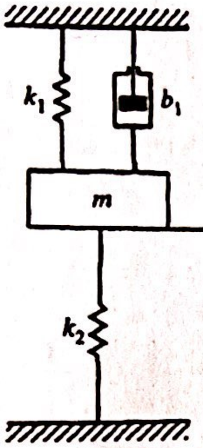
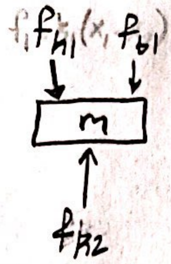
ME 421 Homework #2

Due via Canvas: 11:59pm, Friday, 26th, 2021

1. (20pts) Develop the model of the vertical spring-mass-damper system, assuming the static equilibrium point is the datum point for displacement x(t).





Equations assume initial position is zero and zero gravity

$$f_{k1} = -k_1 (x(t))$$

$$f_{b1} = -b_1 (x'(t))$$

$$f_{k2} = k_2 (x(t))$$

$$m * x''(t) = f_{k1} + f_{b1} - f_{k2}$$

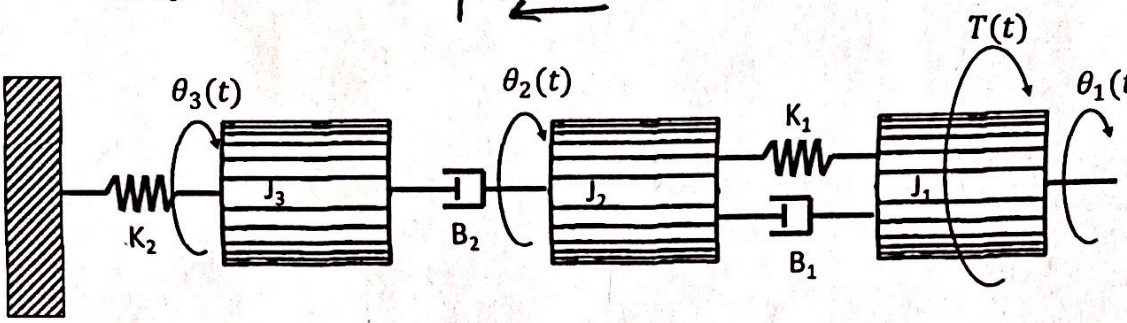
$$m * x''(t) = -k_1 * x(t) - b_1 x'(t) - k_2 x(t)$$

or

$$\boxed{m\ddot{x} = -k_1 x - b_1 \dot{x} - k_2 x}$$

2. (20pts) Obtain a mathematical model of the rotational motion system shown below. The moment of the inertia for the three rotors are J_1, J_2, and J_3. A torque $T(t)$ is applied to J_1. The angular displacements (rotational angles) of the three rotors are marked as θ_1, θ_2, and θ_3.

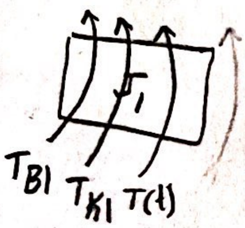
T^{+}



$$T_{k2} = k_2(-\theta_3(t)) \qquad T_{B2} = B_2(\dot{\theta}_3(t) - \dot{\theta}_2(t))$$

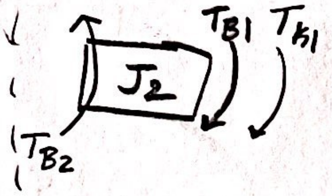
$$T_{k1} = k_1(\theta_2(t) - \theta_1(t)) \qquad T_{B_1} = B_1(\dot{\theta}_2(t) - \dot{\theta}_1(t))$$

Solving Cylinders ↓



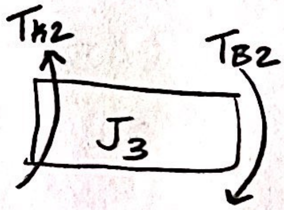
$$J_1\ddot{\theta}_1 = T_{B1} + T_{k1} + T(t)$$

$$J_1\ddot{\theta}_1 = B_1(\dot{\theta}_2 - \dot{\theta}_1) + k_1(\theta_2 - \theta_1) + T$$



$$J_2\ddot{\theta}_2 = T_{B2} - T_{k1} - T_{B1}$$

$$= B_2(\dot{\theta}_3 - \dot{\theta}_2) + k_1(\theta_1 - \theta_2) + B_1(\dot{\theta}_1 - \dot{\theta}_2)$$



$$J_3\ddot{\theta}_3 = T_{k2} - T_{B2}$$

$$= k_2(-\theta_3) + B_2(\dot{\theta}_2 - \dot{\theta}_3)$$

Math Model:

① $J_1\ddot{\theta}_1 = B_1(\dot{\theta}_2 - \dot{\theta}_1) + k_1(\theta_2 - \theta_1) + T$

