

Introduction

Λόγον ζητοῦμεν δὲν ἔστι λόγος¹.

There are two occasions in the history of philosophy in which controversy about the principle of contradiction excited the minds [of philosophers]: the name of Aristotle is linked with one, and the name of Hegel with the other. Aristotle formulated the principle of contradiction as the highest law of thinking and of being. In a dogged fight, sometimes pulsating with anger and scorn, he pursued anyone who did not want to acknowledge this law: Antisthenes and his school, the eristics of Megara, the followers of Heraclitus, and the disciples of Protagoras. He won this fight, and evidently the force of his arguments was so great, or the cause he defended was so just, that for centuries no one dared to deny that paramount principle. Only then did Hegel revive the opinions which Aristotle had buried, and he told us to believe that reality is reasonable and contradictory at the same time. He restored respect for the Greek sophists, and incorporated statements from Heraclitus in his system of logic. This teaching once more sparked off impassioned discussion, in which men tried by means of Aristotle's words to bury even Hegel's.

Today the sound of these disputes has died away, and the principle of contradiction is no longer a subject of present interest. So much the better; for thus we can ponder it without passion. [2]

Yet one must approach this matter with the sharp edge of cautious but strict criticism. The old disputes did not decide the issue; whoever takes a closer look at them will see a fair number of unresolved questions bound up with the problem of contradiction. These questions lie amid the deepest foundations of all our knowledge; from this tangle we can draw the threads which may give us our bearings and guide us in the methodical investigation of reality. If it could resolve just one knot of this tangle, the slightest discovery would have momentous consequences for science. And if this is so, then not only is it worth our while to consider the problem of this principle, but also we should ask with surprise why no one has yet subjected it to critical study.

The history of science gives us the answer to this question. In the ancient dispute about the principle of contradiction, Aristotle's logic won, and despite the unsparing challenges which his logic received, it has persisted for centuries in a scarcely changed form. In practice, scientific research does not appear to have stepped beyond its limits, and in principle both induction and deduction have relied until now on the logical rules of the *Organon*. This logic has proved immensely *useful* for getting knowledge of individual objects and phenomena. Ultimately we should thank this prodigious idea of the Hellenic spirit for the powerful development of the sciences in western lands. We must say that for many centuries no problem has arisen in any one science which would force us to revise the foundations of

¹ Tr. 'We seek a proof of things which have a proof'. This is a deliberate change by Łukasiewicz from Aristotle's statement in *Metaphysics* Γ 6, 1011^a12–13: λόγον γὰρ ζητοῦσιν δὲν οὐκ ἔστι λόγος, 'for they seek a proof of things which have no proof'. Note that the translation of λόγος as 'proof' is Łukasiewicz's own.

Aristotelian logic, and hence to a critical reconsideration of the principle of contradiction. Like Euclidean geometry, this logic has proved to agree with the facts, and on this very agreement [3] depends its immense usefulness and its lasting value; this will continue to be so in the future.

Although the particular sciences have developed vigorously, the general science which Aristotle called ‘first philosophy’ has remained far behind them. This ‘science’ was intended to investigate not particular beings but being in general and its essential properties. It was meant to examine the world as a whole, its past and its future, its origin and its destiny. One must admit openly that this ‘first philosophy’, later called *metaphysics*, has progressed hardly at all beyond the foundations laid by the Stagirite. Hence from the time of Kant we hear repeatedly the opinion that metaphysical problems transcend the ability of the human mind to reason.

But perhaps one might suspect for a moment that the human mind is not so limited, but rather that our manner of treating metaphysical problems is just incompetent. Subtle metaphysical questions undoubtedly call for subtle methods of investigation, and Aristotelian logic, though so useful for getting to know facts, may perhaps be too crude a tool to disclose to us the delicate structure of the essential world beneath the chaos of phenomena.

This idea occurred to Hegel. He believed in the power of knowledge, and he was discouraged by the sceptical results of Kant’s critical philosophy. Kant maintained that the human mind, investigating the world as a whole, necessarily falls into antinomies and gets entangled in contradictions. Hegel accepted this finding, but did not infer from it the conclusion that the essence of the world is unknowable. He merely conceded that contradiction really exists, seeing in it the principle of movement and of life. In this way he created a ‘metaphysical logic’ not based on the principle of [4] contradiction. But this attempt was too radical, too imprecise, too unclear to be understood and accepted. In investigating and reshaping the highest logical and ontological principles, we must proceed as carefully, as precisely, and as critically as possible. We should state with all possible precision, *what is the meaning of these principles, how they should be formulated, how their certainty is guaranteed, how they are related to each other, what consequences follow from each of them, whether we can omit any one of them or replace it with another, whether they can be useful for enquiring into facts*, and so on.

Now Hegel did not perform this task; he rejected the principle of contradiction in words, without proof. So therefore his idea had too little scientific value to weaken our faith in the truth of this principle, or to contribute to the resolution of metaphysical questions. Today Hegel’s fight against [the principle of] contradiction belongs to history. There remains to us a valuable memento of this fight, a lovely treatise by Fr. Gratry, which routed the leader of the pantheists with the words of Aristotle².

² I am thinking of Gratry’s second book of logic, *La Logique du Panthéisme*; see his *Logique* 5, Paris (1868), vol. I, pp. 256–415.

So controversy about the principle of contradiction came to life only briefly in the days when Hegelian philosophy reigned. Otherwise neither in particular sciences nor in ‘general science’ was there any sign through all the centuries of a need to examine this supreme principle. Today as of old we believe that the principle of contradiction is the most certain law of thinking and being. Only a madman could deny it. Its truth is forced upon every one with immediate self-evidence. [5] This principle has not, and cannot have, any need of justification; so Aristotle taught us to believe. Why then is it surprising that no one is concerned about a problem so clear, undoubted, and resolved for ever?

Yet it is bad if there exist in philosophy incontestable principles; worse if these principles are unjustified, and even worse if these incontestable, unjustified principles were once the subjects of heated controversy. How did it happen that a controversial principle, which no one knows how to prove, could be considered so certain that one cannot even contest it freely? What has happened to the scientific criticism of which we are so proud in this critical age?

If I am not mistaken, a time is coming, the third in the history of the principle of contradiction, when these omissions will be put right. In the development of logic this is as necessary as was the revision of the axiom of parallels in the development of geometry. Aristotle created the *beginnings* of logic, and all beginnings are imperfect. This objection does not lower the value of his work of genius; on the contrary, we may rather regret that the Stagirite gave us all at once a too-finished whole, as a result of which the possibility of its development was held back. Centuries have passed, and logic has frozen in its traditional form. It even seemed to Kant that the system of logic created by Aristotle was an integral structure so closed that it could not in principle progress even one step forward.

But even at that time, first Leibniz, and then in the second half of the nineteenth century the English logicians, deepened and broadened traditional formal logic in an unheard of manner. Boole, De Morgan, Jevons, Peirce, Schröder, Russell, Peano [6] — these were the most prominent creators of this new logic. Contemporary symbolic logic is related to the logic of Aristotle in a way similar to that in which contemporary Euclidean geometry is related to Euclid’s *Elements*. It still continues to be *Aristotelian* logic, for it accepts all principles which Aristotle discovered and recognised; but, being at the height of its development, it also knows additional logical laws which Aristotle either did not yet know, or did not formulate explicitly; for example, the principles of identity and of double negation, the laws of logical multiplication and addition, the principles of tautology, absorption, simplification, and so forth.

There must come a time when logicians begin to consider how these principles are related to each other, and take up the investigations ignored by Hegel. Then alone will it be clear what position the principle of contradiction occupies among the other laws of logic, what is the basis for its certainty and value, and how far it is possible to apply it. Then it will be clear whether this principle is truly the highest of all and the cornerstone of all our logic, or whether it is also possible to change or even to abandon it and to create a system

of *non-Aristotelian* logic, just as by changing the axiom of parallels there arose a system of non-Euclidean geometry. No one has yet conducted these investigations, though there are passages in Aristotle's *Metaphysics* and in his logical works which challenge us directly to do so. However, not everyone hears the voice of old books.

These investigations could rightly be called ‘metalogical’. They will not be without value, even if they should only demonstrate that a homogeneous and consistent system of non-Aristotelian logic is impossible in [7] scientific practice. Whatever their result, they will cast light on the foundations of traditional logic, and in this light the ultimate principles lying beneath this and every other science will stand out in clear contours. To cast a critical light on these principles would accomplish no small task, since it would manage not only to base logic on more certain foundations, but also at the same time to refine its entire structure, forging a supple, strong weapon for winning the fight for knowledge of the world.

In recent years mathematicians (Russell, Couturat, Frege, Hilbert, Peano, and others) have begun to work on a similar task of analysis and criticism. They are investigating, in connection with symbolic logic, the foundations of arithmetic and geometry. The results of these investigations have already led people to hold unexpected opinions, for they seem to prove that all of mathematics, in respect to both its form and its content, can be deduced from a few basic logical concepts and premises³. These efforts by mathematicians can serve in several respects as a model of method for the similar work of logicians.

It is not the aim of the present discussion to present ‘metalogical’ investigations in their entirety, or to work out all aspects of the principle of contradiction against the background of contemporary symbolic logic. Before this can happen, we should first turn to Aristotle. We must call to mind some old, unanswered questions, currently forgotten, which are connected with this principle, and associate our new investigations with them. I want to convince the reader that the principle of contradiction [8] is not so unshakeable as it is reputed to be, that it is a thesis which requires proof, and that its proof can be found, at least partially, despite the words of the Stagirite: ‘They seek a proof of things for which there is none’⁴. If I awaken such a belief in the reader, and make in this way out of an uncontested and incontestable, and thus currently dead, problem a living issue of no little weight, this work will have achieved its aim completely. [9]

³ See B. Russell, *The Principles of Mathematics*, vol. I, Cambridge (1903), ch. I.

⁴ *Metaphysics* Γ 6, 1011^a12–13: λόγον ζητοῦσιν δὲ οὐκ ἔστι λόγος.

1. Three formulations of the principle of contradiction

Aristotle formulates the principle of contradiction in an ontological, a logical, and a psychological sense, though he nowhere distinguishes these senses explicitly.

a) He states the ontological principle of contradiction in the sentence: ‘The same cannot both belong and not belong to the same at the same time and in the same respect’¹.

He expresses this principle more briefly in the words: ‘It is impossible for something at the same time to be and not to be’².

Aristotle uses the expressions ‘something belongs to something’ ($\bar{\nu}\pi\alpha\rho\chi\epsilon i \tau i \tau\nu i$) and ‘something does not belong to something’ ($\mu\bar{n} (\bar{o}\bar{u}\bar{x}) \bar{\nu}\pi\alpha\rho\chi\epsilon i \tau i \tau\nu i$) to denote a relation which is not more fully defined, to which in a logical sentence ($\bar{\alpha}\bar{\pi}\bar{\rho}\bar{\phi}\bar{\alpha}\bar{s}\bar{i}\bar{\varsigma}$) the relation of the predicate to the subject corresponds. I think that this relation can be called the relation of inherence, and its terms are the object and the property. *I understand by object anything which is ‘something’ and not ‘nothing’, and by property anything [10] that can be predicated of some object.* The relation of inherence obtains between a given object and a given property if the object possesses the property; that is, if the property can be predicated of the object. If the object does not possess it, there is no relation of inherence. Using these terms, I formulate the principle of contradiction as follows:

No object can at the same time possess and not possess the same property.

This formulation does not change Aristotle’s ideas, but instead of the stylistically awkward pronouns ‘the same’ ($\tau\bar{o} \alpha\bar{u}\bar{t}\bar{o}$) and ‘something’ (τi), it merely introduces the nouns ‘object’ and ‘property’.

The above principle is called *ontological* because it concerns all being ($\tau\bar{o} \bar{\delta}v$), that is, everything that is ‘something’ and not ‘nothing’³.

b) Aristotle states the logical principle of contradiction in the words: ‘It is the most certain of all principles that two contradictory propositions are not true at the same time’⁴.

By contradictory propositions ($\bar{\alpha}\bar{n}\bar{t}\bar{i}\bar{f}\bar{a}\bar{t}\bar{i}\bar{x}\bar{\omega}\bar{s} \bar{\alpha}\bar{n}\bar{t}\bar{i}\bar{x}\bar{e}\bar{i}\bar{m}\bar{e}\bar{n}\bar{a}\bar{i}\bar{v}\bar{a}\bar{r}\bar{\varphi}\bar{e}\bar{s}\bar{i}\bar{s}$), Aristotle understands propositions which affirm ($\bar{\chi}\bar{a}\bar{t}\bar{a}\bar{\varphi}\bar{a}\bar{s}\bar{i}\bar{\varsigma}$) and which deny ($\bar{\alpha}\bar{\pi}\bar{\rho}\bar{\varphi}\bar{a}\bar{s}\bar{i}\bar{\varsigma}$) something of the same object and in the same respect; for example, ‘Callias is just’ and ‘Callias is not just’. The relation between such propositions is called contradiction ($\bar{\alpha}\bar{n}\bar{t}\bar{i}\bar{f}\bar{a}\bar{s}\bar{i}\bar{\varsigma}$). For we read: [11]

To every affirmation there corresponds a contradictory denial, and to every denial [there corresponds a contradictory] affirmation. Then let this be called contradiction, and let the affirmation and denial be called contradictories. I call contradictories

¹ Metaphysics Γ 3, 1005^b19–20: $\tau\bar{o} \gamma\bar{a}\bar{p} \alpha\bar{u}\bar{t}\bar{o} \bar{\alpha}\bar{m}\bar{a} \bar{\bar{\nu}}\bar{\pi}\bar{\alpha}\bar{\rho}\bar{\chi}\bar{e}\bar{i}\bar{n} \tau\bar{e} \kai \mu\bar{n} \bar{\bar{\nu}}\bar{\pi}\bar{\alpha}\bar{\rho}\bar{\chi}\bar{e}\bar{i}\bar{n} \bar{\alpha}\bar{d}\bar{\bar{\nu}}\bar{\eta}\bar{a}\bar{t}\bar{o}\bar{n} \tau\bar{w} \alpha\bar{u}\bar{t}\bar{w} \kai \kai \chi\bar{a}\bar{t}\bar{a} \tau\bar{o} \alpha\bar{u}\bar{t}\bar{o}$.

² Metaphysics B 2, 996^b30: $\bar{\alpha}\bar{d}\bar{\bar{\nu}}\bar{\eta}\bar{a}\bar{t}\bar{o}\bar{n} \bar{\alpha}\bar{m}\bar{a} \bar{e}\bar{i}\bar{n}\bar{a}\bar{i}\bar{v}\bar{a} \kai \mu\bar{n} \bar{e}\bar{i}\bar{n}\bar{a}\bar{i}\bar{v}\bar{a}$.

³ Metaphysics Γ 3, 1005^a22–23: $\bar{\alpha}\bar{p}\bar{a}\bar{s}\bar{i} \gamma\bar{a}\bar{p} \bar{\bar{\nu}}\bar{\pi}\bar{\alpha}\bar{\rho}\bar{\chi}\bar{e}\bar{i}\bar{n} \tau\bar{o}\bar{s}\bar{i}\bar{v}\bar{a}\bar{n}$.

⁴ Metaphysics Γ 6, 1011^b13–14: $\beta\bar{e}\bar{\beta}\bar{a}\bar{i}\bar{o}\bar{t}\bar{a}\bar{t}\bar{h} \bar{\delta}\bar{\bar{\nu}}\bar{\bar{\chi}} \bar{\pi}\bar{a}\bar{s}\bar{w}\bar{n} \tau\bar{o} \mu\bar{n} \bar{e}\bar{i}\bar{n}\bar{a}\bar{i}\bar{v}\bar{a} \bar{\bar{\alpha}}\bar{\eta}\bar{\theta}\bar{e}\bar{e}\bar{i}\bar{s} \bar{\alpha}\bar{m}\bar{a} \bar{\tau}\bar{a}\bar{s} \bar{\bar{\alpha}}\bar{n}\bar{t}\bar{i}\bar{x}\bar{e}\bar{i}\bar{s}\bar{m}\bar{e}\bar{n}\bar{a}\bar{s} \bar{\bar{\varphi}}\bar{a}\bar{s}\bar{i}\bar{s}$.

[propositions which affirm and deny] the same property ($\tauοῦ αὐτοῦ$) of the same object ($\chiατὰ τοῦ αὐτοῦ$)⁵.

Two propositions are contradictories when one ascribes to an object exactly the same property which the second denies. And so one can also state the logical principle of contradiction in this way:

Two propositions of which one ascribes to an object the same property that the second denies cannot be true at the same time.

This principle is called logical because it concerns the truth of propositions, and so of logical facts.

c) Aristotle expresses the psychological principle of contradiction in these words: ‘No one can believe that the same thing is and is not, as Heraclitus said he did according to some; but it is not necessary for a speaker to believe what he says⁶.

Here $\boldsymbol{\text{ὑπολαμβάνειν}}$ does not mean ‘to assume’ or ‘to suppose’, but in contrast to $\lambdaέγειν$ ‘to say’ or ‘to utter a sentence’, it means a mental act usually, though not always, associated with uttering a sentence. This act is conviction or belief. So Schwegler translates [12] $\boldsymbol{\text{ὑπολαμβάνειν}}$ in this passage by ‘glauben’ ‘believe’⁷, and H. Maier states that this word, as well as the noun $\boldsymbol{\text{ὑπόληψις}}$, means in Aristotle the mental state of ‘being convinced’ or ‘regarding something as true’, or a subjective decision connected with a moment of faith, $\boldsymbol{\piστις}$ ⁸.

Whenever we have some belief, we believe in something. For, believing, we always believe that something is or is not, that it is so or not so; in a word, that some object has, or does not have, a certain property. *I call a series of words or other signs stating that some object has, or does not have, a certain property a logical sentence or proposition.* So to each belief, as a mental phenomenon, there corresponds as a logical fact an affirmative or negative proposition, expressed in words or other signs. So Aristotle’s principle, ‘no one can believe that something is, and at the same time believe that the same thing is not’, can also be formulated like this:

Two beliefs to which contradictory propositions correspond cannot exist at the same time in the same mind.

This principle concerns mental phenomena, and so is a psychological principle. [13]

⁵ *De Interpretatione* ch. 6, 17^a32–35: πάσῃ καταφάσει ἔστιν ἀπόφασις ἀντικειμένη καὶ πάσῃ ἀποφάσει κατάφασις. καὶ ἔστω ἀντίφασις τοῦτο, κατάφασις καὶ ἀπόφασις οἱ ἀντικείμεναι. λέγω δὲ ἀντικεῖσθαι τὴν (*scil.* κατάφασιν καὶ ἀπόφασιν) τοῦ αὐτοῦ κατὰ τοῦ αὐτοῦ.

⁶ *Metaphysics* Γ 3, 1005^b23–26: ἀδύνατον γὰρ ὄντινοῦν ταῦτὸν ὑπολαμβάνειν εἶναι καὶ μὴ εἶναι, καθάπερ τινὲς σίονται λέγειν Ὡράκλειτον. οὐκ ἔστι γὰρ ἀναγκαῖον, ἢ τις λέγει, ταῦτα καὶ ὑπολαμβάνειν.

⁷ *Die Metaphysik des Aristoteles*, vol. 2, Tübingen (1847), p. 54.

⁸ *Die Syllogistik des Aristoteles*, vol. 1, Tübingen (1896), p. 46 *sq.*: ‘The first sentence [*i.e.*, the one quoted above from *Metaphysics* Γ 3] refers to the sphere of subjective conviction, or holding that something is true ...’ On p. 104, ‘So in *De Anima* III 4 the practical ability to combine and distinguish ($\deltaιανοεῖσθαι$) and $\boldsymbol{\text{ὑπολαμβάνειν}}$, the subjective decision linked with a moment of $\boldsymbol{\piστις}$, appear separately’.

d) Although Aristotle did not define the differences obtaining among these three principles, certainly he must have felt them. His discussion of the relations between these principles gives evidence in support of this. He attached the greatest importance to his formulation of the ontological principle, which comprises the principle of contradiction in its fullest form. For this reason he mentioned this form of the principle first, and he discussed contradiction not in the *de Interpretatione*, not in the *Analytics* nor in his treatise *On the Soul*, but in book IV of the *Metaphysics*, which begins with these memorable words: ‘There is a science which investigates being as such and its essential properties’⁹.

