# Superposition and Decomposition Principles in Hierarchical Social Accounting and Input-Output Analysis

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## 1 Introduction

Interest in the *structure* of an economy, as manifested in the transactions between industries, factors, and institutions, has not been a prominent feature of the use of input-output and, more generally, social accounting models. Attention, for the most part, has been directed towards issues such as model construction, aggregation, multiplier and impact estimation, and problems of updating. In this paper, attention will be focused more on the nature of the structure revealed in the input-output model rather than on the other issues, although it will be clear that the insights gained might have considerable utility for many of the issues noted earlier. However, the focus on structure provides a stronger rationale for the development of social accounting models, especially at the regional level, and their eventual integration into the development of a consistent set of accounts for nations and regions.

In other papers (Jensen, West, et al, 1988; Jensen, Hewings et al, 1988) some attempts have been made to cast the study of structure in the context of the development of a taxonomy of economies and, potentially, in terms of the articulation of a theory of development of structure over space and time. In essence, these approaches complement some of the important work already undertaken by Chenery (1960), Chenery and Syrquin (1975), Chenery and Watanabe (1958), Chenery et al (1986), and Deutsch et al (1986) in relating production structure and development.

Unlike many other approaches, the focus in this paper will be on the decomposition of the structure of an economy. The analytical techniques to be examined and compared may be applied to any matrix representation of an economy, although their economic meaning and interpretation become more important as the matrix is expanded to include a greater percentage of the transactions which take place in the economy. Although there is a clear link between these approaches and issues of aggregation, no attempt is made here to explore this link. Hence, by increasing the transactions in the matrix under review, one is thinking more of the movement from a simple Leontief set of accounts to the Miyazawa (1976) variety to full social accounts. This is in contrast to simple aggregation issues wherein the matrix is expanded by increasing the number (and hence, detail) of the sectors (see Hewings, 1986). The Theil entropy decomposition procedures are not considered here—they have been reviewed by Jackson et al (1987) and seem to have some important limitations precluding their use in a more general sense.

In the next section of the paper, the decomposition approaches initially pioneered by Pyatt and Round (1979) will be examined and some alternatives proposed. Thereafter, the structural path technique will be presented and its link to the notion of a field of influence and the general issues of error and sensitivity analysis will be demonstrated. The superposition principle will be presented in the following section, together with its algorithm. Some empirical results to demonstrate its use are provided, and in the final section we explore the link between some of the approaches.

## 2 Multiplicative decompositions of social accounting systems

The multiplicative decomposition of social accounting systems was introduced by Pyatt and Round (1979) and has achieved its widest citation in the applications to the Sri Lankan model (Pyatt and Roe, 1977). Subsequently, the procedures have been modified (Pyatt and Round, 1984; 1985) and extended to the problem of interregional and world trade (Round, 1985; 1986; 1988). In this presentation, the basic procedure revealed in Pyatt and Round (1979) is used as a starting point.

The methodology involves the multiplicative decomposition of a matrix multiplier representation of a social accounting system under the assumption of fixed prices. Assume that we have the familiar solution to a Leontief-type system:

$$\boldsymbol{X} = (\mathbf{I}-\mathbf{A})^{-1}\boldsymbol{F}, \tag{1}$$

where $\boldsymbol{X}$ and $\boldsymbol{F}$ are vectors of gross output and final demand, and $\mathbf{A}$ is a matrix derived from the transactions between activities (interindustry flows), factors (for example, labour of various types, income levels, or location), and institutions (households, governments, firms) in coefficient form. The matrix takes the form

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{A}_{13} \\ \mathbf{A}_{21} & \mathbf{A}_{22} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{32} & \mathbf{A}_{33} \end{bmatrix}. \tag{2}$$

The columns (rows) may be referred to as factors, institutions, and activities, respectively; zero submatrices indicate no transactions between these major divisions. Each submatrix then represents a complex of flows (in coefficient form as a percentage of total inputs) both within and between these major divisions. In place of the solution shown in equation (1), Pyatt and Round (1979) suggested the form

$$X = (\mathbf{I}-\mathbf{A})^{-1}F = \mathbf{M}F = \mathbf{M}_3\mathbf{M}_2\mathbf{M}_1 F, \tag{3}$$

where the matrix multipliers, $\mathbf{M}_i$, have specific forms describing 'own direct effects', ($\mathbf{M}_1$), 'own transfer multiplier effects', ($\mathbf{M}_2$), and 'cross multiplier effects', ($\mathbf{M}_3$). The structure of each multiplier, $\mathbf{M}_i$, is heavily dependent on the location of zero *blocks* in the matrix, $\mathbf{A}$, and does not depend on the values of the *individual* input coefficients.

However, the multiplicative decomposition shown in equation (3) is not unique and there may be many alternatives which may provide additional insights into the structure and functioning of an economy. For example, consider the social accounting system portrayed in equation (2), with asterisks replacing the nonzero blocks:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & * \\ * & * & \mathbf{0} \\ \mathbf{0} & * & * \end{bmatrix}. \tag{4}$$

The decomposition may proceed as before (Pyatt and Round, 1979) by presenting matrix $\mathbf{A}$ as a sum of block-diagonal ($\mathbf{D}$) and block-permutation ($\mathbf{A}^*$) matrices:

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & * & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & * \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & * \\ * & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & * & \mathbf{0} \end{bmatrix} = \mathbf{D} + \mathbf{A}^* . \tag{5}$$

The essential property of the block-permutation matrix $\mathbf{A}^*$ is that $(\mathbf{A}^*)^3$ is block-diagonal as well. This property allows the multiplicative decomposition of the inverse matrix to take the form

$$(\mathbf{I}-\mathbf{A})^{-1} = \mathbf{M}_3\mathbf{M}_2\mathbf{M}_1 = \begin{bmatrix} \mathbf{I} & * & * \\ * & \mathbf{I} & * \\ * & * & \mathbf{I} \end{bmatrix} \begin{bmatrix} * & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & * & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & * \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & * & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & * \end{bmatrix}. \tag{6}$$

This is the form developed by Pyatt and Round (1979).

