Hand-in 1

michael Ståhle

February 2017

Exercise 1

a) I will use the exchange rate of SEK/GBP. The exchange rate at time t is defined as $ER_t = number\ of\ swedish\ krona\ per\ one\ GBP\ at\ time\ t$. Then the log return of the exchange rate is $logr = log(\frac{ER_t}{ER_{t-1}})$. A plot of the log returns is presented in figure 1.

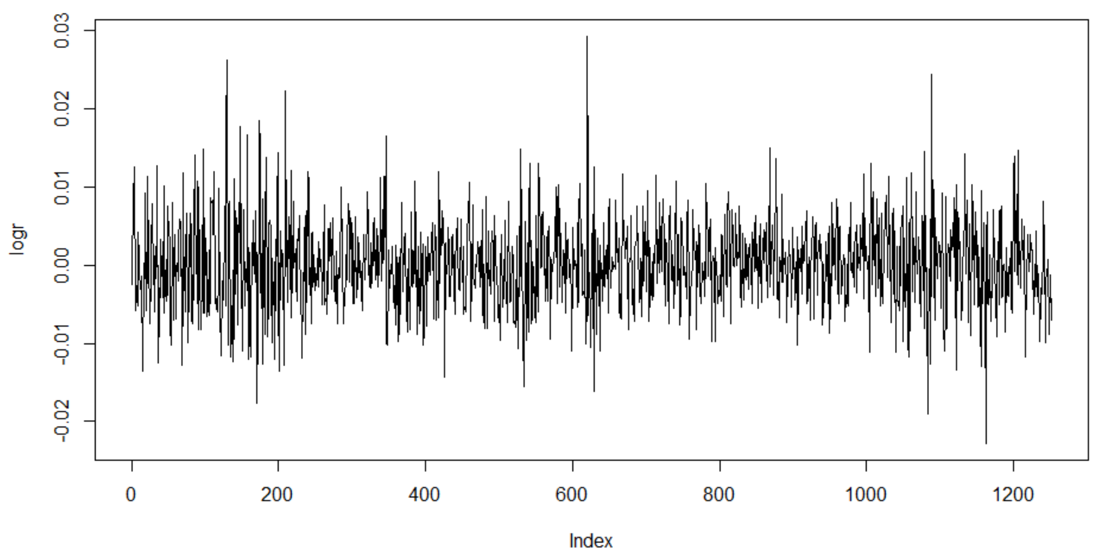

Figure 1: Plotted log returns of GBP/SEK exchange rate against time.

The plot show no direct sign of conditional heteroscedasticity. Define r_t as the log return at time t, then serial correlation for lag l is defined as:

$$\rho_l = \frac{Cov(r_t, r_{t-l})}{Var(r_t)}$$

And the sample serial correlation for lag l is defined as:

$$\hat{\rho}_l = \frac{\sum_{t=l+1}^{T}(r_t - \bar{r})(r_{t-l} - \bar{r})}{\sum_{t=l}^{T}(r_t - \bar{r})^2}$$

Where T is the total number of time steps. Assuming r_t is a iid sequence satisfying $E(r_t^2) < \infty$, then $\hat{\rho}_l$ is asymptotically normal with mean zero and variance $1/T$ for any given l.

To jointly test if there is serial correlation present, a Ljung-Box test is preformed. We have the following hypothesis:

$$H_0 : \rho_1 = ... = \rho_m = 0$$

$$H_a : \rho_i \neq 0 \quad for\ any\ i$$

The test statistic is:

$$Q(m) = T(T+2)\sum_{l=1}^{m}\frac{\hat{\rho_l}^2}{T-l}$$

where $Q(m)$ follows a chi-squared distribution with m degrees of freedom. H_0 is rejected if $Q(m) > \chi_\alpha$ for a given α. Simulation studies suggests that choosing m to $m = ln(T)$ provides a better power performance. In our data set we have $T = 1252$, so $m = ln(1252) \approx 7$. Then the test statistic equals:

$$Q(m) = 5.5686$$

which is not significant. H_0 can't be rejected, so there is no evidence of serial correlation in the log returns for the 8 first lags.

b) Continuing we want to fit the following linear model to the data:

$$r_t = \mu_t + a_t$$

where $\mu_t = E[r_t|F_{t-1}]$ is called the mean equation and we have that $\sigma_t^2 = Var(r_t|F_{t-1})$. a_t is referred to as shocks of the log returns. The insignificant Ljung-Box test suggest that the mean equation is constant. It turns out that the constant mean equation is zero so we have

$$r_t = a_t$$

and $\sigma_t^2 = Var(r_t|F_{t-1}) = Var(a_t|F_{t-1})$. Now we want to fit a volatility model for the log returns. We use the GARCH(m,s) model where we assume that $a_t = \sigma_t\epsilon_t$ and