The ontological principle is the pre-eminent (*χατ' εξοχήν*) principle of contradiction.

Remark: I stress the difference between a proposition as a logical fact, and a belief as a mental phenomenon. Contemporary logicians and psychologists recognise this distinction ever more explicitly. Thus, for example, Meinong distinguishes beliefs or acts of judging from the objects of beliefs, *e.g.*, the fact that something is or is not, or that something is or is not such-and-such. Meinong calls these facts ‘objectives’, and hands over their investigation to a separate science, the so-called ‘theory of objects’ (*Gegenstandstheorie*). This science would also include ‘pure logic’ and mathematics¹⁰. In this work I do not use the term ‘objective’ because a ‘proposition’ as I understand it [14] is something other than an ‘objective’; a proposition is some ‘objective’ expressed in words or other signs. It seems to me that this definition corresponds more closely to the Aristotelian concept of a proposition.

Aristotle also distinguishes a proposition from a belief, pointing out explicitly that a belief (*δόξα, ὑπόληψις*) exists in the soul (*ἐν τῇ ψυχῇ*), and has a corresponding sign (*σύμβολον*) in the sounds of speech (*ἐν τῇ φωνῇ*), and this sign is an affirmation (*χατάφασις*) or a denial (*ἀπόφασις*)¹¹. And so a proposition is a sentence expressed in words, and one which means something. For every sentence means something, but not every sentence is a proposition, only those to which the properties true and false belong¹². But only a sentence which asserts that something is or is not can be true or false¹³.

Combining these remarks into one definition, one can say that, according to Aristotle, *a proposition is a series of words asserting that something is or is not*. How much better this definition is than the usual definitions of a proposition which stray into logic textbooks, and which look in a proposition for a combination of concepts or a mental state of belief! For ‘the golden mountain’ is a combination of concepts, and the fact that ‘the sun shines’ is not a mental phenomenon. [15]

⁹ *Metaphysics* Γ 1, 1003^a21–22: οὐσίαν ἐπιστήμην τις ἡ θεωρεῖ τὸ δῦνον καὶ τὰ τούτων ὑπάρχοντα καθ' αὑτό.

¹⁰ See for example Meinong’s article ‘Über die Stellung der Gegenstandstheorie im System der Wissenschaften’ (On the Position of the Theory of Objects in the Framework of the Sciences), Leipzig (1907).

¹¹ See the quote from the *De Interpretatione*, ch. 14, 24^b1–3 in chapter 3.

¹² *De Interpretatione* ch. 4, 16^b33–17^a3: οὐσία δὲ λόγος ἀπας μὲν σημαντικός ..., ἀποφαντικὸς δὲ οὐ πᾶς, ἀλλ' ἐν φῷ τὸ ἀληθεύεν ἡ φεύδεσθαι ὑπάρχει.

¹³ *De Interpretatione* ch. 1, 16^a16–18: καὶ γὰρ ὁ τραγέλαφος σημαίνει μέν τι, οὔπω δὲ ἀληθὲς ἡ φεῦδος, ἔὰν μὴ τὸ εἶναι ἡ μὴ εἶναι προστεθῇ.

2. The relation of the ontological to the logical principle of contradiction

Someone might doubt whether the above formulations truly represent three different principles, and might rather think that they constitute just one principle expressed in different words. To resolve this doubt one must define when two propositions composed of different words express the same thought.

Every proposition can be reduced to one of two forms: ‘object O has property p ’, or ‘object O does not have property p ’. *The two propositions ‘ O has p ’ and ‘ O' has p' express the same thought in different words, or are synonymous, if O denotes the same object as O' , and p denotes the same property as p' .* For example, the following propositions express the same thought in partially different words: ‘Aristotle was the creator of logic’, and ‘the Stagirite was the creator of logic’. For according to accepted custom, the word ‘Stagirite’ denotes the same person, and hence the same object, as the word ‘Aristotle’. No negative proposition is synonymous with an affirmation, because affirmation means something different from negation; either of them is as simple as the other, and cannot be reduced to the other in any way.

If the propositions ‘ O has p ’ and ‘ O' has p' ’ are synonymous, then the truth of the second follows from the truth of the first, and the truth of the first follows from the truth of the second. Or more briefly, the second follows from the first, and the first follows from the second. For example, if it is true that ‘Aristotle was the creator of logic’, then it is true that ‘the Stagirite was the creator of logic’, and conversely.

Two propositions which are related in such a way [16] that the second follows from the first, and the first follows from the second, are equivalent. Consequently two synonymous propositions are also equivalent. Conversely, it follows that if two sentences are not equivalent, then they are not synonymous. The lack of equivalence is the most certain criterion of not being synonymous.

But conversely, not all equivalent propositions are synonymous. For example, ‘Aristotle was a student of Plato’ and ‘Plato was a teacher of Aristotle’ are equivalent propositions, for the second follows from the first, and the first from the second. However, these propositions are not synonymous, for the word ‘Aristotle’ and the word ‘Plato’ denote different objects, and the words ‘was a student of Plato’ and ‘was a teacher of Aristotle’ denote different properties¹.

The formulations of the principle of contradiction given in chapter 1 are not synonymous propositions. In the ontological principle of contradiction there are references to

¹ The business of distinguishing synonymous propositions from those which are equivalent but not synonymous belongs in my opinion among the logical tasks which are not only the most difficult but also the most important. All knowledge of the world consists of propositions, and thus of signs and symbols. There must be some criterion which would make it possible to determine in any case whether a difference between signs is real (that is, whether it corresponds to an existing real, though equivalent, difference between objects), or is apparent (that is, merely a difference in the signs). Obviously the remarks made in the text do not solve this problem; however, I think they should be sufficient for the aims of the present discussion. See also in the Appendix, § 9 γ.

objects, in the logical to propositions, in the psychological to beliefs. The words ‘object’, ‘proposition’, and ‘belief’ denote different objects. Therefore the propositions in which these words occur do not represent one [17] principle expressed in different words, but are three different principles. These principles, though different, may be equivalent. In fact, one can show that, according to Aristotle, *the ontological and logical principles of contradiction are equivalent propositions*. Aristotle did not state this outright; however, it is implied by his opinion on the relation between true propositions and being.

a) From the ontological principle of contradiction, the logical principle follows. For we read: ‘If the proposition is true that [something] is white or not white, then it must be white or not white’².

The context shows that this example is typical; that is, white (*λευκόν*) stands for any property whatever. Consequently if a proposition is true which ascribes a property to an object, then the object possesses the property. If a proposition is true which denies a property to an object, then the object does not possess the property. So if two contradictory propositions were true at the same time, then the same object would possess some property and at the same time not possess it. But this is not possible on the ground of the ontological principle of contradiction, so contradictory propositions cannot be true at the same time.

b) From the logical principle of contradiction, the ontological principle follows. For we read: ‘If [something] is white or not white, then the proposition would be true affirming or denying this’³. [18]

‘Someone speaks truly who holds that what is disconnected is disconnected, and what is connected is connected’⁴.

Consequently if an object possesses a property or is connected with it, then the proposition is true which ascribes [the property to the object]. If it does not possess it, or is not connected with it, then the proposition is true which denies [that the object has that property]. So if the same object possessed some property and at the same time did not possess it, two contradictory propositions would be true at the same time. However this is not possible on the strength of the logical principle of contradiction; therefore no object can possess and not possess the same property at the same time.

Proofs *a* and *b*, taken together, establish that the ontological and logical principles of contradiction are equivalent propositions.

I hold that this conclusion is correct, for it is a consequence of the correct opinion that being and true propositions correspond to each other. And this opinion depends on the definition of a true proposition: an affirmative proposition is true if it ascribes to an object a property which the object possesses; a negative proposition is true if it denies to an

² *De Interpretatione* ch. 9, 18^a39–^b1: εἰ γὰρ ἀληθὲς εἴτειν ὅτι λευκὸν ή ὅτι οὐ λευκόν ἐστιν, ἀνάγρητε εἶναι λευκὸν ή οὐ λευκόν.

³ *De Interpretatione* ch. 9, 18^b1–2: εἰ ἔστι λευκὸν ή οὐ λευκόν, ἀληθὲς ην φάναι ή ἀποφάναι.

⁴ *Metaphysics* Θ 10, 1051^b3–4: ἀληθεύει μὲν ὁ τὸ διηρημένον οἰόμενος διηρῆσθαι καὶ τὸ συγκείμενον συγκεῖσθαι.

object a property which the object does not possess. Similarly the converse: every object possesses that property which a true proposition ascribes to it, and no object possesses that property which a true proposition denies to it. Aristotle would agree with this definition, since he says: [19] ‘To say of what is that it is, and of what is not that it is not, is true’⁵.

From these definitions follows necessarily the equivalence of the ontological and logical principles of contradiction.

However, Aristotle would have called this logical, and not real, equivalence, for we read: ‘It is not the case that you are white because we think truly that you are white, but it is because you are white that we speak truly when we say so’⁶.

Consequently what is is both the logical reason for the truth of propositions and the real cause of their being uttered. On the other hand, the truth of propositions is merely the logical reason for what is and not its real cause. I presume that Aristotle would have expressed this difference in this way if he had realised it clearly.

3. The relation of the ontological and logical principles of contradiction to the psychological

Aristotle does not treat the psychological principle of contradiction on the same terms as the previous ones. He tacitly assumes that the ontological and logical principles are equivalent, and states emphatically that, being ultimate principles, they cannot be proved. On the other hand, he tries to prove the psychological principle from the logical or ontological principle. This proof falls into two parts

a) The first part is contained in the passage: [20]

If the same object cannot possess contrary properties at the same time, and if beliefs corresponding to contradictory propositions are contrary to each other, then obviously one and the same person cannot believe at the same time that the same thing is and is not. For he would have contrary beliefs at the same time if he were mistaken about this¹.

In this passage one phrase is difficult to interpret: ἐναντία δ' ἔστι δόξα δόξη ἡ τῆς ἀντιφάσεως. For it is well known in formal logic that contrariety (*ἐναντίωσις, oppositio contraria*) and contradiction (*ἀντίφασις, oppositio contradictoria*) are not just different concepts, but mutually exclusive. How then can we reconcile contradiction and contrariety in this passage? Well, the key to understanding this passage is the final section of the *De Interpretatione*, in which Aristotle poses the following question²: [21]

⁵ *Metaphysics* Γ 7, 1011^b26–27: τὸ ... γάρ λέγειν ... τὸ ὃν εἶναι καὶ τὸ μὴ ὃν μὴ εἶναι ἀληθές.

⁶ *Metaphysics* Θ 10, 1051^b6–9: οὐ γάρ διὰ τὸ ἡμᾶς οἰεσθαι ἀληθῶς σὲ λευκὸν εἶναι εἴ σὺ λευκός, ἀλλὰ διὰ τὸ σὲ εἶναι λευκὸν ἡμεῖς οἱ φάντες τοῦτο ἀληθεύομεν.

¹ *Metaphysics* Γ 3, 1005^b26–32: εἰ δὲ μὴ ἐνδέχεται ἄμα ὑπάρχειν τῷ αὐτῷ τάναντίᾳ . . . , ἐναντία δ' ἔστι δόξα δόξη ἡ τῆς ἀντιφάσεως, φανερὸν διὰ ἀδύνατον ἄμα ὑπολαμβάνειν τὸν αὐτὸν εἶναι καὶ μὴ εἶναι τὸ αὐτό· ἄμα γάρ ἂν ἔχοι τὰς ἐναντίας δόξας ὁ διεψευσμένος περὶ τούτου.

² H. Maier’s work (*loc. cit.*, vol. I, p. 155) pointed me towards this. Moreover Alexander of

Is an affirmation contrary to a negation or to an affirmation? *E.g.*, ‘Callias is just’, ‘Callias is not just’, ‘Callias is unjust’; which of these [last two] is contrary [to the first]? For if the [propositions] expressed in words are associated with [beliefs] in thought, and if there the belief is contrary, stating the contrary property, then the corresponding affirmations expressed in words must stand in a similar relation. But if in thought the belief stating the contrary is not contrary, then the [second] affirmation will not be contrary to the [first] affirmation, but only the negation mentioned. We must then consider which true belief is contrary to a false belief, that corresponding to the negative [the contradictory], or that which states the contrary³.

The problem of the contrariety of propositions raised in this passage causes particular difficulty for Aristotle. For according to him, it is primarily *properties* which stand at opposite extremes of a series of properties of the same kind which are contrary to each other; *e.g.*, white and black, or good and bad. Well, propositions are not properties, so that it is difficult to speak of the contrariety of propositions. Wishing to avoid this difficulty, Aristotle transfers the problem to psychological ground. Beliefs in thought correspond to propositions expressed in words, and one can understand beliefs as properties of the mind [22] in which they exist⁴. So beliefs, being properties, can be related as contraries; consequently we can call propositions contraries when they correspond to contrary beliefs. With the help of many arguments, Aristotle tries to show that beliefs are contraries when they correspond to an affirmation and a negation about the same object; for example, to ‘Callias is just’ and ‘Callias is not just’. Therefore we read in the conclusion [of the last passage]: ‘So that if this is the case for belief, and if affirmations and negations in words are signs of beliefs in the soul, it is clear that an affirmation is contrary to a negation’⁵.

It follows from these considerations that:

α) ‘Opinion’ (*δόξα*) here denotes a mental act, existing in the intellect (*ἐν τῇ διανοίᾳ*) or in the soul (*ἐν τῇ ψυχῇ*); to it there corresponds in words (*ἐν τῇ φωνῇ*) as its symbol (*σύμβολον*) an affirmation (*κατάφασις*) or a negation (*ἀπόφασις*). So here *δόξα* has the same meaning as *ὑπόληψις*, belief or conviction. This passage from the *De Anima* confirms this interpretation: ‘It is impossible for someone who has an opinion not to believe in it’⁶.

β) ‘Belief in the contrary’ (*δόξα ἡ τοῦ ἐναντίου*) means the same as ‘belief that the contrary is’ (*δόξα τὸ ἐναντίον εἶναι δοξάζουσα*); it is this belief to which [23] there corresponds

Aphrodisias said the same thing: *ὅτι δὲ ἐναντίαι αἱ δόξαι τῆς ἀντιφάσεως, δέδεικται διὰ πλειόνων ἐπὶ τέλει τοῦ περὶ Ἐρμηνείας* (That the beliefs in a contradiction are contraries is shown at length at the end of the *De Interpretatione*). See *Scholia in Aristotelem*, collected by Brandis, ed. Acad. Bor., [Berlin (1836),] p. 652.

³ *De Interpretatione* ch. 14, 23^a27–39: πότερον δὲ ἐναντία ἔστιν ἡ κατάφασις τῇ ἀποφάσει ἢ ἡ κατάφασις τῇ καταφάσει ...; οἷον ἔστι Καλλίας δίκαιος — οὐκ ἔστι Καλλίας δίκαιος — Καλλίας ἄδικος ἔστι· ποτέρα δὴ ἐναντία τούτων; εἰ γὰρ τὰ μὲν ἐν τῇ φωνῇ ἀκολουθεῖ τοῖς ἐν τῇ διανοίᾳ, ἐκεῖ δὲ ἐναντία δόξα ἡ τοῦ ἐναντίου, ... καὶ ἐπὶ τῶν ἐν τῇ φωνῇ καταφάσεων ἀνάγκη ὅμοιως ἔχειν. εἰ δὲ μηδὲ ἐκεῖ ἡ τοῦ ἐναντίου δόξα ἐναντία ἔστιν, οὐδέ τῇ κατάφασις τῇ καταφάσει ἔσται ἐναντία ἀλλ’ ἡ εἰρημένη ἀπόφασις. ὃστε σκεπτέον ποιά δόξα ἀληθής ψευδεῖ δόξῃ ἐναντία, πότερον ἡ τῆς ἀποφάσεως ἢ ἡ τὸ ἐναντίον εἶναι δοξάζουσα.

⁴ See Maier, *loc. cit.*, vol. I, p. 150.

⁵ *De Interpretatione* ch. 14, 24^b1–3: ὃστε εἴπερ ἐπὶ δόξης οὕτως ἔχει, εἰσὶ δὲ αἱ ἐν τῇ φωνῇ καταφάσεις καὶ ἀποφάσεις σύμβολα τῶν ἐν τῇ ψυχῇ, δῆλον ὅτι καὶ καταφάσει ἐναντία ... ἀπόφασις.

⁶ *De Anima* Γ 3, 428^a20–21: οὐκ ἐνδέχεται ... δοξάζοντα οἵς δοκεῖ μὴ πιστεύειν.

in words a proposition affirming the contrary property. Similarly ‘belief in the negation’ ($\delta\acute{o}\xi\alpha$ ή $\tau\eta\varsigma$ $\grave{\alpha}\pi\varphi\acute{a}\sigma\epsilon\omega\varsigma$) denotes a belief to which there corresponds in words a negation. Therefore the expression $\grave{\epsilon}\nu\alpha\nu\tau\acute{a}\alpha$ δ’ $\acute{e}\sigma\tau\grave{i}$ $\delta\acute{o}\xi\alpha$ $\delta\acute{o}\xi\eta$ ή $\tau\eta\varsigma$ $\grave{\alpha}\pi\varphi\acute{a}\sigma\epsilon\omega\varsigma$ can be translated ‘beliefs to which contradictory propositions correspond are contrary to each other’.

Thanks to this interpretation, the abovementioned difficulty of reconciling contrariety with contradiction disappears. The propositions are contradictories, and the corresponding beliefs are contraries. At the same time, it appears most clearly that Aristotle distinguishes propositions from beliefs. The proof under discussion from the *Metaphysics* can be formulated in the following way:

No object can possess contrary properties at the same time. If some one believed that something is, and at the same time believed that the same thing is not, then that person would have two contrary beliefs at the same time, and hence two contrary properties as well. Therefore no one can believe that something is, and at the same time believe that the same thing is not. In other words, two beliefs corresponding to contradictory propositions cannot exist at the same time in the same mind.

This is the first part of the proof.

b) The second part [of the proof] aims to justify the assertion that no object can possess contrary properties at the same time. Aristotle proves this assertion from the logical principle of contradiction, for we read: [24]

Since contradictory propositions cannot be true of the same object at the same time, clearly also contrary properties cannot belong to the same object at the same time. For one of two contrary properties is a deficiency no less than the other; that is, a deficiency of substance. But deficiency is negation of some definite kind. So if it is impossible at the same time to affirm and to deny truly, it is then impossible for contrary properties to exist at the same time⁷.

The statement ‘one of two contrary properties is a deficiency no less than the other’ is not a sufficiently clear and exact expression of the idea that the possession of one of two contrary properties is connected with not possessing the second contrary property. For example, if something is white, it is not black, and conversely. Spinoza’s famous statement ‘all determination is negation’ (*omnis determinatio est negatio*) involves a similar inexactness. Aristotle expresses himself more correctly in another passage: ‘Every contrariety has the absence (*έτερησις*) of one or the other of the contraries’⁸.

It appears from this that if the same object were to possess contrary properties at the same time, a contradiction would arise. For two propositions would be true at the same time, one of which would ascribe to the object that very property which the second proposition, asserting the contrary property, would implicitly deny.

⁷ Metaphysics Γ 6, 1011^b15–21: ἐπεὶ δὲ ἀδύνατον τὴν ἀντίφασιν ἄμα ἀληθεύεσθαι κατὰ τοῦ αὐτοῦ, φανερὸν ὅτι οὐδὲ τάνατίᾳ ἄμα ὑπάρχειν ἐνδέχεται τῷ αὐτῷ· τῶν μὲν γὰρ ἐναντίων θάτερον στέρησίς ἔστιν οὐχ ἡπτον, οὐσίας δὲ στέρησις. ἡ δὲ στέρησίς ἀπόφασίς ἔστιν ἀπό τυνος ὥρισμένου γένους. εἰ οὖν ἀδύνατον ἄμα καταφάναι καὶ ἀποφάναι ἀληθῶς, ἀδύνατον καὶ τάνατίᾳ ὑπάρχειν ἄμα.

