Chapter 4
Lists

Data Structures and Algorithms

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Outcomes

- **L.O.2.1** - Depict the following concepts: (a) array list and linked list, including single link and double links, and multiple links; (b) stack; and (c) queue and circular queue.
- **L.O.2.2** - Describe storage structures by using pseudocode for: (a) array list and linked list, including single link and double links, and multiple links; (b) stack; and (c) queue and circular queue.
- **L.O.2.3** - List necessary methods supplied for list, stack, and queue, and describe them using pseudocode.
- **L.O.2.4** - Implement list, stack, and queue using C/C++.

Outcomes

- **L.O.2.5** - Use list, stack, and queue for problems in real-life, and choose an appropriate implementation type (array vs. link).
- **L.O.2.6** - Analyze the complexity and develop experiment (program) to evaluate the efficiency of methods supplied for list, stack, and queue.
- **L.O.8.4** - Develop recursive implementations for methods supplied for the following structures: list, tree, heap, searching, and graphs.
- **L.O.1.2** - Analyze algorithms and use Big-O notation to characterize the computational complexity of algorithms composed by using the following control structures: sequence, branching, and iteration (not recursion).

Contents

Sources of Materials

1. We would like to thank **Dr. The-Nhan LUONG**, a former instructor of our Department, for the composing of this document.
2. This document also uses figure, sentences and demo source code from the following sources:
    - The old presentation for course *Data Structures and Algorithms* edited by other members in our Department
    - Book entitled **Data Structures - A Pseudocode Approach with C++ (first edition, 2001)** written by Richard F. Gilberg and Behrouz A. Forouzan

Linear list concepts

Linear list concepts

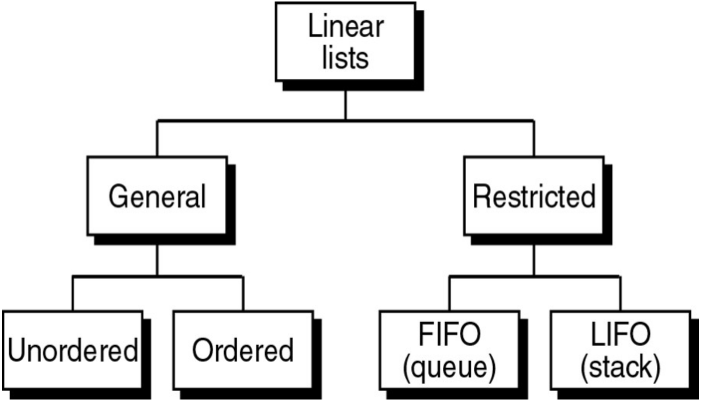
Definition

A linear list is a data structure in which each element has a unique successor.

Example

- Array
- Linked list

Linear list concepts



Linear list concepts

General list:

- No restrictions on which operation can be used on the list.
- No restrictions on where data can be inserted/deleted.

- Unordered list (random list): Data are not in particular order.
- Ordered list: data are arranged according to a key.

Linear list concepts

Restricted list:

- Only some operations can be used on the list.
- Data can be inserted/deleted only at the ends of the list.

- Queue: FIFO (First-In-First-Out).
- Stack: LIFO (Last-In-First-Out).

List ADT

Definition

A list of elements of type T is a finite sequence of elements of T.

Basic operations:

- Construct a list, leaving it empty.
- Insert an element.
- Remove an element.
- Search an element.
- Retrieve an element.
- Traverse the list, performing a given operation on each element.

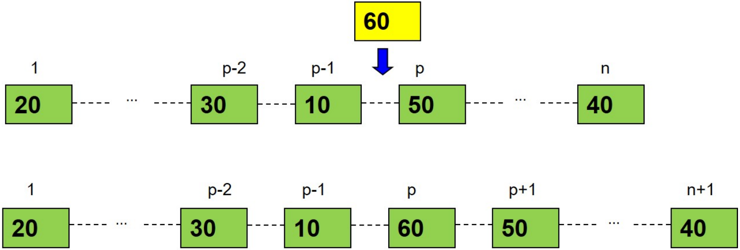
List ADT

Extended operations:

- Determine whether the list is empty or not.
- Determine whether the list is full or not.
- Find the size of the list.
- Clear the list to make it empty.
- Replace an element with another element.
- Merge two ordered list.
- Append an unordered list to another.

Insertion

- **Insert an element at a specified position p in the list**
  - Only with *General Unordered List*.



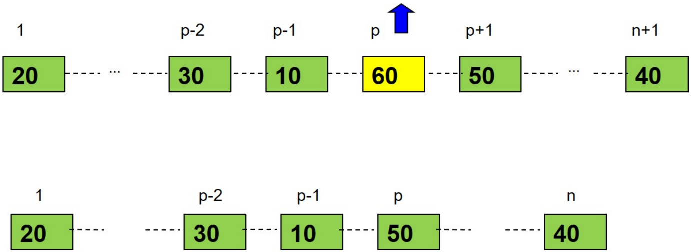
Any element formerly at position p and all later have their position numbers increased by 1.

Insertion

- **Insert an element with a given data**
  - With *General Unordered List*: can be made at any position in the list (at the beginning, in the middle, at the end).
  - With *General Ordered List*: data must be inserted so that the ordering of the list is maintained (searching appropriate position is needed).
  - With *Restricted List*: depend on it own definition (FIFO or LIFO).