An alternative presentation, in the form of an additive decomposition, was suggested by Defourny and Thorbecke (1984):

$$(\mathbf{I}-\mathbf{A})^{-1} = \mathbf{M} = \mathbf{M}_3\mathbf{M}_2\mathbf{M}_1 = \mathbf{I} + (\mathbf{M}_1 - \mathbf{I}) + (\mathbf{M}_2 - \mathbf{I})\mathbf{M}_1 + (\mathbf{M}_3 - \mathbf{I})\mathbf{M}_2\mathbf{M}_1, \tag{7}$$

or, in summary form,

$$\mathbf{M} = \mathbf{I} + \mathbf{T} + \mathbf{O} + \mathbf{C}. \tag{8}$$

This arrangement provides for a similar decomposition of the multiplier effects articulated by Pyatt and Round (1979). However, in this case, the effects are additive and the four terms may be interpreted as the initial injection, **I**; the net contribution of transfer multiplier effects, **T**; the net contribution of open-loop or cross-multiplier effects, **O**; and the net contribution of circular closed-loop effects, **C**. Thus, the formulation has been presented in terms of 'paths' through the system. **T** represents the intradivision (for example, interindustry) multiplier effects net of the initial injection, **I**. These effects are spread throughout the system in two forms—an open-loop effect, **O**, in which transactions in one major division influence another (for example, flows from factors to institutions) and a closed-loop term, **C**, which captures the circular path of multiplier effects of the form

$$\text{interindustry} \longrightarrow \text{factors} \longrightarrow \text{institutions} \longrightarrow \text{interindustry}\,.$$

In the next section, the modifications to this path-theoretic approach made by Defourny and Thorbecke (1984) will be examined. Before this, an alternative to the multiplicative decomposition will be presented. As before, an asterisk indicates a nonzero block; the procedure will retain the same 'own direct multiplier effects' used by Pyatt and Round (1979), but will provide alternatives to the other two multiplier matrices, $\mathbf{M}_2$, and $\mathbf{M}_3$.

Decomposition (5) provides

$$(\mathbf{I}-\mathbf{A})^{-1} = (\mathbf{I}-\mathbf{D}-\mathbf{A}^*)^{-1} = [\mathbf{I}-(\mathbf{I}-\mathbf{D})^{-1}\mathbf{A}^*]^{-1}(\mathbf{I}-\mathbf{D})^{-1}\,. \tag{9}$$

Here

$$\mathbf{I}-\mathbf{D} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & * & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & * \end{bmatrix}, \qquad (\mathbf{I}-\mathbf{D})^{-1} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & * & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & * \end{bmatrix} = \mathbf{M}_1\,. \tag{10}$$

Thus, matrix $\mathbf{A}_1$ may be defined as

$$\mathbf{A}_1 = (\mathbf{I}-\mathbf{D})^{-1}\mathbf{A}^* = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & * & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & * \end{bmatrix}\begin{bmatrix} \mathbf{0} & \mathbf{0} & * \\ * & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & * & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & * \\ * & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & * & \mathbf{0} \end{bmatrix}, \tag{11}$$

which has the same basic block-structure as matrix $\mathbf{A}^*$, and hence

$$(\mathbf{I}-\mathbf{A})^{-1} = (\mathbf{I}-\mathbf{A}_1)^{-1}\mathbf{M}_1\,. \tag{12}$$

Further, the decomposition

$$\mathbf{A}_1 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & * \\ * & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & * & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & * & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} & * \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} = \mathbf{A}_2+\mathbf{A}_3 \tag{13}$$

gives us the block-nilpotent matrix

$$\mathbf{A}_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & * & \mathbf{0} \end{bmatrix},$$

with the property

$$\mathbf{A}_2^2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & \mathbf{0} & \mathbf{0} \end{bmatrix}, \qquad \mathbf{A}_2^3 = \mathbf{0} .$$

Therefore

$$(\mathbf{I}-\mathbf{A}_2)(\mathbf{I}+\mathbf{A}_2+\mathbf{A}_2^2) = \mathbf{I} ,$$

which provides the definition of $\mathbf{M}_2$ as

$$(\mathbf{I}-\mathbf{A}_2)^{-1} = \mathbf{I}+\mathbf{A}_2+\mathbf{A}_2^2 = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ * & \mathbf{I} & \mathbf{0} \\ * & * & \mathbf{I} \end{bmatrix} = \mathbf{M}_2 , \tag{14}$$

and

$$(\mathbf{I}-\mathbf{A}_1)^{-1} = (\mathbf{I}-\mathbf{A}_2-\mathbf{A}_3)^{-1} = [\mathbf{I}-(\mathbf{I}-\mathbf{A}_2)^{-1}\mathbf{A}_3]^{-1}(\mathbf{I}-\mathbf{A}_2)^{-1} . \tag{15}$$

The matrix

$$\mathbf{A}_4 = (\mathbf{I}-\mathbf{A}_2)^{-1}\mathbf{A}_3 = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ * & \mathbf{I} & \mathbf{0} \\ * & * & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{0} & \mathbf{0} & * \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & * \\ \mathbf{0} & \mathbf{0} & * \\ \mathbf{0} & \mathbf{0} & * \end{bmatrix}$$

implies the decomposition

$$(\mathbf{I}-\mathbf{A}_1)^{-1} = (\mathbf{I}-\mathbf{A}_4)^{-1}\mathbf{M}_2 . \tag{16}$$

From this expression, it is easy to see that matrix $(\mathbf{I}-\mathbf{A}_4)^{-1}$ has the form

$$(\mathbf{I}-\mathbf{A}_4)^{-1} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & * \\ \mathbf{0} & \mathbf{I} & * \\ \mathbf{0} & \mathbf{0} & * \end{bmatrix} = \mathbf{M}_3 , \tag{17}$$

and the final decomposition is

$$(\mathbf{I}-\mathbf{A})^{-1} = \mathbf{M}_3\mathbf{M}_2\mathbf{M}_1 = \begin{bmatrix} \mathbf{I} & \mathbf{0} & * \\ \mathbf{0} & \mathbf{I} & * \\ \mathbf{0} & \mathbf{0} & * \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ * & \mathbf{I} & \mathbf{0} \\ * & * & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & * & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & * \end{bmatrix} . \tag{18}$$

If the same major division of **A** is assumed, into factors, institutions, and activities, then the components of equation (18) may be interpreted as follows. As before, matrix $\mathbf{M}_1$ represents the own direct multiplier effects. Matrix $\mathbf{M}_2$ has an interesting triangular structure suggestive of a path of interaction which becomes more complex as one moves from factors to activities. The final matrix, $\mathbf{M}_3$, shows the influence of all major divisions of the social accounting system on production activities.