⁸ Metaphysics I 4, 1055^b18: πᾶσα γὰρ ἐναντίωσις ἔχει στέρησιν θατέρου τῶν ἐναντίων.

This is the second part of the proof. Consequently the whole Aristotelian proof of the psychological principle of contradiction is presented as follows:

If two beliefs corresponding [25] to contradictory propositions existed at the same time in the same mind, then that mind would possess contrary properties at the same time. But from the logical principle of contradiction it follows that no object can possess contrary properties at the same time. Therefore two beliefs corresponding to contradictory propositions cannot exist at the same time in the same mind.

So Aristotle treats the psychological principle of contradiction as a consequence of the logical principle. For the logical principle of contradiction in the above argument justifies the general rule asserting the impossibility of the coexistence of contrary properties. And because the logical principle is equivalent to the ontological principle, it would not be inconsistent with Aristotle's opinions to hold that the psychological principle of contradiction is also a consequence of the ontological principle.

4. A critique of the Aristotelian proof of the psychological principle of contradiction

We must concede that in proving the psychological principle of contradiction, Aristotle argues very cautiously. Clearly he is aware that the simultaneous presence in the same mind of two beliefs corresponding to contradictory propositions would not constitute an open contradiction. Essentially, every belief is a positive mental act. No open contradiction would result if someone had one belief that something is, and a second belief at the same time that the same thing is not. Open contradiction would arise only if some belief existed, [26] and at the same time the same belief did not exist in the same mind.

Aristotle has proved only that two beliefs corresponding to contradictory propositions would contain a contrariety, and so a hidden contradiction, if they existed at the same time in the same mind. Let us consider whether this proof is valid.

a) It is based on the premise that beliefs are properties of the mind in which they exist. Let us accept this assumption. Since beliefs are properties, they can be related as contraries. Let us accept this too. Finally, let us assume that contrary properties exclude each other. Which beliefs are contraries? If we wish to discover them, we must arrange all beliefs about a given object in a series according to some principle, and identify the extreme elements of the series. For the extremes, since they differ most from each other, will be contrary to each other in the sense of Aristotle's definition: 'contrary properties are those which differ most in the same respect'¹.

Aristotle orders the beliefs concerning a given object according to their truth and falsity. Because a series for a given property arises only when there is a gradual difference

¹ De Interpretatione ch. 14, 23^b22–23: τὰ γὰρ ἐναντία τῶν πλεῖστον διαφερόντων περὶ τὸ αὐτό.

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in this property, the Stagirite must accept that some beliefs are truer than others, and some
are falser. And so we read:

‘The belief about each thing [27] which ascribes an essential property to it is truer
[than a belief which ascribes an accidental property]’².

‘The belief that good is not good would be falser than the belief in the contrary (*i.e.*,
that good is bad)’³.

These opinions lead to the conclusion that the relation of contrariety obtains between
the truest and the falsest beliefs about a given thing; for example, ‘good is good’ and ‘good
is not good’. And these are the beliefs which correspond to an affirmative and negative
proposition about the same object.

It is impossible to agree with this argument, for it is impossible to accept that there
are different degrees of truth and falsity. If it is at all possible to speak of the truth of
beliefs, then the belief which corresponds to a true proposition is true. An affirmative
proposition is true if it ascribes to an object a property which the object possesses, without
regard to whether this property is essential or accidental, or to whether the object possesses
it permanently or temporarily. We know of no gradations in the relation of inherence
which might correspond to differences in truth. If we want to recognise the existence of
propositions which are truer or less true than others, then we must change the definition of
a true proposition.

If then there are no gradual differences in truth or in falsity, there are no contrary
beliefs either, and there are no extremes with respect to truth and falsity. Hence Aristotle’s
statement, that [28] beliefs corresponding to contradictory propositions are contraries, is
not justified.

b) The argument in chapter 14 of the *De Interpretatione*, in which Aristotle proves
that beliefs are contraries, is subject to a more fundamental objection. In it, probably for
the first time in the history of philosophy, there appears a confusion (which today is all too
common) between logical and psychological issues.

Aristotle does distinguish propositions from beliefs, but he is not duly aware of the
correct nature of either of these. In particular he is unable to grasp the mental essence of
beliefs, which cannot after all surprise us, since these are extremely subtle phenomena, and
psychologically they have not been completely investigated today. So here lies the source
of his mistakes. Wishing to solve the problem of contrary propositions, Aristotle considers
the problem of contrary beliefs, and hence a psychological problem. In this way he tacitly
accepts the generally mistaken premise that the same relations obtain among propositions
as among beliefs⁴.

² *De Interpretatione* ch. 14, 23^b17: μᾶλλον δὲ ἐκάστου ἀληθῆς ή καθ' ἔαυτό (scil. δόξα).

³ *De Interpretatione* ch. 14, 23^b20–21: μᾶλλον ἀν εἴη φευδῆς τοῦ ἀγαθοῦ ή τῆς ἀποφάσεως ή ή
τοῦ ἐναντίου δόξα.

⁴ Maier sees nothing wrong with this premise, since he says ‘This method is allowed’ (*loc. cit.*,
p. 150).

However this erroneous premise does not entangle him in any difficulty. For, being unable to give a psychological analysis, *Aristotle treats beliefs as propositions*, and in the consideration which he intended to be psychological, he gives a logical analysis.

a) *He ascribes to beliefs relations of a kind which obtain between propositions.* For, asking which of the false beliefs ‘good is bad’ and ‘good is [29] not good’ is contrary to the true belief ‘good is good’, he points out among other things: ‘The belief [that good is not good] is connected with the belief that good is bad; for [if someone believed that good is bad], he should probably believe that it is also not good’⁵.

This opinion is mistaken. It is true only that the propositions ‘good is bad’ and ‘good is not good’ are connected with each other, for the first proposition justifies the second. However, it does not follow that the corresponding beliefs (if anyone could hold such ‘beliefs’ at all) are also connected with each other in this way. Someone could believe that good is bad, but not at all therefore believe that good is not good. Aristotle has falsely transferred the relation of dependence, which obtains between propositions, to the domain of mental phenomena⁶.

β) *Aristotle attributes to beliefs certain properties which, properly understood, belong only to propositions*, such as the properties of truth and falsity.

Truth and falsity are not in the strict sense properties of things or of phenomena, but only of certain *xes* — for the moment we do not know whether these are propositions or beliefs — which mean that something is or is not, or more generally that some [30] object possesses, or does not possess, some property. Moreover, these are relative properties; that is, they belong to the *xes* with respect to their corresponding, or not corresponding, to the fact that an object possesses, or does not possess, some property. Are these *xes* propositions or beliefs?

Propositions mean that something is or is not, that some object possesses or does not possess some property. These propositions also have the relation of corresponding, or not corresponding, to the fact that the object possesses, or does not possess, the property, for they are a series of words or other signs representing these facts. So propositions can be true or false.

Beliefs, as mental phenomena, do not mean that something is or is not. They are feelings of the sort which cannot be defined, but which must be experienced. For example, I am sitting at a table and writing. My mother is in a nearby room. I do not see her, but I feel she is there in the room. I am about to say something to her through the door, which is ajar, or to stand up and go to her, expecting to see her sitting in her chair. This feeling is a belief in the presence of my mother in the room. But it does not mean that ‘my mother

⁵ *De Interpretatione* ch. 14, 23^b25–27: ἡ δὲ (scil. δόξα ἡ τῆς ἀποφάσεως) τοῦ ὅτι κακὸν τὸ ἀγαθὸν συμπεπλεγμένη ἔστι· καὶ γὰρ ὅτι οὐκ ἀγαθὸν ἀνάγκη ἵσως ὑπολαμβάνειν τὸν αὐτόν.

⁶ It is interesting that the very same mistake turns up in some recent psychological textbooks of logic. See, for example, Höfler-Meinong, *Logik*, Vienna (1890), p. 136. In connection with this, see also my article ‘Analiza i konstrukcja pojęcia przyczyny’ (The Analysis and Construction of the Concept of Cause), *Przegląd Filozoficzny* vol. IX (1906), p. 138.

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is in the room', nor does it stand in the relation of corresponding or not corresponding to
this 'fact'; it is merely connected with it by a so-called 'intentional relation'.

Every belief concerns something, refers to something, has some intention. One term of this intentional relation is the act of belief; the second is some real or imaginary state of affairs (Meinong's 'objective') to the effect that something is or is not, that it is this way or that way; in a word, that some object possesses or does not possess some property. If we express this second member of an intentional relation [31] in words or in other signs, a proposition arises which is either true or false, for it represents either a real or an imaginary state of affairs. As a phenomenon, the belief, or the first member of the intentional relation, does not represent any fact, and consequently it is not strictly speaking true or false.

The continual confusion of beliefs with propositions has caused the psychology of knowledge to consist for the most part of logical analyses based on *a priori* premises, and not on experience. Aristotle's argument in chapter 14 of the *De Interpretatione* belongs to such pseudopsychological analyses. Hence this argument cannot justify the psychological thesis that the beliefs which correspond to contradictory propositions are contraries. And since this thesis is a premise of Aristotle's proof of the psychological principle of contradiction, it therefore appears that this proof is not sufficient.

5. A critique of the psychological principle of contradiction

A fallacious argument does not prove that a thesis is wrong. If Aristotle's proof is insufficient, let us see whether there are other arguments which would justify the psychological principle of contradiction.

To do this there is no need to show, as Aristotle tried to do, that beliefs corresponding to contradictory propositions are contraries. It would be sufficient to prove that they exclude each other. The concept of mutually exclusive properties is more extensive than the concept of contrary properties. *We call two properties mutually exclusive with respect to a given class of objects [32] when they cannot belong at the same time to the objects from this class.* This is independent of the question whether the properties are contrary extremes of some series (like 'black' and 'white') or not (like 'grey' and 'white', or 'red' and 'white').

We show that two properties are mutually exclusive either *a priori*, on the basis of premises and definitions accepted in advance, or empirically, on the ground of experience. Thus for example we show *a priori* that, with respect to the class of triangles, the properties 'equilateral' and 'right-angled' are mutually exclusive, for it follows from the axioms and definitions accepted in geometry that no equilateral triangle is right-angled. And we show empirically that, with respect to the class of chemical elements, the properties 'metallic' and 'produced at the anode' are mutually exclusive, for we know on the ground of experience, and only by experience, that in electrolysis metallic elements are always produced at the cathode and not at the anode. Which of these methods of demonstration should we employ in the present case?

a) I maintain that one cannot prove *a priori* that beliefs are mutually exclusive, and consequently that one cannot prove the psychological principle of contradiction *a priori*. Let us suppose that there are some premises and definitions about beliefs from which it is possible to prove the psychological principle of contradiction by deductive reasoning. In spite of everything, such a proof would be neither certain nor *a priori*, for it would always be doubtful whether the concept of belief defined by the accepted definitions corresponded to real belief. For the psychological principle [33] of contradiction deals with *phenomena*, and hence with real facts, and the concepts of such facts cannot be made up arbitrarily, but must contain properties which belong to these facts essentially and permanently. Consequently these must be real, reconstructive concepts, built on the foundation of experience, and not ideal constructs of the mind. The agreement between real concepts and reality is always only a hypothesis, more or less probable; and as a result of this, the inferences based on such concepts can be only probable, and depend ultimately on experience.

b) The psychological principle of contradiction can therefore be at most an empirical law. Such laws are never certain, but merely probable. We come to them by way of inductive inference, looking for some reason expressed in the form of a universal proposition about phenomena of a given kind which always and without exception behave in one way.

But is it possible to treat the principle under discussion as a confirmed empirical law? Husserl speaks of this as follows:

In the same individual [34] or, better, in the same consciousness, even during the smallest stretch of time, contradictory acts of belief cannot coexist. But is this really a *law*? May we really proclaim it with unlimited generality? Where are the psychological inductions which justify our accepting it? Can there never have been, will there never be, people who on some occasion (for example, being confused by fallacious reasoning) hold that opposite laws are true at the same time? Has someone carried out scientific research about whether such a thing, perhaps even a plain contradiction, happens among madmen? What about in a state of hypnosis, in the delirium of fever, and so on? Does this law also hold for animals?¹

We do not yet have an exhaustive answer to these questions. Psychologists are not interested in them, probably because they cannot distinguish the psychological from the logical principle of contradiction, and they do not doubt the truth of the logical principle. However, until all these investigations have been carried out with all the precision and the strict conditions which every empirical investigation requires, and until it appears, based on these investigations, that we do not know of any such states of mind in which the same

¹ E. Husserl, *Logische Untersuchungen (Logical Investigations)*, vol. I, Halle an der Saale (1900), ch. V, p. 82: ‘In demselben Individuum, oder noch besser, in demselben Bewußtsein, können während einer noch so kleinen Zeitstrecke contradictorische Glaubensacte nicht andauern. Aber ist dies wirklich ein *Gesetz*? Dürfen wir es wirklich mit unbeschränkter Allgemeinheit aussprechen? Wo sind die psychologischen Inductionen, die zu seiner Annahme berechtigen? Sollte es nicht Menschen gegeben haben und noch geben, die gelegentlich, z.B. durch Trugschlüsse verwirrt, Entgegengesetztes zu gleicher Zeit für wahr hielten? Hat man wissenschaftliche Forschungen darüber angestellt, ob dergleichen nicht unter den Irrsinnigen und vielleicht sogar bei nackten Widersprüchen vorkomme? Wie steht es mit den Zuständen der Hypnose, des Fieberdeliriums u.s.w.? Gilt das Gesetz auch für Thiere?’ Chapter V of this work by Husserl contains an excellent critique of the views of certain contemporary philosophers (e.g., Mill, F. A. Lange, Sigwart) who confuse the logical and the psychological principles of contradiction.

person believes that something is, and believes at the same time that the same thing is not, until then no one is free to declare that the psychological principle of contradiction is a law of thought. [35]

c) But we are already free to doubt whether these investigations will confirm that principle. It is already possible to cite facts which, in my opinion, cannot be reconciled with that principle *without appealing to additional hypotheses*.

Several philosophers have stated that the same thing can be and not be at the same time. Aristotle mentions Heraclitus, but no explicit statement about contradiction is preserved in the fragments of his work². Hegel, the modern follower of Heraclitus, speaks clearly and unambiguously in favour of this:

Something is in motion not because it is here in this Now and there in another Now, but because in one and the same Now it is here and not here, because it is and at the same time it is not in this Here. One must accept the contradictions which the ancient dialecticians revealed in movement, but it follows from this not that movement does not exist, but rather that movement is actually an *existing* contradiction³.

Those who defend the psychological principle of contradiction could reply to this, together with Aristotle, ‘It is not necessary for a speaker to believe what he says’⁴. But then they must assume either that Hegel said and wrote one thing and thought something else, or that he did not [36] clearly realise what he was saying and writing. In either case they must assume some additional auxiliary hypothesis, which would complicate their proposed theory, and lower its value and probability. Would it not be simpler to assume that Hegel truly believed what he wrote?

d) Finally, I cite an argument from my own inner experience. I have experienced states of mind in which I believed that something is, and at the same time I believed that the same thing is not. Clearly this argument has no scientific value, unless one can manage to evoke similar states in other minds. I shall try to do this.

I keep reading the simple but powerful words of the creed of St. Athanasius, his marvelous song about the Trinity. There is no *open* contradiction in this hymn; there is no *hidden* contradiction either, if we interpret its words in a manner consistent with theology. But someone who submissively allows the poem to affect him in a religious and æsthetic manner, not thinking about theological problems at all, will feel for a moment that he believes in two propositions which *appear* to be contradictory. With a slow, solemn, measured rhythm, in clauses built of balanced phrases, the powerfully eloquent words ring out:

² See H. Diels, *Herakleitos von Ephesos*, Berlin (1901).

³ Hegel, *Wissenschaft der Logik* (*The Science of Logic*), Werke, vol. IV, Berlin (1834), p. 69: ‘Es bewegt sich etwas nur, nicht indem es in diesem Jetzt hier ist, und in einem anderen Jetzt dort, sondern indem es in einem und demselben Jetzt hier und nicht hier, indem es in diesem Hier zugleich ist und nicht ist. Man muß den alten Dialektikern die Widersprüche zugeben, die sie in der Bewegung aufzeigen, aber daraus folgt nicht, daß darum die Bewegung nicht ist, sondern vielmehr daß die Bewegung der *daseyende* Widerspruch selbst ist’.

⁴ *Metaphysics* Γ 3, 1005^b25–26: οὐκ ἔστι γάρ ἀναγκαῖον, ἐπειδὴ ταῦτα καὶ ὑπολαμβάνειν.

For the Father is one person, the Son another, the Holy Spirit another; [37] but the divinity of the Father, and of the Son, and of the Holy Spirit is one; their glory is equal, their majesty co-eternal.

Such as the Father is, so is the Son, and so the Holy Spirit.

The Father is uncreated, the Son uncreated, the Holy Spirit uncreated.

The Father is infinite, the Son infinite, the Holy Spirit infinite.

The Father is eternal, the Son eternal, the Holy Spirit eternal.

And yet there are not three eternal beings, but one eternal being, just as there are not three uncreated beings, nor three infinite beings, but one uncreated being, one infinite being.

Likewise the Father is almighty, the Son almighty, the Holy Spirit almighty; and yet there are not three almighty beings, but one almighty being.

So the Father is God, the Son is God, the Holy Spirit is God; and yet there are not three gods, but one God.

Thus the Father is Lord, the Son is Lord, the Holy Spirit is Lord; and yet there are not three Lords, but one Lord⁵.

If a believer simply accepts these words, reading them in an attentive spirit, and does not analyse their theological content, then he will experience a feeling of unfathomable mystery. For he believes that there are three different divine persons, and that each is truly God, and he believes at the same time that there are not three gods, but just one God, uncreated, infinite, almighty, and eternal. I think that these very acts of belief in apparently contradictory propositions produce [38] this feeling of mystery and awe. Even some theologians, surely under the influence of such states of mind, have sought contradictions in the concept of God; it is sufficient to mention Cardinal Nicholas of Cusa, who looked for a coincidence of opposites (*coincidentia oppositorum*) in God.

I have given this example so that the reader, concentrating on the words of the creed, might produce in himself, just for a moment, a state of mind similar to that which I myself have experienced. Someone who has experienced this feeling will accept the argument as decisive; someone who has not can pass over it.

If I am not mistaken, the remarks above will at least convince the reader that the psychological principle of contradiction is not so certain and undoubted as it is reputed to be. And because an uncertain, doubtful principle can hardly be the basis for logic, we can pass over it without prejudice to further considerations. *The way to the foundations of logic does not lead through psychology.*

6. The unprovability of the ontological and logical principles of contradiction

After disposing of the psychological formulation of the principle of contradiction, there remain the ontological and logical formulations, to which we now turn our attention.

I remind you that the ontological principle of contradiction reads: *no object can at the same time possess and not possess the same property*, and the logical principle states:

⁵ I quote this according to the *Horae diurnae Breviarii Romani* (*The Daily Hours of the Roman Breviary*), Mechelen (1886), pp. 13–14.

two propositions, of which one attributes to an object the same property which the second denies, cannot be true at the same time. We know already that these propositions are equivalent; consequently we must [39] either accept both or reject both. In the following chapters I consider mainly the ontological principle, which, as we know, is the pre-eminent (*κατ' εξοχήν*) principle of contradiction.

Proofs showing that some principle is true can dispose us to accept it. What kinds of proof are there of the ontological or logical principle of contradiction?

This question evidently irritates Aristotle; it throws him off balance, and puts into his mouth words which testify to a certain pique. I shall paraphrase a passage [from the *Metaphysics*] to bring out the emotions which I sense in it¹:

Now we have said that it is impossible for something to be and not to be at the same time, and *by this* we have shown (!) that this is the most certain of all principles. Do you want proofs? There are no proofs! For it is *ignorance* not to know what needs proof and what does not. Of course it is impossible to prove everything, for we would get lost in infinity, and still there would be no proof. Besides, if you are so eager for proofs, [40] all right; there are proofs, just not the usual kinds, but proofs by refutation! Just let someone try to say a word! And if he says nothing, it would be ridiculous to argue with him, just as if you were chatting with a tree².