Removal

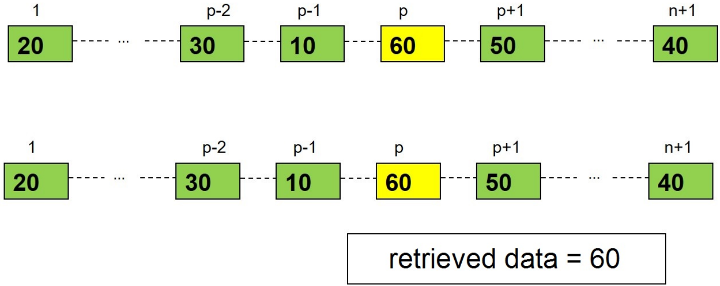
- **Remove an element at a specified position p in the list**
  - With *General Unordered List* and *General Ordered List*.



The element at position p is removed from the list, and all subsequent elements have their position numbers decreased by 1.

Retrieval

- **Retrieve an element at a specified position p in the list**
  - With *General Unordered List* and *General Ordered List*.



All elements remain unchanged.

Removal, Retrieval

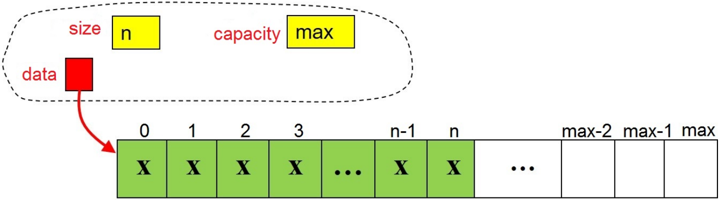
- **Remove/ Retrieve an element with a given data**
  - With *General Unordered List* and *General Ordered List*: Searching is needed in order to locate the data being deleted/ retrieved.

Success of Basic Operations

- **Insertion** is successful when the list is not full.
- **Removal, Retrieval** are successful when the list is not empty.

Array implementation

Dynamically Allocated Array




```
List // Contiguous Implementation of List
  // number of used elements (mandatory)
  size <integer>

  // (Dynamically Allocated Array)
  data <dynamic array of <DataType> >

  capacity <integer>
End List
```

Dynamic Array: Implementation in C++

```
class DynamicArray {
private:
  int size;
  int capacity;
  int *storage;

public:
  DynamicArray() {
    capacity = 10;
    size = 0;
    storage = new int[capacity];
  }
```

Dynamic Array: Implementation in C++

```
DynamicArray(int capacity) {
  this->capacity = capacity;
  size = 0;
  storage = new int[capacity];
}

~DynamicArray() {
  delete[] storage;
}
```

Dynamic Array: Implementation in C++

```
    void setCapacity(int);
    void ensureCapacity(int);
    void pack();
    void trim();

    void rangeCheck(int);
    void set(int, int);
    int get(int);
    void removeAt(int);
    void insertAt(int, int);

    void print();
};
```

Dynamic Array: Implementation in C++

```
void DynamicArray::setCapacity(
int newCapacity) {
  int *newStorage = new int[newCapacity];
  memcpy(newStorage, storage,
sizeof(int) * size);
  capacity = newCapacity;
  delete[] storage;
  storage = newStorage;
}
```

Dynamic Array: Implementation in C++

```
void DynamicArray::ensureCapacity(
int minCapacity) {
  if (minCapacity > capacity) {
    int newCapacity = (capacity*3)/2 + 1;
    if (newCapacity < minCapacity)
      newCapacity = minCapacity;
    setCapacity(newCapacity);
  }
}
```

Dynamic Array: Implementation in C++

```
void DynamicArray::pack() {
  if (size <= capacity / 2) {
    int newCapacity = (size * 3) / 2 + 1;
    setCapacity(newCapacity);
  }
}

void DynamicArray::trim() {
  int newCapacity = size;
  setCapacity(newCapacity);
}
```

Dynamic Array: Implementation in C++

```
void DynamicArray::rangeCheck(int index) {
  if (index < 0 || index >= size)
    throw "Index out of bounds!";
}

void DynamicArray::set(int index,
int value) {
  rangeCheck(index);
  storage[index] = value;
}

int DynamicArray::get(int index) {
  rangeCheck(index);
  return storage[index];
}
```

Dynamic Array: Implementation in C++

```
void DynamicArray::insertAt(int index,
int value) {
  if (index < 0 || index > size)
    throw "Index out of bounds!";

  ensureCapacity(size + 1);

  int moveCount = size - index;
  if (moveCount != 0)
    memmove(storage + index + 1,
storage + index,
sizeof(int) * moveCount);
  storage[index] = value;
  size++;
}
```

Dynamic Array: Implementation in C++

```
void DynamicArray::removeAt(int index) {
  rangeCheck(index);
  int moveCount = size - index - 1;
  if (moveCount > 0)
    memmove(storage + index,
storage + (index + 1),
sizeof(int) * moveCount);
  size--;
  pack();
}
```

Dynamic Array: Using

```
void DynamicArray::print() {
    for (int i=0; i<this->size; i++) {
        cout << storage[i] << " ";
    }
}

int main() {
  cout << "Dynamic Array" << endl;
  DynamicArray* da = new DynamicArray(10);
  da->insertAt(0, 55);
  // ...
  da->print();
  return 0;
}
```

Contiguous Implementation of List

In processing a contiguous list with n elements:

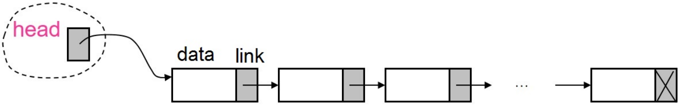
- Insert and Remove operate in time approximately proportional to n (require physical shifting).
- Clear, Empty, Full, Size, Replace, and Retrieve in constant time.