Clearly, the nonuniqueness of the decomposition opens new ways for the alternative explanations of the essence of the mutual interactions in social accounting systems. More recently, Round (1988) has developed appropriate decompositions for the two-region and three-region cases; here, the alternatives are many and, without some simplifications in the structure of the interregional matrices, the problem of finding a solution which can be interpreted from the perspective of theory becomes even more acute. Thus, although the nonuniqueness provides many options, many of the decompositions are not usually connected with the socio-economic content of social accounting systems and, therefore, they may not be very useful ultimately in the analysis undertaken with these models.

### 3 Structural path analytic approaches

Structural path analysis, elaborated by Lantner (1974), Gazon (1976), Crama et al (1984), and others (see Defourny and Thorbecke, 1984) introduces the important concept of economic influence and its transmission within the input–output or social accounting system. Although the previous decomposition would enable the analyst to evaluate the differing contributions each component makes both separately and synergistically to the functioning in the system, one of the major objectives in structural path analysis is to highlight the important interactions. As Byron (1978) has shown, the complexity of the interactions within a social accounting system can be seen to consist of a small set of very important transactions, with the remaining interactions providing a less significant complement. In essence, structural path analysis provides an intermediate step towards the possibility of a consistent approach to importance within a matrix and thus, the potential for uncovering a decomposition which has sound socioeconomic utility while focusing on the analytically important sets of interactions. The analysis to be presented here differs from some earlier attempts to use graph-theoretic concepts in input–output analysis in a number of important ways (see Blin and Cohen, 1976; Blin and Murphy, 1974; Campbell, 1974; 1975; Slater, 1974; 1977). These approaches were mainly focused on measuring associations or clusters, appropriate ordering of sectors, or efficient aggregation schemes.

The operational basis of the structural path analysis is associated with the *influence graph* whose vertices are the elements which comprise the set of 'sectors' in the social accounting system. Thus, the associated coefficients $a_{ij}$ of matrix **A** may be interpreted in a slightly different way than is normally the case. The influence is regarded as the strength of the connection; hence, the magnitude of the direct influence $J^{\mathrm{D}}(j, i)$ transmitted from vertex $j$ to vertex $i$ through the arc $(j \to i)$ is equal to $a_{ij}$. Drawing on the nomenclature of graph theory, we may trace the 'flow-on' effects of individual transactions by defining the appropriate path (a sequence of arcs). An elementary path $\mathrm{p}(j, i)$ from vertex $j$ to vertex $i$ might appear as

$$(j, k_1), (k_1, k_2), ..., (k_r, i)\,, \qquad k_t \neq k_s\,,$$

from which the total influence $J^{\mathrm{T}}\mathrm{p}(j, i)$ of vertex $j$ on vertex $i$ along the elementary path $\mathrm{p}(j, i)$ is given by

$$J^{\mathrm{T}}\mathrm{p}(j, i) = a_{k_1 j} a_{k_1 k_2} \ldots a_{ik_r} \frac{\mathrm{M}[\mathrm{p}(j, i)]}{\Delta}\,, \tag{19}$$

where $\Delta = \det(\mathbf{I}-\mathbf{A})$, and $\mathrm{M}[\mathrm{p}(j, i)]$ denotes the minor of the matrix $(\mathbf{I}-\mathbf{A})$ obtained by removing the rows and columns $j, k_1, k_2, ..., k_r, i$; $\mathrm{M}(\mathrm{p})/\Delta$ is a path multiplier. The global influence of the vertex $j$ on vertex $i$ $J^{\mathrm{G}}(j, i)$ is the component $b_{ij}$ of the associated inverse $\mathbf{B} = (\mathbf{I}-\mathbf{A})^{-1}$. Thus, this inverse (the Leontief inverse in the case of a simple interindustry model) can be referred to as a matrix of global influences.

Building on this notion, structural path analysis provides a *theorem of global influence* (Lantner, 1974); the global influence of vertex $j$ on vertex $i$ is equal to the sum of the total influences of vertex $j$ on vertex $i$ along all elementary paths $\mathrm{p}(j, i)$ joining vertex $j$ to vertex $i$:

$$b_{ij} = J^{\mathrm{G}}(j, i) = \sum_{\mathrm{p}(j,\, i)} J^{\mathrm{T}}\mathrm{p}(j, i)\,. \tag{20}$$

Based on this property, structural path analysis provides a tool for the analytical evaluation of the economic influence of a given vertex (for example, the iron and steel industry, household group, government) through the variety of interindustry interactions associated with the multitude of elementary paths. Defourny and Thorbecke (1984), in an application to a South Korean social accounting matrix, were thus able to identify some of the features of the complexity of the structure which would not be readily apparent through an examination of the multiplier matrix, $\mathbf{M}$, alone. For example, the procedure enabled the analyst to estimate not only the total effect but the path along which the effect 'travelled' through the system. A deficiency of this type of analysis is the existence of a very large number of elementary paths between different vertices. For example, in an $n \times n$ input-output matrix, $\mathbf{A}$, the number, $P$, of all possible elementary paths between two vertices $j$ and $i$ is equal to

$$P = 1+(n-2)+(n-2)(n-3)+\ldots+(n-2)(n-3)\ldots2\times1 = (n-2)!\sum\frac{1}{k!}\,, \tag{21}$$

and, consequently, the number, $N$, of all elementary paths for all pairs of different vertices is equal to

$$N = n! \sum_{k=1}^{n-2} \frac{1}{k!} . \tag{22}$$

In a simple $5 \times 5$ input–output matrix, there are 600 pairs of different vertices and each pair is associated with ten elementary paths, thus yielding 6000 elementary paths. However, this large number is reduced by the existence of zero coefficients (and, hence, zero-path transmissions). On the other hand, the consideration of 'long' paths with more than three arcs is not very efficient because of the very small path multipliers.

