Satellite Attitude Control and Estimation via LQG

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Abstract

The Linear Quadratic Gaussian (LQG) controller has proved to be an useful optimal controller for regulating the state of a dynamical system as well as reference tracking. A LQG system consists of an optimal estimator and a Linear Quadratic Regulator (LQR). Due to the Separation Principle of controller and estimator, the LQG system can be optimal if the estimator and controller are separately optimal. This allows for the independent designs and implementations for both the estimator and the controller. This projects attempts to linearize satellite attitude error dynamics as well as to simulate and implement a LQG controller for attitude reference tracking.

Dynamics Model

Satellite rigid body dynamics follow Euler's Second Law:

$$I\dot{\omega} = -\omega \times I\omega + L$$

Where I is the moment of inertia of the satellite, ω is the angular velocity of the satellite in body coordinates, and L is the applied torque on the satellite in body coordinates. As shown, this dynamical system is highly nonlinear and needs to be linearized before LQG can be applied.

Quaternion Parameterization

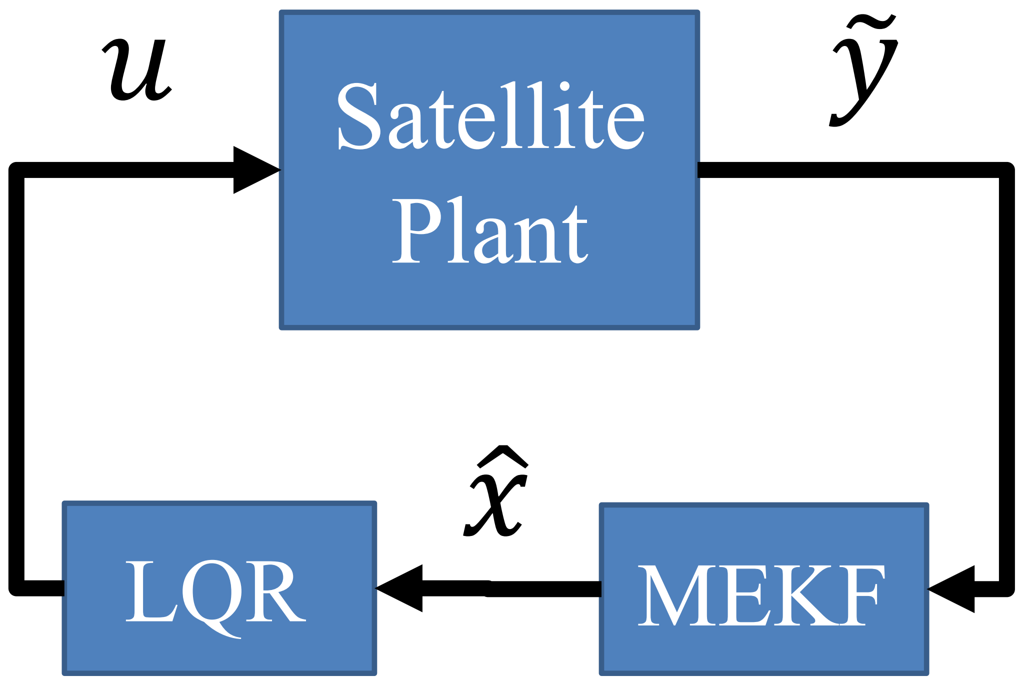
Quaternions are used to track the attitude, or orientation, of a satellite with respect to an inertial frame:

$$q = \begin{bmatrix} \rho \\ q_4 \end{bmatrix}$$

The benefit of quaternions over conventional Euler angles is that they are singularity-free. The time derivative of the quaternion is defined as:

$$\dot{q} = \frac{1}{2}\begin{bmatrix} \omega \\ 0 \end{bmatrix} \otimes q$$

Analysis



Linear Quadratic Regulator

We can reformulate the nonlinear dynamics and the reference tracking problem into linear error dynamics. If we define error quaternion as:

$$\delta q = \begin{bmatrix} \delta\rho \\ \delta q_4 \end{bmatrix} = q \otimes q_{desired}^{-1}$$

The error dynamics can have the following *linear* and *time-invariant* form:

$$\begin{bmatrix} \delta\dot{\rho} \\ \delta\ddot{\rho} \end{bmatrix} = \begin{bmatrix} 0_{3x3} & I_{3x3} \\ 0_{3x3} & 0_{3x3} \end{bmatrix} \begin{bmatrix} \delta\rho \\ \delta\dot{\rho} \end{bmatrix} + \begin{bmatrix} 0_{3x3} \\ I_{3x3} \end{bmatrix} u$$