Singly linked list

Linked List

Definition

A linked list is an ordered collection of data in which each element contains the location of the next element.



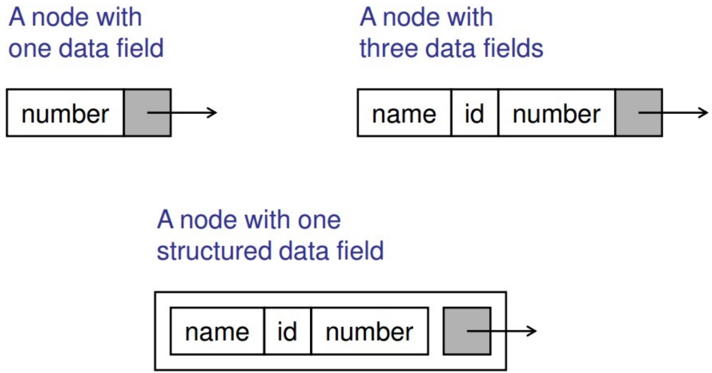
Hình: Singly Linked List

```
list // Linked Implementation of List
  head <pointer>
  count <integer> // number of elements (optional)
end list
```

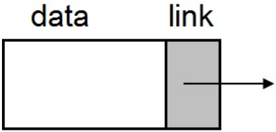
Nodes

The elements in a linked list are called nodes.
A node in a linked list is a structure that has at least two fields:

- the data,
- the address of the next node.



Nodes

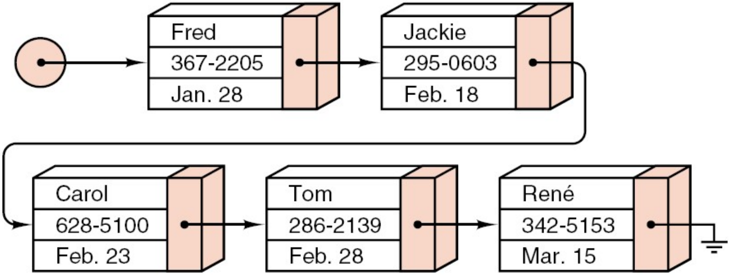


Hình: Linked list node structure

```
node
  data <dataType>
  link <pointer>
end node
```

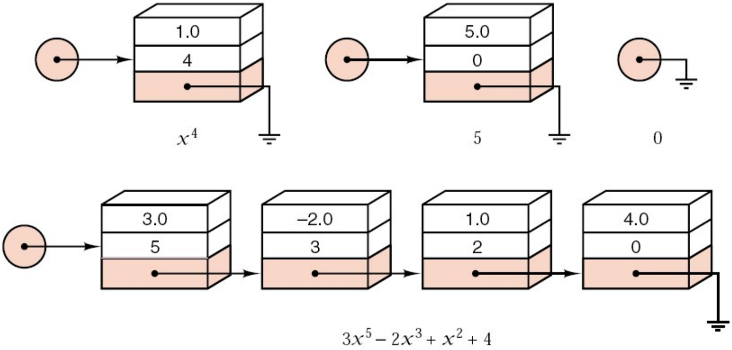
```
// General dataType:
dataType
   key <keyType>
   field1 <...>
   field2 <...>
   ...
   fieldn <...>
end dataType
```

Example



Fred
367-2205
Jan. 28
Jackie
295-0603
Feb. 18
Carol
628-5100
Feb. 23
Tom
286-2139
Feb. 28
René
342-5153
Mar. 15

Example



Hình: List representing polynomial

Implementation in C++

Example

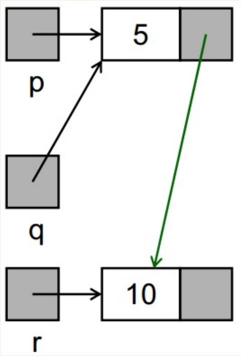
```
node
  data <dataType>
  link <pointer>
end node
```

```cpp
struct Node {
    int data;
    Node *link;
};
```

Implementation in C++

Example

```
#include <iostream>
using namespace std;

struct Node {
    int data;
    Node *link;
};

int main () {
  Node *p = new Node();
  p->data = 5;
  cout<< p->data << endl;
  Node *q = p;
  cout<< q->data << endl;
  Node *r = new Node();
  r->data = 10;
  q->link = r;
  cout<< p->link->data << endl;
}
```



Implementation in C++

Example

```
struct Node {
   int data;
   Node *link;
};
```

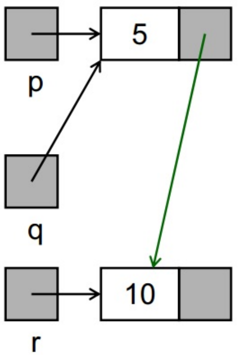
```
struct Node {
   float data;
   Node *link;
};
```

```
template <class ItemType>
struct Node {
   ItemType data;
   Node<ItemType> *link;
};
```

Implementation in C++

Example

```cpp
#include <iostream>
using namespace std;

template <class ItemType>
struct Node {
  ItemType data;
  Node<ItemType> *link;
};

int main () {
  Node<int> *p = new Node<int>();
  p->data = 5;
  cout<< p->data << endl;
  Node<int> *q = p;
  cout<< q->data << endl;
  Node<int> *r = new Node<int>();
  r->data = 10;
  q->link = r;
  cout<< p->link->data << endl;
}
```



