SUPPLEMENTARY HOMEWORK SOLUTIONS

1. Problem 3.7

Since no emissivity exists, we have:

$$\frac{u}{\rho}\frac{dI_\nu}{dz} = -k_\nu I_\nu \tag{1}$$

Rearranging, and assuming we have $u = 1$, we can obtain the equation:

$$\frac{dI_\nu}{I_\nu} = -k_\nu \rho dz = -\frac{\sqrt{z}dz}{4\xi} \tag{2}$$

which can be integrated to find the specific intensity exiting the cloud:

$$\int_{I_0}^{I_\nu} \frac{dI_\nu}{I_\nu} = \int_0^{z_0} -\frac{\sqrt{z}dz}{4\xi} \tag{3}$$

the result of which is:

$$ln\left(\frac{I_\nu}{I_\nu^0}\right) = -\frac{1}{6\xi} z_0^{3/2} \tag{4}$$

from which you can determine the expression for the thickness of the cloud:

$$z_0 = 12\xi^{2/3} \tag{5}$$

2. Problem 4.7

The Maxwell distribution of velocities of particles is:

$$fv = 4\pi \left(\frac{1}{\pi a^2} \right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} \tag{6}$$

The most probable speed v_0 corresponds to the maximum of the derivative of the distribution function $f(v)$. To simplify, a parameter $a^2 = \frac{2kT}{m}$ is defined so that the maxwell distribution becomes:

$$f(v) = \frac{4}{\sqrt{\pi} a^3} v^2 e^{-\frac{v^2}{a^2}} \tag{7}$$

Differencing this expression results in:

$$\frac{df(v)}{dv} = \frac{8}{\sqrt{\pi} a^3} v e^{-\frac{v^2}{a^2}} - \frac{4}{\sqrt{\pi} a^3} v^2 \frac{2v}{a^2} e^{-\frac{v^2}{a^2}} \tag{8}$$

which we can solve to find a value of:

$$v_0 = a = \sqrt{\frac{2kT}{m}} \tag{9}$$

3. Problem 5.10

The mass of the star is given by the following equation (Eq. 5.103):

$$M_* = 4\pi \alpha^3 \rho_c \int_0^{\xi_0} \xi^2 \theta^n(\xi) d\xi \tag{10}$$

For $n = 1$ in consideration, the solution for $\theta(r)$ is (see Ex. 5.3):

$$\theta(\xi) = \frac{\sin \xi}{\xi} \tag{11}$$

Substituting this in with $n = 1$ and $\xi_0 = \pi$ and integrating gives:

$$M_* = 4\pi^2\alpha^3\rho_c \tag{12}$$

Since $R_* = \alpha\xi_0 = \alpha\pi$, the mass of the star is:

$$M_* = \frac{4}{\pi}R_*^3\rho_c \tag{13}$$

4. Problem 5.12

The radiation energy density at a given frequency is given by the following equation (Eq. 3.21):

$$U_\nu = \frac{1}{c}\oint I_\nu d\Omega \tag{14}$$

If $I_\nu = B_\nu$ then we get:

$$U_\nu = \frac{4\pi}{c}B_\nu \tag{15}$$

To find the total radiation energy density one needs to integrate this for all frequencies:

$$U_{rad} = \int_0^\infty U_\nu d\nu = \frac{4\pi\sigma}{c}T^4 \tag{16}$$

The gas energy density is given by the following equation:

$$U_{gas} = n_{tot}\left(\frac{3}{2}kT\right) \tag{17}$$

For a gas insisting of completely ionized hydrogen, the mean atomic weight is $\mu = \frac{1}{2}$ and:

$$n_{tot} = \frac{\rho}{\mu m_H} = \frac{2\rho}{m_H} \tag{18}$$

Therefore:

$$U_{gas} = \frac{3\rho k T}{m_H} \tag{19}$$

If the radiation energy density is equal to the gas energy density, then:

$$\frac{4\pi\sigma}{c} T^4 = \frac{3\rho k T}{m_H} \tag{20}$$

and the temperature is:

$$T^3 = 1 \times 10^{22} \rho \ K^3/(gcm^{-3}) \tag{21}$$

So for $\log \rho = -4$ this gives $\log T = 6$. According to the results shown in Figure 5.10, these conditions are those of an ideal gas since they fall within the appropriate boundaries found in the figure.