Mathematical Model: Multi-Period MECWLP

1. Sets and Indices

- I: Set of postcode districts (customers), indexed by i.
- J: Set of candidate facility locations, indexed by j.
- K: Set of suppliers, indexed by k.
- T: Set of time periods (years), indexed by $t \in \{1, \dots, 10\}$.

2. Parameters

- b_{it}: Demand of customer i in period t.
- f_j: One-off construction cost for a warehouse at location j.
- g_j: Annual operating cost for warehouse j, incurred every year the facility is in operation.
- Cap_j: Maximum throughput capacity of warehouse j.
- C_{kj}^{SK}: Unit transport cost from supplier k to warehouse j.
- C_{ji}^{KC}: Unit transport cost from warehouse j to customer i.
- V_{cap}: Maximum capacity of the delivery vehicle.

3. Decision Variables

- $y_{jt} \in \{0, 1\}$: 1 if a warehouse at location j is constructed in period t, 0 otherwise.
- $z_{jt} \in \{0, 1\}$: 1 if a warehouse at location j is operating in period t, 0 otherwise.
- $x_{kjt}^{SK} \geq 0$: Quantity transported from supplier k to warehouse j in period t.
- $x_{jit}^{KC} \geq 0$: Quantity transported from warehouse j to customer i in period t.
- $n_{jit} \in \mathbb{Z}^+$: Number of vehicle trips required from j to i in period t.

4. Objective Function

Minimize the total cost, consisting of one-time construction costs, cumulative annual operating costs, and total transportation costs across the 10-year horizon:

$$\min \quad \sum_{j \in J} \sum_{t \in T} f_j y_{jt} + \sum_{j \in J} \sum_{t \in T} g_j z_{jt} + \sum_{t \in T} \left(\sum_{k \in K} \sum_{j \in J} C_{kj}^{SK} x_{kjt}^{SK} + \sum_{j \in J} \sum_{i \in I} C_{ji}^{KC} x_{jit}^{KC} \right) \tag{1}$$

5. Constraints

- **Demand Satisfaction:** Total flow through the network must meet each customer's demand for every period t:

$$\sum_{j \in J} x_{jit}^{KC} = b_{it} \quad \forall i \in I, t \in T \tag{2}$$

- **Flow Conservation:** Incoming supply from all suppliers must match outgoing deliveries to all customers at each warehouse location:

$$\sum_{k \in K} x_{kjt}^{SK} = \sum_{i \in I} x_{jit}^{KC} \quad \forall j \in J, t \in T \tag{3}$$

- **Capacity and Operational Linkage:** Total warehouse throughput cannot exceed its capacity and requires the facility to be operational ($z_{jt} = 1$):

$$\sum_{i \in I} x_{jit}^{KC} \leq Cap_j z_{jt} \quad \forall j \in J, t \in T \tag{4}$$

- **Vehicle Trip Requirements:** Ensures the number of trips is sufficient to cover the total quantity transported:

$$n_{jit} \geq \frac{x_{jit}^{KC}}{V_{cap}} \quad \forall i \in I, j \in J, t \in T \tag{5}$$

- **Gradual Build-up Logic:** A warehouse is operational in period t if it was constructed in that year or any preceding year:

$$z_{jt} = \sum_{\tau=1}^{t} y_{j\tau} \quad \forall j \in J, t \in T \tag{6}$$

- **Single Construction Constraint:** Each candidate location can have a warehouse built at most once during the planning horizon:

$$\sum_{t \in T} y_{jt} \leq 1 \quad \forall j \in J \tag{7}$$

- **Variable Domains:**

$$y_{jt}, z_{jt} \in \{0, 1\} \quad \forall j \in J, t \in T \tag{8}$$

$$x_{kjt}^{SK}, x_{jit}^{KC} \geq 0 \quad \forall i, j, k, t \tag{9}$$

$$n_{jit} \in \mathbb{Z}^{+} \quad \forall i, j, t \tag{10}$$