If someone proclaims an opinion with great emphasis and self-assurance, but giving no proofs, if he gets angry instead of arguing, then evidently he has no sufficiently strong arguments, and surely he feels that his position is shaky; so he puts on a bold face to avoid betraying himself. The passage quoted from the *Metaphysics* makes just such an impression on me. But let us analyse it logically.

For the moment I shall pass over the contradiction, real or apparent, involved in the fact that Aristotle at first considers the principle of contradiction as unprovable, but a few lines later he grants that a proof is possible, saying that ‘one can prove even that this is impossible by refutation’³, and he labours over this proof for several chapters. In this section I am concerned only with his opinion that the principle of contradiction, as an ultimate principle, cannot be proved.

Aristotle maintains that you cannot prove everything. Here he has in mind not *singular* propositions about specific facts, but principles or *universal* propositions. If one wishes to prove some such principle, it is necessary to show that another proposition is

¹ Tr. In the original the Greek passage from the original precedes the paraphrase in the text. I have made a slight insertion in this sentence to allow the Greek text to appear in a footnote, as Łukasiewicz directed in his own translation of this chapter. The original text reads ‘I shall paraphrase this passage to bring out ...’, and the alteration is exactly that made by Łukasiewicz in his translation.

² *Metaphysics* Γ 4, 1006^a3–15: ἡμεῖς δὲ νῦν εἰλήφαμεν ὡς ἀδυνάτου ὄντος ἀμά εἶναι καὶ μὴ εἶναι, καὶ δὰ τούτου ἐδείξαμεν ὅτι βεβαιοτάτη αὕτη τῶν ἀρχῶν πασῶν. ἀξιοῦσι δὴ καὶ τοῦτο ἀποδεικνύναι τινὲς δι' ἀπαιδευσίαν. ἔστι γὰρ ἀπαιδευσία τὸ μὴ γιγνώσκειν τίνων δεῖ ζητεῖν ἀπόδειξιν καὶ τίνων οὐ δεῖ. ὅλως μὲν γὰρ ἀπάντων ἀδύνατον ἀπόδειξιν εἶναι. εἰς ἀπειρον γὰρ ἀν βαδίζοι, ὥστε μηδ' οὔτως εἶναι ἀπόδειξιν ... ἔστι δ' ἀποδεῖξαι ἐλεγχτικῶς καὶ περὶ τούτου ὅτι ἀδύνατον, ἀν μόνον τι λέγη ὁ ἀμφισβητῶν ἀν δὲ μηδὲν, γελοῖον τὸ ζητεῖν λόγον πρὸς τὸν μηδενὸς ἔχοντα λόγον ..., ὅμοιος γὰρ φυτῷ ὁ τοιοῦτος.

³ *Metaphysics* Γ 4, 1006^a11–12: ἔστι δ' ἀποδεῖξαι ἐλεγχτικῶς καὶ περὶ τούτου ὅτι ἀδύνατον.

true which is a premise for this one. The truth of this premise must again be proved from another premise, and so forth. This series of conclusions and premises must end in some *ultimate* principle, which is not the consequence of any other premise, but is true by itself. [41] For if we accepted that a series of reasons could go on to infinity, we would obtain propositions whose truth was merely *hypothetical*; for example, proposition p_1 is true if proposition p_2 is true, proposition p_2 is true if proposition p_3 is true, and so forth. At some point we must say *categorically* that some proposition p_n in this series is true *by itself*, *without reference to other propositions*. Such a proposition is an *ultimate* principle (*πρώτη ἀρχή*) and indemonstrable (*ἀναπόδεικτος*). Hence we read: ‘We say that not all knowledge is demonstrable, but the knowledge of propositions in which the predicate belongs to the subject immediately (these are $\tau\alpha \; \dot{\alpha}\mu\epsilon\sigma\alpha$) is indemonstrable’⁴.

Aristotle calls ultimate principles $\dot{\alpha}\mu\epsilon\sigma\alpha$ because there is in them no middle term ($\tau\delta \; \mu\acute{e}s\sigma\sigma\acute{o}n$) between the subject and the predicate. Let us suppose that in some ultimate principle, ‘ A is B ’, there existed a middle term C which would allow us to construct the propositions ‘ A is C ’ and ‘ C is B ’. But this would make it possible to prove this principle from the premises ‘ A is C ’ and ‘ C is B ’, and so it would not be ultimate.

We can agree with the above reasoning, but with the reservation that by ‘principle’ we should not understand only universal propositions; the importance of this reservation will be clear later. So we can accept that there are some ultimate principles, that is, *propositions which cannot be proved from other propositions, but which are true by themselves*. [42] Now we must look for these principles.

According to Aristotle, one of these principles, the most certain (*βεβαιοτάτη*) and the most easily accessible to knowledge (*γνωριμωτάτη*), about which no mistake is possible (*Metaphysics* Γ 3, 1005^b12: *περὶ ἣν διαφευσθῆναι ἀδύνατον*) is the principle of contradiction. But where is the proof of this? Well, the Stagirite does not give a proof, and is content to say only that ‘if it is not necessary to look for a proof of some principles, then it would be hard to name a more suitable principle [than the principle of contradiction]’⁵. That is, no principle has a better reason for being indemonstrable than the principle of contradiction.

But what if there were such a principle? What if it were possible not just to affirm but also to *prove* that this principle which we do not yet know is true by itself and unprovable, and so ultimate, although it is not the principle of contradiction? And finally, what if, because of the discovery of this essential, ultimate principle, it were to be shown that the principle of contradiction needs a proof?

Aristotle does not think it is necessary to prove the assertion that the principle of contradiction is an ultimate principle. So he accepts at the same time two ultimate, indemonstrable principles: the principle of contradiction, and the principle which states that the principle of contradiction is an ultimate principle.

⁴ *Posterior Analytics* A 3, 72^b18–20: *ἡμεῖς δέ φαμεν οὕτε πᾶσαν ἐπιστήμην ἀποδεικτικὴν εἶναι, ἀλλὰ τὴν τῶν ἀμέσων ἀναπόδεικτον.*

⁵ *Metaphysics* Γ 4, 1006^a10–11: *εἰ δέ τινων μὴ δεῖ ζητεῖν ἀπόδειξιν, τίνα ἀξιοῦσιν εἶναι μᾶλλον τοιαύτην ἀρχὴν οὐχ ἀνέχοιεν εἰπεῖν.*

His position is weak, for we shall soon be convinced that it cannot manage to stand up under the attacks of logical criticism. [43]

7. The principle of contradiction and the principle of identity

Among universal propositions, there is a principle which has a stronger claim than that of the principle of contradiction to be an ultimate principle: the principle of *identity*.

Nowhere does Aristotle formulate this as a separate logical or ontological law¹. It can be specified in two ways according to the relations of inherence and non-inherence: *Every object possesses a property which it possesses*, and *no object possesses a property which it does not possess*. These two formulations can also be expressed in conditional sentences: *If an object possesses a property, then it possesses it*, and *if an object does not possess a property, then it does not possess it*. These are ontological formulations, since they contain the concepts of object and property.

Logicians usually hold that the principle of identity is an affirmative but worthless formulation of the principle of contradiction which does not say anything². This opinion [44] probably originates from the fact that the principle of contradiction is often stated in the form given by Leibniz: ‘*a* is not *non-a*’³, which is the negative form of the corresponding affirmative formula accepted as the principle of identity, ‘*a* is *a*’.

¹ See Maier, *loc. cit.*, vol. I, p. 101, remark 1; compare the passage cited below in chapter 8 from *Metaphysics* Γ 7, 1011^b26–27.

² See for example Ueberweg, *Logik*, Bonn (1882), p. 232: ‘Als tautologischer Satz ist die Formel $A = A$ nichtssagend und keineswegs die nothwendige positive Ergänzung zu dem Satze des Widerspruchs. Denn daß der einmal als wahr erkannte Gedanke nicht durch einen widersprechenden wieder aufgehoben werde, ist eine berechtigte logische Anforderung, daß er aber sich selbst gleich und also immer wieder wahr sei, ist eine überflüssige Bemerkung’. ‘As a tautological proposition, the formula $A = A$ says nothing, and it is in no way a necessary positive supplement to the principle of contradiction. For that a thought once recognised as true will not be canceled by one which contradicts it, is a justified logical claim, but that it is equal to itself, and so is always still true, is a superfluous remark’. Sigwart, *Logik*, Freiburg in Breisgau (1889), vol. I, p. 186: ‘Es ist also vollkommen naturgemäß, daß Aristoteles den Satz des Widerspruchs allein als Princip heraushebt und seine positive Kehrseite nur gelegentlich zum Ausdruck bringt, wie auch lange Zeit unter dem Principium identitatis der aristotelische Satz des Widerspruchs verstanden wurde’. ‘So it is completely natural that Aristotle has singled out the law of contradiction alone as a principle, and only incidentally brings its positive reverse to expression, as also for a long time the Aristotelian law of contradiction was known as the principle of identity’. This last remark of Sigwart’s applies among others to Trendelenburg, who in his *Logische Untersuchungen (Logical Investigations)*, vol. I, Leipzig (1862), p. 31, discusses the principle of contradiction under the name ‘principle of identity’. Finally I give an example of a quote from the most recent treatises *not* about logic, to point out how very widespread this opinion is. Dr. H. Kleinpeter writes in his book *Die Erkenntnistheorie der Naturforschung der Gegenwart (The Theory of Knowledge in Contemporary Science)*, Leipzig (1905), p. 103: ‘Sagt man, der Satz [des Widerspruchs] bestehe darin, daß die beiden Urteile “*A* ist *B*” und “*A* ist nicht *B*” nicht gleichzeitig bestehen können ..., so sagt er das Nämliche aus, wie das Identitätsprinzip’. ‘If one says that the law [of contradiction] consists in this, that the two judgments “*A* is *B*” and “*A* is not *B*” cannot exist at the same time, then it says the same thing as the identity principle’.

³ See for example Höfler-Meinong, *Logik*, Vienna (1890), p. 135: ‘Der Satz des Widerspruches pflegt häufig so ausgesprochen zu werden: *A* ist nicht *non-A*’. ‘The law of contradiction is often expressed like this: *A* is not *non-A*’.

It is surprising how obstinately some opinions are held in science, not just formulated inexactly and unjustified, but plainly false, merely, in my opinion, because others repeat uncritically what someone once said. This is how matters stand with respect to the opinion about the relation of the principle of contradiction to the principle of identity.

a) The principle of identity is said to be ‘an affirmative formulation of the principle of contradiction’. This statement, whose clarity and accuracy leave much to be desired (see in the above quote from Sigwart the even fuzzier catchword ‘positive Kehrseite’), apparently means just that both principles express the same thought in different words, the first in an affirmative sentence, and the second in a [45] negation; that is, it apparently means that they are synonymous. Well, I have already mentioned in chapter 2 that no affirmative proposition is synonymous with a negation, for affirmation means something different from negation. An affirmative proposition can be at most *equivalent* to a negation, but equivalence and synonymity are two different concepts. So we cannot accept that the formulas ‘*a* is *a*’ and ‘*a* is not non-*a*’ are synonymous, or consequently that they express the same principle.

b) However, the question of the synonymity of these formulas is of no significance for our problem, since the formula ‘*a* is not non-*a*’ does not represent the principle of contradiction at all. Scientific logic, which employs in its investigations unambiguously defined symbols, and which avoids the unclear, faltering expressions of everyday life, regards this formula as an inexact expression of the *principle of double negation*⁴. It can be shown that none of these principles, identity, contradiction, and double negation, expresses the same thought, as a result of which they all have different meanings⁵.

c) In order to justify this assertion adequately, I shall put all these principles together in the form of implications, since in this form they approach [46] most closely to the exact formulas of symbolic logic:

The principle of identity: If *O* has *p*, then *O* has *p*.

The principle of double negation: If *O* has *p*, then *O* cannot not have *p*.

The principle of contradiction: If *O* is an object, then *O* cannot have and not have *p* at the same time.

Some doubt might be suggested by the problem of whether the third of these implications is synonymous with the principle of contradiction expressed in categorical form: ‘no object can possess and not possess the same property at the same time’. Now in my opinion, every universal proposition, affirmative or negative, represents a combination of two propositions; ‘all *A* is *B*’ means that ‘if something is *A*, then it is *B*’, and ‘no *A* is *B*’ means that ‘if something is *A*, then it is not *B*’. The *equivalence* of these forms is not in doubt; their *synonymity* follows from the meaning of the expressions ‘all’ and ‘no’.

⁴ See E. Schröder, *Vorlesungen über die Algebra der Logik (Exakte Logik)* (*Lectures on the Algebra of Logic (Exact Logic)*), Leipzig (1890), vol. I, p. 350.

⁵ Even Schröder agrees that the formula ‘*a* is not non-*a*’ is not an expression of the principle of contradiction; see *loc. cit.*, vol. I, p. 182, § 23. I shall show in chapter 10 that the principles enumerated above are not even equivalent to each other.

The words ‘all A ’ do not represent *one* object, but an indefinite *number* of objects A_1, A_2, \dots, A_n . So someone who asserts that ‘all A is B ’ expresses in an abbreviated form a *series* of propositions: ‘ A_1 is B ’, ‘ A_2 is B ’, …, ‘ A_n is B ’, in which A_1, A_2, \dots, A_n exhaust the range of A . This means that whichever A_x we consider, it is B ; or that ‘if something is A , then it is B ’. Likewise the sentence ‘no A is B ’ expresses in an abbreviated form a *series* of propositions: ‘ A_1 is not B ’, ‘ A_2 is not B ’, …, ‘ A_n is not B ’, in which A_1, A_2, \dots, A_n exhaust the range of A . This means that whichever A_x we consider, it is [47] not B ; or that ‘if something is A , then it is not B ’. The concept of consequence involved in the implication is contained in the words ‘all’ and ‘no’.

Here I give the principle of contradiction in conditional form, since in this form it is easier to grasp the properties which distinguish this principle from the principles of identity and double negation. However, these same differences also characterise the *categorical* form of the principle of contradiction, so that the following considerations would still be correct, even if the hypothetical and categorical forms were not synonymous. The properties which distinguish the principle of contradiction from the principles of identity and double negation are the following:

α) In the antecedent of the principle of contradiction there occurs the term ‘object’, which does not occur in the antecedents of the principles of identity and double negation. If we were to formulate these principles with the help of this term, we would get only *particular* cases of them: ‘if O is an object, then O is an object’, and ‘if O is an object, then O cannot not be an object’. Clearly these cases are different from the *universal* principles of identity and double negation.

On the other hand, it would be possible to express the principle of contradiction with the help of an indefinite property p' : ‘If O has p' , then O cannot at the same time have p and not have p' ⁶. Yet I doubt this new formulation is synonymous with the older one, for it would mean that if anything whatever can be predicated of some object, then that object could not at the same time possess and not possess the same property. So this formulation would be more extensive than that usually accepted. But I pass over the problem of whether the principle of contradiction would express [48] the same thought in this new form as in the older one, for the main difference between the principles under discussion is not in their antecedents, but in their consequents.

β) In the consequent of the implication which expresses the principle of contradiction, and likewise in its categorical form, there are terms without which it is impossible to formulate *this* principle, but which are completely unnecessary for stating the *principles* of identity and double negation. These terms are ‘and’ and ‘at the same time’, and they represent the concept of *logical multiplication*. There is a second difference, directly connected with the first, and no less important: in any logical multiplication, there must

⁶ Tr. In the original text, the prime mark is missing immediately before the colon; this is clearly a mistake. The German translation ingeniously omits the prime mark from the antecedent of the implication, but then the surrounding text does not make sense. The present text is Łukasiewicz’s own correction of the error.

be at least two multiplied factors, and so two propositions. In fact, the consequent of the principle of contradiction contains two propositions united by the words ‘and’ and ‘at the same time’, and one of these two propositions is the negation of the second. On the other hand, neither the consequent of the principle of identity nor that of the principle of double negation contains two propositions joined together by the relation of multiplication, and I see no way in which, by means of any analysis whatever, one could get out of the terms contained in these principles two propositions which would constitute a logical product.

Because of these differences, the principle of contradiction expresses a new thought, distinct from the thoughts contained in the other two principles. The principle of contradiction cannot be formulated without its containing two propositions of which one is the negation of the second, and which together constitute a logical product. The principle of identity, as well as the principle of double negation, can be formulated without containing two propositions which constitute a logical product. The principle of identity can be expressed without [49] using the concept of negation, while the principle of contradiction cannot be formulated without making use of this concept. As long as ‘two’ means something different from ‘one’, and as long as negation and logical multiplication have a different meaning from that of the absence of negation and of multiplication, *the principle of identity will be different from the principle of contradiction.*

This shows that the assertion maintained by so many logicians, that the principle of identity is just an affirmative formulation of the principle of contradiction, makes no sense. Moreover, this is not surprising, for no logician has ever taken the trouble to present the assertion adequately or to justify it in any way. Amid fundamental logical problems, they are spouting hackneyed rubbish⁷.

Even Aristotle is not free from a similar charge. Trusting in the dialectic strength of his argument, he throws this challenge before us: ‘If someone does not want to concede the principle of contradiction without proofs, let him suggest another principle which would have a better reason to pass as ultimate’. There is such a principle, different from the principle of contradiction, simpler, more certain, and more intelligible. Consequently, should not that principle be recognised as ultimate instead? [50]

⁷ Wundt may be the only logician who does not identify the principle of identity with the principle of contradiction, though even he has not grasped precisely how these two propositions are related. See *Logik*, vol. I, Stuttgart (1893), p. 564: ‘Es [ist] möglich sich des Satzes vom Widerspruch auch zum Ersatz des Identitätsgesetzes zu bedienen (?), wie es die ältere Logik (In the third edition, Wundt mistakenly adds ‘nach dem Vorbilde des Aristoteles.’) durchgängig gethan hat, während es dagegen umgekehrt nicht möglich ist, aus dem Identitätsgesetz auf den Satz des Widerspruchs zu schließen. Denn das erstere würde auch dann gültig sein, wenn die Funktion der Verneinung nicht existierte’. ‘It is also possible for the law of contradiction to serve as a substitute for the law of identity, as it did throughout older logic (In the third edition, Wundt mistakenly adds ‘on the model of Aristotle.’), while conversely it is not possible to infer the law of contradiction from the law of identity. For the latter would still hold if there existed no negation function’.

8. The ultimate principle

The principle of identity is not ultimate, because it can be proved from another proposition. Its justification is the *definition of a true proposition*.

An affirmative proposition is true if it ascribes to an object a property which the object possesses; a negative proposition is true if it denies to an object a property which the object does not possess. These are definitions. It follows immediately from them that, if an object possesses a property, then it is true that it possesses it, and so it possesses it. And if an object does not possess a property, then it is true that it does not possess it, and so it does not possess it. These sentences are true because this is how I define truth. And so it is not the principle of identity which is ultimate, but the definition of a true proposition, on which that principle depends.

Let us consider this statement in detail.

α) Like every definition, the definition of a true proposition is a *singular* proposition. For *every definition contains, openly or covertly, a word which states a singular fact*: ‘*I define*’, ‘*I denote*’, ‘*I call*’, ‘*I understand*’. For example, ‘by a true proposition *I understand* an affirmative proposition which ascribes to an object a property which the object possesses’, or ‘*I denote* the relation of inherence “*O* has *p*” by the letter *p*’¹, or ‘*I call* a circle a closed curve in a plane, every point of which is equidistant from the centre’. The words ‘*I understand*’, ‘*I denote*’, ‘*I call*’ can be omitted if the sentences which ought to contain them are designated explicitly or implicitly as definitions. Instead of expressing myself like this: ‘*I understand* by a true proposition’ and so on, [51] I say more briefly: ‘An affirmative proposition is true if’, and so forth; and I add ‘and this is a definition’. If the added phrase is not there, and if it cannot be inferred, a definition is changed into a universal proposition which has the appearance of an ultimate principle.

