A Category-Theoretic Framework for Wildfire Suppression Planning

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Abstract

Wildfire suppression is a complex coordination problem involving ground crews, helicopters, fixed-wing aircraft, retardant, and time-sensitive environmental dynamics. While existing fire suppression doctrine provides tactics and modern AI systems can generate candidate actions, neither ensures that those actions compose into a coherent, resource-feasible suppression plan. Category theory provides a formal language for representing wildfire suppression as a network of design problems (DPs) with monotone provides/requires relationships. Using the MCDP solver, we construct a compositional model and compute minimal resource allocations required to reach target containment performance. This document formalizes the posets, design problems, and compositional structure needed to model wildfire suppression in a mathematically explicit way.

Contents

1 Introduction

The purpose of this project is to build a category-theoretic model that checks whether a candidate Course of Action (COA) is structurally coherent and resource-feasible. Real fire suppression efforts often fail not because actions do not exist, but because individual actions do not compose into a safe, resource-feasible, domain-consistent plan.

In this framework, category theory is not used to generate plans; it is used to determine whether proposed plans are valid with respect to explicit structure: resources, situation parameters, and containment objectives. The MCDP (Monotone Co-Design Problems) solver then evaluates resource trade-offs and identifies minimal feasible allocations that meet a desired containment performance. The tradespace visualized later is the direct result of this structure: each Pareto point corresponds to a feasible composition of air, ground, and logistics subsystems that satisfies all monotone constraints.

2 Problem Statement

Given a wildfire situation described by fire size and spread index, and given resources such as crew-hours, helicopter hours, airtanker hours, and retardant, we want to:

1. determine whether a candidate sequence of suppression actions forms a valid, compositional plan, and
2. compute the minimal resources required to achieve a desired containment probability.

The category-theoretic viewpoint enforces that every COA is represented as a composition of design problems with compatible provides/requires interfaces. MCDP is then used to solve for Pareto-minimal resource allocations consistent with the required level of performance.

3 Posets

We model resources, functionalities, situation parameters, and figures of merit as partially ordered sets (posets).

Definition 1 (Poset)**.** *A partially ordered set (poset) is a pair $(P, \leq)$ where $\leq$ is a binary relation that is reflexive, antisymmetric, and transitive.*

3.1 Resource posets

Resources are quantities the incident commander can allocate. We model them as posets where "more" is never worse:

- Crew-hours: $R_{\text{crew}} = (\mathbb{R}_{\geq 0}, \leq)$.
- Helicopter flight-hours: $R_{\text{hel}} = (\mathbb{R}_{\geq 0}, \leq)$.

- Airtanker flight-hours: $R_{\text{air}} = (\mathbb{R}_{\geq 0}, \leq)$.
- Retardant volume: $R_{\text{ret}} = (\mathbb{R}_{\geq 0}, \leq)$.

The full resource space is the product poset

$$R = R_{\text{crew}} \times R_{\text{hel}} \times R_{\text{air}} \times R_{\text{ret}},$$

ordered componentwise:

$$(r_1, r_2, r_3, r_4) \leq (r_1', r_2', r_3', r_4') \iff r_i \leq r_i' \text{ for all } i.$$

3.2 Functionality posets

Functionalities represent what the suppression effort accomplishes. For this version we focus on a single primary functionality:

- Containment probability: $F_{\text{cont}} = ([0, 1], \leq)$, where higher values indicate better containment performance.

3.3 Situation posets

Situation parameters describe the external conditions that drive fire behavior:

- Fire size: $S_{\text{size}} = (\mathbb{R}_{\geq 0}, \leq)$.
- Spread index: $S_{\text{spread}} = (\mathbb{R}_{\geq 0}, \leq)$.

