ASTR 127 LECTURE NOTES JAN 20, 2015

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1. Questions?

2. Examples

2.1. **Blackbody Thought Experiment.** What is the source function for Blackbody radiation? Suppose we have a box of optically thick gas at a constant temperature T. The material in the box, as well as the radiation field are in thermodynamic equilibrium, so there is no net flow of energy between the gas particles and the radiation field. That is, each absorption is exactly balanced by an emission. In this case, the intensity of the radiation is $I_\nu = B_\nu$. Since we are in a constant radiation field we have:

$$\frac{dI}{d\tau} = 0 \ . \tag{1}$$

Therefore we have:

$$0 = B - S \tag{2}$$

or

$$S_\nu = B_\nu. \tag{3}$$

2.2. **Earth's Atmosphere Sight Distance.** How far could you see if the Earth's atmosphere had the same opacity of the solar photosphere? For this we need to know the density of the Earth's atmosphere, and the opacity of the solar photosphere:

$$\rho_E = 1.2\ kg\ m^{-3} \qquad \kappa_{Sol} = 0.03\ m^2\ kg^{-1} \tag{4}$$

Start with the intensity gradient:

$$\frac{dI}{ds} = -I\kappa\rho \tag{5}$$

Then solve for the intensity:

$$\int \frac{dI}{I} = -\int_0^s \kappa\rho ds \tag{6}$$

Which gives us:

$$I = I_0 e^{-\int_0^s \kappa\rho ds} \tag{7}$$

We then have the characteristic travel distance for a photon before it get absorbed:

$$s = \frac{1}{\kappa\rho} \tag{8}$$

Plugging in the numbers we get:

$$s = \frac{1}{(0.03)(1.2)} = 27.7 \text{ meters} \tag{9}$$

2.3. **Stefan-Boltzmann Derivation.** Consider a spherical blackbody of radius R and temperature T. We have the flux:

$$F_\nu = 2\pi \int_{-1}^{1} I_\nu u du \tag{10}$$

For a spherical blackbody, $I_\nu = B_\nu$ so we get:

$$F_\nu = \pi B_\nu \tag{11}$$

Which is the specific flux. We then want to integrate this over all frequencies.

$$F = \pi \int_0^\infty \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \tag{12}$$

With the substitution, $x = \frac{h\nu}{kT}$ we have:

$$F = \frac{2\pi}{c} \int_0^\infty \frac{x^3 k^3 T^3}{h^2} \frac{1}{e^x - 1} \frac{kT}{h} dx \tag{13}$$

Which can be simplified to:

$$F = \frac{2\pi k^4 T^4}{c^2 h^3} \int_0^\infty \frac{x^3}{e^x - 1} dx \tag{14}$$

That integrates to become:

$$F = \frac{2\pi^5 k^4}{15 h^3 c^2} T^4 \tag{15}$$

Now we have the flux:

$$F = \sigma T^4 \tag{16}$$

Where σ is the Stefan-Boltzmann constant, and is the collection of all the constants from the previous equation. From this we then obtain the luminosity which is the flux multiplied by the surface area of the star:

$$L = 4\pi R^2 \sigma T^4 \ . \tag{17}$$

2.4. **Eddington-Barbier Relation.** We want to find the flux emitted from an atmosphere as a function of the optical depth. We start with the equation of radiation transfer for a plane-parallel atmosphere:

$$u\frac{dI}{d\tau} = I - S \tag{18}$$

Divide by u:

$$\frac{dI}{d\tau} = \frac{I}{u} - \frac{S}{u} \tag{19}$$

Then we multiply both sides by $e^{-\tau/u}$ and collect both terms with the intensity on the left side.

$$\frac{dI}{d\tau}e^{-\tau/u} - \frac{I}{u}e^{-\tau/u} = -\frac{S}{u}e^{-\tau/u} \tag{20}$$

We can condense the left side of the equation:

$$\frac{d}{d\tau}(Ie^{-\tau/u}) = -\frac{S}{u}e^{-\tau/u} \tag{21}$$

Then we can integrate both sides to get:

$$Ie^{-\tau/u}|_{\tau_1}^{\tau_2} = -\int_{\tau_1}^{\tau_2} \frac{S}{u}e^{-\tau/u}d\tau \tag{22}$$

Then we set τ_1 to the surface, and τ_2 to infinity. This simplifies the equation to:

$$I = \int_0^\infty \frac{S}{u} e^{-\tau/u} d\tau \tag{23}$$

We can then Taylor expand the source function as a function of τ:

$$S_\nu = a_\nu + b_\nu \tau \tag{24}$$

Then plugging it into the previous equation to get:

$$I = \int_0^\infty \frac{a_\nu + b_\nu \tau}{u} e^{-\tau/u} d\tau \tag{25}$$

And finally integrating to get:

$$I_\nu = a_\nu + b_\nu u \tag{26}$$

To get the flux we want to take the first moment of I:

$$F_\nu = \int_0^{2\pi} \int_0^1 I u du d\phi \tag{27}$$

Which results in:

$$F_\nu = \pi(a + b\frac{2}{3}) \tag{28}$$

Which is equivalent to:

$$F_\nu = \pi S_\nu(\tau = \frac{2}{3}) \tag{29}$$

In other words, we can define the surface of a star wherever the optical depth is equal to 2/3. That is to say, the flux we see is the same as the flux emitted at an optical depth of 2/3. Another piece of astrophysics we can find from this derivation is limb darkening. One way to think about it is this: If the surface of the star is at $\tau = \frac{2}{3}$, then when we see the disk of the sun, we are looking at constant depth. However, when we are looking toward the edge of the sun, we are looking at a larger solar radius. Because the temperature decreases toward larger radii, we are looking at a cooler temperature, and thus are receiving less flux. Another explanation is to look at the equation:

$$I = a + bu \tag{30}$$

and to translate this to the disk radius of the sun. It predicts the same effect, decreasing flux toward the edge of the stellar disk.