Week 1: GRE-Style Question Solutions

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Q1: Acceleration on an Incline

A 2.0 kg block slides down a frictionless incline of $\theta = 30^\circ$. The acceleration is given by:

$$a = g \sin\theta = 9.8 \cdot \sin(30^\circ) = 4.9\,\mathrm{m/s}^2$$

Q2: Projectile Motion (Integral Approach)

Start from vertical velocity component:

$$v_y(t) = v_0 \sin\theta - gt$$

Time to Apex

At maximum height, $v_y = 0$. Solve:

$$0 = v_0 \sin\theta - gt_{\text{apex}} \Rightarrow t_{\text{apex}} = \frac{v_0 \sin\theta}{g}$$

Time of Flight

$$t_{\text{flight}} = 2t_{\text{apex}} = \frac{2v_0 \sin\theta}{g}$$

Maximum Height

Integrate vertical velocity:

$$y(t) = \int v_y(t)dt = \int (v_0 \sin\theta - gt)dt = v_0 \sin\theta \cdot t - \frac{1}{2}gt^2$$

Evaluate at $t = t_{\text{apex}}$:

$$H = \frac{(v_0 \sin\theta)^2}{2g}$$

Horizontal Range Derivation

Horizontal velocity is constant:

$$v_x = v_0 \cos\theta$$

Then range is:

$$R = v_x \cdot t_{\text{flight}} = v_0 \cos\theta \cdot \frac{2v_0 \sin\theta}{g} = \frac{2v_0^2 \sin\theta \cos\theta}{g}$$

Using identity $\sin(2\theta) = 2\sin\theta\cos\theta$:

$$\boxed{R = \frac{v_0^2 \sin(2\theta)}{g}}$$

Q3: Newton-Raphson for Solving $x(t) = 5$

Given:

$$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$$

Let $f(t) = x(t) - 5$. Apply Newton-Raphson:

$$t_{n+1} = t_n - \frac{f(t_n)}{f'(t_n)}$$

$$f(t) = x_0 + v_0 t + \frac{1}{2}at^2 - 5, \quad f'(t) = v_0 + at$$

Choose initial guess t_0, iterate until $|f(t)| < \epsilon$.

Q4: Variable Force $F(t) = 5t$

Using Newton's 2nd Law: $F = ma \Rightarrow a(t) = F(t)/m = 5t$

Integrate acceleration to get velocity:

$$v(t) = \int a(t)dt = \int 5t\, dt = \frac{5}{2}t^2 + C$$

If initial velocity is 0, then:

$$v(t) = \frac{5}{2}t^2$$

Q5: Tangential and Normal Acceleration for $y = \sin x$

Let $\vec{r}(x) = x\hat{i} + \sin x\hat{j}$. Then:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \cos x\frac{dx}{dt}\hat{j}$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = \frac{d^2x}{dt^2}\hat{i} - \sin x\left(\frac{dx}{dt}\right)^2\hat{j} + \cos x\frac{d^2x}{dt^2}\hat{j}$$

Decompose $\vec{a}$ into tangential and normal components.