STA9715 - Test 3 - Formula Sheet

Advanced Inequalities

- Chernoff - (Sub-)Gaussian: if X has mean μ and variance at most σ^2, then $\mathbb{P}(|X-\mu| > t) \leq 2e^{-t^2/2\sigma^2}$ and $\mathbb{P}(X > \mu + t) \leq e^{-t^2/2\sigma^2}$
- Chernoff - Bounded: if X takes values in the range $[a, b]$ with mean μ, then $\mathbb{P}(|X-\mu| > t) \leq 2e^{-2t^2/(b-a)^2}$ and $\mathbb{P}(X > \mu + t) \leq e^{-2t^2/(b-a)^2}$

Moment Generating Functions

- Moment Generating Function: $\mathbb{M}_X(t) = \mathbb{E}[e^{tX}]$
- MGF to Moments: $\mathbb{E}[X^k] = \mathbb{M}_X^{(k)}(0)$
- MGF of linear transforms: $\mathbb{M}_{aX+b} = \mathbb{E}[e^{t(aX+b)}] = e^{tb}\mathbb{M}_X(ta)$
- MGF of sums of independent RVs: $\mathbb{M}_{X+Y}(t) = \mathbb{M}_X(t)\mathbb{M}_Y(t)$
- If $\mathbb{M}_X(t) = \mathbb{M}_Y(t)$, then X and Y have the same distribution.
- Standard Normal MGF: $\mathbb{M}_Z(t) = e^{t^2/2}$

Limit Theory

- Convergence in Probability: $X_n \xrightarrow{P} X_*$ means $\mathbb{P}(|X_n - X_*| > \epsilon) \xrightarrow{n\to\infty} 0$
- Convergence in Distribution: $X_n \xrightarrow{d} X_*$ means $\mathbb{E}[f(X_n)] \xrightarrow{n\to\infty} \mathbb{E}[f(X_*)]$ for 'reasonable' functions $f(\cdot)$.
- Law of Large Numbers: if $X_1, X_2, \dots$ are IID random variables with mean μ and finite variance, $\overline{X}_n \xrightarrow{P} \mu$ for $\overline{X}_n = \frac{1}{n}\sum_{i=1}^n X_i$.
- Central Limit Theorem: if $X_1, X_2, \dots$ are IID random variables with mean μ and finite variance σ^2, then $\sqrt{n}\left(\frac{\overline{X}_n - \mu}{\sigma}\right) \xrightarrow{d} \mathcal{N}(0,1)$. More usefully: $\overline{X}_n \xrightarrow{d} \mathcal{N}(\mu, \sigma^2/n)$
- Delta Method: if $X_n \xrightarrow{d} \mathcal{N}(\mu, \sigma^2)$, then $g(X_n) \xrightarrow{d} \mathcal{N}(g(\mu), \sigma^2 g'(\mu)^2)$ for any differentiable $g(\cdot)$
- Glivenko-Cantelli Theorem ('Fundamental Theorem of Statistics'): the sample CDF $\hat{F}_n(\cdot)$ converges to the true CDF $F_X(\cdot)$ at all points where $F_X(\cdot)$ is continuous
- Massart's Inequality: $\mathbb{P}(\max_x |\hat{F}_n(x) - F_X(x)| > \epsilon) \leq 2e^{-2n\epsilon^2}$ for any distribution and any $\epsilon > 0$

Key Statistical Distributions

- Gaussian / Normal - See above. Standardizing, CLT, Delta Method.
- χ^2_k sum of squares of k IID Standard Normals. Arises from 'goodness of fit' type statistics (*e.g.*, sum of squared errors in OLS)
- t_k - (Student's) t distribution with k degrees of freedom. $t_k \stackrel{d}{=} Z/\sqrt{\chi^2_k/k}$ where $Z \perp\!\!\!\perp \chi^2_k$. t_1 is a Cauchy; t_∞ is a standard normal. Can replace Z with other normal distributions. Arises in testing with unknown variance.
- χ^2_2 is an Expo(1/2) distribution with mean 2
- Gamma distribution: sum of n exponential distributions with mean $1/\theta$ is $\Gamma(n,\theta)$ distributed. $\Gamma(n/2, 2) \xrightarrow{d} \chi^2_n$
- Beta distribution = Gamma ratio. $X \sim \Gamma(\alpha,\theta), Y \sim \Gamma(\beta,\theta) \implies X/(X+Y) \sim B(\alpha,\beta)$. Distribution on $[0,1]$, used for probabilities.