Node implementation in C++

```
template <class ItemType>
class Node {
  ItemType data;
  Node<ItemType> *link;

  public:
    Node(){
      this->link = NULL;
    }

    Node(ItemType data){
      this->data = data;
      this->link = NULL;
    }
};
```

Linked list implementation in C++

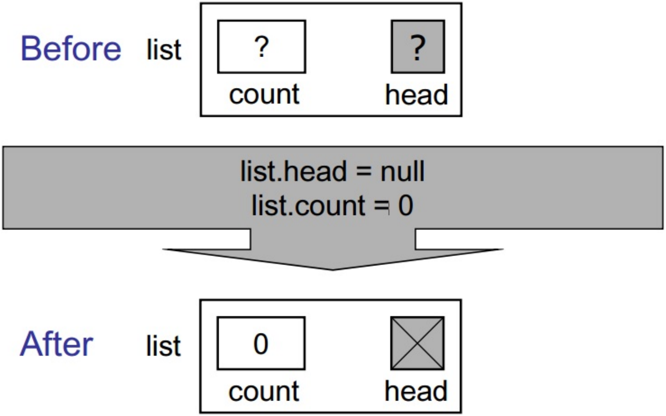
```
template <class List_ItemType>
class LinkedList {
  Node<List_ItemType> *head;
  int count;

  public:
    LinkedList();
    ~LinkedList();
};
```

```
list
  head <pointer>
  count <integer>
end list
```

Linked list operations

- Create an empty linked list
- Insert a node into a linked list
- Delete a node from a linked list
- Traverse a linked list
- Destroy a linked list

Create an empty linked list



Create an empty linked list

Algorithm createList(ref list <metadata>)
Initializes metadata for a linked list
Pre: `list` is a metadata structure passed by reference
Post: metadata initialized
list.head = null
list.count= 0
return
End createList

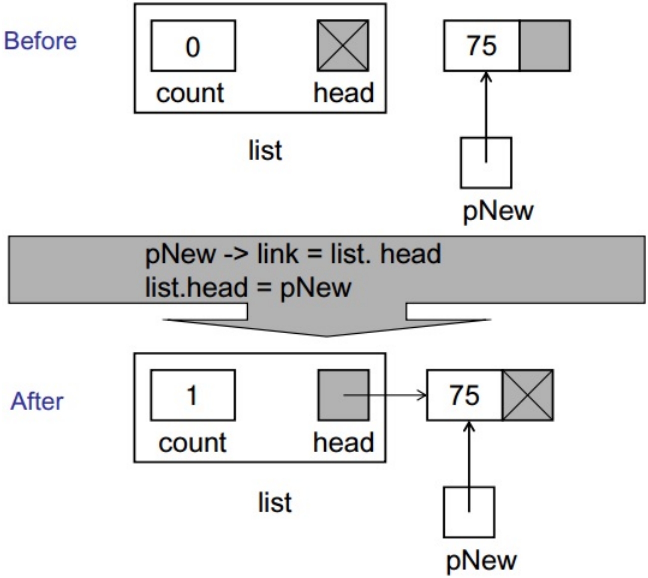
Create an empty linked list

```
template <class List_ItemType>
class LinkedList {
  Node<List_ItemType> *head;
  int count;

  public:
    LinkedList();
    ~LinkedList();
};

template <class List_ItemType>
LinkedList<List_ItemType>::LinkedList(){
  this->head = NULL;
  this->count = 0;
}
```

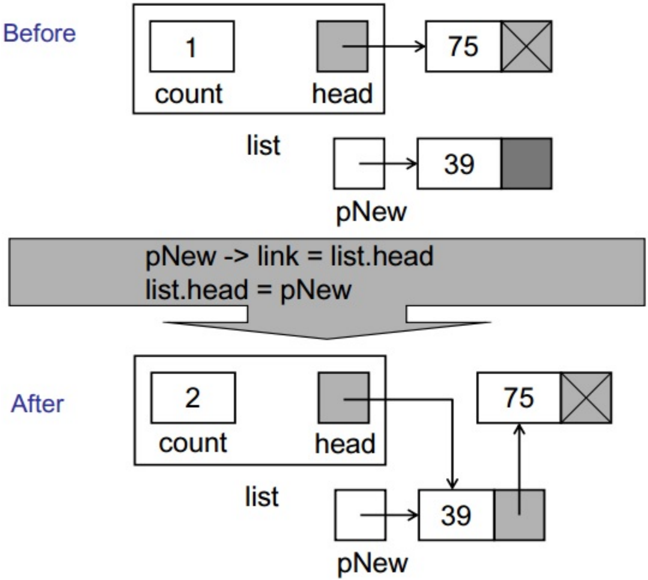
Insert a node into a linked list

1. Allocate memory for the new node and set up data.
2. Locate the pointer p in the list, which will point to the new node:
   - If the new node becomes the first element in the List: `p is list.head`.
   - Otherwise: `p is pPre->link`, where `pPre` points to the predecessor of the new node.
3. Point the new node to its successor.
4. Point the pointer p to the new node.

