mc_sim

Monte Carlo Simulation

BL-TVP provides two new approaches to the synthetic control framework: time varying parameters and Bayesian shrinkage. To better understand which addition is contributing and how, I perform Monte Carlo simulations comparing *BL-TVP* to @brodersen_inferring_2015. I compare each model with and without time varying parameters. The four variations are:

1) the original @brodersen_inferring_2015 model (*CI*),
2) @brodersen_inferring_2015 with time varying coefficients (*CI-TVP*),
3) The proposed model without time varying coefficients (*BL*),
4) *BL-TVP*.

BL is a simplification of *BL-TVP* in which $\sqrt{\theta_j} = 0$ for all j. *CI* and *CI-TVP* are run using the R package `CausalImpact` [@brodersen_inferring_2015]. *BL* is run using the `BayesReg` R package [@makalic_high-dimensional_2016].

The simulation is based off of @kinn_synthetic_2018. Assume the following data generating process:

$$
\begin{aligned}
y_{j,t}(0) &= \xi_{j,t} + \psi_{j,t} + \epsilon'_{j,t} \qquad \text{j=1,..,J} \\
y_{0,t}(0) &= \sum_{j=1}^{J} w_{j,t}(\xi_{j,t}) + \epsilon'_{1.t}
\end{aligned}
$$

for t=1,..,T where ξ_{jt} is the trend component, $\psi_{j,t}$ is a seasonality component, and $\epsilon'_{jt} \sim N(0,\sigma^2)$. Specifically, $\xi_{jt} = c_j t + z_j$ where $c_j,\ z_j \in \mathbb{R}$. This will allow for each observation to have a unit-specific time varying confounding factor and a time-invariant confounding factor. Seasonality will be represented as $\psi_{j,t} = \gamma_j sin\left(\frac{\pi t}{\rho_j}\right)$. The explicit data generating process is:

$$
\begin{aligned}
y_{j,t}(0) &= c_j t + z_j + \gamma_j sin\left(\frac{\pi t}{\rho_j}\right) + \epsilon'_{j,t} \qquad \text{j=1,..,J} \\
y_{0,t}(0) &= \sum_{j=1}^{J} w_{j,t}\left(c_j t + z_j + \gamma_j sin\left(\frac{\pi t}{\rho_j}\right)\right) + \epsilon'_{1.t}
\end{aligned}
$$

The treatment begins at period T_0. The treatment effect is set to 0 for all periods.

This paper proposes testing two scenarios: (i) deterministic continuous varying coefficients with no treatment effect, and (ii) constant coefficients with no treatment effect. These scenarios will provide insight on the point prediction accuracy (via mean squared error) and the probability interval size. Within each scenario, the pre-treatment time length and donor pool will be varied. The full time frame will be 34 periods (e.g. $T = 34$).

Deterministic Continuous Varying Coefficients

To simulate continuous varying coefficients, $c_{1,t}$ and $c_{2,t}$ are defined .75 and .25 respectively. All other $c_{j,t}$ are randomly drawn from U[0,1]. In order to avoid $y_{1,t}$ and $y_{2,t}$ from crossing, set $z_1 = 25$ and $z_2 = 5$. Finally, define $w_{1,t} = .2 + .6\frac{t}{T}$ and $w_{2,t} = 1 - w_{1,t}$ in the time varying case.

To summarize, the parameters of this simulation are:

1) $c_{1,t} = .75$, $c_{2,t} = .25$, and $c_{j,t} \sim U[0,1]$ for all $j \notin \{1,2\}$.
2) $z_1 = 25$, $z_2 = 5$ and z_j is sampled from $\{1,2,3,4,...,50\}$.
3) $\epsilon'_{j,t} \sim N(0,1)$.
4) $T = 34$.
5) $\gamma_j = 4$.
6) $\rho_j = 20$.
7) $w_{1,t} = .2 + .6\frac{t}{T}$, $w_{2,t} = 1 - w_{2,t}$, and $w_{j,t} = 0$ for all else (Time Varying).