4 One-Step Wildfire Decision Structure

Before detailing the figures of merit and individual design problems, it is useful to make explicit the logical structure of a single suppression decision step. We consider a one-step decision structure that maps:

$$(\text{situation}, \text{resources}, \text{COA}) \longrightarrow \text{outcomes}.$$

4.1 Inputs

At a given planning step, the incident commander observes or estimates:

- a situation state $(S_{\text{size}}, S_{\text{spread}})$ capturing fire size and spread potential;
- a resource budget $r \in R$ representing available crew-hours, aviation hours, and retardant;
- a candidate Course of Action (COA), which specifies how the available resources will be deployed (for example, how many crew-hours go to direct line construction versus indirect line, which drops are used for point protection, and so on).

In the category-theoretic framing, the COA is implicitly encoded by the choice of which design problems are activated and how their interfaces are wired. Different COAs correspond to different compositions of the same primitive building blocks.

4.2 Internal mapping

The one-step decision structure factors into a sequence of internal maps:

1) **Assessment:** an assessment process that maps the current situation (possibly improved by reconnaissance resources) to effective fire size and spread parameters.

2) **Suppression production:** a mapping from resource allocations to fireline length and suppression effects (for example, installed control line in meters and abstractThat's why it looks broken in the suppression index).

3) **Effect combination:** a mapping that converts line and suppression into an effective control metric, such as total effective line relative to the perimeter that must be secured.

4) **Containment performance:** a mapping from effective control and situation parameters to containment probability and other outcome metrics.

Each of these maps is implemented by one or more design problems, which we make explicit in the next sections.

4.3 Outputs

The output of the one-step structure is a tuple of figures of merit, including at minimum:

- containment probability at the end of the planning step,
- time to containment (if achievable),
- resource usage profile.

This one-step structure can, in principle, be iterated or embedded into a larger dynamic decision process, but even at a single step it provides a compositional, checkable mapping from situation and resources to performance.

4.4 Diagram

Figure 1 summarizes the one-step decision structure as a composition of design problems.

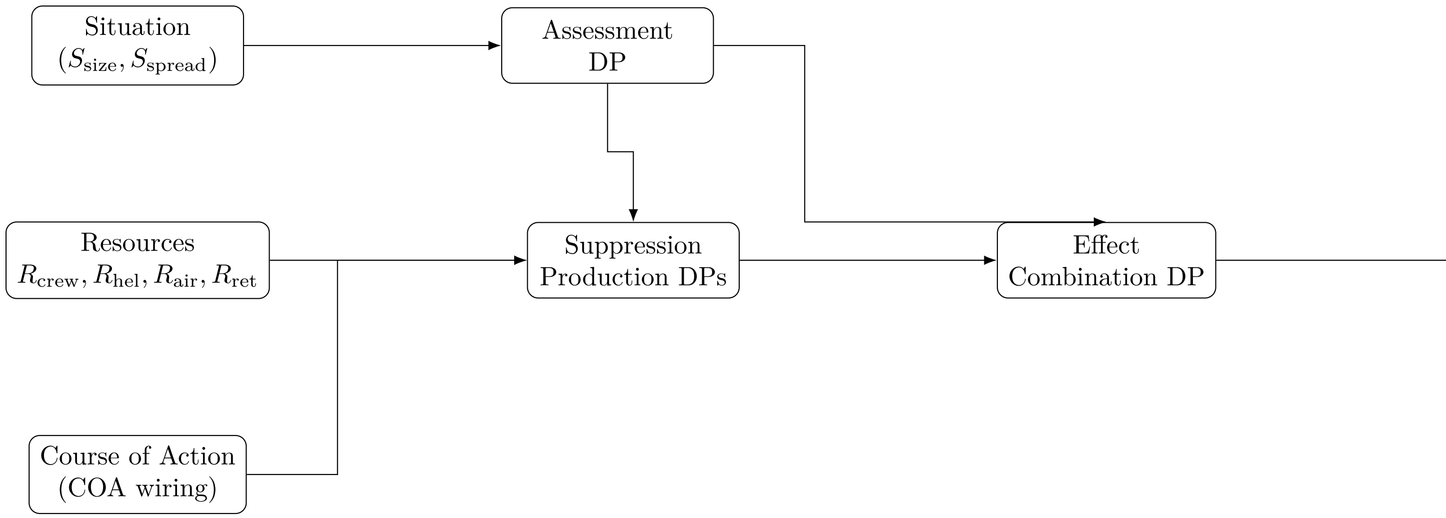

Figure 1: One-step wildfire decision structure as a composition of design problems from situation and resources to outcome figures of merit.

