HW #2 SOLUTIONS

1. Problem 4.1

Using the expression for the specific intensity, one can calculate the average intensity:

$$J_\nu = \frac{1}{2}\int_{-1}^{1} I_\nu du = \frac{1}{2}\int_{-1}^{1}(a_\nu + b_\nu u)du = a_\nu \tag{1}$$

and the K integral:

$$K_\nu = \frac{1}{2}\int_{-1}^{1} I_\nu u^2 du = \frac{1}{2}\int_{-1}^{1}(a_\nu u^2 + b_\nu u^3)du = \frac{a_\nu}{3} \tag{2}$$

Comparing these results, we find that:

$$J_\nu = 3K_\nu \tag{3}$$

2. Problem 4.2

In a grey plane-parallel atmosphere assuming radiative equilibrium, the average intensity at optical depth τ is (Eqs. 4.5, 4.20):

$$J(\tau) = S(\tau) = 3H\left[\tau + \frac{2}{3}\right] \tag{4}$$

where $S(\tau)$ and H are the local source function and the integrated Eddington flux, the latter being constant throughout the atmosphere. The source function at depth

$\tau = \frac{2}{3}$ is:

$$S(\tau = 2/3) = 3H\left[\frac{2}{3} + \frac{2}{3}\right] = 4H \tag{5}$$

from which we can obtain H:

$$H = \frac{S(\tau = 2/3)}{4} \tag{6}$$

3. Problem 4.6

The opacity of the atomic line i -¿ j is given by Eq. (4.77):

$$k_\nu \rho = f_{ij}\frac{\pi e^2}{m_e c}\phi_\nu n_i\left(1 - e^{-\frac{h\nu_0}{kT}}\right) \tag{7}$$

where ϕ_ν is the line profile. Assuming that the atomic populations obey the Boltzmann equation, the population of level i is:

$$\frac{n_i}{n_{ion}} = \frac{g_i}{U_{ion}}e^{-\frac{h\nu_0}{kT}} \tag{8}$$

Combining these two equations gives:

$$k_\nu \rho = g_i f_{ij}\frac{\pi e^2}{m_e c}\phi_\nu \frac{n_{ion}}{U_{ion}}e^{-\frac{h\nu_0}{kT}}\left(1 - e^{-\frac{h\nu_0}{kT}}\right) \tag{9}$$

4. Problem 4.9

The equivalent width of an atomic line can be calculated using:

$$W_\lambda = \int 1 - \frac{F_\lambda}{F_c} d\lambda \tag{10}$$

We can relate the monochromatic flux to the flux in the continuum by the following expressions:

$$F_\lambda = F_c\left(1 - \frac{\lambda}{4\text{Å}}\right) \qquad 0 < \lambda < 3\text{Å} \tag{11}$$

$$F_\lambda = F_c\left(\frac{\lambda}{4\text{Å}} - \frac{1}{2}\right) \qquad 3 < \lambda < 6\text{Å} \tag{12}$$

Then these are inserted into the integral to get:

$$W_\lambda = \int_0^3 \left(1 - 1 + \frac{\lambda}{4}\right) d\lambda + \int_3^6 \left(1 - \frac{\lambda}{4} + \frac{1}{2}\right) d\lambda \tag{13}$$

Which is evaluated to become:

$$W_\lambda = 2.25\text{Å} \tag{14}$$

5. Problem 4.12

The rotational broadening of a spectral line is a result of the Doppler shifts of the various regions of a star's disc cut to stellar rotation. It also depends on the angle of inclination of the rotation axis relative to the line-of-sight. This creates broadening on the order of:

$$\frac{V \sin i}{c} = \frac{\Delta\lambda}{\lambda_0} \tag{15}$$

where λ_0 is the central wavelength of the spectral line, V is the velocity of rotation along the star's equator and i is the angle of inclination. In this case, $\lambda_0 = 5000\text{Å}$, one can find the velocity that produces a rotational broadening similar to the observed broadening $\Delta\lambda = 0.2\text{Å}$:

$$V \sin i = \frac{\Delta\lambda}{\lambda_0} \times c = \frac{0.02\text{Å}}{5000\text{Å}} \times 2.998 \times 10^5 km/s = 12 km/s \tag{16}$$