There is a passage in Aristotle which looks like a formulation of the principle of identity, though it is merely the definition of a true proposition: ‘To say that what is, is, and what is not, is not, is true’².

Fr. Gabryl treats this sentence as an expression of the principle of identity³, and that is what it would in fact be, had Aristotle not forewarned us that he intended to give in these words a definition of truth. For he writes immediately before this: ‘It is obvious [that there can be nothing between two contradictory propositions] if we first define what truth and falsity are’⁴.

¹ Tr. This definition is by no means clear. In his translation, Łukasiewicz may have changed the final ‘*p*’ to ‘*p*’; but as this is by no means clear in the manuscript, and certainly not done in some places, I have allowed the original to stand, despite its obscurity.

² Metaphysics Γ 7, 1011^b26–27: τὸ ... γὰρ λέγειν ..., τὸ δὲ εἶναι καὶ τὸ μὴ δὲν μὴ εἶναι ἀληθές.

³ Metafizyka ogólna, czyli nauka o bycie (General Metaphysics, or the Science of Being), Cracow (1903), p. 105.

⁴ Metaphysics Γ 7, 1011^b25: δῆλον δὲ (scil. ὅτι οὐδὲ μεταξὺ ἀντιφάσεως ἐνδέχεται εἶναι οὐδέν) πρῶτον μὲν ὄρισαμένους τί τὸ ἀληθές καὶ ψεῦδος.

So we ought to distinguish a definition from the principle which follows from it. A definition is always a *singular* proposition which asserts the fact that someone defines, denotes, or calls some object in a certain way. The principle based on a definition is always a *universal* proposition which predicates of the defined object the property which the definition ascribes to it.

b) Every definition is a true proposition, for [52] it predicates of the definition's *author* a property which belongs to him. For if someone says or writes 'by circle I understand a curved line', or 'I denote the relation of inherence "*O* has *p*" by the letter *p*', he creates the fact of which he speaks or writes. Of course, just by saying or writing these words, he confirms that he understands by 'circle' a curved line, or that he denotes the relation by the letter *p*.

But here there may be an objection: the author of a definition could *lie*, and while calling a curved line a circle, he could believe that a circle is something else. I concede that such cases might happen. But definitions do not exist, as some people wish them to do, in order to express someone's act of belief or a decision of his will, but to fix the meaning of words or of some other signs that can be perceived by the senses, with the aim of understanding each other and apprehending real facts. Hence it makes no difference whether someone who gives some definition believes what he says or not, or whether he truly wishes to define a given sign as he says, or is just pretending to do so. It matters only in what *words* he expresses the definition, and how he *uses* the defined form in the statements he pronounces. So the words 'I understand', 'I denote', 'I define', and so on must always refer to the *signs*; that is, to the sentences which contain them, and not to *mental acts*. Thus *every definition is related to a fact which arises together with it, and which is contained in it; and hence every definition is true*.

But not every definition is practical and useful. A definition is impractical if it defines some expression of established scientific meaning in a way [53] different from that generally accepted. A definition is not useful if it gives a meaning to some word or sign, or creates a concept, which does not correspond to reality and cannot apply to reality in any way, even indirectly. The assertion that the concept which some definition creates does correspond to reality is always just a hypothesis which must be verified by experience.

c) *The definition of a true proposition is an ultimate principle, for it is true in itself and cannot be proved from another proposition.*

The definition of a true proposition is true, for every definition is true; and it is true in itself, for its truth does not depend on another proposition, but only on its own truth. If someone says, 'I understand by a true proposition one which ascribes to an object a property which the object possesses', this ascribes to him the property of 'understanding something by a true proposition', and he actually possesses this property, for he confirms this by the very act of enunciating the definition. Therefore this definition is true by the definition of truth which it contains in itself. This is the only case in which some proposition can be true *in itself* in the exact and literal sense of the term. If this property is predicated of other

propositions, the expression ‘true in itself’ is not used in a strictly determined way.

The definition of a true proposition cannot be proved from another proposition. For let us assume that some true proposition is its premise; then the truth of this premise would have to depend in part on the truth of the definition of a true [54] proposition, and hence the proof would contain a *petitio principii*.

d) Besides the definition of a true proposition, there is no other ultimate principle.

α) Other definitions are not ultimate, because they depend on the truth of the definition of a true proposition.

β) Propositions about facts are not ultimate, whether the facts are given directly in external or internal experience. Indeed, propositions such as ‘it is thundering’ or ‘my head aches’ cannot be proved, and in this respect are like an ultimate principle. Yet they differ, because they are not true in themselves, for they do not state facts which they contain, but they concern phenomena which exist *beyond* the propositions.

γ) Finally, universal propositions are not ultimate principles. We know already from chapter 7 that every universal proposition, whether affirmative, like ‘all A is B’, or negative, like ‘no A is B’, represents a *combination* of two propositions. For the sentence ‘all A is B’ means ‘if something is A, then it is B’, and the sentence ‘no A is B’ means ‘if something is A, then it is not B’. The truth of such a combination involves nothing in the combination itself, but always requires a proof, which can depend either on a definition or on experience.

The principle of contradiction is a universal proposition, and it expresses the relation that if something is an object, then it cannot possess and not possess the same property at the same time. The truth of this relation involves nothing in the relation itself, but calls for proof unconditionally. Of course, even those philosophers who regard the principle of contradiction as immediately evident base its truth not on the principle itself, but merely on a feeling that it is evident, [55] and thus on a *mental fact*, which they experience in connection with this principle. *So the principle of contradiction is not an ultimate principle, and someone who accepts it, or wants to encourage others to recognise it, must prove it.*

9. Aristotle's proof by refutation

It seems that no one has felt a stronger need to prove the principle of contradiction than Aristotle himself; however, he was unable to do so, and could not reconcile this feeling with his conviction that the principle of contradiction, being an ultimate principle, could not be proved. Consequently he found himself in an awkward position: he involved himself in contradiction in the very act of considering the principle of contradiction.

The way out of such awkward situations is known: look for some *verbal* distinction which could hide the contradiction by covering it up. Mephistopheles rightly says, ‘for just where concepts fail, a *word* adjusts things in the nick of time’¹.

¹ Denn eben wo Begriffen fehlen, da stellt ein *Wort* zur rechten Zeit sich ein.

So here too a *word* has saved Aristotle from trouble, namely the adverb ἐλεγχτικῶς, ‘by refutation’. The principle of contradiction cannot be proved; there is indeed a proof, yet it is not a proper one, but merely a proof ‘by refutation’. How does a proof by refutation differ from a proper proof? Let us listen to what Aristotle says: [56]

I say that a proof by refutation differs from a [proper] proof, because someone proving [the principle of contradiction properly] would seem to assume in advance [what was to be proved; that is, he would commit a *petitio principii*]; but if someone else were to do this, there would be a proof by refutation and not a proper proof².

This seems to me to be what these words mean: Someone who proves the principle of contradiction in a proper manner does so badly, for he commits *petition principii*; but if someone else commits this error, then a proof by refutation is possible, and everything is in good shape. To put it briefly, a proof by refutation differs from a proper proof in this way: in the present case the first proof is good, and the second is bad. Is it not clear that this is a precisely stated difference between a proof by refutation and a proper proof?

To this day Aristotle's authority is so great that the most recent commentators treat this sentence seriously, and do not see evasion in it. Schwegler writes:

And so the impossibility of accepting that something is and at the same time is not cannot be proved directly, only apagogically (ἐλεγχτικῶς) [*sic!*]; if someone doubts this impossibility (that is, if he doubts the principle of contradiction), make him prove the contrary thesis³. [57]

And Maier says: ‘Although it (i.e., the principle of contradiction) cannot be properly proved, yet it is possible to convey to the mind a kind of proof by refuting the contrary doctrine, and especially by demonstrating the absurd consequences which follow from this’⁴.

No interpretation of the passage from the *Metaphysics* will ever both eliminate the contradiction contained in it and be itself free from contradiction. In order to expose this contradiction, let us consider what Aristotle understands by ἐλεγχός elsewhere.

a) Ἐλεγχός is a syllogism whose conclusion contradicts a given thesis. Such a syllogism arises when an opponent is induced to accept propositions from which there follows a consequence that contradicts a thesis which he defends. For we read:

If the opponent agrees with everything, a refutation may arise. If the thesis is contrary to the conclusion, a refutation must arise, for refutation is syllogism of a contradiction.

² *Metaphysics* Γ 4, 1006^a15–18: τὸ δ' ἐλεγχτικῶς ἀποδεῖξαι λέγω διαφέρειν καὶ τὸ ἀποδεῖξαι, ὅτι ὁ ἀποδεικνύων μὲν ἀν δόξαιν αἰτεῖσθαι τὸ ἐν ἀρχῇ, ἄλλου δὲ τοῦ τοιούτου αἰτίου ὅντος ἐλεγχός ἀν εἴη καὶ οὐκ ἀπόδειξις.

³ *Die Metaphysik des Aristoteles*, Tübingen (1847), vol. III, p. 162: ‘Also nicht direct, nur apagogisch (ἐλεγχτικῶς) lässt sich die Unmöglichkeit der Annahme, daß etwas zugleich sey, und nicht sey, erweisen, indem man nämlich demjenigen, der an dieser Unmöglichkeit, d. h. also am Satze des Widerspruchs zweifelt, den Beweis des Gegentheils zuschiebt’. The worthwhile point among this pile of mistakes is Schwegler's admission that the principle of contradiction can be proved in some way after all.

⁴ *Loc. cit.*, vol. I, p. 47: ‘So wenig sich dasselbe (*scil.* das Prinzip des Widerspruchs) nun aber eigentlich begründen lässt, so lässt sich doch ein gewisser Nachweis in dem Sinn führen, daß die entgegengesetzte Lehre widerlegt und namentlich die aus ihr folgenden absurdum Consequenzen dargethan werden’.

But if the opponent agrees with nothing, a refutation [58] cannot arise, for there would not be a syllogism if all the terms were negative⁵.

Let us assume, for example, that our opponent does not accept the principle of contradiction. If we encourage him to accept propositions from which this principle follows by a syllogism, an ἔλεγχος or refuting syllogism arises. The syllogism is a refutation only accidentally (*κατὰ συμβεβηκός*), since it happened by accident that someone who at first had denied its conclusion later accepted its premises. A refuting syllogism is at the same time a refuting *proof* of its conclusion, for example, of the principle of contradiction, if its conclusion is this principle itself. And because every valid syllogism with true premisses is a proper proof of its conclusion, therefore *a refuting syllogism also proves its conclusion in a proper manner*. And so for this reason Maier says, 'As an inference, a refuting syllogism is as good as a proving syllogism'⁶.

So, according to Aristotle himself, if a refuting proof is a proper proof, since it is a syllogism, then does not Aristotle get involved in a contradiction when he asserts that the principle of contradiction is unprovable, but that it can be proved by refutation? And the Stagirite not only asserts this, but even actually proves this reputedly unprovable principle! And, even though it might appear that his proofs are not convincing, he nevertheless held that they were, since he concludes his refuting arguments with the words: [59] 'If this is so, then *it is proved* that it is impossible to affirm contradictory propositions at the same time'⁷.

b) Aristotle proves the principle of contradiction not only *by refutation* but also *apagogically*. An apagogic proof (*ἡ εἰς τὸ ἀδύνατον ἀπαγωγή, reductio ad absurdum*) arises when we choose as our starting point a sentence which contradicts a given thesis, and we show that the syllogistic consequences of this thesis are absurd. It follows from the falsity of the consequences that the starting point of the proof was false, and hence the thesis which it contradicts is true.

The difference between refuting and apagogic proofs is particularly important for the present problem. Let us state this difference precisely.

A *refuting* proof of a given proposition *B* consists of this: we look for some premise *A* (or two premises, as in a syllogism) which is a basis for concluding *B*. Then we establish that premise *A* is a true proposition, and we force our opponent to accept this proposition. Whoever accepts a premise must accept its conclusion. So we get this schema:

⁵ Prior Analytics B 20, 66^b6–13: πάντων μὲν γὰρ συγχωρουμένων ... ἐγχωρεῖ γίγνεσθαι ἔλεγχον ... εἰ τὸ κείμενον εἴη ἐναντίον τῷ συμπεράσματι, ἀνάγκη γίγνεσθαι ἔλεγχον· ὁ γὰρ ἔλεγχος ἀντιφάσεως συλλογισμός. εἰ δὲ μηδὲν συγχωροῖτο, ἀδύνατον γίγνεσθαι ἔλεγχον· οὐ γὰρ ἡνὶ συλλογισμὸς πάντων τῶν δρῶν στερητικῶν ὄντων.

⁶ Loc.cit., vol. II, p. 359: 'Er (scil. der Elenchus) ist ein Syllogismus, der das Gegenteil eines vorliegenden Satzes erschließt ... Als Schluß deckt er sich mit dem beweisenden Syllogismus'. (It (i.e., the refuting syllogism) is a syllogism which infers the opposite of an existing proposition ... As an inference it is as good as the proving syllogism.)

⁷ Metaphysics Γ 4, 1007^b17–18: εἰ δὲ τοῦτο, δέδεικται ὅτι ἀδύνατον ἄμα κατηγορεῖσθαι τὰς ἀντιφάσεις.

If A is true, then B is true.

A is true.

Therefore B is true.

In formal logic this is known as reasoning by *modus ponens*.

An *apagogic* proof of a proposition B consists of this: we accept temporarily that proposition B is false, [60] and show that proposition A would then have to be false. Next we establish that A is true despite this conclusion, and we force our opponent to accept this proposition, and hence to reject the inferred conclusion B . But anyone who rejects a conclusion cannot accept its premise, and so must accept in the given case that B is not false. So we get this schema:

If B is false, then A is false.

A is not false.

Therefore B is not false, but true.

In formal logic this is known as reasoning by *modus tollens*.

Consequently, refuting and apagogic proofs are ultimately reduced to two known forms of reasoning. Now inference by *modus ponens* does not depend on the principle of contradiction, unless perhaps this principle is used to prove the connection between the premise A and the conclusion B . On the other hand, inference by *modus tollens* always supposes the principle of contradiction, as we shall convince ourselves in chapter 12. So someone who proved this principle apagogically would commit *petitio principii*, as Aristotle notes, and obviously could not convince his opponent.

From this it appears that Schwegler in the above quotation was quite mistaken in confusing refuting and apagogic proofs, and it is plainly absurd when he explains that *reductio ad absurdum* consists of making one's opponent prove the contrary thesis. Schwegler's quoted comment is in tune with the cited text of Aristotle in a harmonious whole of improbable blunders. [61]

How powerful a suggestion is, if it comes from a mind of genius and is expressed in peremptory words flowing from deep conviction! Aristotle asserted categorically that the principle of contradiction is the ultimate principle of thought and of being, and nearly everyone has believed him and trusted his words until the present day. Up to now I have tried to show that it is at least doubtful whether this principle is a law of thought; then I have shown that it is not an ultimate law at all, but needs to be proved; finally I have stated that Aristotle himself tries to prove it. All of *Metaphysics* book Γ chapter 4 is dedicated to this proof. These matters are not transparent; the course of his thought is muddled. Aristotle is the first among philosophers to reach the deepest foundations of logic and ontology. To untangle these proofs, to present them very clearly and to appraise them critically is the task of the following chapters.

10. The principle of contradiction and the principle of double negation

In a proof by refutation, the opponent is forced to accept propositions which lead to a conclusion that he refused to concede. The propositions whose acceptance Aristotle forces upon opponents of the principle of contradiction are *definitions*. For we read:

The starting point against all such [62] objections is not the demand that the opponent should assert that something is or is not, but that he should name at least one word which has some meaning both for him and for others. And when someone gives such a word, a proof will be given, for something will already be defined¹.

Here Aristotle wants not just the word itself but *the determination of its meaning*, and hence its *definition*; this follows both from the subsequent course of his argument as well as from the parallel passage in *Metaphysics* Γ 7, in which he discusses the principle of the excluded middle:

‘The starting point agains all [opponents who do not accept the principle of contradiction without a proof] must begin from a definition. And a definition arises when they must denote something’².

It is therefore necessary to force the opponent to give some meaningful word, such as ‘man’, and to say a sentence ‘By this word I understand such and such’; for example, ‘By the word “man” I understand a two-footed animal’. Then he has given the basis for a proof by refutation.

In the argumentation which follows these introductory remarks (*Metaphysics* Γ 4, 1006^a28–1007^b18), one can distinguish in my opinion two proofs: one is more complex, connected with the concept of essence and substance, and I shall treat this proof in the next chapter; the second is simpler, inserted into the considerations of the [63] first proof, and is not connected with the concept of substance. This second proof reads:

So if something can truly be called a man, it must be a two-footed animal, for this is what the word man ($\tauὸ\ \alphaὐθωπος$) meant. And if it must be a two-footed animal, then it cannot not be, for to be necessary means to be impossible not to be. Therefore it is impossible to say truly that the same thing is a man and is not a man (or a two-footed animal)³.

This argumentation can be changed into a general form if we put letters instead of concepts: By O I understand something which is p . So O must be p , and hence it cannot not be p , based on the definition of the word ‘must’. The conclusion: O cannot be p and not be p at the same time.

¹ *Metaphysics* Γ 4, 1006^a18–25: ἀρχὴ δὲ πρὸς ἀπαντα τὰ τοιαῦτα οὐ τὸ ἀξιοῦν ἢ εἶναι τι λέγειν ἢ μὴ εἶναι ..., ἀλλὰ τὸ σημαίνειν γέ τι καὶ αὐτῷ καὶ ἄλλῳ· τοῦτο γὰρ ἀνάγκη, εἰπερ λέγοι τι. ... ὃν δέ τις τοῦτο διδῷ, ἔσται ἀπόδειξις· ἦδη γάρ τι ἔσται ὄρισμένον.

² *Metaphysics* Γ 7, 1012^a21–23: ἀρχὴ δὲ πρὸς ἀπαντας τούτους ἐξ ὄρισμοῦ. ὄρισμὸς δὲ γίγνεται ἐκ τοῦ σημαίνειν τι ἀνάγκαιον εἶναι αὐτούς.

³ *Metaphysics* Γ 4, 1006^b28–34: ἀνάγκη τοίνυν, εἴ τι ἔστιν ἀληθὲς εἰπεῖν, ὅτι ἀνθρωπος, ζῷον εἶναι δίπουν· τοῦτο γὰρ ἦν δὲ σημαίνει τὸ ἀνθρωπος. εἴ δὲ ἀνάγκη τοῦτο, οὐκ ἐνδέχεται μὴ εἶναι τὸ αὐτὸ ζῷον δίπουν· τοῦτο γάρ σημαίνει τὸ ἀνάγκη εἶναι, τὸ ἀδύνατον εἶναι μὴ εἶναι [ἀνθρωπον]. οὐκ ἕτερα ἐνδέχεται ἀλλα ἀληθὲς εἶναι εἰπεῖν τὸ αὐτὸ ἀνθρωπον εἶναι καὶ μὴ εἶναι ἀνθρωπον.

This inference consists of two premises: the proposition ‘ O must be p ’, which is based on the definition of the term ‘ O ’, and the proposition ‘if O must be p , then it cannot not be p ’, which is based on the definition of the words ‘must be’. This second premise represents the *principle of double negation*. Therefore Aristotle is trying to prove the principle of contradiction from the principle of double negation. What should we think of this proof?

Apparently it seems very convincing. The premises are true, for they are based on [64] definitions. There is no *petitio principii*, since the premises are different from the principle of contradiction. The error is just this: *the premises do not prove the conclusion in question*.

From the premises ‘ O must be p ’ and ‘if O must be p , then it cannot not be p ’, there follows by *modus ponens* only the conclusion ‘ O cannot not be p ’, and therefore ‘ O , which must be p , cannot not be p ’. This is just the principle of double negation. We know already that the principle of contradiction and the principle of double negation are not synonymous: see chapter 7. Hence in this first proof by refutation, Aristotle has proved at most the principle of double negation, but he has *not* proved the principle of contradiction.