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{m}\alpha_i a_{t-i}^2 + \sum_{j=1}^{s}\beta_j\sigma_{t-j}^2$$

$$\epsilon_t \sim iid, \quad E[\epsilon_t] = 0, \quad Var(\epsilon_t) = 1.$$

In order to estimate the model we assume standard normal distribution for ϵ_t. After trying different orders i.e different values on m and s we conclude that the best model is a GRACH(1,1) model. It was the only model where all the Garch effects were significant. The AIC values between models with different orders are negligible. The result is presented in Table 1.

Table 1: Estimated coefficients of the GARCH(1,1) model

```
Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     2.271e-04   1.528e-04    1.486  0.13737
omega  6.553e-07   3.909e-07    1.676  0.09367 .
alpha1 4.339e-02   1.518e-02    2.858  0.00426 **
beta1  9.368e-01   2.447e-02   38.283  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Both μ and ω are insignificant at the 5% level but both garch effects are significant.

c) First we look at the serial correlation in the residuals and squared residuals. This is preformed by Ljung-Box tests presented in Table 2.

Table 2: Residual diagnostics of the GARCH(1,1) model

```
Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  56.62202  5.065948e-13
 Shapiro-Wilk Test  R    W      0.9931334 1.493618e-05
 Ljung-Box Test     R    Q(10)  12.56863  0.2487999
 Ljung-Box Test     R    Q(15)  15.63779  0.406524
 Ljung-Box Test     R    Q(20)  16.71283  0.6715342
 Ljung-Box Test     R^2  Q(10)  9.839098  0.4547215
 Ljung-Box Test     R^2  Q(15)  16.8702   0.3266827
 Ljung-Box Test     R^2  Q(20)  17.76039  0.6031892
 LM Arch Test       R    TR^2   9.797032  0.6337616
```

All the Ljung-Box tests are insignificant which means that the null hypothesis of zero autocorrelation can't be rejected.

Now we examine the justification of the normal assumption by looking at qq-plot of standardized residuals presented in Figure 2.

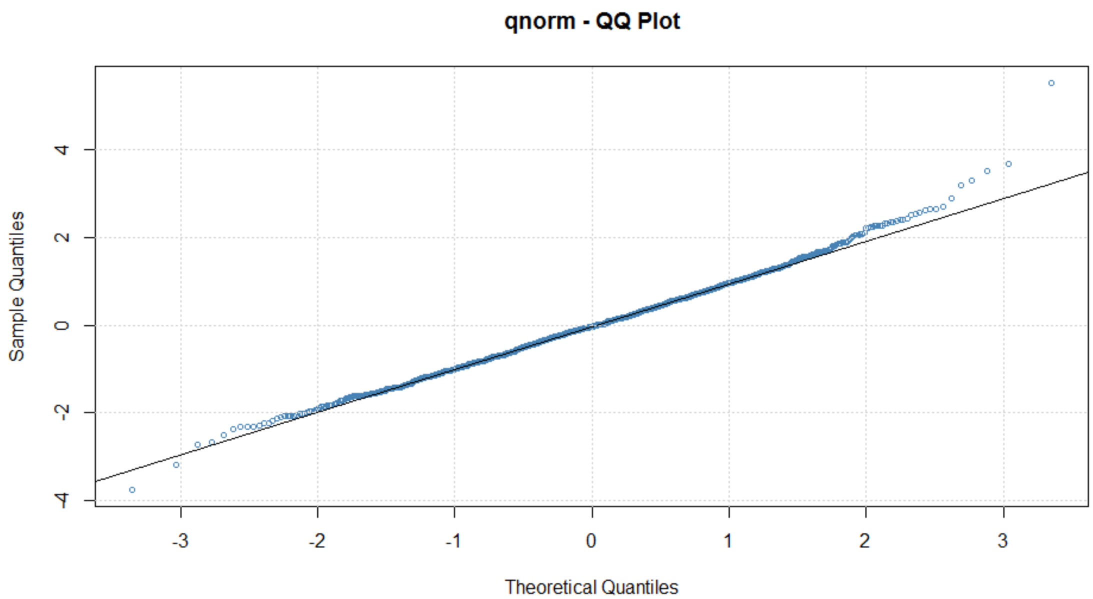

Figure 2: qq-plot of the standardized residual from the GARCH(1,1) model

The qq-plot in Figure 2 reveals that the models residuals fits the normal distribution very well in the middle part and the left tail of the distributions. The fit is not optimal in the right tail nut it is reasonable.