Labwork 1: Gradient Descent

April 29, 2025

1 Introduction

Gradient descent: definition
Gradient descent is an optimization algorithm used to minimize a function by iteratively moving in the direction of the steepest descent, as defined by the negative of the gradient.

2 Implementation

The implementation of gradient descent in this lab minimizes the function $f(x)$. The algorithm starts with an initial value of x, a learning rate r, and a convergence threshold ϵ. It iteratively updates x using the formula $x = x - r \cdot f'(x)$, where $f'(x)$ is the gradient of $f(x)$. The process continues until $f(x)$ becomes less than or equal to ϵ, at which point the algorithm returns the final values of x and $f(x)$.

3 Evaluation

Test with $y = x^4$
Output of my program:

Enter initial x: 2
Enter learning rate r: 0.05
Enter convergence threshold epsilon: 5
Final x: 0.3999999999999999, Final f(x): 0.025599999999999977

How ever, for the learning rate of 0.2, the program gives:
OverflowError: (34, 'Numerical result out of range')

The reason for this is that the learning rate is too high, causing the algorithm to overshoot the minimum and produce an overflow error.

4 Conclusion

In this labwork, I implemented the gradient descent algorithm to minimize the function $f(x) = x^4$. The result is that the algorithm converged to a minimum value of $f(x)$ at $x \approx 0.4$ with a final function value of approximately 0.0256.

An interesting finding is that the choice of learning rate significantly affects the convergence speed and stability of the algorithm. A smaller learning rate leads to slower convergence, while a larger one may cause divergence.