But although the principles above are not synonymous, they may be *equivalent*, and even if they are not equivalent, they could be connected by the relation of inference *in one direction*. So let us extrapolate Aristotle’s considerations further, and see whether the principle of contradiction can follow from the principle of double negation.

Logic teaches us that, if a proposition B follows from a proposition A , then there cannot be any circumstances in which A would be true and B would be false. And if there were to be found only one case of the coexistence of the truth of A with the falsity of B , then B could not follow from A .

There are cases in which the principle of double negation is true, but to which the principle of contradiction does not apply; let us say bluntly that it is false. To find these strange cases we must reach out into the realm of *contradictory objects*. Wooden irons ($\sigma\deltaηρόξυλον$), square circles, and round squares are classical examples [65] of contradictory objects. Some regard these queer combinations of words as empty sounds, stripped of meaning. For my part, I think they are not merely empty sounds like ‘abracadabra’ or ‘mohatra’, but they do in fact have meaning. We can say of a round square that it is round, a square, a contradictory object, and so on, but we can say nothing about abracadabra, for this word means nothing.

I admit that these artificially constructed examples of contradictory objects have little meaning. Nevertheless, we know of other examples in the history of science which do not appear quite so absurd. A student of geometry undoubtedly understands what ‘a square constructed with straightedge and compass, and equal in area to a circle of radius 1’ is. How many people at all times throughout history have tried in vain to construct such a square! Only in the nineteenth century did Hermite and Lindemann show that such a square is as contradictory an object as a ‘round square’. For if it is constructed with a straightedge and compass, it must have sides which *can* be expressed in terms of an algebraic number (rational or irrational, like $\sqrt{2}$), but if it is equal in area to a circle with radius 1, it must

have sides which *cannot* be expressed in terms of an algebraic number (its side = $\sqrt{\pi}$, and π is a transcendental number). Hence such a square — let us use the letter S to denote it briefly — is a contradictory object, and it does mean something, it is something; it is an object.

The principle of double negation is undoubtedly true when applied to square S : S must have a ; that is, it must have sides which can be expressed in terms of an algebraic number. ‘Must have’ means ‘cannot [66] not have’. So S cannot not have a . Whoever accepts the principle of double negation must grant that these propositions are true. But despite this, it is not true that S cannot at the same time both have and not have a ; on the contrary, S does have a , and at the same time does not have a . It is for this very reason that S is a contradictory object, and that squaring the circle is an insoluble problem.

In the same way we could show that the principle of contradiction does not follow from the principle of identity either. The principle of identity asserts that if S has a , then it has a ; and also, if it does not have a , then it does not have a . We cannot infer from these propositions that S cannot at the same time have a and not have a . So the principle of contradiction does not follow either from the principle of double negation or from the principle of identity. Hence it is evident *a fortiori* that each of these principles is not synonymous with the principle of contradiction, and not even equivalent to it.

However, this whole argument is correct only under the condition that contradictory objects are *something*, that they are objects. If by ‘object’ you were to understand only a non-contradictory object, then square S would not be an object; it would be nothing. And since it would not be subject to the principle of contradiction, it would not constitute an exception to it. For this principle applies only to objects, and thus to everything which is something and not nothing.

The attentive reader will certainly be aware of the result to which the present enquiry is leading.

11. The principle of contradiction and the ‘essence’ of things

The second proof by refutation involves the concept of ‘essence’ and substance. [67]

Here again the point of departure is a definition. Aristotle asks that his opponent mention some word and determine its meaning. But this word must denote some *one* thing, for we read:

‘For not to denote some one thing is to denote nothing, and if words denote nothing, then the possibility of understanding each other ceases’¹.

We learn from the following passage what Aristotle had in mind when he said that a word ought to denote some one thing:

¹ *Metaphysics* Γ 4, 1006^b7–9: τὸ γὰρ μὴ ἔν τι σημαίνειν οὐδὲν σημαίνειν ἐστίν, μὴ σημαινόντων δὲ τῶν ὀνομάτων ἀνήργηται τὸ διαλέγεσθαι πρὸς ἄλληλους.

‘If the word “man” denotes some one thing, then let this be a two-footed animal. I say that to denote some one thing is this: if this is a man, or as long as something is a man, then its essence will be humanity’².

Aristotle uses the words ‘τὸ ἀνθρώπῳ’ to denote the *essence* of man, which is one, unchangeable, distinct from matter, unable to be perceived by the senses, and accessible to knowledge only by a *concept*³. To denote some one thing is hence to denote [68] the *conceptual essence* of a given object. A word has meaning only if it indicates something which is one in its essence. If someone calls something a man which in its essence could just as well be a wall or a ship, then he is not using the word ‘man’ in a determinate way, and it is impossible to communicate with him. *For the essence of every object is some one thing.* This sentence, which Aristotle has not formulated explicitly, is the foundation of his entire proof. The proof runs like this:

Then let a word be given which denotes something, and which denotes some one thing. It is impossible for humanity to denote what non-humanity denotes, if the word ‘man’ denotes some one thing. And so the same thing cannot be and not be [a man], except as a homonym, as if we were to call something a man which others would call a non-man. But the problem is not whether it is possible for the same thing to be and not to be a man at the same time with respect to its name, but with respect to the fact⁴.

This proof is not formulated clearly or accurately enough. I wish first to make it more precise and to put it into a general form, like the previous proof, by inserting letters instead of concepts.

The proof supposes that a word *O* means something, or denotes some object. This premise [69] is forced upon an opponent who does not accept the principle of contradiction, by asking him to name some meaningful word. The opponent must do this if he wants to debate at all. When this happens, the conditions for a proof by refutation are fulfilled. This proof consists of a hypothetical syllogism with reasoning by *modus ponens*:

First premise: If the word *O* denotes something, it must denote something which is one in its essence.

Second premise: If *O* denotes something which is one in its essence, then it cannot denote anything which in its essence is *p* and not *p* at the same time.

Conclusion: If *O* denotes something, it cannot denote anything which in its essence is *p* and not *p* at the same time.

² *Metaphysics* Γ 4, 1006^a31–34: εἰ τὸ ἀνθρωπὸς σημαίνει ἐν, ἔστω τοῦτο τὸ ζῷον δίπουν. λέγω δὲ τὸ ἐν σημαίνειν τοῦτο· εἰ τοῦτ’ ἔστιν ἀνθρωπὸς, ἀνὴ τὸ ἀνθρωπὸς, τοῦτ’ ἔσται τὸ ἀνθρώπῳ εἶναι.

³ See Trendelenburg, *Aristotelis de Anima libri tres* (*Aristotle’s three books about the soul*), Jena (1833), p. 471. He explains the expression τὸ τινὶ εἶναι by an example: ‘τὸ μεγέθει εἶναι universam esse notionem, qua res constituitur, a materia avocatam, universa cogitatione conceptam, τὸ μέγεθος vero ad singula quaeque pertinere, quae sub sensu cadant’. ‘τὸ μεγέθει εἶναι is a universal notion by which a thing is constituted, separate from matter, conceived by a universal thought; indeed τὸ μέγεθος applies to each individual thing which falls under the senses’.

⁴ *Metaphysics* Γ 4, 1006^b11–22: ἔσθω δὴ ... σημαῖνόν τι τὸ ὄνομα καὶ σημαῖνον ἐν. οὐ δὴ ἐνδέχεται τὸ ἀνθρώπῳ εἶναι σημαίνειν ὅπερ μὴ εἶναι ἀνθρώπῳ, εἰ τὸ ἀνθρωπὸς σημαίνει ... ἐν καὶ οὐκ ἔσται εἶναι καὶ μὴ εἶναι τὸ αὐτὸ ἄλλ’ ἢ καθ’ ὅμωνυμίαν, ὃσπερ ἀν εἰ δν ἡμεῖς ἀνθρωπὸν καλοῦμεν, ἄλλοι μὴ ἀνθρωπὸν καλοῖεν. τὸ δ’ ἀπορούμενον οὐ τοῦτο ἔστιν, εἰ ἐνδέχεται τὸ αὐτὸ ἄνα εἶναι καὶ μὴ εἶναι ἀνθρωπὸν τὸ ὄνομα, ἀλλὰ τὸ πρᾶγμα.

Supposition: *O* denotes something.

Therefore: The word *O* cannot denote something which in its essence is *p* and not *p* at the same time.

Proof of the first premise: If a word does not denote something which is one in its essence, then it denotes nothing. (*τὸ γὰρ μὴ ἐν τῷ σημαίνειν οὐδὲν σημαίνειν ἔστιν*—‘for not to mean some one thing is to mean nothing’.) Then the possibility of mutual understanding ceases.

Proof of the second premise: If a word denotes something which in its essence is *p* and not *p* at the same time, then it does not denote something which is one in its essence. For to be a man in its essence means something other than not to be a man. (See *Metaphysics* Γ 4, 1007^a27–29: ‘If something were in its essence not a man, then it would not be a man, but *something else*’⁵). [70]

I have tried to present Aristotle’s argument in its strictest form, in which it concerns above all words in speech. Clearly the argument in this form is not convincing, for:

a) Words which do not denote something one in its essence can be meaningful. For example, let us suppose that the word ‘centaur’ denotes an animal which is a man and which at the same time is not a man but a horse. Of course this word denotes something non-existent, but it is not devoid of meaning. For everyone understands what ‘centaur’ means; indeed, one must understand what it means if one is able to assert that such a being is merely a product of the imagination and does not really exist.

The statement is wrong which Aristotle seems to have professed, or which is usually attributed to him in connection with the present consideration; namely, that words in speech which have some meaning cannot denote contradictory objects, because they must be *unambiguously determined*. A ‘square constructed with straightedge and compass and equal in area to a circle of radius 1’ is undoubtedly an unambiguously determined expression and has some meaning; nevertheless it denotes an object with contradictory properties.

b) The above proof does not justify the ontological principle of contradiction. For even if every word in speech were to denote something one in its essence and non-contradictory, still it would not follow from this that *reality* contains no contradictions. Human speech might not reproduce reality accurately. Moreover, Aristotle understands his proof quite otherwise: for him it concerns not *words* but [71] *objects*. Words in speech must denote objects with a *single* essence for this reason alone, because only objects of this kind actually exist. Therefore the Stagirite says, ‘the same thing cannot be and not be a man, except through words which are homonyms, as when we call something a man which others call a non-man. But the problem is not whether it is possible for the same thing to be and not to be a man at the same time with respect to its name, but with respect to the fact’.

⁵ εἰ δ’ ἔσται τι ἡ ὅπερ μὴ ἀνθρώπῳ εἶναι ἡ ὅπερ μὴ εἶναι ἀνθρώπῳ, ἄλλο τι ἔσται.

From this it appears that Aristotle’s proof, which does take a spoken word as its starting point, nevertheless concerns not words but the objects which they denote. Hence his proof could be formulated more properly in the following way:

The premise runs: a word O means something, or denotes some object; therefore O is an object, or O is something. There follows a hypothetical syllogism and reasoning by *modus ponens*:

First premise: If O is an object, then it must be something one in its essence.

Second premise: If O is something one in its essence, then it cannot in its essence be p and not be p at the same time.

Conclusion: If O is an object, then it cannot in its essence be p and not be p at the same time.

Supposition: O is an object.

Therefore: O cannot in its essence be p and not be p at the same time.

The first and second premises are proved in a way analogous to that in the previous formulation.

This is a second and perhaps more correct formulation of Aristotle’s argument by refutation. But [72] in this formulation also the argument is not convincing, for it is subject to the following objections:

c) Even if it were correct, it would prove the principle of contradiction only for a limited range of objects; it would concern only the essence of things and not their accidental properties. O , being a man, would have to be some one thing in his essence; hence he could not at the same time not be a man, but he could be white and not white at the same time, for the essence of O is not whiteness, but humanity.

d) This argument relies on the premise that there is in objects some conceptual essence distinct from their accidental properties, some real universal. *This premise is the metaphysical basis of Aristotle’s entire logic.* Even Socrates maintained that true knowledge relies on concepts; hence something real must correspond to concepts if true knowledge of reality can exist. Plato created out of concepts ideas which exist beyond the world. Aristotle placed Platonic ideas in concrete, individual objects.

What are these real universals? These, which Aristotle calls the essence of things, are *groups of properties* which always appear together; for example, there are some properties which constitute the human organism, like having two feet. In order to explain why these properties always occur together, we assume that there lies underneath them some one thing which holds them together, some *substantial being*, about which we know nothing else at all. This substance and the properties which always occur together — these are the essence of a given object. And Aristotle admits that the concept of substance is at issue here, since he says: [73]

‘In general, those who speak like this [i.e., who reject the premises of the above proof] destroy substance and essence’⁶.

‘For the opponent indicated one thing, and that was the substance of something’⁷.

‘Therefore there must be something denoting the substance, and, if so, it is shown that it is impossible to accept contradictory propositions at the same time’⁸.

Hence ultimately the second proof of the principle of contradiction depends on the concept of substance. That Aristotle chose this very proof not only may follow from his metaphysical views, but also probably is connected with his fight against the Megarians. For the Megarians did not accept the principle of contradiction; they denied that there was a difference between substance and accident. They said that Socrates is a man, and at the same time not a man, since he is white, educated, and so on; and to be white (or to be whiteness) means not to be a man⁹. Because of this, Aristotle tried to show that having the *accident* ‘white’ means something other than being white in its essence, which is being whiteness. Every object can have many accidents; there is no contradiction in this. But the essence or substance of each object must be one, and cannot contain a contradiction.

In a series of [74] arguments, Aristotle tried to prove that substantial beings exist. I pass over these arguments, for in my opinion the history of the concept of substance shows clearly that the proposition which asserts that substantial beings exist, or that there is some ‘essence’ of a thing different from its accidental properties, can be only a more or less probable *hypothesis*, but never certain. Hence the conclusions which depend on this hypothesis can also be only probable, so that the second proof by refutation, even if it were correct, would only prove that the principle of contradiction is *probable*.

e) Let us nevertheless suppose that every object truly has some single essence and substance; that is, let us accept all the essential premises of the proof. It appears that this proof contains a *petitio principii*, because the justification of the premises of the hypothetical syllogism relies on reasoning by *modus tollens*, which supposes the principle of contradiction. Aristotle does not prove the first premise directly, that ‘if a word *O* denotes something, then it must denote something one in its essence’ (or ‘if *O* is an object, then it must be something one in its essence’), but only apagogically, maintaining that if *O* did not denote something one in its essence (or if *O* were not something one in its essence), then it would denote nothing (or it would be nothing). Similarly the proof of the second premise is apagogic. I doubt that these premises could be proved directly, and in any case I find nothing in Aristotle which points in this direction.

To summarise my last three difficulties: the first objection (c) demonstrates that Aristotle’s proof does not establish [75] that the principle of contradiction is a *universal* law;

⁶ Metaphysics Γ 4, 1007^a20–21: ὅλως δ' ἀναιροῦσιν οἱ τοῦτο λέγοντες οὐσίαν καὶ τὸ τί ἦν εἶναι.

⁷ Metaphysics Γ 4, 1007^a25–26: ἐν γὰρ ἦν ὁ ἐσήμανε, καὶ ἦν τοῦτο τινος οὐσία.

⁸ Metaphysics Γ 4, 1007^b16–18: ἔσται ἄρα τι καὶ ὡς οὐσίαν σημαῖνον. εἰ δὲ τοῦτο, δέδευκται δι τι ἀδύνατον ἄμα κατειγορεῖσθαι τὰς ἀντιφάσεις.

⁹ See Maier, *loc.cit.*, vol. II, part 2, p. 7, remark 1.

the second (*d*) that it does not show that the principle of contradiction is *certain* as a law of logic; and finally the third objection (*e*) uncovers a *formal mistake* in the argument. All these objections taken together bear witness that Aristotle’s second argument by refutation is also not convincing.

12. The apagogic proofs of the principle of contradiction

As we know already from chapter 9, apagogic proof depends on reasoning by *modus tollens*, the form of which can be represented in the following manner:

If B is false, then A is false.

A is not false.

Therefore B is not false, or B is true.

All argumentation of this kind supposes the principle of contradiction. For we can assert that the denial of the falsity of proposition B follows from denying that proposition A is false only because in the contrary case a *contradiction* would arise. For if the denial of the falsity of A could coexist with the falsity of B , then it would also have to coexist with the falsity of A , since the falsity of A follows from the falsity of B . Thus proposition A would be false, and at the same time it would not be false, which contains a contradiction.

I shall explain this reasoning with the help of an example. let B denote the proposition that a number N is divisible by 3, and A the proposition that N is divisible by 6. If B is false (or if N is not divisible by 3), then A must be false [76] (that is, N cannot be divisible by 6). Let us moreover suppose that N is divisible by 6, or that A is not false. If we were to accept that B is false, or that N is not divisible by 3, we would have to accept as well the consequence of this proposition; that is, to accept that N is not divisible by 6. Hence N would then be, and at the same time not be, divisible by 6, which makes a contradiction. To eliminate the contradiction, let us assume that if A is not false (or if N is divisible by 6), then B cannot be false (that is, N must be divisible by 3).

Therefore reasoning by *modus tollens* relies on the principle of contradiction. If someone does not accept this principle, or if he just wishes to prove it, obviously he cannot use this kind of argument. Hence we can already state in advance that apagogic proofs of the principle of contradiction have no convincing force. However, the three such proofs which Aristotle gives deserve our attention; let us examine them more closely.

a) The first proof, which is repeated several times, reads:

‘If all the contradictory propositions about the same object were true at the same time, then it is clear that all things would be one. For the same thing would be a ship and a wall and a man’¹.

Here the expression ‘all the contradictory propositions’ is striking. Why *all* of them? Someone who doubts the principle of contradiction, and even more someone who just [77]

¹ *Metaphysics* Γ 4, 1007^b18–21: εἰ ἀληθεῖς αἱ ἀντιφάσεις ὅμα κατὰ τοῦ αὐτοῦ πᾶσαι, δῆλον ὅτι ἄπαντα ἔσται ἐν. ἔσται γὰρ τὸ αὐτὸν καὶ τριήρης καὶ τεῖχος καὶ ἄνθρωπος.

demands a proof for it, need not assume that *all* contradictory propositions are true at the same time. An opponent of the principle of contradiction considers whether just *one* case is found in which only *one* pair of contradictory propositions would be true at the same time, for this one case would be completely sufficient to overthrow it. Based on this supposition, Aristotle proves that if someone accepted that all contradictory propositions are true at the same time, it would lead to an awkward conclusion. This may be so, but such an argument is tilting at windmills.

In this whole passage, Aristotle's reasoning is seriously weak. We read, for example, that if everything were one, then nothing would truly exist². How does he reach this conclusion? If it is true that *O* is and is not a man at the same time, then *O* is not *truly* a man, for this follows from the definition of a true proposition. Later on we read this strange sentence:

'If it is true to say that a man is not a man, then it is clear that he is not a ship. And if this [the denial that he is a ship] is true, then the affirmation [that he is a ship] is also true'³.

I do not understand how it is possible to infer consequences of this kind from the rejection of the principle of contradiction. Did Aristotle think that, if someone does not accept this principle, then he can freely ignore all the rules of reasoning? [78]

b) The second apagogic proof is only an outline. From the rejection of the principle of contradiction there follows, in Aristotle's opinion, the following conclusion:

'In addition to this, everyone would be speaking truly, and everyone would be lying, and each would agree himself that he is lying'⁴.

This conclusion is connected with the definition of a false proposition which Aristotle accepted:

'It is false to say of what is that it is not, and of what is not that it is'⁵.

Hence it follows that if *O* is *p*, then the proposition that *O* is not *p* is false, and if *O* is not *p*, then the proposition that *O* is *p* is false. So anyone who said that *O* is and is not *p* at the same time would regard both of these propositions as true and as false simultaneously, and he would have to acknowledge this if only he accepted the above definition of a false proposition.

This conclusion convinces no one that he cannot reject the principle of contradiction, since properly it just expresses the rejection of this principle. Moreover, we should point out that *in Aristotle's formulation* this conclusion does not follow at all from rejecting the principle. If someone did not accept the principle of contradiction, he need not attribute

² Metaphysics Γ 4, 1007^b26: ὥστε μηδὲν ἀληθῶς ὑπάρχειν.