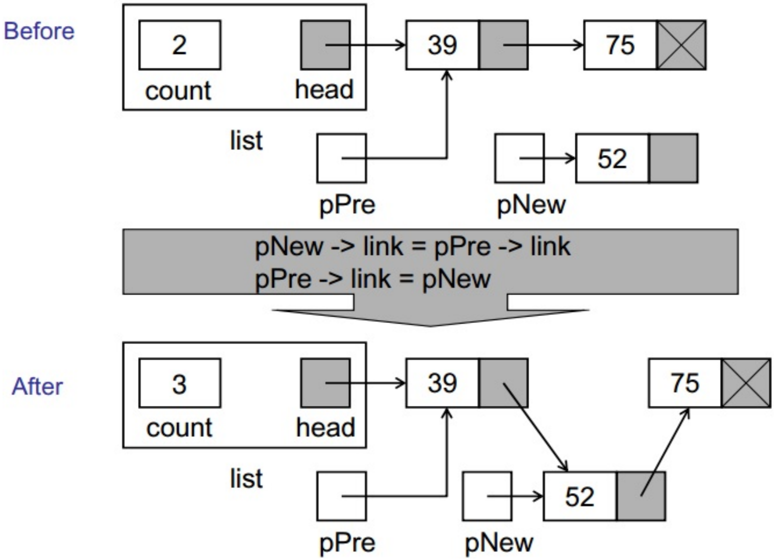
Insert into an empty linked list



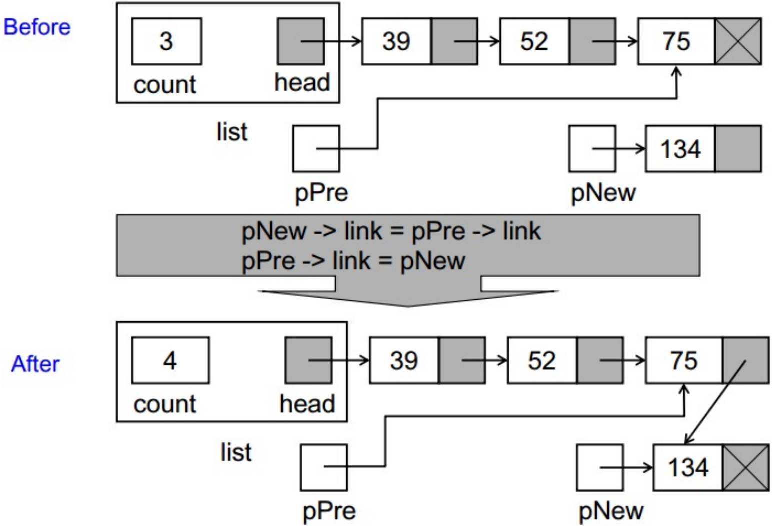
Insert at the beginning



Insert in the middle



Insert at the end



Insert a node into a linked list

- Insertion is successful when allocation memory for the new node is successful.
- There is no difference between insertion at the beginning of the list and insertion into an empty list.

```
pNew->link = list.head
list.head = pNew
```

- There is no difference between insertion in the middle and insertion at the end of the list.

```
pNew->link = pPre->link
pPre->link = pNew
```

Insert a node into a linked list

Algorithm insertNode(ref list <metadata>,
val pPre <node pointer>,
val dataIn <dataType>)

Inserts data into a new node in the linked list.

Pre: `list` is metadata structure to a valid list
`pPre` is pointer to data's logical predecessor
`dataIn` contains data to be inserted

Post: data have been inserted in sequence

Return true if successful, false if memory overflow

Insert a node into a linked list

```
allocate(pNew)
if memory overflow then
    return false
end
pNew -> data = dataIn
if pPre = null then
    // Adding at the beginning or into empty list
    pNew -> link = list.head
    list.head = pNew
else
    // Adding in the middle or at the end
    pNew -> link = pPre -> link
    pPre -> link = pNew
end
list.count = list.count + 1
return true
End insertNode
```

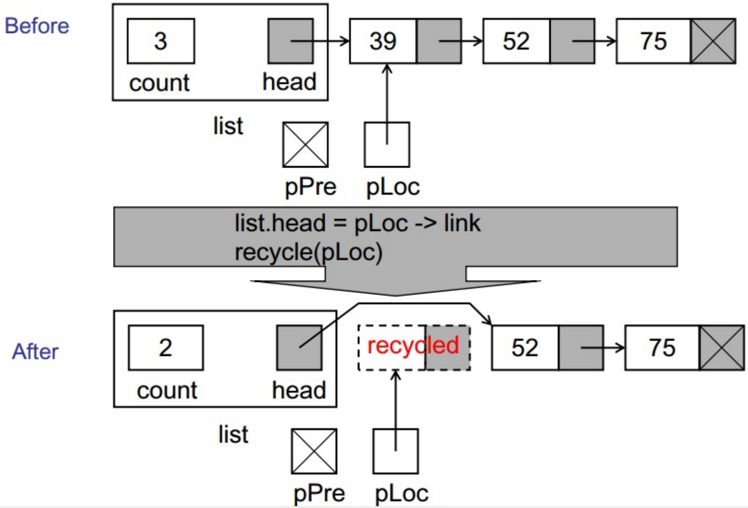
Insert a node into a linked list

```
template<class List_ItemType>
int LinkedList<List_ItemType>::InsertNode(
Node<List_ItemType> *pPre,
List_ItemType value) {
  Node<List_ItemType> *pNew
= new Node<List_ItemType>();
  if (pNew == NULL)
    return 0;
  pNew->data = value;
  if (pPre== NULL){
    pNew->link = this->head;
    this->head = pNew;
  } else {
    pNew->link = pPre->link;
    pPre->link = pNew;
  }
  this->count++;
  return 1;
}
```