5 Model Scope, Interface, and Decision Step

This work models wildfire suppression at the level of a single decision step. The core object constructed in this project is a compositional co-design problem, expressed using the Monotone Co-Design Problems (MCDP) framework. This section makes explicit the interface exposed by the one-step MCDP, how it differs from a full multi-step decision model, and how the two compose.

5.1 Posets and ordering assumptions

The model relies on three classes of quantities, each represented as a partially ordered set (poset):

- Fire state poset S, capturing quantities such as fire size, spread conditions, and accumulated damage;
- Resource poset R, representing allocatable resources (e.g., crew-hours, aviation effort, retardant);
- Performance poset F, representing suppression outcomes (e.g., area controlled, response time).

Each of these quantities is modeled as a poset, enabling monotone composition and dominance reasoning. Resource and performance posets are ordered so that "less resource" and "better performance" are preferred, while the state poset is ordered according to worsening fire conditions.

5.2 The one-step MCDP interface

The one-step wildfire suppression model constructed in this work is *not* a state-transition model. Instead, it is a monotone relation from resources to performance, parameterized by the current fire state.

Formally, the co-design model can be written as a mapping

$$\mathcal{F} : S \longrightarrow \mathrm{Rel}(R, F),$$

where $\mathrm{Rel}(R, F)$ denotes the set of monotone relations between the resource poset R and the performance poset F. For a fixed fire state $s \in S$, this induces a relation

$$\mathcal{F}_s \subseteq R \times F.$$

For a fixed fire state s, the MCDP solver computes the Pareto-minimal resource allocations that achieve required performance levels. In other words, the MCDP answers the question:

> Given a fire state s and available resources, what are the feasible and Pareto-minimal system-level suppression packages?

5.3 External state-transition dynamics

A full multi-step wildfire decision model requires an explicit state-to-state mapping. We introduce an external state-transition function

$$\Phi : S \times R \longrightarrow S,$$

which maps the current fire state and chosen resource allocation to the next fire state.

The map Φ captures fire spread, damage accumulation, and suppression effects. It may be empirical, simulated, stochastic, or learned, and is intentionally outside the scope of the MCDP formulation. The MCDP does not produce next-state fire dynamics; it constrains feasible actions given a state.

5.4 Sequential composition over time

At each decision step t, the overall decision loop proceeds as:

1. Observe the current fire state $s_t \in S$
2. Solve the one-step co-design problem $\mathcal{F}_{s_t}$
3. Select a feasible resource allocation $r_t \in R$ from the resulting Pareto set
4. Update the fire state via $s_{t+1} = \Phi(s_t, r_t)$

This induces the following conceptual pipeline:

$$s_t \longrightarrow \mathcal{F}_{s_t} \longrightarrow r_t \longrightarrow s_{t+1}.$$

The same co-design problem is re-solved at each time step with updated parameters, yielding a sequence of Pareto-feasible decisions rather than a single static plan.

5.5 Monotonicity across steps

The MCDP relation $\mathcal{F}_s$ is monotone with respect to resources and performance by construction. The state-transition map Φ is assumed to be monotone with respect to both state severity and resource effort. Under these monotonicity assumptions, feasibility and dominance are preserved under sequential composition.

This separation clarifies the role of MCDP in a larger decision loop: the co-design model enforces structural feasibility and resource trade-offs at each step, while state evolution and damage accumulation are handled by an external dynamic model.