One area of potential future development is the possible linkage of this approach with the network equilibrium methodologies. An earlier suggestion was made by Hewings (1982) of a possible link between input–output analysis and the notion of critical paths in transportation systems. Nagurney (1987) has extended this work in a general approach to the matrix estimation problem, revealing the utility of a network approach. Nagurney's approach may provide for a way of reducing the inefficiency of the structural path analysis for very large systems.

**4 Sensitivity and error analysis of input–output systems**

In the last two years, a significant step has been made in the analysis of the structure of transmissions of influence in the context of error and sensitivity analysis of input–output and social accounting systems (Hewings et al, 1988; Sonis and Hewings, 1988a). The transmission of influence has been specified in the form of the influence of changes in all the direct coefficients on the components of the Leontief inverse. The compact formula which has been proposed provides the structure of change in the global influence, $b_{ij}$, inflicted by the changes in all direct influences, $a_{ij}$.

The earlier attempts to elaborate the error and sensitivity analysis were associated with the work of Sherman and Morrison (1950), Theil (1957), Bullard and Sebald (1977), Byron (1978), Jensen and West (1980), Hewings and Romanos (1981) and Hewings (1984a; 1984b), among others. The new approach which has the capability of addressing the problem of transfer of influence of changes in a more general way (see Sonis and Hewings, 1988b) can be presented in the following form. Let $\mathbf{A} = \|a_{ij}\|$ be a matrix of direct input coefficients, and $\mathbf{E} = \|\varepsilon_{ij}\|$ a matrix of incremental changes in the direct input coefficients. The associated Leontief inverse matrices will be $\mathbf{B} = (\mathbf{I}-\mathbf{A})^{-1} = \|b_{ij}\|$ and $\mathbf{B}(E) = (\mathbf{I}-\mathbf{A}-\mathbf{E})^{-1} = \|b_{ij}(\varepsilon)\|$. The following formula for the global influence of change may be presented as

$$b_{ji}(\varepsilon) = \left[ b_{ji} + \frac{1}{\Delta} \sum_{k=1}^{n-1} (-1)^k \sum_{\substack{i_r \neq i_s, i \\ j_r \neq j_s, j}} \operatorname{sign}\begin{pmatrix} j_1 \ldots j_k j \\ i_1 \ldots i_k i \end{pmatrix} \mathbf{M}\begin{pmatrix} j_1 \ldots j_k j \\ i_1 \ldots i_k i \end{pmatrix} \varepsilon_{i_1 j_1} \ldots \varepsilon_{i_k j_k} \right]$$
$$\times \left[ 1 - \sum_{i_1, j_1} b_{j_1 i_1} \varepsilon_{i_1 j_1} + \frac{1}{\Delta} \sum_{k=2}^{n-1} (-1)^k \sum_{\substack{i_r \neq i_s \\ j_r \neq j_s}} \operatorname{sign}\begin{pmatrix} j_1 \ldots j_k \\ i_1 \ldots i_k \end{pmatrix} \right.$$
$$\left. \times \mathbf{M}\begin{pmatrix} j_1 \ldots j_k \\ i_1 \ldots i_k \end{pmatrix} \varepsilon_{i_1 j_1} \ldots \varepsilon_{i_k j_k} \right]^{-1} , \tag{23}$$

where $\Delta = \det(\mathbf{I} - \mathbf{A})$,

$$\mathbf{M}\begin{pmatrix} j_1 \ldots j_k \\ i_1 \ldots i_k \end{pmatrix}$$

is a minor derived from $\det(\mathbf{I} - \mathbf{A})$ by removal of rows $i_1, \ldots, i_k$, and of columns $j_1, \ldots, j_k$,

$$\operatorname{sign}\begin{pmatrix} j_1 \ldots j_k \\ i_1 \ldots i_k \end{pmatrix} = (-1)^z ,$$

$$z = i_1 + \ldots + i_k + j_1 + \ldots + j_k + \sigma(i_1, \ldots, i_k) + \sigma(j_1, \ldots, j_k) ,$$

and $\sigma(i_1, \ldots, i_k)$ is the (odd or even) index of the permutation $i_1, \ldots, i_k$. Should the change take place in only one direct coefficient,

$$\varepsilon_{ij} = \begin{cases} \varepsilon , & i = i_1 , \quad j = j_1 , \\ 0 , & i \neq i_1 , \quad \text{or } j \neq j_1 , \end{cases}$$

then the associated field of influence in matrix form may be approximated by the expression

$$\mathbf{f}(\varepsilon) = \frac{\mathbf{B}(\varepsilon) - \mathbf{B}}{\varepsilon} . \tag{24}$$

Furthermore, an approximate formula may be derived for consideration of the change in two direct coefficients

$$\varepsilon_{ij} = \begin{cases} \varepsilon_1 = \varepsilon_{i_1 j_1} , & i = i_1 , \quad j = j_1 , \\ \varepsilon_2 = \varepsilon_{i_2 j_2} , & i = i_2 , \quad j = j_2 , \\ 0 , & \text{otherwise} , \end{cases}$$

may be seen to be derived as a decomposition of the changes in the sum of the individual influences of each error $\mathbf{f}(\varepsilon_1)$ and $\mathbf{f}(\varepsilon_2)$, the field of cross-interactions between errors

$$b_{j_1 i_1} \mathbf{f}(\varepsilon_2) + b_{j_2 i_2} \mathbf{f}(\varepsilon_1) ,$$

and the field of synergetic interactions which may be obtained from

$$\mathbf{f}(\varepsilon_1)\left(\frac{1+b_{j_1 i_1}}{\varepsilon_2}+b_{j_2 i_2}\right)+\mathbf{f}(\varepsilon_2)\left(\frac{1+b_{j_2 i_2}}{\varepsilon_1}+b_{j_1 i_1}\right)-\frac{\mathbf{B}(\varepsilon_1\varepsilon_2)-\mathbf{B}}{\varepsilon_1\varepsilon_2}\,.$$

An example of the application of this technique is provided in section 7.