Since the system is reachable, the LQR gain $[L_1 \quad L_2]$ is constant and can be chosen offline. After plugging in formulas on the left, the full control policy becomes:

$$\mathbf{L} = [\omega\times]J\omega + 2J\left[\Xi^T(\mathbf{q}_d)\Xi(\mathbf{q})\right]^{-1}\left\{\frac{1}{4}(\omega^T\omega)\Xi^T(\mathbf{q}_d) - \Xi^T(\dot{\mathbf{q}}_d)\Omega(\omega) - \Xi^T(\ddot{\mathbf{q}}_d) - L_1\Xi^T(\mathbf{q}_d) - L_2\left[\frac{1}{2}\Xi^T(\mathbf{q}_d)\Omega(\omega) + \Xi^T(\dot{\mathbf{q}}_d)\right]\right\}\mathbf{q}$$

Multiplicative Extended Kalman Filter

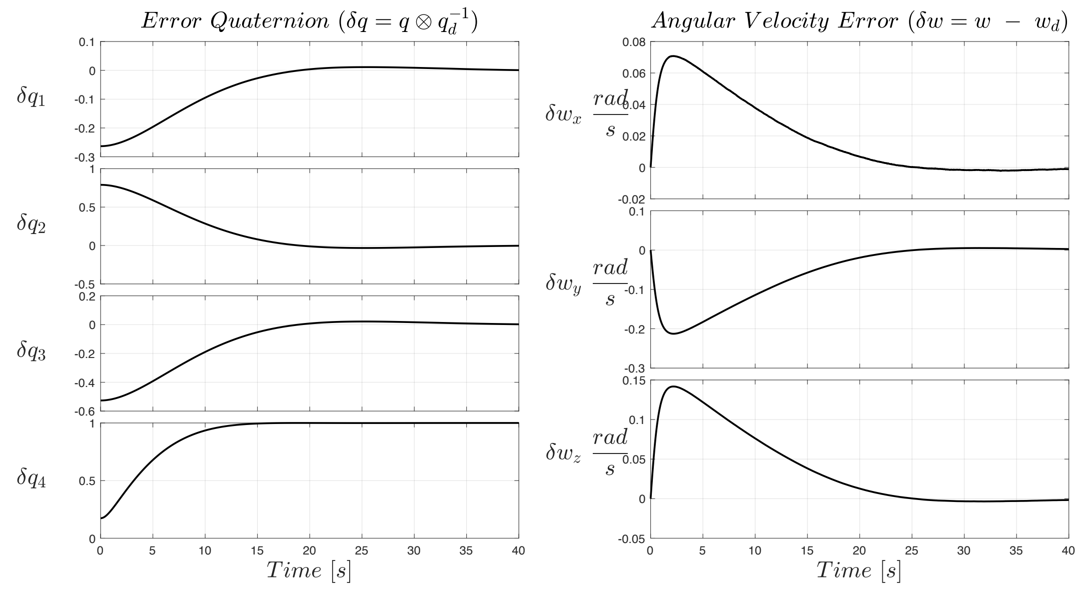
We can reformulate the system using error dynamics and linearize about the most recent estimate:

$$\delta\dot{\tilde{x}} = \begin{bmatrix} -[\hat{\omega}(t)\times] & -I_{3x3} \\ 0_{3x3} & 0_{3x3} \end{bmatrix}\delta\tilde{x} + \begin{bmatrix} -I_{3x3} & 0_{3x3} \\ 0_{3x3} & I_{3x3} \end{bmatrix} w(t)$$