6 Figures of Merit (FOMs)

In wildfire suppression, a "good" outcome must be defined before we can evaluate whether a candidate COA is viable. Figures of Merit (FOMs) quantify effectiveness, cost, and performance of a suppression plan. Each FOM is represented as a poset, making it compatible with the monotone structure required by MCDP.

6.1 Containment effectiveness

The primary objective is to contain the fire. We model containment effectiveness as a probability

$$F_{\text{cont}} \in [0, 1],$$

ordered by the usual $\leq$. Higher values are better, so the FOM poset is $([0, 1], \leq)$. This FOM is the final output of the full compositional chain of design problems.

6.2 Time to containment

Time required to achieve control is also a relevant performance metric. Smaller time is better, so we use the reverse order:

$$F_{\text{time}} = (\mathbb{R}_{\geq 0}, \geq).$$

This expresses that if a strategy can achieve containment with less time, it dominates one that takes longer.

6.3 Resource use

Resource cost is measured componentwise:

$$R_{\text{use}} = (R_{\text{crew}}, R_{\text{hel}}, R_{\text{air}}, R_{\text{ret}}) \in R,$$

with the product order $(\leq, \leq, \leq, \leq)$. Lower resource use is preferable. In MCDP terms, resource vectors are minimized monotonically while respecting the required containment threshold.

6.4 Multi-objective structure

These FOMs form a multi-objective performance space

$$(F_{\text{cont}}, F_{\text{time}}, R_{\text{use}}),$$

with mixed orders. MCDP represents this space as an antichain, ensuring that the output of a solve is a Pareto frontier: a set of resource allocations that cannot be improved in any one dimension without worsening another. The explicit definition of FOM posets is necessary for the MCDP solver to evaluate the trade-offs between containment performance and resource cost.

7 Design Problems (DPs)

Each design problem (DP) relates resources to functionalities via a monotone mapping. In this first version of the model, we keep the mappings intentionally simple but structurally explicit.

7.1 DP 1: Construct fireline

Crew-hours produce fireline length linearly:

$$F_{\text{line}} = p_{\text{crew}}\, R_{\text{crew}},$$

where $p_{\text{crew}} > 0$ is a productivity coefficient (for example, meters of effective fireline per crew-hour). This map is monotone in R_{crew}.

7.2 DP 2: Drop retardant

Retardant volume provides suppression effect:

$$F_{\text{supp}} = p_{\text{ret}}\, R_{\text{ret}},$$

where $p_{\text{ret}} > 0$ converts gallons of retardant into an abstract suppression index. This map is monotone in R_{ret}.

7.3 DP 3: Combine fireline and suppression

We define effective line by combining constructed line and suppression:

$$F_{\text{line}}^{\text{eff}} = F_{\text{line}} + \alpha\, F_{\text{supp}},$$

with $\alpha > 0$ describing the suppression-to-line equivalence (how many meters of "equivalent line" one unit of suppression provides). This is monotone in both F_{line} and F_{supp}.

7.4 DP 4: Required line for containment

The amount of line required to contain a fire depends on its size and spread:

$$L_{\text{req}} = c_1 \sqrt{S_{\text{size}}}\,\big(1 + c_2 S_{\text{spread}}\big),$$

where $c_1, c_2 > 0$ are scaling constants. This function is monotone in both S_{size} and S_{spread}.

We define the surplus (or deficit) of effective line:

$$x = F_{\text{line}}^{\text{eff}} - L_{\text{req}}.$$

7.5 DP 5: Achieve containment

Containment probability increases linearly with surplus line and is clipped to the unit interval:

$$F_{\text{cont}} = \min\big(1, \max(0, ax + b)\big),$$

where $a > 0$ sets the gain and b sets the threshold. This is a monotone map in $F_{\text{line}}^{\text{eff}}$ and anti-monotone in L_{req}, as expected.