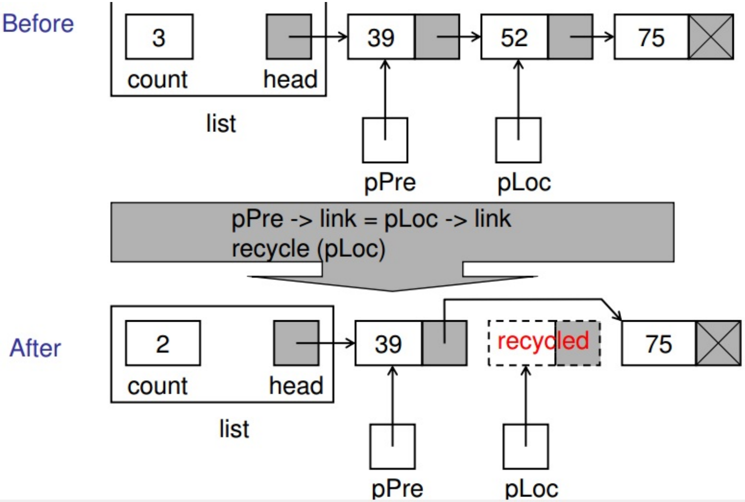
Delete a node from a linked list

1. Locate the pointer p in the list which points to the node to be deleted (pLoc will hold the node to be deleted).
   - If that node is the first element in the List: `p is list.head`.
   - Otherwise: `p is pPre->link`, where `pPre` points to the predecessor of the node to be deleted.
2. `p` points to the successor of the node to be deleted.
3. Recycle the memory of the deleted node.

Delete first node



General deletion case



Delete a node from a linked list

- Removal is successful when the node to be deleted is found.
- There is no difference between deleting the node from the beginning of the list and deleting the only node in the list.

```
list.head = pLoc->link
recycle(pLoc)
```

- There is no difference between deleting a node from the middle and deleting a node from the end of the list.

```
pPre->link = pLoc->link
recycle(pLoc)
```

Delete a node from a linked list

Algorithm deleteNode(ref list <metadata>,
val pPre <node pointer>,
val pLoc <node pointer>,
ref dataOut <dataType>)

Deletes data from a linked list and returns it to calling module.

Pre: `list` is metadata structure to a valid list
`pPre` is a pointer to predecessor node
`pLoc` is a pointer to node to be deleted
`dataOut` is variable to receive deleted data

Post: data have been deleted and returned to caller

Delete a node from a linked list

```
dataOut = pLoc -> data
if pPre = null then
    // Delete first node
    list.head = pLoc -> link
else
    // Delete other nodes
    pPre -> link = pLoc -> link
end
list.count = list.count - 1
recycle (pLoc)
return
End deleteNode
```

Delete a node from a linked list

```
template<class List_ItemType>
List_ItemType LinkedList<List_ItemType>::
DeleteNode(Node<List_ItemType> *pPre,
Node<List_ItemType> *pLoc) {
  List_ItemType result = pLoc->data;

  if (pPre== NULL){
    this->head = pLoc->link;
  } else {
    pPre->link = pLoc->link;
  }

  this->count--;
  delete pLoc;
  return result;
}
```

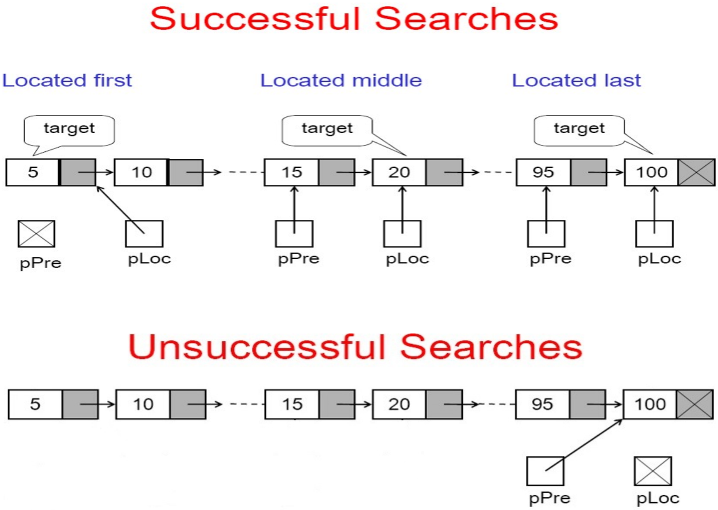
Searching in a linked list

- Sequence Search has to be used for the linked list.
- Function Search of List ADT:

  ```
  <ErrorCode> Search (val target <dataType>,
  ref pPre <pointer>, ref pLoc <pointer>)
  ```

  Searches a node and returns a pointer to it if found.

Searching in a linked list



Searching in a linked list

Algorithm Search(val target <dataType>,
ref pPre <node pointer>,
ref pLoc <node pointer>)

Searches a node in a singly linked list and return a pointer to it if found.

Pre: `target` is the value need to be found

Post: `pLoc` points to the first node which is equal target, or is NULL if not found.
`pPre` points to the predecessor of the first node which is equal target, or points to the last node if not found.

