Complete the following programming exercises in Matlab. Submit a record of your work using the 'diary' function, which generates a text file showing command line input and the resulting output during a session. Just type 'diary' at the beginning of your session, and then 'diary off' to write your record to a file.

Where you are asked to make plots, use the 'print' command to save the graphical output to file. To save a plot, you should run the command,

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print('filename', '-dpdf').
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For this assignment, please submit the plot files alongside your diary. The filenames should be "problem03.pdf", "problem04.pdf", and "problem05.pdf". You should submit your diary file, the plots, and answers to questions not covered by the diary file to the bcourses site under the assignments tab.

1. Consider the symmetric 5×5 matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 0 & 2 \\ 1 & 1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 1 & 2 \\ 2 & 2 & 0 & 2 & 1 \end{pmatrix}$$

(i) Create this matrix in Matlab.

(ii) Calculate the eigenvalues $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\}$ of $\mathbf{A}$ using Matlab's 'eig' function.

(iii) The command '[U D] = eig(A)' computes the *right* eigenvectors $\{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3, \mathbf{c}_4, \mathbf{c}_5\}$ of $\mathbf{A}$, which are stored as columns of $\mathbf{U}$. Using this function, find the right eigenvector of $\mathbf{A}$ which has the largest eigenvalue. Store it as a column vector $\mathbf{c}_1$.

(iv) Using matrix multiplication, show that $\mathbf{c}_1$ is indeed a right eigenvector of $\mathbf{A}$, with eigenvalue λ_1, i.e.,

$$\mathbf{A} \cdot \mathbf{c}_1 = \lambda_1 \mathbf{c}_1 \tag{1}$$

(v) Show that the eigenvectors computed by Matlab are normalized, i.e., $\mathbf{c}_1^{\mathrm{T}} \cdot \mathbf{c}_1 = 1$.

(vi) Because $\mathbf{A}$ is symmetric, right eigenvectors $\mathbf{c}_i$ and $\mathbf{c}_j$ are also orthogonal, $\mathbf{c}_i^{\mathrm{T}} \cdot \mathbf{c}_j = 0$, for $i \neq j$. Show that this is true for the case $i = 1$, $j = 2$.

(vii) The transpose of Eq. 1 reads

$$\mathbf{c}_1^{\mathrm{T}} \cdot \mathbf{A} = \lambda_1 \mathbf{c}_1^{\mathrm{T}},$$

where we have made use of the fact that $\mathbf{A}^{\mathrm{T}} = \mathbf{A}$. The left and right eigenvectors of a symmetric matrix are thus identical. Show that this is true by computing $\mathbf{c}_1^{\mathrm{T}} \cdot \mathbf{A}$.

2. Next consider the *asymmetric* matrix $\mathbf{B}$:

$$\mathbf{B} = \begin{pmatrix} 1 & 1 & 1 & 1 & 2 \\ 1 & 2 & 1 & 0 & 2 \\ 0 & 1 & 2 & 1 & 0 \\ 1 & 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

(i) Create this matrix and compute its eigenvalues.

(ii) Because $\mathbf{B}$ is not symmetric, its right and left eigenvectors are different, although the set of eigenvalues is identical:

$$\mathbf{B} \cdot \mathbf{c}_1 = \lambda_1 \mathbf{c}_1, \qquad \mathbf{d}_1^{\mathrm{T}} \cdot \mathbf{B} = \lambda_1 \mathbf{d}_1^{\mathrm{T}},$$

Show that this is true by computing $\mathbf{c}_1$ and $\mathbf{d}_1$, where λ_1 is the largest eigenvalue. [Note that the 'eig' function in Matlab generates *right* eigenvectors by default. To determine the left eigenvectors, note that the transpose of the left eigenvalue equation reads $\mathbf{B}^{\mathrm{T}} \cdot \mathbf{d}_1 = \lambda_1 \mathbf{d}_1$.]

(iii) Show that the right eigenvectors of $\mathbf{B}$ are not orthogonal to one another. Specifically, compute $\mathbf{c}_1^{\mathrm{T}} \cdot \mathbf{c}_2$, where λ_2 is the second largest eigenvalue.

(iv) For an asymmetric matrix, an orthogonality condition instead holds for a product of left and right eigenvectors:

$$\mathbf{d}_i^{\mathrm{T}} \cdot \mathbf{c}_j = 0$$

for $i \neq j$. Show that this is true for the case $i = 1$, $j = 2$.

3. Consider the parametric equations:

$$\begin{aligned} x &= \cos(t) \\ y &= \sin(3t) \end{aligned}$$

Using Matlab, make a plot of y as a function of x for values of t between 0 and 2π.

4. Consider the more exotic parametric equations:

$$\begin{aligned} x &= \sin(t) \left[e^{\cos(t)} - 2\cos(4t) - \sin^5\left(\frac{t}{12}\right) \right] \\ y &= \cos(t) \left[e^{\cos(t)} - 2\cos(4t) - \sin^5\left(\frac{t}{12}\right) \right] \end{aligned}$$

(i) Plot y as a function of x for values of t between 0 and 12π.

(ii) Does your plot resemble a specific animal? If so, what animal?

5. Make a single plot showing both of the following functions of x:

$$\begin{aligned} f(x) &= \frac{1}{2} x^2 \\ g(x) &= \frac{1}{2} + e^{-x^2/2} \end{aligned}$$

Show $f(x)$ in blue, and show $g(x)$ in red. Your plot should span a range of x from -4 to 4.