HW #3 SOLUTIONS

1. Problem 5.3

Start with the energy per unit time generated inside a shell between radii r and $r + dr$ (Eqn. 5.21):

$$dL = 4\pi r^2 \rho(r)\epsilon(r)dr \tag{1}$$

All of the energy produced by the star comes from the region where $r < 0.2R_*$, so we obtain the luminosity by integrating:

$$L_* = \int_0^{0.2R_*} 4\pi r^2 \rho_c \left(1 - \frac{r}{R_*}\right)\epsilon_c \left(1 - \frac{r}{0.2R_*}\right)dr \tag{2}$$

Which becomes:

$$L_* = 4\pi\rho_c\epsilon_c \int_0^{0.2R_*} r^2 \left(1 + \frac{5r^2}{R_*^2} - \frac{6r}{R_*}\right)dr \tag{3}$$

Which finally results in:

$$L_* = 2.35 \times 10^{-3}\pi\rho_c\epsilon_c R_*^3 \tag{4}$$

2. Problem 5.7

By assuming the white dwarf is composed of pure carbon, we can determine the number density of carbon:

$$n_C = \frac{\rho}{m_C} = 5 \times 10^{28}\ cm^{-3} \tag{5}$$

For these high temperatures, we can assume that all of the carbon atoms are completely ionized, so the electron density is:

$$n_e = 6n_C = 3 \times 10^{29}\ cm^{-3} \tag{6}$$

Then, assuming the opacity is dominated by Thompson scattering, the opacity is:

$$\sigma_e = \frac{n_c \sigma_T}{\rho} = 0.2\ cm^2 g^{-1} \tag{7}$$

For the given temperature and density, the conduction opacity in the center of a white dwarf is:

$$k_{\text{cond}} = 5 \times 10^{-5}\ cm^2 g^{-1} \tag{8}$$

This value is much smaller than the scattering opacity. The energy therefore flows preferably through the mode of transport with the smaller opacity and conduction completely dominates energy transport in the center of this white dwarf.

3. Problem 5.13

Assuming the condition of hydrostatic equilibrium, the pressure can be obtained by integrating the corresponding equation:

$$\frac{dP(r')}{dr'} = -\rho(r')g(r') \tag{9}$$

from R_* to r.

$$\int dP(r') = -\int_{R_*}^{r} \rho g dr' \tag{10}$$

In this case the density is constant, but the gravitational acceleration varies with radius according to:

$$g(r) = \frac{GM(r)}{r^2} \tag{11}$$

Where the mass enclosed is:

$$M(r) = \rho V(r) = \frac{4}{3}\pi r^3 \rho \tag{12}$$

Therefore:

$$g(r) = \frac{4}{3}\pi G \rho r \tag{13}$$

So the integral becomes:

$$P(r) = -\int_{R_*}^{r} \frac{4}{3}\pi G \rho^2 r' dr' \tag{14}$$

Which give the solution:

$$P(r) = \frac{2}{3}\pi G \rho^2 (R_*^2 - r^2) \tag{15}$$