7.6 DP 6: Assess fire (structural only)

In this first version we treat the assessment DP as providing situation parameters without an explicit equation:

$$R_{\text{hel}} \longrightarrow (S_{\text{size}}, S_{\text{spread}}).$$

Helicopter hours could be used in later versions to reduce uncertainty in these estimates, but here the DP is included to emphasize that assessment is an explicit part of the provides/requires structure.

8 Compositional Structure

We model a single suppression cycle as a compositional graph of design problems. At a high level:

$$\text{Assess Fire} \longrightarrow (\text{Construct Fireline} \parallel \text{Drop Retardant}) \longrightarrow \text{Achieve Containment}.$$

An illustrative dependency chain is:

$$\begin{array}{ccccccc} \text{Assess Fire} & \longrightarrow & \text{Construct Fireline} & \longrightarrow & \text{Combine Effects} & \longrightarrow & \text{Achieve Containment} \\ & & & & \searrow \;\; \uparrow & & \\ & & & & \text{Drop Retardant} & & \end{array}$$

The provides/requires relationships ensure that only structurally valid compositions are considered. In the MCDP representation, the composed DP relates the resource tuple $r \in R$ and situation parameters $(S_{\text{size}}, S_{\text{spread}})$ to the output FOMs $(F_{\text{cont}}, F_{\text{time}}, R_{\text{use}})$.

The following Figure 2 shows the full compositional structure of the wildfire suppression design problem used in this work.

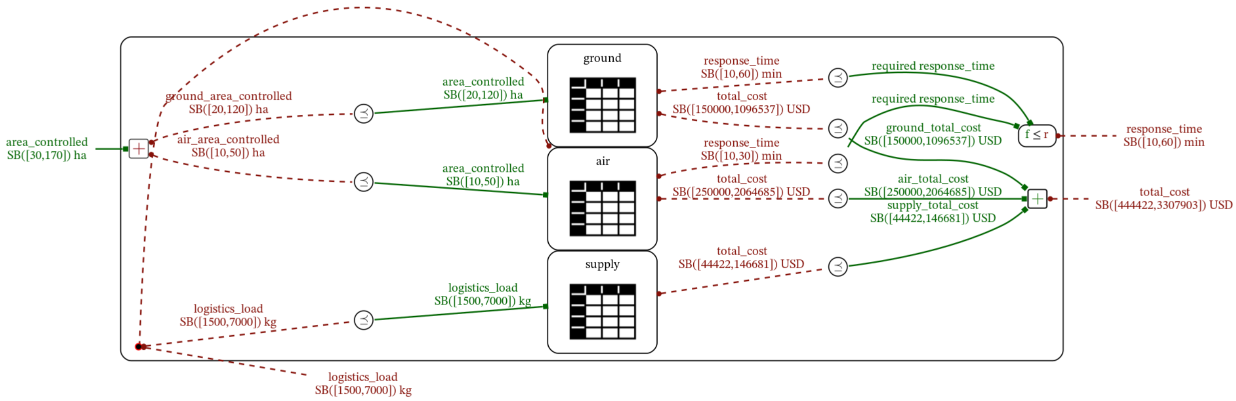

Figure 2: System-level co-design structure for wildfire suppression. Air, ground, and supply subsystems are composed through monotone provides/requires interfaces over partially ordered resource and function spaces. The diagram shows how area control, response time, logistics load, and total cost are aggregated into a single system-level design problem.

9 MCDP Query Formulation

Let

$$r = (R_{\text{crew}}, R_{\text{hel}}, R_{\text{air}}, R_{\text{ret}}) \in R$$

be a resource allocation, and let $(S_{\text{size}}, S_{\text{spread}})$ be a given situation. Via the chain of design problems defined above, we obtain a resulting containment probability

$$F_{\text{cont}} = F_{\text{cont}}(r; S_{\text{size}}, S_{\text{spread}}).$$

A basic MCDP query is:

$$\text{Find all minimal } r \in R \text{ such that } F_{\text{cont}}(r; S_{\text{size}}, S_{\text{spread}}) \geq \alpha,$$

for a chosen target containment level $\alpha \in (0, 1)$. "Minimal" here is with respect to the product order on R (Pareto minimality): no component of r can be reduced without violating the containment constraint. The result of the solve is a Pareto frontier of feasible resource allocations.