**5 A superposition principle for the derivation of the structure of socioeconomic systems**

In earlier sections, a review of alternative methods for the decomposition of the influences associated with change in the Leontief inverse was provided. In this part of the paper, we provide a different approach which focuses attention on the matrices of intermediate flows rather than the inverse matrix, although the relationship between the two matrices in the form of the input–output model provides the necessary linkage. Further developments on the linkage between this approach and some of the others is provided in section 8. In the new approach we examine the flows in terms of their *hierarchical structure*, drawing upon the superposition principle (see Sonis, 1980; 1982; 1985; 1986).

In the superposition principle the socioeconomic accounting system is considered to comprise a decentralised set of subsystems (industrial sectors, components of final demand, etc) which are acting according to different often conflicting and noncommensurable extreme tendencies or trends. In a sense, these tendencies may be regarded analogously as objectives in a multiobjective framework; the intersectoral flows matrix in the input–output model may therefore be regarded as the *resultant* or the 'weighted' sum of these tendencies. As Sonis (1982) has demonstrated, the decomposition of flows viewed in this fashion may be regarded as an inverted problem of multiobjective programming in which the overall challenge is to find the weights associated with various sets of flows in the system. These sets of flows are extracted hierarchically (the most important first) and thus provide a way of decomposing the interactions which differs from the Pyatt and Round (1979) and Defourny and Thorbecke (1984) approaches.

In developing the hierarchical decomposition, consider that the actual set of intersectoral flows, $\mathbf{X}$, may be derived geometrically from the convex multidimensional polyhedron whose inner point is $X$. Thus, the problem is reduced to one of finding the location of $\mathbf{X}$. The mechanics of the solution may be summarised as follows. Assume that a collection of weights were to be hung on the vertices of the convex polyhedron in such a way that the centre of gravity coincided with $\mathbf{X}$. Analytically, this is found by the Minkovski–Karatheordory theorem: every $\mathbf{X}$ within a convex polyhedron can be represented in the form of the additive weighted sum of several vertices, $\mathbf{X}_1, \mathbf{X}_2, ..., \mathbf{X}_k$, with the weights (barycentric coordinates) $p_1, p_2, ..., p_k$, such that

$$\mathbf{X} = p_1\mathbf{X}_1 + p_2\mathbf{X}_2 + \ ... \ + p_k\mathbf{X}_k \ , \tag{25}$$

where

$$0 \leqslant p_i \leqslant 1 \ , \qquad p_1 + p_2 + ... + p_k = 1 \ .$$

It is possible to prove that, in the input-output case, the vertices are the accounting matrices $(\mathbf{X}_1, \mathbf{X}_2, ...)$ of a specific form: in each column of such a matrix there is only one nonzero coefficient. The choice of this vertex corresponds to the 'everything or nothing' principle of the economic transactions. Of course, such an extreme tendency can only enter with some partial weight—given the multiple objectives in the system—although, as will be noted, the simpler the system, the larger the initial weights and the smaller the number of hierarchies. As with the other decompositions, the one shown in equation (25) is not unique; the choice usually made is a hierarchical viewpoint which is close to the 'principal component' statistical analysis technique. **X** will be decomposed into the extreme tendencies, $\mathbf{X}_1, \mathbf{X}_2, ...$ such that the weight $p_1$ will be the largest one, and

$$1 > p_1 \geqslant p_2 \geqslant ... \geqslant p_k > 0 \ .$$

This hierarchical rule provides the possibility for using the sequential sums

$$p_1 \ , \qquad p_1 + p_2 \ , \qquad p_1 + p_2 + p_3 \ , \qquad ... \ , \qquad p_1 + p_2 + ... + p_s$$

as measures of the appropriateness of partial decompositions

$$\begin{aligned} \mathbf{X} &= p_1\mathbf{X}_1 + p_2\mathbf{X}_2 + ... + p_s\mathbf{X}_s + p_{s+1}\mathbf{Y}_s \\ &= \mathbf{X}_a + p_{s+1}\mathbf{Y}_s \end{aligned} \tag{26}$$

where $p_{s+1} = 1 - p_1 - p_2 - ... - p_s$, and $\mathbf{Y}_s$ is a negligible residual. Thus, one can now interpret the approximate decomposition

$$\mathbf{X} \approx p_1\mathbf{X}_1 + p_2\mathbf{X}_2 + ... + p_s\mathbf{X}_s = \mathbf{X}_a \ , \tag{27}$$

as a reflection of the hierarchical structure of the system under consideration. Clearly, **X** could reflect the intersectoral flows in an input-output table or the broader set of social accounts within a social accounting system. In either case, the system may be specified at the single-economy (region or nation) or multieconomy level. A link between this type of decomposition and the one provided by Pyatt and Round (1979) is suggested in section 8.

## 6 The superposition algorithm

The algorithm presented below was first introduced to the field of regional science in an application which sought to identify push-pull factors in migration (Sonis, 1980). In the development of the algorithm, the procedure seeks to identify successive states of the system beginning with the state, $\mathbf{X}_1$ which is closest to the actual state $\mathbf{X}$; in other words, the vertex of the polyhedron which is closest to $\mathbf{X}$. In most cases, $\mathbf{X}_1$ will only be a simplified representation of $\mathbf{X}$ since only one nonzero component can be included in each column of $\mathbf{X}_1$. This component will be located in the place in which the largest interindustry transactions occurs in each column. Unless one were dealing with a very sparse matrix (that is, an economy with few intersectoral transactions), $\mathbf{X}_1$ would thus include only a fraction of the detailed *distribution* of the total interindustry transactions associated with purchases made by that industry. This can be seen by the fact that, although the column totals of $\mathbf{X}$ and $\mathbf{X}_1$ are the same, the latter matrix has only one entry per column. The value of the weight, $p_1$, associated with the flows $\mathbf{X}_1$, is found in the following manner. The column sum (or row sum, since the methodology can be applied in either direction) of the intermediate transactions, $x_{.j}$, for each column is placed in the cell with the largest flow; this will be referred to as $x_{ij}^1$. The weight, $p_1$, is chosen from the set of numbers satisfying the inequality