³ Metaphysics Γ 4, 1007^b32–35: εἰ ἀληθὲς εἰπεῖν τὸν ἀνθρωπὸν ὅτι οὐκ ἀνθρωπός, δῆλον ὅτι καὶ οὐ τριήρης εἰ δ' αὕτη (scil. ἡ ἀπόφασις τῆς τριήρους ὑπάρχει), καὶ ἡ κατάκρασις.

⁴ Metaphysics Γ 4, 1008^a28–30: πρὸς δὲ τούτῳ (scil. δῆλον) ὅτι πάντες ἀν ἀληθεύοιεν καὶ πάντες ἀν ψεύδοιντο, καὶ αὐτὸς αὐτὸν ὁμολογεῖ ψεύδεσθαι.

⁵ Metaphysics Γ 7, 1011^b26–27: τὸ μὲν γὰρ λέγειν τὸ ὃν μὴ εἶναι ἢ τὸ μὴ ὃν εἶναι, ψεῦδος.

contradictory properties to *all* objects, and he need not regard *every* proposition as true and as false at the same time, which Aristotle seems to accept. Here there occurs the same mistake which we saw in the first apagogic proof. [79]

c) The third apagogic proof is connected with human behaviour in practice. Aristotle says:

From this [human behaviour] it follows most clearly that no one thinks this way, neither those who say so nor any others. For why does he go to Megara, and not sit quietly believing that he is going there? And why does he not go some morning strait into a well or a chasm, if it so happens, but he seems to be cautious, as if he does not think that falling in is equally good and not good? Therefore it is clear that he believes one case is better and the other worse⁶.

This proof, which would undoubtedly be accepted eagerly by the followers of the now fashionable pragmatism, is no stronger than the previous ones. First, Aristotle seems to assume that someone who rejected the principle of contradiction could not perform any action. Yet men do act, even those who deny this principle, and they treat one course of action as better and a second as worse; they do not think that one and the same course of action is good and not good at the same time.

In my opinion, there is no connection at all between activity and the acceptance or non-acceptance of the principle of contradiction. Very often we act [80] mechanically, prompted by some external influences, in reaction to which we move. *Conscious* action, based on a decision of the will, can also occur without accepting the principle of contradiction. I might believe that motion involves contradiction, and that when I go to Megara, I am at the same time not going to Megara. Nevertheless I decide to perform an act of motion, and as I go I avoid the well, so that I do not fall into it. For not falling into the well is both a good thing and, at the same time, not good, since according to our supposition everything may contain a contradiction; hence I am free to choose the second contingency instead of the first. A suicide will choose the first.

Secondly, Aristotle seems to accept that someone who does not accept the principle of contradiction must treat an affirmation and its negation as identical. Well, someone could think that whenever he walks, he is not walking; but it does not follow from this that walking is the same as not walking, or that going to Megara and staying at home are completely the same.

But the chief flaw in this proof, as in the two previous ones, is a shifting of the subject (*μετάβασις εἰς ἄλλο γένος*, or *ignoratio elenchi*). Aristotle tries to show that someone who accepted contradiction *everywhere* would come to a conclusion which could make speech and action impossible. But this is not so! You cannot force someone who merely has doubts about the principle of contradiction, or who demands proof of it, to assert that *everything* is

⁶ Metaphysics Γ 4, 1008^b12-19: ὅθεν καὶ μάλιστα φανερόν ἐστιν ὅτι οὐδεὶς οὔτω διάκειται οὔτε τῶν ἄλλων οὔτε τῶν λεγόντων τὸν λόγον τοῦτον. Διὰ τί γάρ βαδίζει Μέγαράδε ἀλλ' οὐχ ἡσυχάζει οἰόμενος βαδίζειν; οὐδὲ εὑθέως ἔωθεν πορεύεται εἰς φρέαρ ή εἰς φάραγγα, ἐάν τύχῃ, ἀλλὰ φαίνεται εὔλαβούμενος, ὡς οὐχ ὁμοίως οἰόμενος μὴ ἀγαθὸν εἶναι τὸ ἔμπεσεῖν καὶ ἀγαθόν; δῆλον ἂρα ὅτι τὸ μὲν βέλτιον ὑπολαμβάνει τὸ δ' οὐ βέλτιον.

contradictory. In that case there is a shift in the controversial problem, and the arguments lose their power to convince us.

Consequently, ignoring the formal flaws which affect every apagogic proof of the principle of contradiction, [81] none of the three cited arguments stands up to criticism. And since Aristotle gave no other proofs of the principle of contradiction besides the arguments by refutation and the apagogic proofs, it follows from this that, *in spite of his attempts to prove this principle, he did not manage to do so.*

13. *Ignoratio elenchi* in Aristotle's proofs

The shifting of the problem in the Aristotelian proofs of the principle of contradiction (this error is called *ignoratio elenchi*) is so unusual that it deserves a separate discussion. Aristotle's original intention was to prove that no object can possess and not possess the same property at the same time. At the beginning of chapter 4 of *Metaphysics* Γ, which contains his proofs, the Stagirite says explicitly, 'We have now accepted that it is impossible for something to be and not to be at the same time'¹.

Later he adds: 'And it is possible to prove by refutation that it is impossible [to reject] this principle, if only the opponent would say something'².

The first two proofs by refutation, although they are inadequate, actually aim to prove this principle in its full extent, for they end with the words: [82] 'And if this is so, then it is shown that it is impossible to assert contradictory propositions at the same time'³.

But the deeper we dig while considering the passage under discussion, the more the original goal of his considerations shifts, and in the end Aristotle's arguments are heading very clearly towards justifying the thesis that *not all objects contain a contradiction*. This thesis is not only completely different from the principle of contradiction, but even contradicts it, if we note that it involves admitting that contradictory objects exist. But before we show how such a strange and unexpected thought could occur to Aristotle, let us first investigate the traces of its origin.

Already at the beginning of the proofs by refutation, we find a brief sentence inserted, to which Aristotle does not seem to give much weight:

'First, then, it is clear that this at least is true: the words "to be" and "not to be" have a definite meaning, and thus not everything would be like this and not like that. Furthermore, if the word "man" denotes one thing, ...'⁴

¹ *Metaphysics* Γ 4, 1006^a3–4: ἡμεῖς δὲ νῦν εἰλέγραμεν ὡς ἀδυνάτου ὄντος ἄμα εἶναι καὶ μὴ εἶναι.

² *Metaphysics* Γ 4, 1006^a11–13: ἔστι δ' ἀποδεῖξαι ἐλεγχτικῶς καὶ περὶ τούτου ὅτι ἀδύνατον, ἐὰν μόνον τι λέγη ὁ ἀμφισβητῶν.

³ *Metaphysics* Γ 4, 1007^b17–18: εἰ δὲ τοῦτο, δέδειχται ὅτι ἀδύνατον ἄμα κατηγορεῖσθαι τὰς ἀντιφάσεις.

⁴ *Metaphysics* Γ 4, 1006^a28–31: πρῶτον μὲν οὖν δῆλον ὡς τοῦτό γ' αὐτὸς ἀληθές, ὅτι σημαίνει τὸ ὄνομα τὸ εἶναι ἢ μὴ εἶναι τοδέ· ὁστ' οὐκ ἀν πᾶν οὕτως καὶ οὐχ οὔτως ἔχοι. ἔτι εἰ τὸ ἄνθρωπος σημαίνει ἔν,

So here for a moment there already appears the conclusion that not everything can be this way and not that way, or that not every object contains a contradiction. Apparently this conclusion satisfies Aristotle, but it does not interest him more deeply; for he immediately passes [83] on to the second argument, as the fragment of the next sentence testifies. However, we must admit that the above thought is not yet inconsistent with the principle of contradiction, but could in fact pass as the first step towards justifying it. For by showing in particular cases that at least *some* objects are not contradictory, he clears the way for the universal proposition that *no* object contains a contradiction.

This hitherto innocent thought gradually assumes a somewhat different appearance in the sections dedicated to the apagogic arguments. For we read:

Furthermore, either all objects are thus [that is, contradictory], and everything is white and not white, and exists and does not exist, and likewise for all other affirmations and negations; or they are not, and some objects behave like this, while others do not. And if not all were contradictory, these would need to be accepted as exceptions. But if all objects are contradictory, ...⁵

Here again I have deliberately quoted a fragment of the next [84] sentence to show that Aristotle has finished with the previous thought. This thought cannot easily be reconciled with the principle of contradiction. This is how I understand it: 'Either everything is contradictory, or not everything is. Whoever affirms that not everything is contradictory must accept, in addition to the existence of contradictory objects, the existence of non-contradictory objects as well'. And that is all. Would it have been enough for Aristotle to state the fact that at least some objects, though not all, are non-contradictory? It seems to follow from the conclusion of the above quotation that this and nothing else is what he is thinking:

But if all [objects are contradictory], then again either there are objects of which any affirmation can also be denied, and of which any denial can also be affirmed, or there are objects of which any affirmation can be denied, but of which some denial cannot be affirmed. And if so, then there would be something which truly would not exist, and this opinion would be certain⁶.

I understand this sentence in the following way: if there were objects about which whatever could be denied could not be affirmed, then there would be an object about which only negative propositions would be true. Such an object would not, of course, exist, but it would also contain no contradiction, for contradiction arises only when affirmation and negation exist at the same time. And again it seems that [85] this *one* object, not existing but also not contradictory, would be enough for Aristotle, and besides this all other objects could contain a contradiction.

Here the shifting of the problem is clearly visible. The statement that *at least some* objects are not contradictory is consistent with the principle of contradiction, but the assertion

⁵ *Metaphysics* Γ 4, 1008^a7–12: ἔτι ήτοι περὶ ἀπαντά οὕτως ἔχει, καὶ ἔστι καὶ λευκὸν καὶ οὐ λευκὸν καὶ ὄν καὶ οὐκ ὄν, καὶ περὶ τὰς ἄλλας φάσεις καὶ ἀποφάσεις ὅμοιοτρόπως, ή οὖ, ἀλλὰ περὶ μέν τινας, περὶ τινας δ' οὐ. καὶ εἰ μὲν μὴ περὶ πάσας, αὗται ἀν εἰεν ὅμολογούμεναι. εἰ δὲ περὶ πάσας, ...

⁶ *Metaphysics* Γ 4, 1008^a12–16: εἰ δὲ περὶ πάσας, πάλιν ήτοι καθ' ὅσων τὸ φῆσαι καὶ ἀποφῆσαι καὶ καθ' ὅσων ἀποφῆσαι καὶ φῆσαι, ή κατὰ μὲν ὅν φῆσαι καὶ ἀποφῆσαι, καθ' ὅσων δὲ ἀποφῆσαι οὐ πάντων φῆσαι. καὶ εἰ μὲν οὕτως, εἴη ἂν τι παγίως οὐκ ὄν, καὶ αὕτη βεβαῖα δόξα.

that some objects are indeed non-contradictory, but only *some* of them, while others could be contradictory, expresses a thesis which directly contradicts the principle of contradiction.

This almost improbable thought of Aristotle's is revealed most clearly at the end of his proofs of the principle of contradiction. We read:

And further, let us even suppose that everything is this way and not that; yet 'more' and 'less' are present in the nature of things, for we would not say that two and three are even in the same way, or that someone who thought four was five would be wrong in the same way as someone who thought four was a thousand. And if they are not wrong in the same way, then clearly one of them is less wrong, so that he speaks more truly. And since what is truer is closer to truth, [86] there would be something [absolutely] true to which the truer would be closer. And even if [absolute truth] did not exist, there would at least be something surer and truer, and we would get rid of this foolish thought which prevents us from defining anything reasonably⁷.

When we read these words, we feel as if Aristotle, straining hopelessly in his thought, were seeking an Archimedean 'δός μοι ποῦ στῶ' (Give me a place to stand)! If only he could find just *one* truth, certain and not mixed with any error, free from all contradiction! If only he could find just this certainty, that there exist some probable propositions which approach the truth! This would certainly restore his faith in the power of reason, and free him from the waves of contradictions which seem to be flooding the whole world!

The deepest logical questions sometimes awaken unusual states of mind. For a moment, there flashes a light of evident, unshakeable, utmost truth. We fly towards the light to see it in its full brilliance. In vain! The closer we come, the darker it gets; we lose the way, we wander through complex corridors of thought, and slowly we sink into despair. Was there perhaps no light? Is everything an illusion? Again and again this thought returns and torments us endlessly. Fear overpowers us in our mysterious, dark isolation. We would be happy to catch sight of just a *spark* from the flame that flashed a moment ago, which would disperse the darkness and rid us of the nightmare that confuses our thoughts and steals our reason away.

I sense a trace of such a feeling in Aristotle. He is almost breathless when he cries out in his conclusion: [87] 'and we would get rid of this mad notion which prevents our defining anything reasonably'⁸. How the beginning differs from the end! There proud self-assurance and angry contempt for the opponent, here a desperate effort to save at least the tattered remnants. It would seem that the Stagirite enters the battle sure of his strength and fully expecting to win. Like arrows from a quiver, he draws out proof after proof, but soon he notices that his arrows are powerless. He has used up his arguments, but none of them have proved the principle which is so dear to him. Therefore, with the remnants of his strength

⁷ Metaphysics Γ 4, 1008^b31–1009^a5: ἔτι εἰ δτι μάλιστα πάντα ούτως ἔχει καὶ οὐχ ούτως, ἀλλὰ τό γε μᾶλλον καὶ ἥπτον ἔνεστιν ἐν τῇ φύσει τῶν ὄντων οὐ γάρ ἂν ὁμοίως φήραιμεν εἶναι τὰ δύο ἄρτια καὶ τὰ τρία, οὐδὲ ὁμοίως διέψευσται ὁ τὰ τέτταρα πέντε οἰόμενος καὶ ὁ χίλια. εἰ οὖν μὴ ὁμοίως, δῆλον δτι ἀτέρος ἥπτον, ὡστε μᾶλλον ἀληθεύει. εἰ οὖν τὸ μᾶλλον ἐγγύτερον, εἴη γ' ἂν τι ἀληθεύει οὐ ἐγγύτερον τὸ μᾶλλον ἀληθές, καὶν εἰ μὴ ἔστιν, ἀλλ' ἥδη γέ τι ἔστι βεβαιότερον καὶ ἀληθινώτερον, καὶ τοῦ λόγου ἀπηλλαγμένοι ἂν εἴημεν τοῦ ἀκράτου καὶ κωλύοντός τι τῇ διανοίᾳ ὅρίσαι.

⁸ Metaphysics Γ 4, 1009^a3–5: καὶ τοῦ λόγου ἀπηλλαγμένοι ἂν εἴημεν τοῦ ἀκράτου καὶ κωλύοντός τι τῇ διανοίᾳ ὅρίσαι.

and faith, he defends his final position: there is only *one* non-contradictory object and only *one* truth free from contradiction.

Could it have been otherwise? Could his pride and self-assurance have been only a sham? One might sometimes be prepared to suppose that Aristotle, foreseeing with his acute, deep mind the practical and ethical importance of the principle of contradiction, deliberately expressed it as unshakeable dogma, and wished to replace the lack of real arguments with his own powerful 'Sic volo, sic iubeo' (Thus do I wish, thus do I command). But in the depths of his heart he was unsure of the matter. He concealed this thought, but the discussion excited him, and so a cry of despair slipped out against his will.

It is difficult to analyse today a mental phenomenon which existed for a brief moment centuries ago. This alone seems certain: it would be a mistake to think that, in shifting the point at issue in his proofs of the principle of contradiction, Aristotle wished only to make concessions as far as possible to the opponents of this principle. It will soon be apparent that he actually had sufficient [88] reason to doubt the universal value of the principle of contradiction, but evidently he did not have sufficient courage to acknowledge this openly.

14. The character of Aristotle's proofs

When we discuss the Aristotelian proofs of the principle of contradiction, we must remember that they have for the most part a *polemic* character. The second proof by refutation, based on the concept of substance, is, as Maier rightly supposes¹, directed against the Megarians, who did not recognise the difference between substance and accident. Similarly, the third apagogic proof as well is obviously directed against the eristics of Megara, as the very choice of an example testifies: 'to go to Megara'². But Aristotle was also undoubtedly defending the principle of contradiction against the school of Antisthenes: it is clear that the word ἀπαιδευσία³ points to them again, a word which both Plato and Aristotle used repeatedly in connection with the name of Antisthenes⁴. Aristotle treats all these eristics with anger and with contempt; he calls their doctrine mad (*ἄχρατος λόγος*) and says they proclaimed it only for the sake of argument⁵.

Aristotle sees more serious opponents of the principle of contradiction in the followers of Protagoras's sensualist theory of knowledge and in the followers of related trends, [89] which he treats in chapters 5 and 6 of *Metaphysics* Γ. According to this theory, our entire knowledge is based on perceiving things with the senses. Now both perception itself and perceived phenomena contain many contradictions. What one takes as sweet seems bitter to another, and the same phenomenon appears different even to the same person, depending

¹ See *loc.cit.*, vol. II, part 2, p. 7, remark 1.

² See Maier, *loc.cit.*, vol. II, part 2, p. 8, remark 1.

³ Tr. See *Metaphysics* Γ 4, 1006^a6.

⁴ See Maier, *loc.cit.*, vol. II, part 2, p. 11, remark 3, and p. 15, remark 2.

⁵ *Metaphysics* Γ 5, 1009^a21: λόγου χάριν λέγουσιν.

on the conditions under which it is perceived. For this reason there may be different, even contradictory, propositions about the same thing, and all these propositions are true, for they are based on sense perception. Besides, the phenomena perceived by the senses change continually; they flow, as Heraclitus says; hence they never truly exist, but in every moment they come to be and pass away. Moreover, since the same phenomenon produces contrary phenomena, then each of them must contain contrary seeds, and hence contradictory seeds. Thus the whole sensory world is full of contradiction.

Aristotle takes a surprising position with respect to this theory. He rejects as erroneous its fundamental premise, that human knowledge is based only on sensory experience; however, he seems to accept those consequences of the theory which are the most dangerous for the principle of contradiction. I cite the following passage for consideration: [90]

This opinion, that contradictories and opposites can exist at the same time, has come from the senses to those who have real difficulties [with the principle of contradiction], for they see opposites arising from the same thing. For if it is impossible for something that does not exist to come to be, then the given thing and its opposite as well must both have existed previously, so that Anaxagoras and Democritus say that everything is mixed in everything, and Democritus says that every particle contains the empty and the full alike, even though one of them is a being and the other is not a being. To those who have got their opinions in this way we shall reply that in a way they are right, but in another way they are ignorant. For the word 'being' has two meanings, so that in one sense it is possible for something to come to be from non-being, but in the other sense it is not possible, and the same thing can be both a being and not a being at the same time, but not with respect to the same kind of being. For it is possible for the same thing to have opposite properties potentially, but not actually⁶.

The last sentence of this passage is immensely important for our problem, since it contains [91] a *restriction* of the principle of contradiction expressed in very explicit words. In Aristotle's opinion, *potential beings* (*τὰ δυνάμει ὄντα*) do not come under this principle, because they can possess contrary properties at the same time, and hence contradictory properties as well. *The principle of contradiction concerns only actual beings* (*τὰ ἐντελεχείᾳ ὄντα*). But what is this potential being, which is not subject to the principle of contradiction? We find the answer to this question in the following passage:

The cause of this opinion [*i.e.*, rejection of the principle of contradiction] is that they were looking for truth in beings, but they accepted only perceptible things as beings. However, among these there are many things of an indeterminate nature, which belong to the kind of being of which we have been speaking. Therefore they speak plausibly, but not truly⁷.

