r_t = \mu + a_t$ where μ is constant. However when fitting a ARMA(1,1) to the data both the auto regressive and moving average components are significant. Hence the model is $r_t = \mu_t + a_t$ where μ_t follows an ARMA(1,1). The summary of the results are presented in table 1.

Table 1: Results of fitting a ARMA(1,1) model

```
Coefficient(s):
           Estimate  Std. Error  t value Pr(>|t|)
ar1        0.698165    0.295809    2.360   0.0183 *
ma1       -0.637004    0.319633   -1.993   0.0463 *
intercept  0.010877    0.004412    2.465   0.0137 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

sigma^2 estimated as: 0.008567
log likelihood:       611.16
AIC Criterion:        -1214.32
```

The AIC value is far from zero and the qq-plot in Figure 3 are not satisfying.

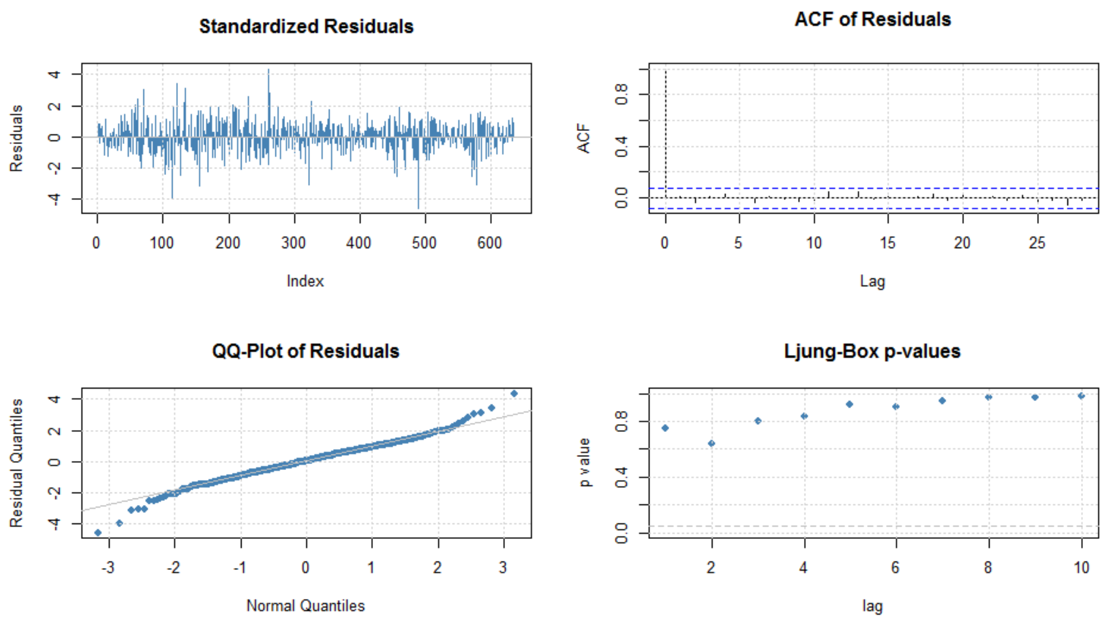

Figure 3: Residual diagnostics for ARMA(1,1)

The ACF and Ljung Box p-values shows that there is no auto correlation present in the residuals but when looking at the ACF and Ljung Box test for the squared residuals, a clear dependence is present. The corresponding p-value for the Ljung-Box test is 0.00054. This is a sign of garch effects i.e. conditional hetroscedasticity. When fitting a ARMA(1,1)-GARCH(1,1) model to data, the ARMA effects becomes insignificant which is line with results in **a**. Hence the final model for the log returns is $r_t = \mu + a_t$ where $a_t = \sigma_t \epsilon_t$ and $\sigma_t^2 = \omega_0 + \alpha_1 a_{t-1}^2 + \alpha_2 a_{t-2}^2 + \beta_1 \sigma_{t-1}^2$. The results are presented in table 2.

Table 2: Results of fitting a GARCH(1,1) model

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu      0.0148031   0.0032281    4.586 4.53e-06 ***
omega   0.0002526   0.0001285    1.966   0.0493 *
alpha1  0.1362666   0.0326798    4.170 3.05e-05 ***
beta1   0.8441072   0.0321609   26.246  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

All the coefficients are significant at the 5% level. The Ljung-Box tests for the residuals and squared residuals can't reject the null hypothesis of zero autocorrelation. The AIC value is -1.989 which is substantially closer to zero than in the ARMA(1,1) model. The qq-plot for the residuals is presented in Figure 4.

