

# Local Search & Optimization

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## Agenda

**Local Search**

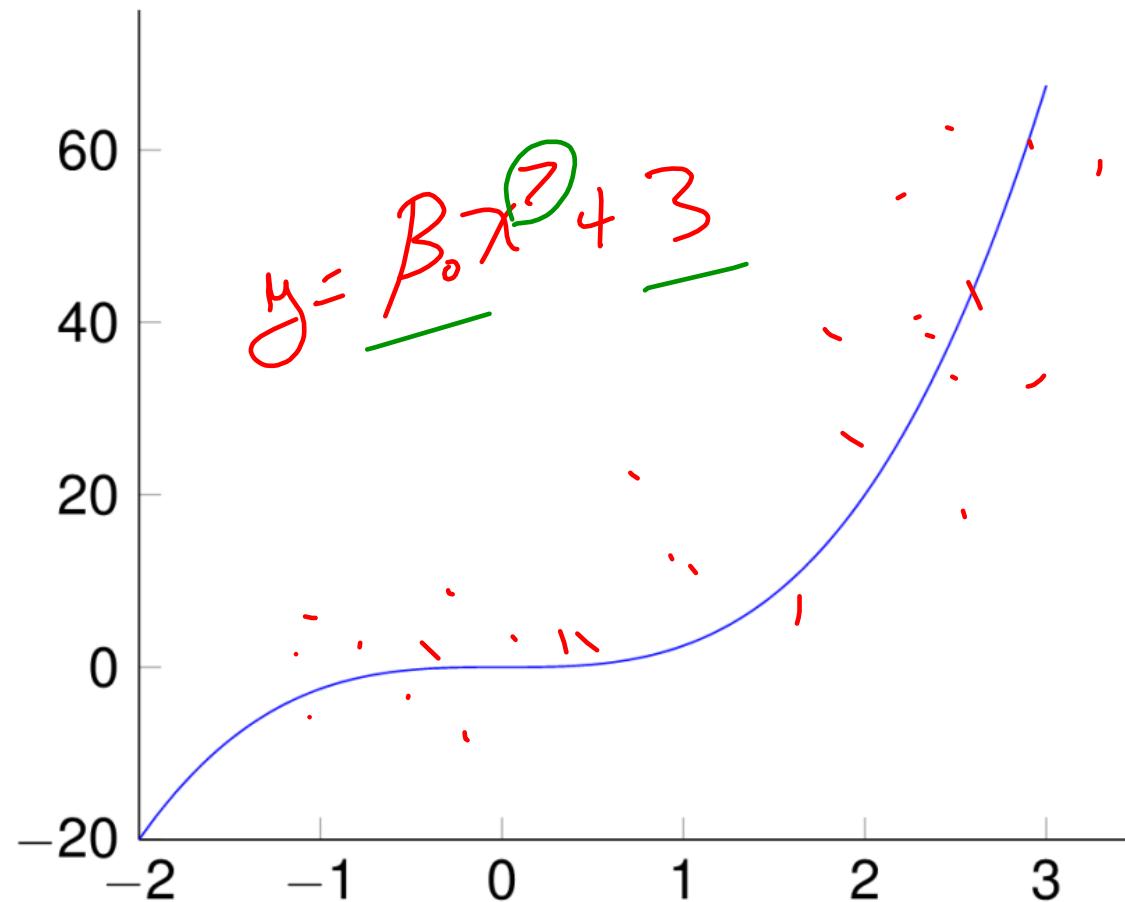
**Approaches**

**Basics of Probability**

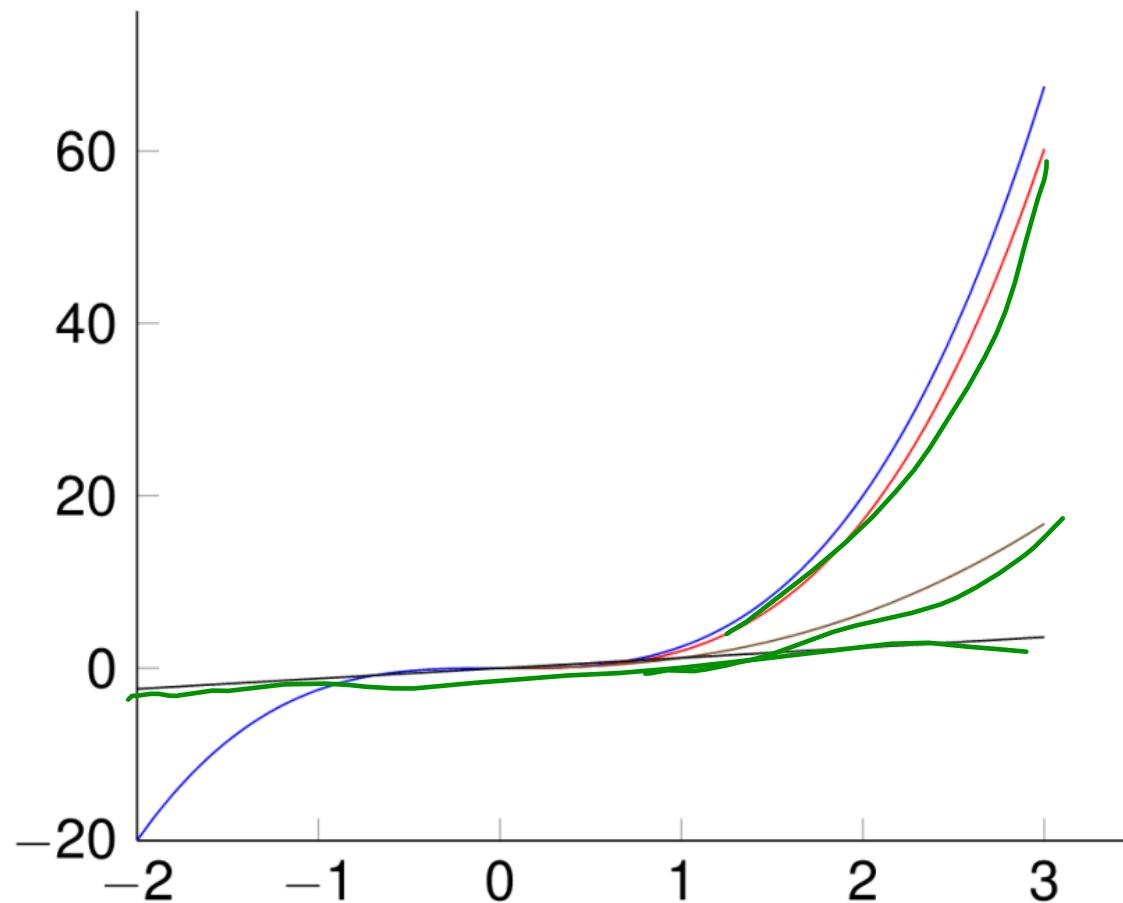
**Bandits**

# Paths Local Search

## Function Approximation



## Function Approximation (2)



## Key points.

- ▶ History no longer matters.  
History no longer matters.
- ▶ We use a solution space.
- ▶ Along with a set of possible modifications.
- ▶ Each solution has a potential fitness
- ▶ We then seek the best fit solution.  
We then seek the best fit solution.

## Problem Definition

1. The solution representation.  
*Ref*  
*Eval*  
*Mot*
2. The available *actions* or modification operations.
3. The Transition Model which defines what each action does:
$$f : S_i \oplus A_j \rightarrow S_k \quad (1)$$
4. The fitness function (sometimes evaluation function).  
*f*  
*S<sub>i</sub>*  
*A<sub>j</sub>*  
*S<sub>k</sub>*