Return `found` or `notFound`

Searching in a linked list

```
pPre = NULL
pLoc = list.head
while (pLoc is not NULL) AND (target != pLoc ->data) do
    pPre = pLoc
    pLoc = pLoc ->link
end
if pLoc is NULL then
    return notFound
else
    return found
end
End Search
```

Searching in a linked list

```
template<class List_ItemType>
int LinkedList<List_ItemType>::Search(
List_ItemType value,
Node<List_ItemType>* &pPre,
Node<List_ItemType>* &pLoc){
  pPre = NULL;
  pLoc = this->head;
  while (pLoc != NULL && pLoc->data != value){
    pPre = pLoc;
    pLoc = pLoc->link;
  }
  return (pLoc != NULL);
  // found: 1; notfound: 0
}
```

Traverse a linked list

Traverse module controls the loop: calling a user-supplied algorithm to process data

Algorithm Traverse(ref <void> process (ref Data <DataType>))
Traverses the list, performing the given operation on each element.
Pre: `process` is user-supplied
Post: The action specified by `process` has been performed on every element in the list, beginning at the first element and doing each in turn.

pWalker = list.head
while *pWalker not null* **do**
    process(pWalker -> data)
    pWalker = pWalker -> link
end
End Traverse

Traverse a linked list

```
template<class List_ItemType>
void LinkedList<List_ItemType>::Traverse() {
  Node<List_ItemType> *p = head;
  while (p != NULL){
    p->data++; // process data here!!!
    p = p->link;
  }
}
template<class List_ItemType>
void LinkedList<List_ItemType>::
        Traverse2(List_ItemType *&visit){
  Node<List_ItemType> *p = this->head;
  int i = 0;
  while (p != NULL && i < this->count){
    visit[i] = p->data;
    p = p->link;
    i++;
  }
}
```

Destroy a linked list

Algorithm destroyList (val list <metadata>)
Deletes all data in list.

Pre: `list` is metadata structure to a valid list
Post: all data deleted
while *list.head not null* **do**
    dltPtr = list.head
    list.head = this.head -> link
    recycle (dltPtr)
end
No data left in list. Reset metadata
list.count = 0
return
End destroyList

Destroy a linked list

```
template<class List_ItemType>
void LinkedList<List_ItemType>::Clear(){
  Node<List_ItemType> *temp;
  while (this->head != NULL){
    temp = this->head;
    this->head = this->head->link;
    delete temp;
  }
  this->count = 0;
}

template<class List_ItemType>
LinkedList<List_ItemType>::~LinkedList(){
  this->Clear();
}
```

Linked list implementation in C++

```
template<class List_ItemType>

class LinkedList{

  Node<List_ItemType>* head;
  int count;

protected:
  int InsertNode(Node<List_ItemType>* pPre,
List_ItemType value);

  List_ItemType DeleteNode(Node<List_ItemType>* pPre,
Node<List_ItemType>* pLoc);

  int Search(List_ItemType value,
Node<List_ItemType>* &pPre,
Node<List_ItemType>* &pLoc);

};
```

Linked list implementation in C++

```
template<class List_ItemType>
class LinkedList{

protected:
  // ...

public:
  LinkedList();
  ~LinkedList();

  void InsertFirst(List_ItemType value);
  void InsertLast(List_ItemType value);
  int InsertItem(List_ItemType value, int position);

  void DeleteFirst();
  void DeleteLast();
  int DeleteItem(int postion);
  int GetItem(int position, List_ItemType &dataOut);

  void Traverse();
  LinkedList<List_ItemType>* Clone();
  void Print2Console();
  void Clear();
};
```

Linked list implementation in C++

How to use Linked List data structure?

```
int main(int argc, char* argv[]) {
  LinkedList<int>* myList =
new LinkedList<int>();
  myList->InsertFirst(15);
  myList->InsertFirst(10);
  myList->InsertFirst(5);
  myList->InsertItem(18,3);
  myList->InsertLast(25);
  myList->InsertItem(20,3);
  myList->DeleteItem(2);
  cout << "List␣1:" << endl;
  myList->Print2Console();
```

Linked list implementation in C++

How to use Linked List data structure?

```
  // ...

  int value;
  LinkedList<int>* myList2 = myList->Clone();
  cout << "List␣2:" << endl;
  myList2->Print2Console();
  myList2->GetItem(1, value);
  cout << "Value␣at␣position␣1:␣" << value;

  delete myList;
  delete myList2;
  return 1;
}
```

Sample Solution: Insert

```
template <class List_ItemType>
int LinkedList<List_ItemType>::InsertItem(
List_ItemType value, int position) {

  if (position < 0 || position > this->count)
    return 0;

  Node<List_ItemType> *newPtr, *pPre;
  newPtr = new Node<List_ItemType>();
  if (newPtr == NULL)
    return 0;

  newPtr->data = value;

  if (head == NULL) {
    head = newPtr;
    newPtr->link = NULL;
  } else if (position == 0) {
    newPtr->link = head;
    head = newPtr;
  }
```

Sample Solution: Insert

```
  else {
    // Find the position of pPre
    pPre = this->head;

    for (int i = 0; i < position-1; i++)
      pPre = pPre->link;

    // Insert new node
    newPtr->link = pPre->link;
    pPre->link = newPtr;
  }

  this->count++;
  return 1;
}
```

