MATH 310L112
Introduction to Mathematical Reasoning
Assignment #5

Michael Wise

March 4th, 2020

Exercise 6: Suppose $a, b, c \in \mathbb{Z}$. If $a \mid b$ and $a \mid c$, then $a \mid (b + c)$.

Proof. Let $a \mid b$ and $a \mid c$ for $a, b, c \in \mathbb{Z}$. We can say $b = ka$ and $c = la$ for some $k, l \in \mathbb{Z}$. Then, $b + c = ka + la = a(k + l)$. We have just shown that their sum is a multiple of a and thus, $a \mid (b + c)$. $\square$

Exercise 14: If $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd. (Try cases.)

Proof. Suppose $n \in \mathbb{Z}$. Therefore, n can be even or odd. Then either $n = 2a$ or $n = 2a + 1$ for some $a \in \mathbb{Z}$. Let's consider both of these cases.

Case 1. $n = 2a$ for some $a \in \mathbb{Z}$. Then

$$\begin{aligned} 5n^2 + 3n + 7 &= 5(2a)^2 + 3(2a) + 7 \\ &= 20a^2 + 6a + 6 + 1 \\ &= 2(10a^2 + 3a + 3) + 1. \end{aligned}$$

Since $(10a^2 + 3a + 3)$ is just an integer, $5n^2 + 3n + 7$ is odd.

Case 2. $n = 2a + 1$ for some $a \in \mathbb{Z}$. Then

$$\begin{aligned} 5n^2 + 3n + 7 &= 5(2a+1)^2 + 3(2a+1) + 7 \\ &= 5(4a^2 + 4a + 1) + 6a + 3 + 7 \\ &= 20a^2 + 26a + 14 + 1 \\ &= 2(10a^2 + 13a + 7) + 1. \end{aligned}$$

Since $(10a^2 + 13a + 7)$ is just an integer, $5n^2 + 3n + 7$ is odd.

In each case we get that $5n^2 + 3n + 7$ is odd, as desired. □

Exercise 16: If two integers have the same parity, then their sum is even.

Proof. Suppose $x, y \in \mathbb{Z}$ have the same parity. Therefore, either x, y are both odd or x, y are both even. We have two cases:

Case 1. x, y are both odd. Then, $x = 2a + 1$ and $y = 2b + 1$ for some $a, b \in \mathbb{Z}$. Then

$$\begin{aligned} x + y &= 2a + 1 + 2b + 1 \\ &= 2a + 2b + 2 \\ &= 2(a + b + 1). \end{aligned}$$

Since $(a + b + 1)$ is an integer, by definition, $x + y$ is even.

Case 2. x, y are both even. Then, $x = 2a$ and $y = 2b$ for some $a, b \in \mathbb{Z}$. Then

$$\begin{aligned} x + y &= 2a + 2b \\ &= 2a + 2b \\ &= 2(a + b). \end{aligned}$$

Since $(a + b)$ is an integer, by definition, $x + y$ is even.

Because $x + y$ is even in both cases, we are done. $\square$

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% PREAMBLE
\documentclass[12pt]{article}
\usepackage{amssymb, amsmath, amsthm}
   % libraries of additional mathematics commands
\usepackage[paper=letterpaper, margin=1in]{geometry}
   % sets margins and space for headers
\usepackage{setspace, listings}
   % allow for adjusted line spacing and printing source code
\title{MATH 310L112\\
       Introduction to Mathematical Reasoning\\
       Assignment \#5}
\author{Michael Wise}
\date{March 4th, 2020}
% END PREAMBLE

\begin{document}
\maketitle
%\thispagestyle{empty}
\begin{description}

\item[Exercise 6:] Suppose $a,b,c \in \mathbb{Z}$. If $a \mid b$ and $a \
   mid c$, then $a \mid (b + c)$.


\begin{spacing}{2}
\begin{proof}
Let $a \mid b$ and $a \mid c$ for $a,b,c \in \mathbb{Z}$. We can say $b=ka
   $ and $c=la$ for some $k,l \in \mathbb{Z}$. Then, $b + c = ka + la = a(
   k+l)$. We have just shown that their sum is a multiple of $a$ and thus,
    $a \mid (b + c)$.
\end{proof}
\end{spacing}
\vspace{.001in}
\item[Exercise 14:] If $n \in \mathbb{Z}$, then $5n^2 + 3n + 7$ is odd. (
   Try cases.)

\begin{spacing}{2}
\begin{proof}
Suppose $n \in \mathbb{Z}$. Therefore, $n$ can be even or odd. Then either
    $n=2a$ or $n=2a+1$ for some $a \in \mathbb{Z}$. Let's consider both of
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\begin{align*}
5n^2 + 3n + 7 &= 5(2a)^2 + 3(2a) + 7 \\
&= 20a^2 + 6a + 6 + 1 \\
&= 2(10a^2 + 3a + 3) + 1.
\end{align*}
Since $(10a^2 + 3a + 3)$ is just an integer, $5n^2 + 3n + 7$ is odd.
\newline
Case 2. $n=2a+1$ for some $a \in \mathbb{Z}$. Then
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5n^2 + 3n + 7 &= 5(2a+1)^2 + 3(2a+1) + 7 \\
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\newline
In each case we get that $5n^2 + 3n + 7$ is odd, as desired.
\end{proof}
\end{spacing}

\item[Exercise 16:] If two integers have the same parity, then their sum
   is even.

\begin{spacing}{2}
\begin{proof}
Suppose $x,y \in \mathbb{Z}$ have the same parity. Therefore, either $x,y$
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\begin{align*}
x+y &= 2a+1 + 2b+1 \\
&= 2a + 2b + 2 \\
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Because $x+y$ is even in both cases, we are done.
\end{proof}
\end{spacing}
\end{description}
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% The commands in this section print the source code starting
% on a new page. Comment out or delete if you do not want to
% include the source code in your document.
%
\newpage
\lstset{
   basicstyle=\footnotesize\ttfamily,
   breaklines=true,
   language=[LaTeX]{TeX}
   }
\lstinputlisting{Assignment5.tex} % Change to correct filename
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
\end{document}
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