10 Toy Numerical Example

To illustrate the model, we fix simple parameter values:

- $p_{\text{crew}} = 100$ meters of line per crew-hour,
- $p_{\text{ret}} = 1$ (arbitrary suppression units per gallon),
- $\alpha = 0.01$ meters per suppression unit,
- $c_1 = 50$, $c_2 = 0.5$,
- $a = 0.02$, $b = 0.5$.

Consider a fire with:

$$S_{\text{size}} = 25 \text{ acres}, \qquad S_{\text{spread}} = 1.$$

Then the required effective line is:

$$L_{\text{req}} = c_1 \sqrt{S_{\text{size}}}\,(1 + c_2 S_{\text{spread}}) = 50\sqrt{25}(1 + 0.5 \cdot 1) = 50 \cdot 5 \cdot 1.5 = 375 \text{ meters}.$$

Case 1: Insufficient resources

Suppose we allocate:

$$R_{\text{crew}} = 3 \text{ crew-hours}, \qquad R_{\text{ret}} = 0.$$

Then

$$F_{\text{line}} = p_{\text{crew}} R_{\text{crew}} = 100 \cdot 3 = 300 \text{ m},$$

$$F_{\text{supp}} = p_{\text{ret}} R_{\text{ret}} = 0,$$

$$F_{\text{line}}^{\text{eff}} = 300 + 0.01 \cdot 0 = 300 \text{ m},$$

$$x = F_{\text{line}}^{\text{eff}} - L_{\text{req}} = 300 - 375 = -75.$$

Containment probability:

$$ax + b = 0.02 \cdot (-75) + 0.5 = -1.5 + 0.5 = -1.0,$$

so after clipping,

$$F_{\text{cont}} = 0.$$

This allocation is not sufficient for containment.

Case 2: Increased crew-hours

Now let

$$R_{\text{crew}} = 4 \text{ crew-hours}, \qquad R_{\text{ret}} = 0.$$

Then

$$F_{\text{line}} = 100 \cdot 4 = 400 \text{ m}, \qquad F_{\text{supp}} = 0,$$

$$F_{\text{line}}^{\text{eff}} = 400.$$

Surplus:

$$x = 400 - 375 = 25.$$

Containment probability:

$$ax + b = 0.02 \cdot 25 + 0.5 = 0.5 + 0.5 = 1.0,$$

so

$$F_{\text{cont}} = 1.$$

Between these two cases, we see that increasing crew-hours can move the system from certain failure ($F_{\text{cont}} = 0$) to certain success ($F_{\text{cont}} = 1$). In a full MCDP analysis, we would explore many such combinations and extract the Pareto frontier of minimal resource allocations.

11 Discussion

This first version of the model is intentionally simple, but it already illustrates the core ideas:

- resources, functionalities, and situations are modeled as posets;
- suppression tactics are modeled as monotone design problems;
- actions compose through provides/requires relationships;
- containment is treated as a function of effective line versus required line;
- MCDP can be used to identify minimal resource allocations that achieve target containment levels.

Category theory is not used to generate tactics. Instead, it provides a framework to check whether a candidate plan is structurally coherent and to compose the effects of multiple design problems in a principled way. The explicit one-step decision structure and FOM definitions also make it straightforward to connect this model to more detailed simulation and optimization tools.

12 Conclusion

We have constructed a category-theoretic model of wildfire suppression using posets and design problems, and we have defined a simple but fully specified chain from resources to containment probability. This framework enables explicit, order-theoretic analysis of resource trade-offs via MCDP. Future work will expand on this with richer functionality, uncertainty, dynamic feedback between time steps, explicit modeling of multiple divisions, and potential application to other domains such as military command, where an adversary plays a role analogous to a spreading wildfire.

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