$$\mathbf{A} - p\mathbf{X}_1 \geqslant \mathbf{0}\ . \tag{28}$$

In coordinate form, this implies the choice of

$$\begin{aligned} p_1 &= \max\{p \,|\, x_{ij} - p x_{ij}^1 \geqslant 0\ , \quad \text{for all } i,j\} \\ &= \min\left\{\frac{x_{ij}}{x_{ij}^1}\ , \quad \text{for all } x_{ij}^1 \neq 0\right\}\ , \end{aligned}$$

where $x_{ij} \in \mathbf{X}$ and $x_{ij}^1 \in \mathbf{X}_1$.

Thus, $p_1$ is found in two steps. First, the largest flow, $x_{ij}$, is identified in each column. The ratio of this flow to total intermediate flows in that column $(x_{ij}/x_{.j})$ provides the set from which $p_1$ will be chosen. The smallest of these coefficients is assigned to the value of $p_1$. The total intermediate flow in each column, $x_{.j}$, is placed in the $ij$ cell which had the largest interindustry flow in that column. This matrix with only $n$ entries now becomes matrix $\mathbf{X}_1$; the allocations in this matrix represent all-or-nothing flows (extreme tendencies), since the total intermediate flows in each column are allocated to only one cell. Since the system is far more complex than this pattern, only a portion of those flows can be realised; the weight, $p_1$, provides a measure of the degree to which this first-cut approximation of the interindustry flows may be represented by only $n$ flows. Hence, the interindustry flows may now be presented in the following form:

$$\mathbf{X} = p_1\mathbf{X}_1 + \mathbf{R}_1 \tag{29}$$

where $\mathbf{R}_1$ represents a residual of all flows not accounted for by the first extreme tendency. By defining $\mathbf{R}_1$ as

$$\mathbf{R}_1 = (1-p_1)\mathbf{Y}_1 , \tag{30}$$

we may write equation (29) as

$$\mathbf{X} = p_1\mathbf{X}_1 + (1-p_1)\mathbf{Y}_1 . \tag{31}$$

Since $\mathbf{X}$, $\mathbf{X}_1$, and $p_1$ are known, it is possible to find $\mathbf{Y}_1$ from the expression

$$\mathbf{Y}_1 = \left(\frac{1}{1-p_1}\right)\mathbf{X} - \left(\frac{p_1}{1-p_1}\right)\mathbf{X}_1 . \tag{32}$$

The matrix $\mathbf{Y}_1$ includes a zero component in the place corresponding to the choice of the component defining the weight $p_1$ in the expression (29). This zero component may be considered to be a 'bottleneck' which causes the actual state, $\mathbf{X}$, to be deflected from an extreme state in which *all* flows are concentrated in this cell. Geometrically, the existence of a zero component in the state $\mathbf{Y}_1$ means that the point $\mathbf{Y}_1$ lies on the face of a convex polyhedron, that is, in the subpolyhedron of lesser dimension.

At this point, the first level in the hierarchy of flows has been identified; thereafter, the algorithm proceeds to operate on successive residual flows matrices, $\mathbf{Y}_1, \mathbf{Y}_2, ..., \mathbf{Y}_s$ until all flows are exhausted. To illustrate the process, $\mathbf{Y}_1$ is now decomposed into

$$\mathbf{Y}_1 = q_1\mathbf{X}_2 + (1-q_1)\mathbf{Y}_2 , \tag{33}$$

and the total system now becomes

$$\begin{aligned} \mathbf{X} &= p_1\mathbf{X}_1 + (1-p_1)\mathbf{Y}_1 \\ &= p_1\mathbf{X}_1 + (1-p_1)[q_1\mathbf{X}_2 + (1-q_1)\mathbf{Y}_2] \\ &= p_1\mathbf{X}_1 + p_2\mathbf{X}_2 + (1-p_1-p_2)\mathbf{Y}_2 , \end{aligned} \tag{34}$$

where $p_2 = (1-p_1)q_1$, and the second residual, $\mathbf{Y}_2$, now includes two fixed zero coordinates.

The weights $p_1, ..., p_s$ provide information about the degree to which the total set of interindustry transactions might be represented by a significantly reduced number of transactions. Since each component of the decomposition, $p_i\mathbf{X}_i$ includes only $n$ nonzero components, the first $s$ components of the decomposition will not be represented by more than $sn$ transactions. The number will probably be smaller than this value, $sn$, by reason of the fact that flows in some cells which are not bottleneck cells may appear in more than one level of the hierarchy. Thus, successive additions (superpositions) of the hierarchies may be used to provide a 'reduced form' representation of the transactions flows; associated direct coefficients may be calculated in the normal manner and the usual input–output derived as follows

$$\boldsymbol{Q} = (\mathbf{I}-\mathbf{A}_a)^{-1}\boldsymbol{F} \tag{35}$$

where $\boldsymbol{Q}$ is the estimate of total output, $\boldsymbol{F}$ observed final demand and

$\mathbf{A}_a$ the direct coefficients matrix derived from the transactions associated with the first $s$ levels in the hierarchy.

In the next section, some empirical results are presented from applications to input-output tables for the State of Washington and for Brazil. Thereafter, some preliminary attempts will be made to suggest linkages between the superposition decomposition and the one suggested by Pyatt and Round (1979).

## 7 Empirical results

In this section, a small sample of some empirical applications of the superposition principle will be provided. Further detail on the Brazilian data may be found in Fonseca et al (1987). Table 1 (see over) provides a summary of the estimated weights for the first ten levels (extreme tendencies) in the hierarchy of intersectoral flows for Brazil and Washington, respectively. In the Brazilian case, one may note evidence of increasing complexity in the pattern of flows; in 1959, the first ten tendencies accounted for about 83% of the flows, whereas the same number of levels described only 76% in 1975. This finding would seem to confirm a priori expectations that increasing levels of development would be associated with an increase in the intermediation of production.