The data generating process for the time varying coefficient case can be rewritten in recursive form:

$$
\begin{aligned}
y_{0,t}(0) &= \sum_{j=1}^{J} w_{j,t}\left(c_j t + z_j + \gamma_j sin\left(\frac{\pi t}{\rho_j}\right)\right) + \epsilon'_{1.t} \\
w_{1,t} &= w_{1,t-1} + \frac{.6}{T} \\
w_{2,t} &= w_{2,t-1} - \frac{.6}{T} \\
w_{j,t} &= w_{j,t-1} \qquad\qquad\qquad\qquad j \notin \{1,2\}
\end{aligned}
$$

with initial conditions:

$$
\begin{aligned}
w_{1,0} &= .2 \\
w_{2,0} &= .8 \\
w_{j,0} &= 0 \quad j \notin \{1,2\}
\end{aligned}
$$

Constant Coefficients

The setup for constant coefficients is identical to deterministic continuous varying coefficients except point (6) is replaced by (6'):

(6') $w_{1,t} = .2$, $w_{2,t} = 1 - w_{2,t}$, and $w_{j,t} = 0$ for all else (Time Invariant).

Model Testing and Comparison

I will compare the mean squared error (MSE) in pre and post treatment. Mean squared error encompasses the paper's main goal of estimation. However, an empirical researcher also needs to understand the inference aspect. Since these models are Bayesian, inference derives from the posterior predictive distribution. This is easily calculated from each iteration of the Gibbs sampler. I summarize the results using the 95% confidence interval (CI Spread) spread and post treatment coverage of the 95% probability interval (95% PI). Each measurement is defined as:

$$\text{MSE} \equiv \frac{1}{T-T_0}\sum_{t=T_0}^{T}\left(y_{0,t}-\hat{y}_{0,t}\right)^2$$

$$\text{CI Spread} \equiv \frac{1}{T}\sum_{t=1}^{T}\left(\hat{y}_{0,t}^{.975}-\hat{y}_{0,t}^{.025}\right)$$

$$95\%\ \text{PI} \equiv \frac{1}{T-T_0}\sum_{t=T_0}^{T} I\left(y_{0,t}\in\left[\hat{y}_{0,t}^{.025},\hat{y}_{0,t}^{.975}\right]\right)$$

where $\hat{y}_{0,t}(0)$ is the median of the posterior predictive density created by each model specification, $\hat{y}_{0,t}^{.025}(0)$ and $\hat{y}_{0,t}^{.975}(0)$ are the 2.5^{th} and 97.5^{th} quantiles of the posterior estimations. The results for mean squared error are presented in Table 1, posterior predictive density is summarized in Table 3, and confidence interval spread is summarized in Table 2.

Results

Mean Squared Error

CI-TVP creates a perfect pre-treatment fit in all eight simulation studies. This result is in line with the concerns of overfitting discussed in @brodersen_inferring_2015. This becomes evident when focusing on the mean squared error in the post treatment. *BL* and *BL-TVP* maintain smaller post-treatment mean squared errors in both time varying parameters and time invariant parameters when $T_0 = 17$. When parameters are time invariant, *BL* and *BL-TVP* have a post-treatment mean squared error magnitudes smaller than both version of *CI*. With time varying parameters, *BL* performs worse than *CI-TVP* but significantly better than *CI*. *BL-TVP* produces a post-treatment mean squared error 6 times smaller than *CI*. When *BL-TVP* ranked first or second smallest for all four simulations in which $T_0 = 17$.

When $T_0 = 5$ and $J = 17$, *CI*, *BL*, and *BL-TVP* all perform similarly in terms of mean squared error. *CI-TVP* produces an post treatment mean squared error 3-4 times larger than the other models. In the case of dynamic coefficients, no model is able to recreate a good counterfactual. There simply is not enough data to identify the complex data generating process. In the event of $T_0 = 5$, $J = 5$ and constant weights, time invariant models greatly outperform time varying models. This would be a situation in which the assumption of constant parameters is easier to argue. However, no model performs well with dynamic weights in this setting.

BL-TVP had a lower post treatment mean squared error compared *CI* 6 of the 8 simulations. Similarly, *BL-TVP* had a lower post treatment mean squared error compared *CI-TVP* 6 of the 8 simulations.