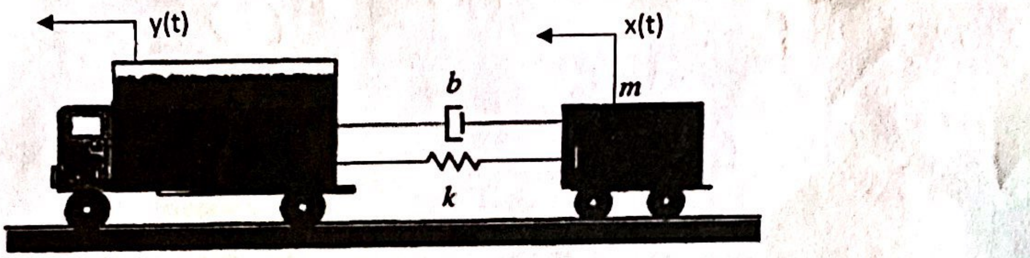
② $J_2\ddot{\theta}_2 = B_2(\dot{\theta}_3 - \dot{\theta}_2) + k_1(\theta_1 - \theta_2) + B_1(\dot{\theta}_1 - \dot{\theta}_2)$

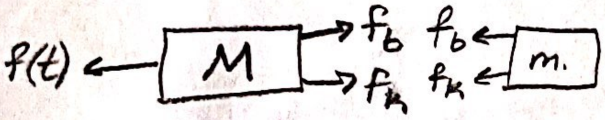
③ $J_3\ddot{\theta}_3 = B_2(\dot{\theta}_2 - \dot{\theta}_3) - k_2(\theta_3)$

3. (20pts) A cart of mass m is attached to a truck of mass M using a spring of stiffness k and a damper of constant b, as shown in the figure below. The traction force applied to the truck is f(t). Both the truck and the cart begin to move from a parking position. Assume the road friction can be ignored.

(a) (10pts) Develop the transfer function of the system with f(t) as input and x(t) as output.

(b) (10pts) Develop the transfer function of the system with f(t) as input and y(t) as output.





$f_b = B(\dot{y}_1 - \dot{x}_2)$

$f_k = k(y_1 - x_2)$

① $M\ddot{y} = f - f_b - f_k = f - B(\dot{y} - \dot{x}) - k(y - x)$

a.) ② $m\ddot{x} = f_b + f_k = B(\dot{y} - \dot{x}) + k(y - x)$

$\mathcal{L}\{①\} = Ms^2 Y(s) + BsY(s) + kY(s) = F(s) + BsX(s) + kX(s) \leftarrow ③$

$\mathcal{L}\{②\} = ms^2 X(s) + BsX(s) + kX(s) = BsY(s) + kY(s) \leftarrow ④$

④ → $Y(s) = \frac{(ms^2 + Bs + k)X(s)}{(Bs + k)} \xrightarrow{\text{Put in ③}} \frac{(Ms^2 + Bs + k)(ms^2 + Bs + k)X(s)}{(Bs + k)} = F(s) + (Bs + k)X(s)$

$F(s) = \left(\frac{(Ms^2 + Bs + k)(ms^2 + Bs + k) - (Bs + k)^2}{(Bs + k)}\right)X(s)$

a.) $\boxed{\frac{X(s)}{F(s)} = \frac{(Bs + k)}{(Ms^2 + Bs + k)(ms^2 + Bs + k) - (Bs + k)^2} = G(s)}$

b.) ④ → $X(s) = \dfrac{(Bs+k)Y(s)}{ms^2+Bs+k}$ substitute into

③ → $(Ms^2+Bs+k)Y(s) = F(s) + (Bs+k)X(s)$

$$\downarrow$$

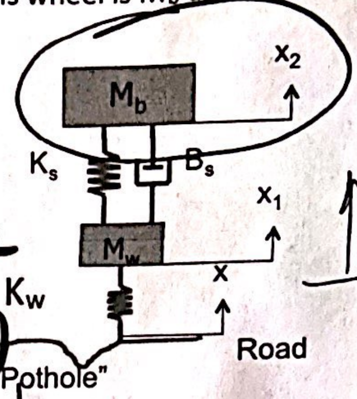
$$(Ms^2+Bs+k)Y(s) = F(s) + \frac{(Bs+k)^2 Y(s)}{ms^2+Bs+k}$$

$$\left((Ms^2+Bs+k) - \frac{(Bs+k)^2}{ms^2+Bs+k}\right)Y(s) = F(s)$$

$$\frac{(Ms^2+Bs+k)(ms^2+Bs+k)-(Bs+k)^2}{ms^2+Bs+k} = \frac{F(s)}{Y(s)}$$

$$\boxed{G(s) = \frac{Y(s)}{F(s)} = \frac{ms^2+Bs+k}{(Ms^2+Bs+k)(ms^2+Bs+k)-(Bs+k)^2}}$$

4. (40pts) The equivalent model for one wheel of a pickup truck is illustrated in the following figure. The mass of the vehicle that is distributed on this wheel is M_b and the mass of the wheel is M_w. The suspension spring has a spring constant K_s and the tire has a spring constant of K_w. The damping constant of the shock absorber is B_s.