⁶ *Metaphysics* Γ 5, 1009^a22–36: ἐλήλυθε δὲ τοῖς διαποροῦσιν αὕτη ἡ δόξα ἐκ τῶν αἰσθητῶν, ἡ μὲν τοῦ ἀμα τὰς ἀντιφάσεις καὶ τάναντία ὑπάρχειν, ὅρῶσιν ἐκ ταύτοις γίγνομενα τάναντία. εἰ οὖν μὴ ἐνδέχεται γίγνεσθαι τὸ μὴ ὄν, προϋπήρχεν ὁμοίως τὸ πρᾶγμα ἀμφώ ὄν, ὕσπερ καὶ Ἀναξαγόρας μεμίχθαι πάντα ἐν παντὶ φησι καὶ Δημόκριτος· καὶ γάρ οὗτος τὸ κενὸν καὶ τὸ πλήρες ὁμοίως καθ' ὅτιοῦν ὑπάρχειν μέρος, καίτοι τὸ μὲν ὄν τούτων εἶναι, τὸ δὲ μὴ ὄν. πρὸς μὲν οὖν τοὺς ἐκ τούτων ὑπολαμβάνοντας ἔροῦμεν, ὅτι τρόπον μέν τινα ὅρθως λέγουσι τρόπον δέ τινα ἀγνοοῦσιν. τὸ γάρ ὄν λέγεται διχῶς, ὥστ' ἔστιν ὄν τρόπον ἐνδέχεται γίγνεσθαι τι ἐκ τοῦ μὴ ὄντος, ἔστι δ' ὄν οὐ, καὶ ἀμα τὸ αὐτὸ εἶναι καὶ ὄν καὶ μὴ ὄν, ἀλλ' οὐ κατὰ ταύτο ὄν. δυνάμει μὲν γάρ ἐνδέχεται ἀμα ταύτο εἶναι τὰ ἔναντια, ἐντελεχείᾳ δ' οὐ.

⁷ *Metaphysics* Γ 5, 1010^a1–5: αἴτιον δὲ τῆς δόξης τούτοις ὅτι περὶ τῶν ὄντων μὲν τὴν ἀληθείαν ἔσκοπουν, τὰ δ' ὄντα ὑπέλαβον εἶναι τὰ αἰσθητὰ μόνον. ἐν δὲ τούτοις πολλὴ ἡ τοῦ ἀορίστου φύσις ἔνυπάρχη, καὶ ἡ τοῦ ὄντος οὔτως ὕσπερ εἴπομεν. διὸ εἰκότως μὲν λέγουσιν, οὐκ ἀληθῆ δὲ λέγουσιν.

From this it would follow that *the objects of perception, phenomena, are potential beings*. For though Aristotle has not ventured to state this definitely and explicitly, having been satisfied to refer diplomatically to a previous passage, yet the word ‘indeterminate’ (*ἀόριστον*) leaves no room to doubt under which kind of being these objects should be reckoned. For the Stagirite has already pointed out before this: [92]

‘What exists potentially and not actually is indeterminate’⁸.

So we come to the following conclusion: *the objects of perception, as potential beings, may possess contrary properties, and hence contradictory properties, at the same time*. The sensory world, which is always changing and contains the seeds of contrary phenomena, is not subject to the principle of contradiction. If someone accepts, as the sensualists do, that only what can be perceived by the senses exists, then he can quite rightly reject this principle, and he has no need at all to suppose that non-contradictory beings exist.

Hence it is now easy to understand why Aristotle shifts his original point of view in his proofs, and tries with every effort of his thought to justify the thesis that besides contradictory objects there must also exist an absolute, non-contradictory truth. In the realm of the sensory world he might lose his case, but that world does not really exist, and the one, true, eternal, unchangeable, non-contradictory being is the *essence* of things, and the *substance* which lies under every concrete object. We do not perceive substances with the senses, but we know them with the help of reason. And therefore the arguments of the sensualists are convincing in so far as they concern the sensory world, but they do not constitute the whole truth, since besides phenomena and matter, there exist substance and form, accessible only to the understanding, and free from all contradiction.

Aristotle expresses this thought many times when he attacks the sensualists. Thus, for example, the first passage cited above ends with the words: [93]

‘And so we shall demand that they accept as well that beings have some other substance, which is not subject to change, nor to passing away, nor to coming to be’⁹.

And in connection with the second passage quoted above, Aristotle says:

‘So it is evident that we shall say here the same thing we said before: for they must accept and believe that some unchangeable nature exists’¹⁰.

The second argument by refutation, which I regard as the most important of all, takes on its proper meaning only in this light. It is significant that in presenting the argument, Aristotle points out emphatically that it is necessary to accept something *definite* (*τὶ ὡρισμένον*), which is something one in its essence, if there is to be any possibility of understanding and thinking. The definite things cannot be sensed objects, ‘which contain a great deal of an undetermined nature’ (*ἐν οἷς πολλὴ ἡ τοῦ ἀορίστου φύσις ἐνυπάρχει*);

⁸ Metaphysics Γ 4, 1007^b28–29: τὸ γὰρ δυνάμει ὃν καὶ μὴ ἐντελεχείᾳ τὸ ἀορίστον ἔστιν.

⁹ Metaphysics Γ 5, 1009^a36–38: ἔτι δ' ἀξιώσομεν αὐτοὺς ὑπολαμβάνειν καὶ ὅλην τινὰ οὐσίαν εἶναι τῶν ὄντων, ή̄ οὔτε κίνησις ὑπάρχει οὔτε φθορὰ οὔτε γένεσις τὸ παράπαν.

¹⁰ Metaphysics Γ 5, 1010^a32–35: ἔτι δὲ δῆλον ὅτι καὶ πρὸς τούτους ταῦτα τοῖς πάλαι λεχθεῖσιν ἐροῦμεν· ὅτι γὰρ ἔστιν ἀκίνητός τις φύσις δεικτέον αὐτοῖς καὶ πειστέον αὐτούς.

they must be substantial beings, which constitute the essence of things. The original type of these beings is pure form, completely free from contradictory matter, and this form is the essence of God¹¹. We know these beings, or substantial forms, by means of *concepts*, and the signs of these concepts are *words* which are determined [94] unambiguously by means of definitions. *This unambiguity of words, which depends on the existence of uniform substantial beings, is the ultimate basis of the principle of contradiction, and consequently should be recognised as the highest law of true conceptual thinking, and of true essential being.*

So all this seems to support the thesis that Aristotle *limited the significance of the principle of contradiction to substantial beings*. Who knows whether the Stagirite was thinking of this very limitation when, immediately after the famous formulation of this principle, 'It is impossible for the same to belong and not to belong to the same at the same time and in the same respect'¹², he added the following words: 'and whatever other reservations we might add to avoid logical difficulties, let them be added'¹³.

What could these other reservations be, given that the principle is formulated extremely cautiously with the addition of 'at the same time' (*αὐτα*), 'in the same respect' (*κατὰ τὸ αὐτό*), and so on, stating unambiguously that the principle applies to the *same* property, which cannot belong and not belong to the *same* object in the *same* respect at the *same* time?

It should be stated that *the Aristotelian principle of contradiction is not just an ontological principle, but also has metaphysical significance*. For it quite clearly depends on the metaphysical premise that substances exist, and it is very closely connected with this premise. But precisely this metaphysical [95] significance weakens its value, for metaphysical premises never have the certainty of logical laws. Aristotle himself surely felt that this was the weak point of his position. Did he perhaps doubt whether there really do exist any permanent substantial beings beyond the mutable world of the senses? Could the sensualists be right? If so, the eristics of Megara would not be so far from the truth, and his entire finely crafted construction would collapse into the abyss. But this is impossible! We would be unable to think or to speak or to do anything! Hence Aristotle clings frantically to his absolute truth, that is, to his substantial beings, as if to his last lifeline.

It appears from the above considerations that Aristotle has not solved the problem of the principle of contradiction at all. We need a new and better justification of this principle. We need it even more because, while Aristotle applied his principle primarily to substantial beings, and drew the guarantee of its truth to some degree from a supersensory world, we

¹¹ So therefore Fr. Gratty could express the opinion that the existence of God refutes the (in his opinion) pantheist and atheist doctrine which denies the principle of contradiction; see *Logique*, vol. I, p. 317, note.

¹² *Metaphysics* Γ 3, 1005^b19–20: τὸ γὰρ αὐτὸ ἄμα ὑπάρχειν τε καὶ μὴ ὑπάρχειν ἀδύνατον τῷ αὐτῷ καὶ κατὰ τὸ αὐτό.

¹³ *Metaphysics* Γ 3, 1005^b20–22: καὶ ὅσα ἄλλα προσδιορισάμεθ' ἂν, ἔστω προσδιωρισμένα πρὸς τὰς λογικάς δυσχερείας.

apply it today to the sensory world as well, to every phenomenon, and even to illusions. As a result of the all-powerful reign of the trend in favour of empiricism, which currently permeates every branch of human knowledge, we are inclined to attach more importance to objects of *experience* than to beings like substance which transcend experience, and which, right or wrong, we consider as creations of the human mind. For this reason, and in this respect, it has become necessary to revise the principle of contradiction which we have taken from Aristotle. [96]

15. The principle of contradiction and the principle of the syllogism

Logical and ontological principles are not only more certain than metaphysical principles, but also more general, for they apply both to the metaphysical beings which make up the essence of the world as well as to objects of experience and to creations of the human mind which do not really exist; they apply generally to everything which is something and not nothing. If Aristotle's principle of contradiction were just a metaphysical law, then we would probably be right to expect it to have no great logical or ontological significance.

Yet the Stagirite affirms that the principle of contradiction is both the most certain and the highest principle of all. It is an *ultimate* principle not only because it needs no proof, but also in that it is the logical basis of all other principles. For we read:

'Hence everyone who proves anything reduces his proof to this ultimate principle, for this is the natural foundation of all other axioms'¹.

This sentence is inaccurately expressed, for we do not know whether Aristotle regards the principle of contradiction as a *sufficient* basis for all other axioms.

The difference between a sufficient and a necessary basis is well known; it is this: *If two propositions A and B are related to each other in such a way that* [97] *B follows from A, or A is a reason for B, then the truth of A is a sufficient basis for the truth of B, and the truth of B is necessary for the truth of A.* For if the premise is true, then the conclusion must be true; hence the truth of the premise is a *sufficient* condition for the truth of the conclusion; but it is not a necessary condition, since the conclusion can be true though the premise is false. Secondly, if the conclusion is false, then the premise must also be false; hence the truth of the conclusion is a *necessary* condition for the truth of the premise, but it is not a sufficient condition, since the premise can be false despite the truth of the conclusion.

I shall not go into the problem of whether the principle of contradiction is a sufficient basis for all other principles, although it is not difficult to prove that an affirmative response to this question would definitely be a mistake. In this chapter I wish only to show that *even*

¹ *Metaphysics* Γ 3, 1005^b32–34: διὸ πάντες οἱ ἀποδεικνύντες εἰς ταύτην ἀνάγουσιν ἐσχάτην δόξαν· φύσει γάρ ἀρχὴ καὶ τῶν ὄλλων ἀξιωμάτων αὕτη πάντων.

according to Aristotle himself, the principle of contradiction is not a necessary condition for one of the most important rules of reasoning, namely the principle of the syllogism. In other words, the principle of the syllogism and syllogistic reasoning would still be valid even if the principle of contradiction were fallacious.

We learned not long ago that Aristotle actually maintained this. Under the influence of the belief that the principle of contradiction is an ultimate principle and the highest basis for thought, a certain passage in the [Posterior] *Analytics* was not properly understood. The English author i. Husic called attention to the passage in an [98] article in *Mind*². Indeed, although Waitz, for example, did not understand this passage at all, H. Maier had explained it completely correctly before Husic did³, but Maier was unable to appreciate the fundamental importance which it could have for all of Aristotle's logic, so that his remarks, made without due emphasis, were lost among the other particulars of his vast work. This discussion refers to the following passage:

No proof [*i.e.*, syllogism] supposes that it is impossible to affirm and to deny at the same time, unless the conclusion must show this as well. This is shown by accepting that it is true to predicate [99] the major term of the middle term, but that to deny it is not true. As for the middle term, and the minor as well, it makes no difference to assume that it is and is not. For if something is given of which it is true to say that it is a man and also not a man, and if it is only true that man is an animal and not a non-animal, then it will be true to say that Callias, even if he is not Callias, is nevertheless an animal and not a non-animal. The reason is that the major term is predicated not only of the middle term but also of other terms, since it is more extensive, so that it makes no difference to the conclusion if the middle term is both the same and not the same⁴.

I have tried to translate this rather difficult passage as faithfully and as clearly as possible. I interpret it as follows: Let us denote the major term ($\tau\ddot{o} \pi\rho\tilde{\omega}\tauov$) by the letter A (animal), the middle term ($\tau\ddot{o} \mu\epsilon\sigmaov$) by B (man), and the minor term ($\tau\ddot{o} \tau\rho\acute{e}tov$) by C (Callias). We obtain the syllogism:

B is A	Man is an animal.
C is B	Callias is a man.
<hr/>	
C is A	Callias is an animal.

The syllogism supposes the principle of contradiction only if the conclusion has to state explicitly that C is A and not at the same time non- A . Then the major premise must

² Isaac Husic, ‘Aristotle on the Law of Contradiction and the Basis of the Syllogism’, *Mind*, New Series, vol. XV (1906), pp. 215–222. Husic’s presentation contains several inaccuracies despite his mainly correct idea; for example, the first two sentences of the passage from the *Posterior Analytics* cited below are translated badly.

³ *Loc. cit.*, vol. II, pp. 238-9, remark 3.

⁴ Posterior Analytics A 11, 77^a10–21: τὸ δὲ μὴ ἐνδέχεσθαι ἀμαρτίαν καὶ ἀποφάναι οὐδεμία λαμβάνει ἀπόδειξις, ἀλλ’ ἡ ἐὰν δέηται καὶ τὸ συμπέρασμα οὗτως. δείκνυται δὲ λαβοῦσι τὸ πρῶτον κατὰ τοῦ μέσου, ὅτι ἀληθές, ἀποφάναι δ’ οὐκ ἀληθές. τὸ δὲ μέσον οὐδὲν διαφέρει εἶναι καὶ μὴ εἶναι λαβεῖν, ὡς δ’ αὕτως καὶ τὸ τρίτον. εἰ γὰρ ἐδόθη, καθ’ οὗ ἀνθρωπὸν ἀληθὲς εἴτειν, εἰ καὶ μὴ ἀνθρωπὸν ἀληθές, ἀλλ’ εἰ μόνον ἀνθρωπὸν ζῷον εἶναι, μὴ ζῷον δὲ μή, ἔσται γὰρ ἀληθές εἴπειν Καλλίαν, εἰ καὶ μὴ Καλλίαν, ὅμως ζῷον, μὴ ζῷον δ’ οὐ. αἴτιον δ’ ὅτι τὸ πρῶτον οὐ μόνον κατὰ τοῦ μέσου λέγεται ἀλλὰ καὶ καθ’ ὅλου διὰ τὸ εἶναι ἐπὶ πλειόνων, ὥστ’ οὐδὲ εἰ τὸ μέσον καὶ αὐτό ἔστι καὶ μὴ αὐτό, πρὸς τὸ συμπέρασμα οὐδὲν διαφέρει.

assert that B is A and not at the same time non- A . This is what the first two sentences of the passage mean.

The next two sentences say that the syllogism is possible even if C were B and not B at the same time, [100] or if C were C and not C at the same time. Under these suppositions not only does the conclusion ‘ C is A ’ remain true, but also the additional clause ‘ C is not at the same time non- A ’ can still be true as well, provided this additional clause occurs in the major premise. Hence the following syllogistic forms are valid:

- $\alpha)$ B is A (and at the same time not non- A).

$$\begin{array}{c} C \text{ is } B \text{ and not } B. \\ \hline C \text{ is } A \text{ (and at the same time not non-}A\text{).} \end{array}$$

- $\beta)$ B is A (and at the same time not non- A).

$$\begin{array}{c} C, \text{ which is not } C, \text{ is } B. \\ \hline C \text{ is } A \text{ (and at the same time not non-}A\text{).} \end{array}$$

Syllogism α is valid, since C is B . That C is not B as well not only does not impair the conclusion ‘ C is A ’, but also need not influence the additional clause ‘ C is at the same time not non- A ’. For the term A has a broader extension than B , and hence also covers objects which are not B . Let us suppose that, in the example Aristotle gives, Callias is a man, and he is at the same time a horse, and hence not a man. Well, the horse is also an animal, so that in this special case the term A (animal) belongs also to things which are non- B . Hence it is possible in the conclusion to assert without contradiction that Callias is an animal, and not a non-animal.

Syllogism β is valid, since here too C is B . That C is also not C at the same time does not impair the conclusion, and need not influence the additional clause, since B has a broader extension than C . Hence Callias, who is not Callias but, for example, Socrates, nevertheless does not stop being a man, and consequently is an animal, and not a non-animal.

This is what the passage cited from the *Analytics* means. [101] It is a pity that Aristotle has obscured his interesting reasonings by needlessly involving the additional clause with the conclusion, and that he has given an explanation which, if we take it strictly, is wrong. It is generally true that the major term A has a broader extension than the middle term B ; hence A usually covers some non- B s as well, but it need not include all of them. Consequently it may happen that some non- B s included under term C do not belong to the extension of A . In the example given, if Callias (C) is a man (B) and is at the same time a rock, and hence is not a man and is non- B , then term A (animal) does not include the term non- B in this special case, and in the conclusion it cannot be said that Callias is an animal and not a non-animal. For this reason I need in my interpretation the expression that the contradictory nature of C does not impair the conclusion, and *need not* influence the additional clause, since I wish thereby to point out that it *may* influence it.

Yet this problem is not relevant to the relation between the principle of contradiction and the principle of the syllogism. On the other hand, what is important is that Aristotle

concedes that, *despite the contradiction* which the minor premise contains, the conclusion ‘*C* is *A*’ follows from the truth of the premises ‘*B* is *A*’ and ‘*C* is *B*’ alone. For the relation of *subsumption*, expressed in the premises with the help of the word ‘is’, is a *transitive* relation; that is, it possesses the property that if it obtains between the classes of objects *A* and *B* and between *B* and *C*, then it must obtain between *A* and *C*. It is precisely this on which the principle of the syllogism depends: *quidquid de omnibus valet, valet etiam de quibusdam et de singulis* (whatever holds for all holds also for some and for each). Hence if the premises of a syllogism are true, its [102] conclusion must be true.

Let us now suppose that some premises of a syllogism contain a contradiction; for example, ‘*C* is *B* and not *B* at the same time’. For a contradiction to arise, both of these propositions must be true at the same time. And since the proposition ‘*C* is *B*’ is true, and the second premise ‘*B* is *A*’ is true, then the conclusion ‘*C* is *A*’ must be true. The principle of the syllogism remains valid, even though the principle of contradiction ceases to be valid. From this it appears that *the principle of contradiction is not necessary as a basis for the principle of the syllogism*.

Modern symbolic logic confirms this result completely. Moreover, even a superficial acquaintance with this logic can convince us that there are many other laws and principles of reasoning which do not depend on the principle of contradiction⁵. We can show the same thing even without symbolic logic, though with less precision, with the assistance of examples of reasoning taken from everyday life. In order to make clear how much our thinking is independent of the principle of contradiction, and at the same time to take up the most effective fight against the deeply rooted opinions about the universal importance of this principle, I shall take the liberty of putting together some examples of arguments in which I assume for the moment that the principle of contradiction is invalid.

16. Non-Aristotelian logic

As far as I know, no one has yet created logical *fictions*. But this is the means of scientific investigation [103] which can best make clear the importance of laws, causes, or properties of objects being studied. For in fiction, for example, we throw out some laws which govern a given range of phenomena, and we try to discover what would happen *without them*, and through this we recognise most clearly to what extent the excluded laws influence a series of phenomena. So let us for once use fiction in logical investigations too.

Let us imagine a society living in the same world as we do, but having a differently organised mind. Let us in fact suppose that members of this society would regard every negative proposition as true. Hence is would always and everywhere be true for them that the sun is not shining, man does not die, two times two is not four, and so on, even when the sun was shining, people were dying, and the concepts two, four, multiplication, and equality meant the same for them as they do for us. It is surely difficult to enter into the

⁵ See the Appendix, ‘The principle of contradiction and symbolic logic’, § 9 δ.

spirit of this kind of thinking, but I believe that the following considerations will enable us to some extent to understand the mental attitude of these fictitious beings.

If the sun is shining, then it shines only