In the Washington case, the pattern was less clear; the first tendency accounted for a smaller percentage of the total flows than was the case in Brazil. Furthermore, there was no clear pattern evident between 1963 and 1972. Figures 1 and 2 (see over) show the pattern of flows associated with the first and the first ten tendencies for the State of Washington in 1963.

Although the pattern for the combined levels (1-10) reveals a very strong diagonal arrangement with an associated lower bordering effect, the pattern for individual hierarchies is less clear. The diagonal effect, though, is most prominent in the patterns for the state for all years (1963, 1967, and 1972) and for the Brazilian economy. Partly, this may reflect the aggregation scheme chosen (and the number of sectors portrayed), although the degree of self-importance (that is, intrasectoral flows) was not pronounced.

## 8 Links between decompositions: preliminary statement

Returning to equation (34), we have

$$\mathbf{X} = p_1\mathbf{X}_1 + p_2\mathbf{X}_2 + (1 - p_1 - p_2)\mathbf{Y}_2 \, .$$

Define the set of input coefficients as

$$\mathbf{A} = \mathbf{X}\hat{\mathbf{X}}^{-1} \, . \tag{36}$$

Then, using the approximation of the transactions flows derived from the first $s$ levels in the hierarchy, we obtain

$$\mathbf{A}_a = (p_1\mathbf{X}_1 + p_2\mathbf{X}_2 + \ldots + p_s\mathbf{X}_s)\hat{\mathbf{X}}^{-1} \, . \tag{37}$$

Then, the associated Leontief inverse is

$$\mathbf{B}_a = (\mathbf{I}-\mathbf{A}_a)^{-1} \tag{38}$$

$$\begin{aligned}\mathbf{B} &= (\mathbf{I}-\mathbf{A})^{-1} = (\mathbf{I}-\mathbf{A}_a - p_{s+1}\mathbf{Y}_s\hat{\mathbf{X}}^{-1})^{-1} \\ &= \{(\mathbf{I}-\mathbf{A}_a)[\mathbf{I}-(\mathbf{I}-\mathbf{A}_a)^{-1}p_{s+1}\mathbf{Y}_s\hat{\mathbf{X}}^{-1}]\}^{-1} \\ &= [\mathbf{I}-p_{s+1}(\mathbf{I}-\mathbf{A}_a)^{-1}\mathbf{Y}_s\hat{\mathbf{X}}^{-1}]^{-1}(\mathbf{I}-\mathbf{A}_a)^{-1}\end{aligned} \tag{39}$$

$$= (\mathbf{I}-p_{s+1}\mathbf{B}_a\mathbf{Y}_s\hat{\mathbf{X}}^{-1})^{-1}\mathbf{B}_a\,. \tag{40}$$

**Table 1.** Value of the weights for the first ten hierarchical levels in Brazil, and the State of Washington, in various years.

| Brazil | | | State of Washington | | |
|---|---|---|---|---|---|
| level | weight | cumulative weight | level | weight | cumulative weight |
| 1959 | | | 1963 | | |
| 1 | 0.196 | 0.196 | 1 | 0.142 | 0.142 |
| 2 | 0.135 | 0.332 | 2 | 0.117 | 0.260 |
| 3 | 0.112 | 0.444 | 3 | 0.089 | 0.349 |
| 4 | 0.103 | 0.548 | 4 | 0.070 | 0.420 |
| 5 | 0.068 | 0.617 | 5 | 0.052 | 0.472 |
| 6 | 0.062 | 0.680 | 6 | 0.043 | 0.516 |
| 7 | 0.049 | 0.729 | 7 | 0.043 | 0.560 |
| 8 | 0.038 | 0.768 | 8 | 0.038 | 0.598 |
| 9 | 0.036 | 0.804 | 9 | 0.035 | 0.633 |
| 10 | 0.026 | 0.831 | 10 | 0.033 | 0.667 |
| 1970 | | | 1967 | | |
| 1 | 0.182 | 0.182 | 1 | 0.170 | 0.170 |
| 2 | 0.134 | 0.317 | 2 | 0.084 | 0.255 |
| 3 | 0.118 | 0.435 | 3 | 0.084 | 0.339 |
| 4 | 0.064 | 0.500 | 4 | 0.074 | 0.413 |
| 5 | 0.063 | 0.564 | 5 | 0.065 | 0.479 |
| 6 | 0.056 | 0.620 | 6 | 0.065 | 0.544 |
| 7 | 0.041 | 0.662 | 7 | 0.056 | 0.601 |
| 8 | 0.040 | 0.702 | 8 | 0.037 | 0.638 |
| 9 | 0.031 | 0.734 | 9 | 0.029 | 0.668 |
| 10 | 0.025 | 0.759 | 10 | 0.029 | 0.698 |
| 1975 | | | 1972 | | |
| 1 | 0.179 | 0.179 | 1 | 0.145 | 0.145 |
| 2 | 0.150 | 0.329 | 2 | 0.128 | 0.273 |
| 3 | 0.083 | 0.412 | 3 | 0.094 | 0.368 |
| 4 | 0.083 | 0.496 | 4 | 0.081 | 0.450 |
| 5 | 0.067 | 0.563 | 5 | 0.055 | 0.505 |
| 6 | 0.053 | 0.616 | 6 | 0.052 | 0.558 |
| 7 | 0.044 | 0.661 | 7 | 0.038 | 0.597 |
| 8 | 0.040 | 0.702 | 8 | 0.029 | 0.626 |
| 9 | 0.036 | 0.739 | 9 | 0.029 | 0.655 |
| 10 | 0.026 | 0.765 | 10 | 0.029 | 0.698 |