Sample Solution: Delete

```
template <class List_ItemType>
int LinkedList<List_ItemType>::DeleteItem(int position){
  if (position < 0 || position > this->count)
    return 0;

  Node<List_ItemType> *dltPtr, *pPre;

  if (position == 0) {
    dltPtr = head;
    head = head->link;
  } else {
    pPre= this->head;
    for (int i = 0; i < position-1; i++)
      pPre = pPre->link;
    dltPtr = pPre->link;
    pPre->link = dltPtr->link;
  }

  delete dltPtr;
  this->count--;
  return 1;
}
```

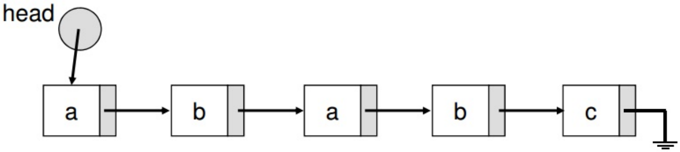
Sample Solution: Clone

```
template <class List_ItemType>
LinkedList<List_ItemType>*
LinkedList<List_ItemType>::Clone(){
  LinkedList<List_ItemType>* result =
new LinkedList<List_ItemType>();

  Node<List_ItemType>* p = this->head;

  while (p != NULL) {
    result->InsertLast(p->data);
    p = p->link;
  }

  result->count = this->count;
  return result;
}
```

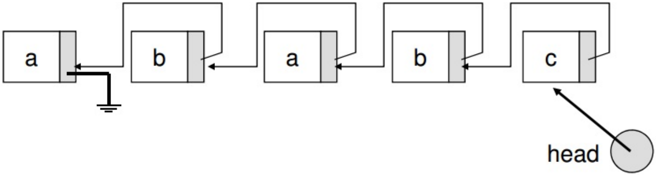
Reverse a linked list

Exercise

```
template <class List_ItemType>
void LinkedList<List_ItemType>::Reverse(){
  // ...
}
```

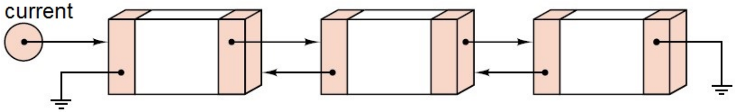


Result:



Other linked lists

Doubly Linked List

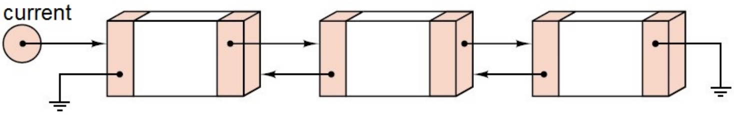


Hình: Doubly Linked List allows going forward and backward.

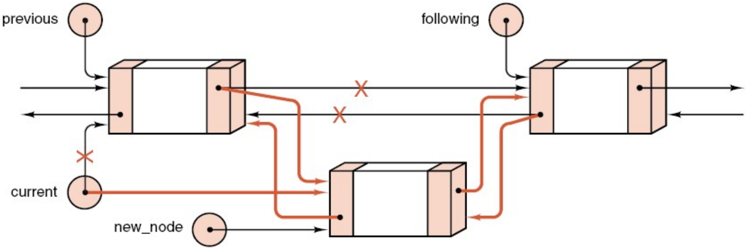
```
node
  data <dataType>
  next <pointer>
  previous <pointer>
end node
```

```
list
  current <pointer>
end list
```

Doubly Linked List

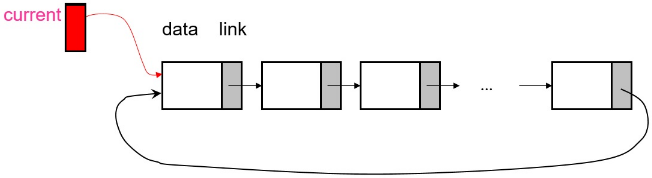


Hình: Doubly Linked List allows going forward and backward.



Hình: Insert an element in Doubly Linked List.

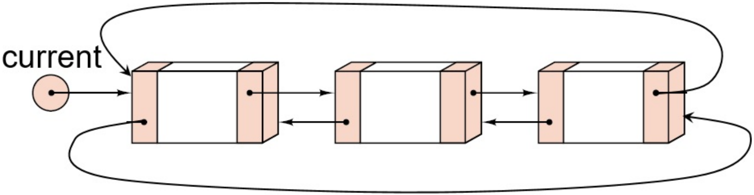
Circularly Linked List




```
node
  data <dataType>
  link <pointer>
end node
```

```
list
  current <pointer>
end list
```

Double circularly Linked List




```
node
  data <dataType>
  next <pointer>
  previous <pointer>
end node
```

```
list
  current <pointer>
end list
```

Comparison of implementations of list

Arrays: Pros and Cons

- **Pros**:
  - Access to an array element is fast since we can compute its location quickly.
- **Cons**:
  - If we want to insert or delete an element, we have to shift subsequent elements which slows our computation down.
  - We need a large enough block of memory to hold our array.

Linked Lists: Pros and Cons

- **Pros**:
  - Inserting and deleting data does not require us to move/shift subsequent data elements.
- **Cons**:
  - If we want to access a specific element, we need to traverse the list from the head of the list to find it which can take longer than an array access.

Comparison of implementations of list

- **Contiguous storage is generally preferable when**:
  - the entries are individually very small;
  - the size of the list is known when the program is written;
  - few insertions or deletions need to be made except at the end of the list; and
  - random access is important.
- **Linked storage proves superior when**:
  - the entries are large;
  - the size of the list is not known in advance; and
  - flexibility is needed in inserting, deleting, and rearranging the entries.