95% Confidence Interval Spread

BL-TVP probability interval spread is close in magnitudes to *BL* with $T_0 = 17$. *Bl* and *BL-TVP* maintain tighter probability intervals than *CI* and *CI-TVP* when $T_0 = 17$. *Bl-TVP* produces slightly larger probability intervals to *BL* when the data generating process consists of constant parameters and slightly smaller probability intervals when the data generating process consists of time varying parameters. However, the differences are miniscule. *BL-TVP* maintains smaller probability intervals than *CI-TVP* in all cases except $T_0 = 5$, $J = 5$.

95% Coverage

All models achieve optimal coverage in the post treatment period with constant coefficients. However, only *CI-TVP* consistently covers 100% of the post treatment period.

Table 1: Simulation Study of Point Estimates

			Pre-Treament MSE				Post-Treatment MSE			
T_0	J	Coefficient Type	CI	CI-TVP	BL	BL-TVP	CI	CI-TVP	BL	BL-TVP
17	17	Constant	0.671	**0**	0.567	0.244	5.886	7.168	**1.762**	2.586
17	17	Dynamic	7.178	**0**	2.061	0.478	428.95	109.199	162.279	**77**
17	5	Constant	0.779	**0**	0.811	0.684	6.391	8.422	**3.077**	4.264
17	5	Dynamic	9.482	**0**	6.454	4.962	463.192	288.33	281.424	**172.792**
5	17	Constant	0.104	**0**	0.113	0.029	**5.814**	21.644	6.178	7.32
5	17	Dynamic	0.312	**0**	0.122	0.018	1516.166	**1028.543**	1549.114	1511.55
5	5	Constant	0.169	**0**	0.394	0.05	8.642	29.385	**5.14**	20.414
5	5	Dynamic	0.419	**0**	0.514	0.036	1529.019	**1473.107**	1596.394	1497.134

* Median results of 100 monte carlo simulations with T=34.
† Each simulation of BL-TVP is run 3000 times with a 1500 burn-in.
‡ All other models are run according to presets.
§ The preset Causal Impact model was used as described in Brodersen et al. 2015.
¶ Cells with lowest MSE per simulation and period are bolded.

Table 2: Simulation Study of Probability Interval Spread Over Whole Sample

T_0	J	Coefficient Type	CI	CI-TVP	BL	BL-TVP
17	17	Constant	6.689	11.816	**6.286**	8.367
17	17	Dynamic	31.389	49.367	25.522	**21.118**
17	5	Constant	6.571	13.72	**5.565**	6.833
17	5	Dynamic	31.786	33.298	25.04	**21.013**
5	17	Constant	**16.725**	81.003	19.475	54.798
5	17	Dynamic	40.017	263.908	**39.313**	77.629
5	5	Constant	**14.781**	31.431	16.953	47.386
5	5	Dynamic	37.758	44.776	**35.414**	61.973

* Median results of 100 monte carlo simulations with T=34.
† Each simulation of BL-TVP is run 3000 times with a 1500 burn-in.
‡ All other models are run according to presets.
§ The preset Causal Impact model was used as described in Brodersen et al. 2015.
¶ Cells with lowest confidence interval spread per simulation are bolded

Table 3: Simulation Study of Coverage of Models

			Pre-Treament Coverage				Post-Treatment Coverage			
T_0	J	Coefficient Type	CI	CI-TVP	BL	BL-TVP	CI	CI-TVP	BL	BL-TVP
17	17	Constant	1	1	0.882	1.000	1.000	1.000	1.000	1.000
17	17	Dynamic	1	1	1.000	1.000	0.471	1.000	0.706	0.765
17	5	Constant	1	1	0.706	0.765	1.000	1.000	0.882	0.941
17	5	Dynamic	1	1	0.824	0.765	0.500	1.000	0.265	0.412
5	17	Constant	1	1	1.000	1.000	1.000	1.000	1.000	1.000
5	17	Dynamic	1	1	1.000	1.000	0.276	1.000	0.241	0.828
5	5	Constant	1	1	1.000	1.000	1.000	1.000	1.000	1.000
5	5	Dynamic	1	1	1.000	1.000	0.276	0.397	0.207	0.414

* Median results of 100 monte carlo simulations with T=34.

† Each simulation of BL-TVP is run 3000 times with a 1500 burn-in.

‡ All other models are run according to presets.

§ The preset Causal Impact model was used as described in Brodersen et al. 2015.

¶ Coverage is defined using a 95% probability interval.