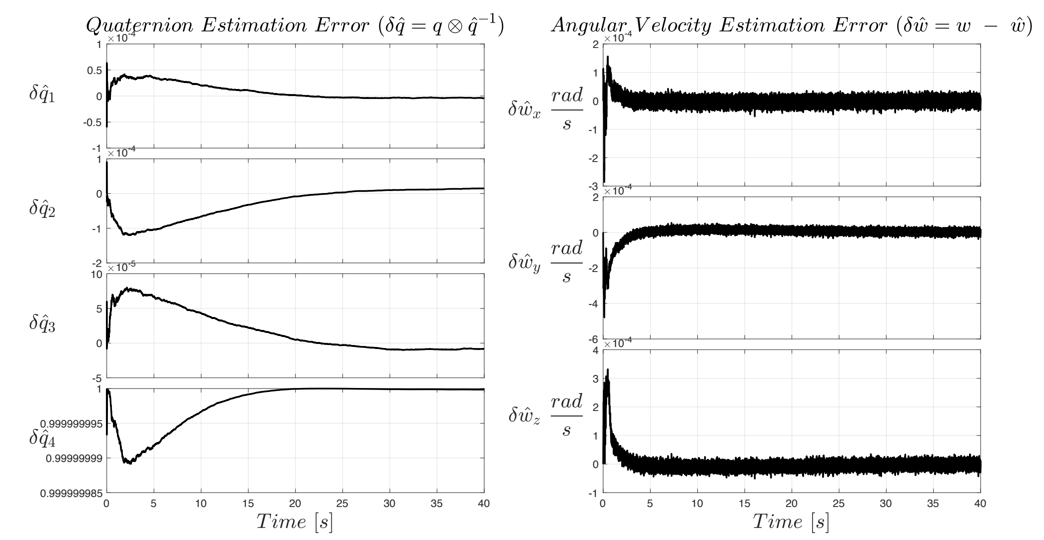
The system is driven by sensor noise w(t). Propagation step can be performed by integrating the nonlinear state equations and the continuous Riccati equation. Update step can be performed by incorporating new measurements with Kalman gain.

Simulation Results

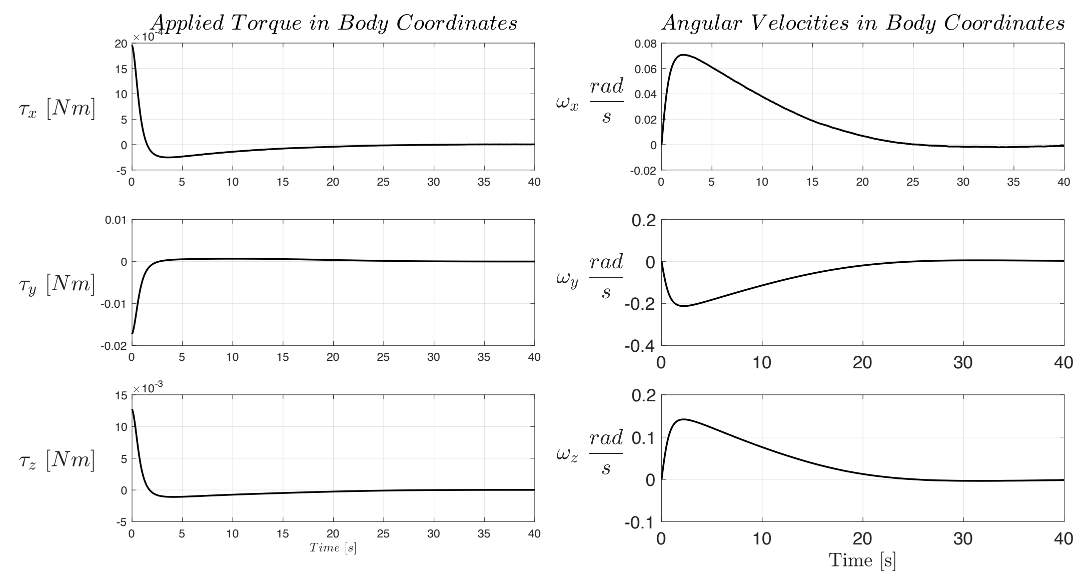
The LQG controller and satellite rigid body dynamics are simulated in MATLAB using Runge-Kutta 4th order integrator. System is assumed to be observable, with artificial white gaussian noise applied to attitude and gyroscope measurements.



As shown, the quaternion and angular velocity errors converge to zero. Note a quaternion error of 0 corresponds to $[\delta q_1 \quad \delta q_2 \quad \delta q_3 \quad \delta q_4]^T = [0 \quad 0 \quad 0 \quad 1]^T$.



The state estimation errors also converge to 0 over time. The commanded torque is given by:



[1] J. Crassidis and J. Junkins, *Optimal estimation of dynamic systems*. Boca Raton, Fla.: Chapman and Hall/CRC, 2012.