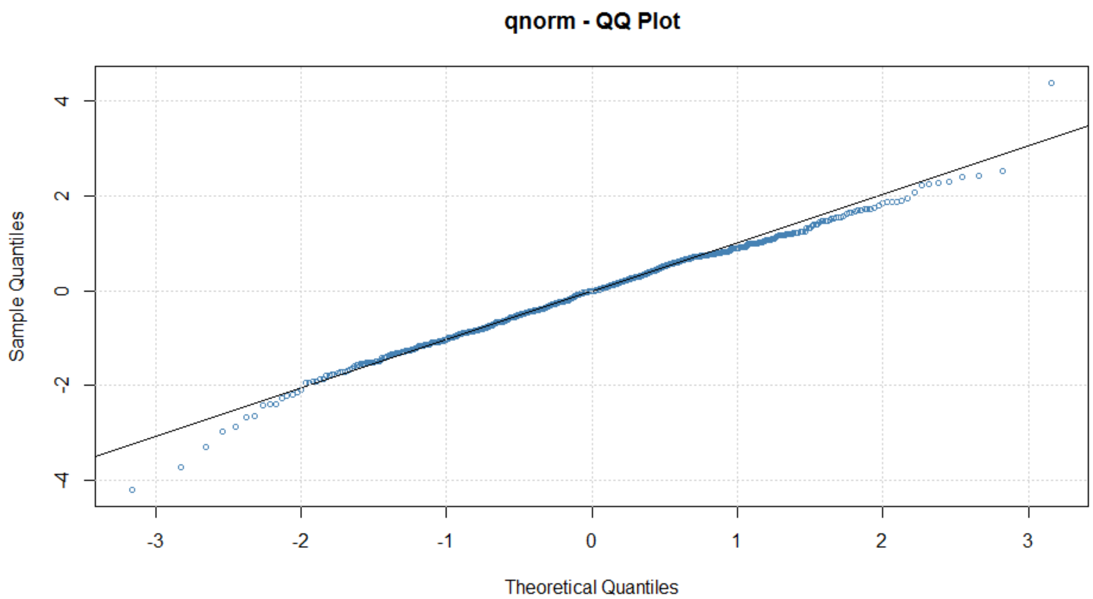

Figure 4: qq-plot for standardized residuals for GARCH(1,1)

From the qq-plot in Figure 4 we see that the standardized residual assembels a normal distribution in the middle and in the right tail, but the left tail of the residual distribution is to heavy compared to the standard normal distribution.

c) The estimates of the GARCH(1,1) model with t innovations is presented in table 3.

Table 3: Estimates of GARCH(1,1) with t innovation

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     1.383e-02   3.244e-03    4.262 2.03e-05 ***
omega  2.802e-04   1.440e-04    1.946 0.051601 .
alpha1 1.117e-01   3.150e-02    3.545 0.000393 ***
beta1  8.623e-01   3.357e-02   25.686  < 2e-16 ***
shape  1.000e+01   2.860e+00    3.496 0.000472 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The result is similar to the estimated coefficients with the standard normal assumption. The qq-plot for the standardized residuals are presented in Figure 5

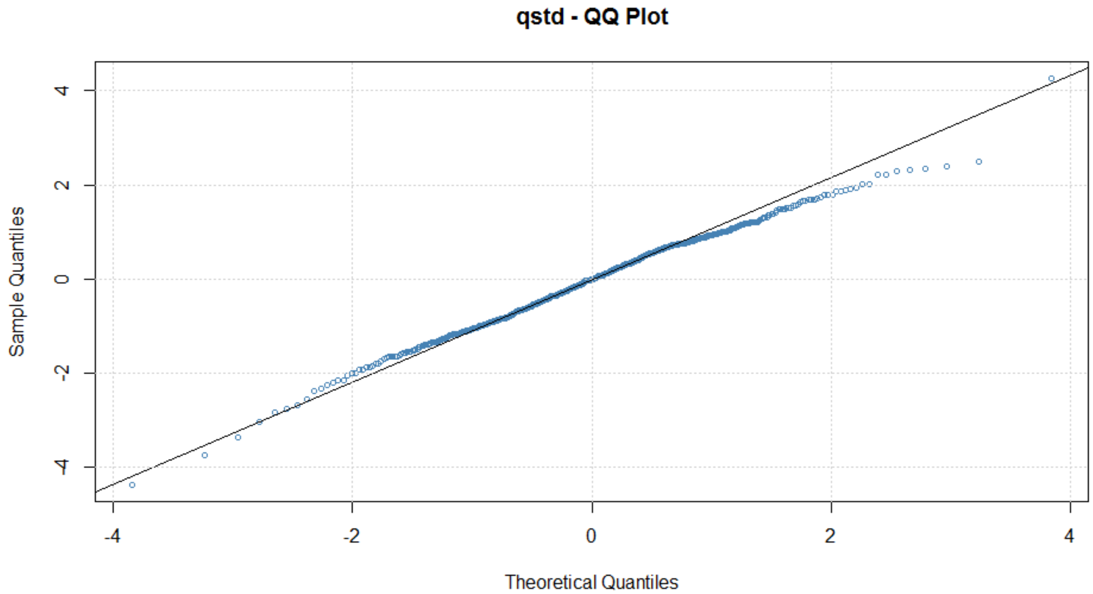

Figure 5: qq-plot for standardized residuals for GARCH(1,1) with t innovation

Opposite to the qq-plot in Figure 4 the distribution of the residuals has a good fit in the left tail and in the middle but the right tail is to light.

d) Based on the qq-plots in Figure 4 and 5 the distribution for the log returns is skewed since the distribution for the standarized residuals $\tilde{a}_t = \frac{r_t - \mu}{\sigma_t}$ are skewed.