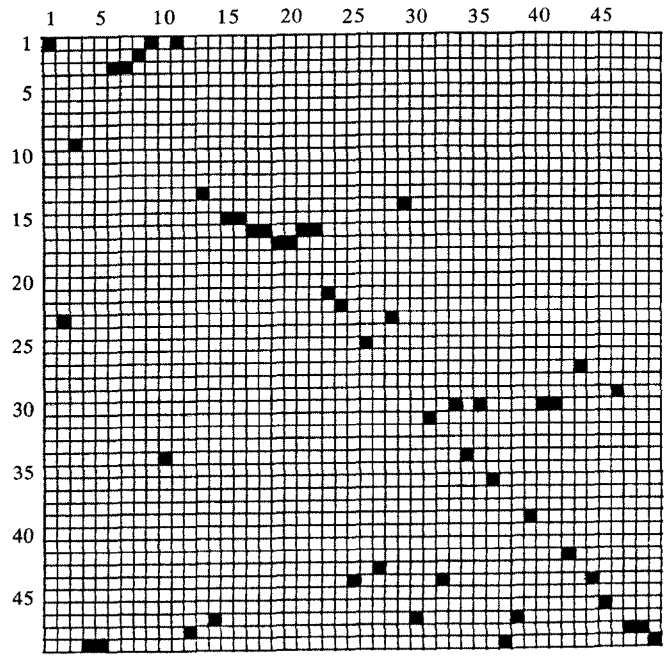


**Figure 1.** Location of flows associated with the first tendency: Washington, 1963.

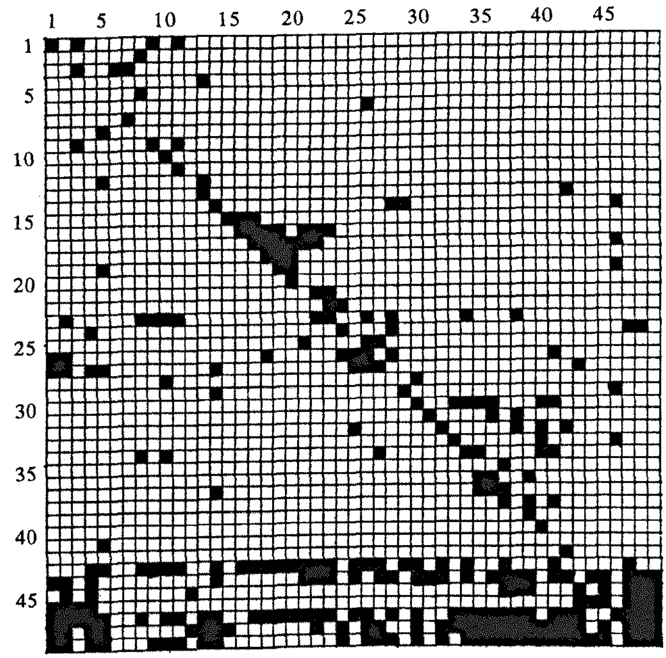


**Figure 2.** Location of flows associated with the first ten tendencies: Washington, 1963.

For small parameter, $p_{s+1}$,

$$\mathbf{B} = (\mathbf{I}+p_{s+1}\mathbf{B}_a\mathbf{Y}_s\hat{\mathbf{X}}^{-1}+p_{s+1}\mathbf{Y}_s\hat{\mathbf{X}}^{-1}(\mathbf{B}_a\mathbf{Y}_s\hat{\mathbf{X}}^{-1}+\ldots)\mathbf{B}_a \tag{41}$$

$$\approx \mathbf{B}_a+p_{s+1}\mathbf{B}_a\mathbf{Y}_s\hat{\mathbf{X}}^{-1}\mathbf{B}_a\,.$$

This decomposition may now be compared with the earlier versions of Pyatt and Round (1979) and Defourny and Thorbecke (1984); in fact, these decompositions may themselves be further evaluated in terms of this hierarchical procedure. In essence, the hierarchical decomposition, by focusing on the most important flows, provides a 'reduced form' approach to the decomposition of the multipliers.

The superposition approach may be used as a vehicle to suggest appropriate partitions of the transactions matrix for the input-output or social accounting table. A link with the Defourny and Thorbecke (1984) approach may be inferred by considering the relationship between the flows associated with the extreme tendencies and the flows on the most important arcs in the system. In fact, the linkage between these decompositions and notions of analytical importance (see Byron, 1978; Hewings, 1984a; 1984b; Sonis and Hewings, 1988a; 1988b) is one of the more interesting research problems deriving from this work. To return to the empirical data discussed in section 7, the extraction of appropriate decompositions would seem to require an objective which reaches beyond mere mathematical convenience. In particular, some of the cells shown in figure 2 as having entered at one of the first ten levels in the hierarchy provide no additional information about their analytical importance. One possibility would be to extract cells hierarchically on this basis; although this would require modification of the algorithm, it might prove a way of enriching the insights to be gained from this technique as well as the two earlier ones.

## 9 Conclusions

In this paper, we have compared three decomposition techniques which have been used or proposed for the identification of the structure of social accounting systems. At the present time, each approach would appear to offer some attractive advantages; research in the future should be directed more towards ways in which these techniques might be merged rather than in attempts to reveal their superiority. In particular, the broader issues of coefficient change in social accounting systems, the role of technological change, and diffusion of innovations will require flexible approaches to facilitate their analysis and interpretation. The issues associated with estimation and updating might be addressed in the context of decompositions designed to reveal the subsets of the system on which the greatest attention to accuracy should be placed.

In the search for some new forms of structural representation, care should be taken to address some of the issues raised by Robinson and

Roland-Holst (1987). In particular, their concerns focus on the degree of compatibility between different model paradigms—macroeconomic models, input-output, social accounting, and computable general equilibrium—since, as they note, different initial conditions and adjustment properties lead to differences in the derived multiplier effects. Although decomposition approaches will not 'solve' this problem in the classic sense of the word, they may provide for some measure of reconciliation. This may be effected through attention being focused on the elements or character set of the economy in which the flexibility afforded by a non fix-price adjustment process would yield the greatest improvement in the more traditional fix-price models.

Finally, it should be noted that these approaches may be applied to other social systems represented in matrix form, such as transportation (commodity and passenger) flows and demographic accounts.

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