

$$\textcircled{1} \omega_0 = 0 \quad t_0 = 0 \rightarrow \omega_0 = C_2 e^{-t/\tau} + C_1$$

$$0 = C_2 e^0 + C_1 \rightarrow C_1 = -C_2 \rightarrow \omega_0 = -C_2 e^{-t/\tau} + C_2$$

So... $\omega_0 = C_1 (1 - e^{-t/\tau})$

$$J_{eq}(\dot{\omega}_m) + b_{eq}(\omega_m) = \frac{K_t}{R_a} V_a(s)$$

↓ Laplace transform

$$J_{eq}(s\omega(s) - \omega(0)) + b_{eq}\omega(s) = \frac{K_t}{R_a} V_a(s) \rightarrow V_a(s) = \frac{1}{s}$$

$$\omega(s) = \frac{\frac{K_t}{R_a}}{J_{eq}s + b_{eq}} \cdot \frac{1}{s} = \frac{A}{s} + \frac{B}{J_{eq}s + b_{eq}}$$

$$\frac{K_t}{R_a} = A_{b_{eq}} \rightarrow A = \frac{K_t}{R_a b_{eq}} \quad ; \quad B = -\frac{K_t J_{eq}}{R_a b_{eq}}$$

$$\frac{\frac{K_t}{R_a b_{eq}}}{s} - \frac{\frac{K_t J_{eq}}{R_a b_{eq}}}{J_{eq}s + b_{eq}} \rightarrow \frac{K_t}{R_a b_{eq}} \left(\frac{1}{s + \frac{b_{eq}}{J_{eq}}} \right)$$

Laplace Inverse:

$$\frac{K_t}{R_a b_{eq}} - \frac{K_t}{R_a b_{eq}} e^{-t \frac{b_{eq}}{J_{eq}}} \rightarrow \text{Max Velocity} = \frac{K_t}{R_a b_{eq}} = C_1 \quad ; \quad \tau = \frac{J_{eq}}{b_{eq}}$$

$$(1 - e^{-t/\tau}) \left(\frac{K_t}{R_a b_{eq}} \right) = \omega \text{ @ } 63.2\% \text{ where } t \approx \tau$$

$$b_{eq} = (1 - e^{-1}) \left(\frac{K_t}{R_a \omega} \right)$$