(a) (15pts) Derive the transfer function of the system with x(t) as input and x_2(t) as output.

$$G(s) = \frac{X_2(s)}{X(s)} \Rightarrow ?$$ unit impulse response of G(s)

(b) (15pts) Using MATLAB <u>plot the displacement of the truck $x_2(t)$ for $0<t<20$s when the truck runs through a pothole of depth of 0.1m for each set of model parameters given below.</u> Include the **MATLAB CODE** and the plot in your homework submission. <u>Label the x-axis and y-axis of your plot.</u>

gives what is $X_2(t)$, when $x(t) = \delta(t)$

Assume

Set 1. M_w = 50kg, M_b = 400kg, K_w = 200,000 N/m, K_s = 20,000 N/m and B_s = 1,000 N-s/m.
Set 2. M_w = 50kg, M_b = 600kg, K_w = 200,000 N/m, K_s = 20,000 N/m and B_s = 1,000 N-s/m.
Set 3. M_w = 50kg, M_b = 600kg, K_w = 200,000 N/m, K_s = 20,000 N/m and B_s = 10, 000 N-s/m.

<u>Hint: use the impulse response function in MATLAB. You can use the MATLAB code "ExampleMatlabCodeSimulatingSystemResponses" on Canvas.</u>

(c) (10pts) Based on the plots in (a) comment on how weight of the car body Mb AND the damping constant of the shock absorber Bs affect the amplitude and duration of the vertical vibration of the car.

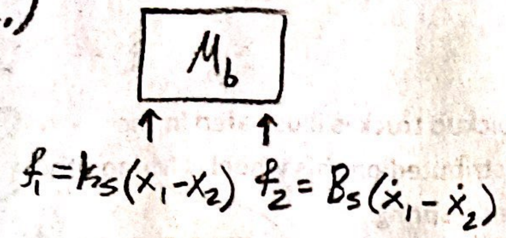
$x(t) = \delta(t)$

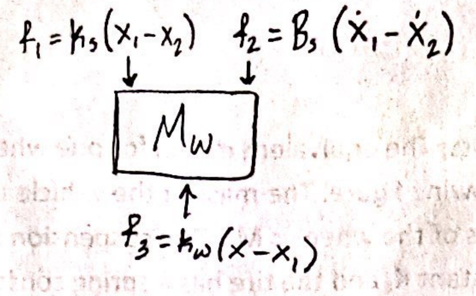
1m

$x(t) = -0.1 \cdot \delta(t) \Rightarrow X_2(t) = ?$

0.1m

4.) a.)





$$M_b \ddot{x}_2 = f_1 + f_2 = k_s(x_1 - x_2) + B_s(\dot{x}_1 - \dot{x}_2) \leftarrow ①$$

$$M_w \ddot{x}_1 = f_3 - f_1 - f_2 = k_w(x - x_1) - k_s(x_1 - x_2) - B_s(\dot{x}_1 - \dot{x}_2) \leftarrow ②$$

$$\mathcal{L}\{①\} = M_b s^2 X_2(s) + k_s X_2(s) + B_s X_2(s) = k_s X_1(s) + B_s X_1(s)$$

$$= (M_b s^2 + k_s + B_s) X_2(s) = (k_s + B_s) X_1(s)$$

$$X_1(s) = \frac{(M_b s^2 + k_s + B_s*s)}{(k_s + B_s*s)} X_2(s) \leftarrow ③$$

$$\mathcal{L}\{②\} = M_w s^2 X_1(s) + k_s X_1(s) + (B_s*s) X_1(s) + k_w X_1(s) = k_w X(s) + k_s X_2(s) + (B_s*s) X_2(s)$$

$$X_1(s)(M_w s^2 + k_s + B_s*s + k_w) = k_w X(s) + (k_s + B_s*s) X_2(s) \leftarrow ④$$

③ into ④

$$\left(\frac{(M_b s^2 + k_s + B_s*s)(M_w s^2 + k_s + B_s*s + k_w)}{(k_s + B_s*s)(k_w)}\right) X_2(s) = \cancel{(k_w)} X(s) + \frac{(k_s + B_s*s)}{k_w} X_2(s)$$

$$\left(\frac{(M_b s^2 + k_s + B_s*s)(M_w s^2 + k_s + B_s*s + k_w)}{(k_s k_w + B_s k_w *s)} - \frac{(k_s + B_s*s)(k_s + B_s*s)}{k_w(k_s + B_s*s)}\right) X_2(s) = X(s)$$

$$\boxed{G(s) = \frac{X_2(s)}{X(s)} = \frac{(k_s + B_s*s) k_w}{(M_b s^2 + k_s + B_s*s)(M_w s^2 + k_s + B_s*s + k_w) - (k_s + B_s*s)^2}}$$