3. (a) Write down the formulas of the Viterbi algorithm using $\mathbf{z}_i$ and $(\hat{x}_i, \hat{y}_i), i = 0, 1, \ldots, N-1$. Your answer should contain the initialization of the messages and the recursion of the messages in the Viterbi algorithm. (1 **pt**)

Initialization

$$w_1(z_1) = \log(p(z_1)p(o_1|z_1))$$

Recursion

$$w_i(z_i) = \log(p(o_i|z_i)) + \max_{z_{i-1}} \Big\{ \log(p(z_i|z_{i-1})) + w_{i-1}(z_{i-1}) \Big\}$$

$$\phi_i(z_i) = \text{argmax}_{z_{i-1}} \Big[\log(p(z_{i-1}|z_i)) + w_{i-1}(z_{i-1}) \Big]$$

(b) After you run the Viterbi algorithm on the data in test_missing.txt, write down the last 10 hidden states of the most likely sequence (i.e., $i = 90, 91, 92, \ldots, 99$) based on the MAP estimate. (3 **pt**)

Last 10 hidden states in the MAP estimate:
(11, 5, 'down')
(11, 6, 'down')
(11, 7, 'down')
(11, 7, 'stay')
(11, 7, 'stay')
(10, 7, 'left')
(9, 7, 'left')
(8, 7, 'left')
(7, 7, 'left')
(6, 7, 'left')

4. Compute and compare the error probabilities of $\{\tilde{\mathbf{z}}_i\}$ and $\{\check{\mathbf{z}}_i\}$ using the data in test_missing.txt. The error probability of $\{\tilde{\mathbf{z}}_i\}$ is 0.04. The error probability of $\{\check{\mathbf{z}}_i\}$ is 0.03. (1 **pt**)

5. Is sequence $\{\check{\mathbf{z}}_i\}$ a valid sequence? If not, please find a small segment $\check{\mathbf{z}}_i, \check{\mathbf{z}}_{i+1}$ that violates the transition model for some time step i. You answer should specify the value of i as well as the corresponding states $\check{\mathbf{z}}_i, \check{\mathbf{z}}_{i+1}$. (1 **pt**)

No, the transition from i=64 to 65 is invalid.
Z64: {(3, 7, 'stay'): 0.500532232730999, (2, 7, 'left'): 0.499467767269001}

Z65: {(3, 7, 'stay'): 0.47346666745026494, (2, 7, 'stay'): 0.499467767269001,
(3, 6, 'up'): 0.02706556528073409}