ASTR 127 LECTURE NOTES JAN 13, 2015

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1. Administrative Topics

1.1. **Overview of the discussion.** This is the discussion for ASTR 127: Stellar Atmospheres, Interiors, and Evolution.

1.2. **What to do during the discussion section?**

- Review of the lectures
- Talk about relevant, but not required topics
- Do example problems from the textbook, etc.
- Go over previous homework
- Do activities
  - Worksheets
  - Problem solving
  - Group work

1.3. **Email Policy.** Give me 24 hours to respond to emails, although I may respond earlier. After 24 hours feel free to email me again. Don't email me questions about what we did in lecture, because I don't go to the lectures.

1.4. **Office Hours.**

- Thursdays 3pm - 4pm
- Fridays 2pm-3pm
- Or by appointment
- Location: PAB 1-704A

2. Timescales

Timescales are important to know because they can help us determine the causes for different aspects of stellar evolution/structure. For example, If you calculate the lifetime of a star as if it were a ball of hydrogen on fire, you would get a lifetime much shorter than the age of the sun currently. Here are three different timescales in order of shortest to longest:

2.1. **Dynamical Timescale.**

Start with the acceleration due to gravity:

$$a = \frac{d^2r}{dt^2} = -\frac{GM}{r^2} \tag{1}$$

From the chain rule, we know that:

$$\frac{d^2r}{dt^2} = \frac{d}{dt}v(r) = \frac{dr}{dt}\frac{d}{dr}v = \frac{1}{2}\frac{d}{dr}v^2 \tag{2}$$

Therefore:

$$\frac{d}{dr}v^2 = -\frac{2GM}{r^2} \tag{3}$$

Integrate both sides:

$$\int dv^2 = -\int \frac{2GM}{r^2}dr \tag{4}$$

With initial conditions of $r = R$, $v = 0$, the velocity becomes:

$$v = \frac{dr}{dt} = \left[2GM(\frac{1}{r} - \frac{1}{R})\right]^{\frac{1}{2}} \tag{5}$$

Solving for the time interval dt we have:

$$dt = \frac{-dr}{\left[2GM(\frac{1}{r} - \frac{1}{R})\right]^{\frac{1}{2}}} \tag{6}$$

Rearranging this and integrating we then get:

$$t_{ff} = (\frac{R}{2GM})^{\frac{1}{2}} \int_R^0 \frac{dr}{(1 - \frac{R}{r})^{\frac{1}{2}}} \tag{7}$$

With the substitution $x = \frac{r}{R}$ and $dx = \frac{dr}{R}$ we obtain:

$$t_{ff} = (\frac{R^3}{2GM})^{\frac{1}{2}} \int_1^0 \frac{dx}{(1 - \frac{1}{x})^{\frac{1}{2}}} \tag{8}$$

The integral evaluates to $\frac{\pi}{2}$, then we have a final answer of:

$$t_{ff} = \left(\frac{3\pi}{32G\rho}\right)^{\frac{1}{2}} \tag{9}$$

This result (Equation 9) is the free fall timescale, sometimes called the dynamical timescale. It measures the timescale for hydrostatic equilibrium to react to changes in gravity, pressure, etc. For the sun it has a value of about 1000s A more simple derivation relies on dimensional analysis.

$$t_{dyn} = \frac{\text{radius}}{\text{characteristic velocity}} = \frac{R}{v_{esc}} \approx \frac{1}{\sqrt{G\rho}} \tag{10}$$

2.2. **Kelvin Helmholtz Timescale.** The Kelvin Helmholtz timescale, sometimes called the thermal timescale, is the time for all currently stored up thermal energy to be radiated away. Its calculation is straightforward:

$$t_{KH} = \frac{U}{L} \tag{11}$$

Where U is the stored thermal energy, and L is the luminosity. From the virial theorem, we know the relationship between the thermal energy and the gravitational energy: $U = \frac{\Omega}{2}$. This leaves us with the formula for the timescale.

$$t_{KH} \approx \frac{GM^2}{2RL} \tag{12}$$

This has a value of roughly 10^7 years for the sun. This is also the timescale for how quickly a star can contract before nuclear fusion starts: its pre-main sequence lifetime.

2.3. **Nuclear Timescale.** The longest timescale is the nuclear timescale. This is the timescale for a star to burn all of its hydrogen through nuclear fission. We can estimate this time by stating:

$$t_{nuc} = \frac{qXM \cdot 6 \times 10^{18}\ erg\ g^{-1}}{L} \tag{13}$$

Where q is the fraction of the star's mass that is in the core, X is the fraction of star that is hydrogen, and $6 \times 10^{18}\ erg\ g^{-1}$ is the amount of energy released by one gram of fusing hydrogen. For the sun this turns out to be roughly 10^{10} years. This is also an estimation of the main sequence lifetime of a star. This makes sense because the main sequence of a star is the period of time that it is burning hydrogen in its core.