## Curve Fitting Problem

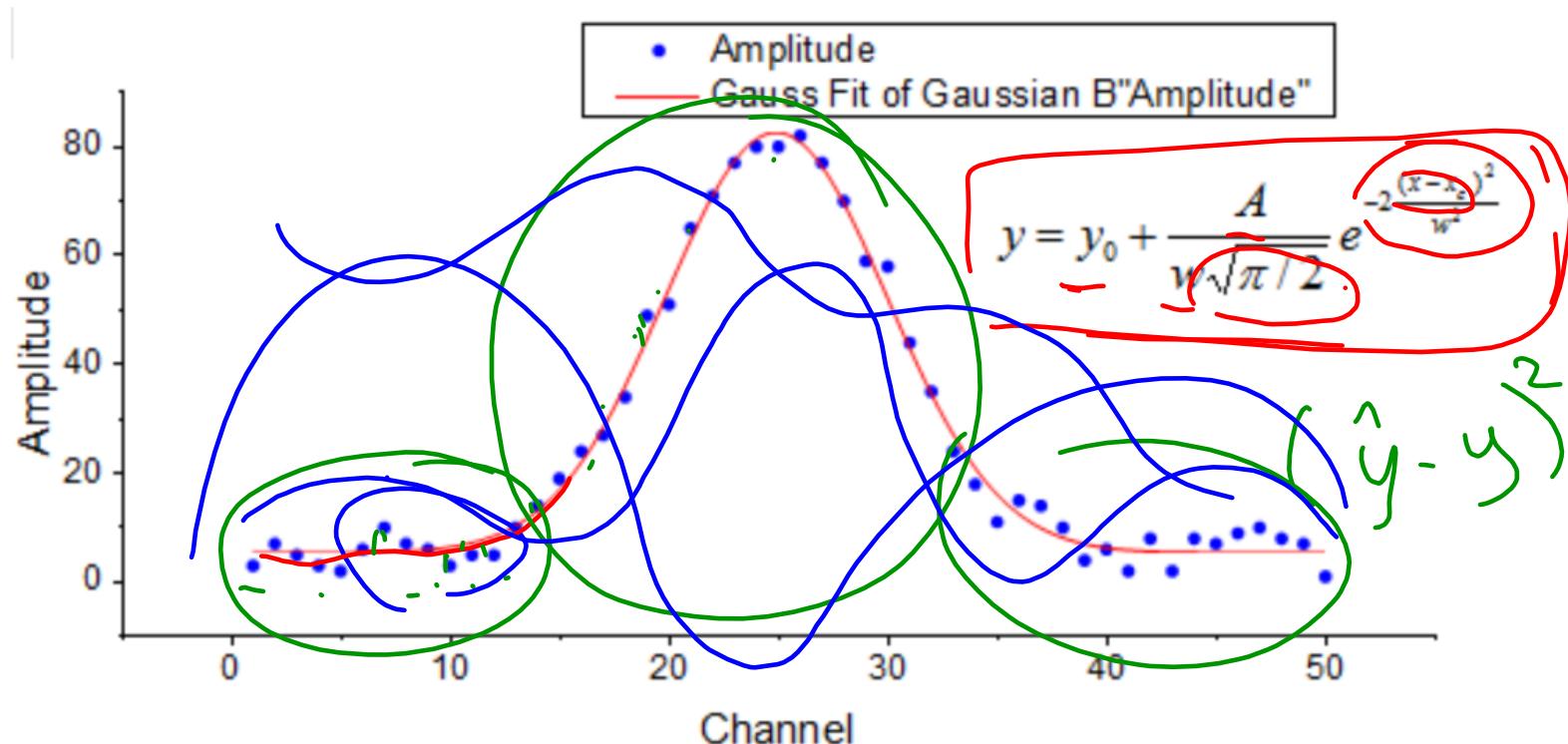


Image Credit: Originlab.com

## Problem Definition

1. *Solution representation.*

*Eg wr*



2. *modification operations.*

*Increase wt*

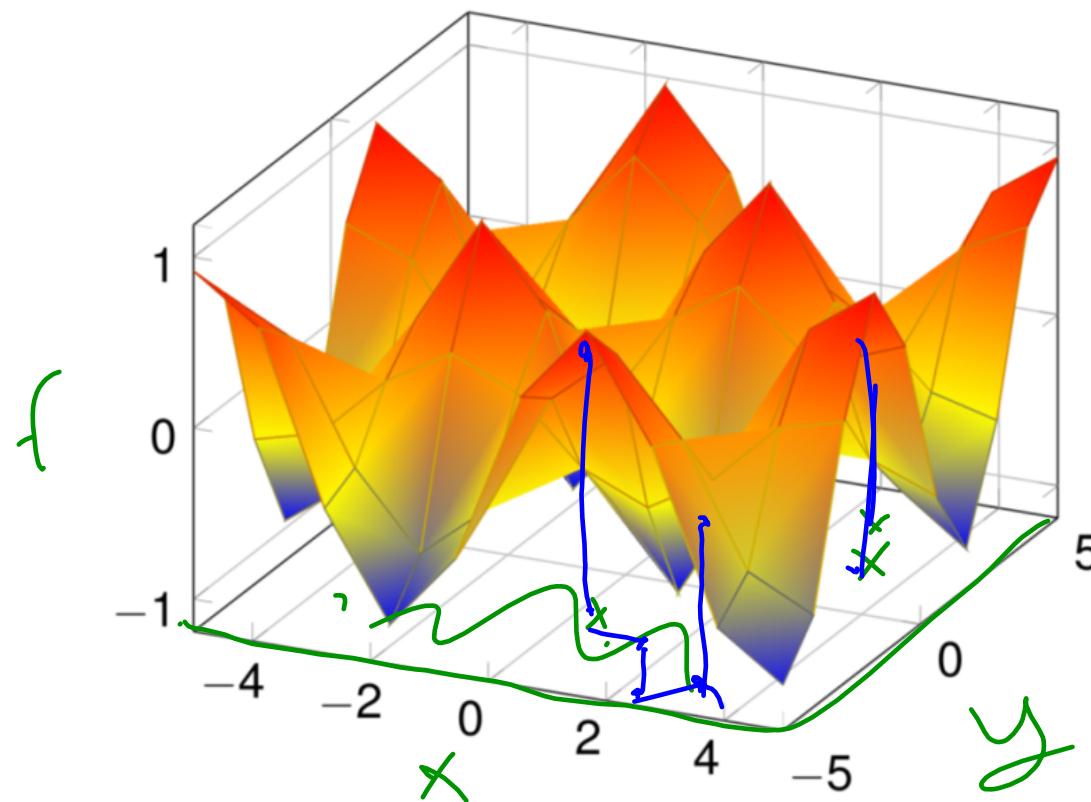
*Drop*

*Add*

3. *Fitness function.*

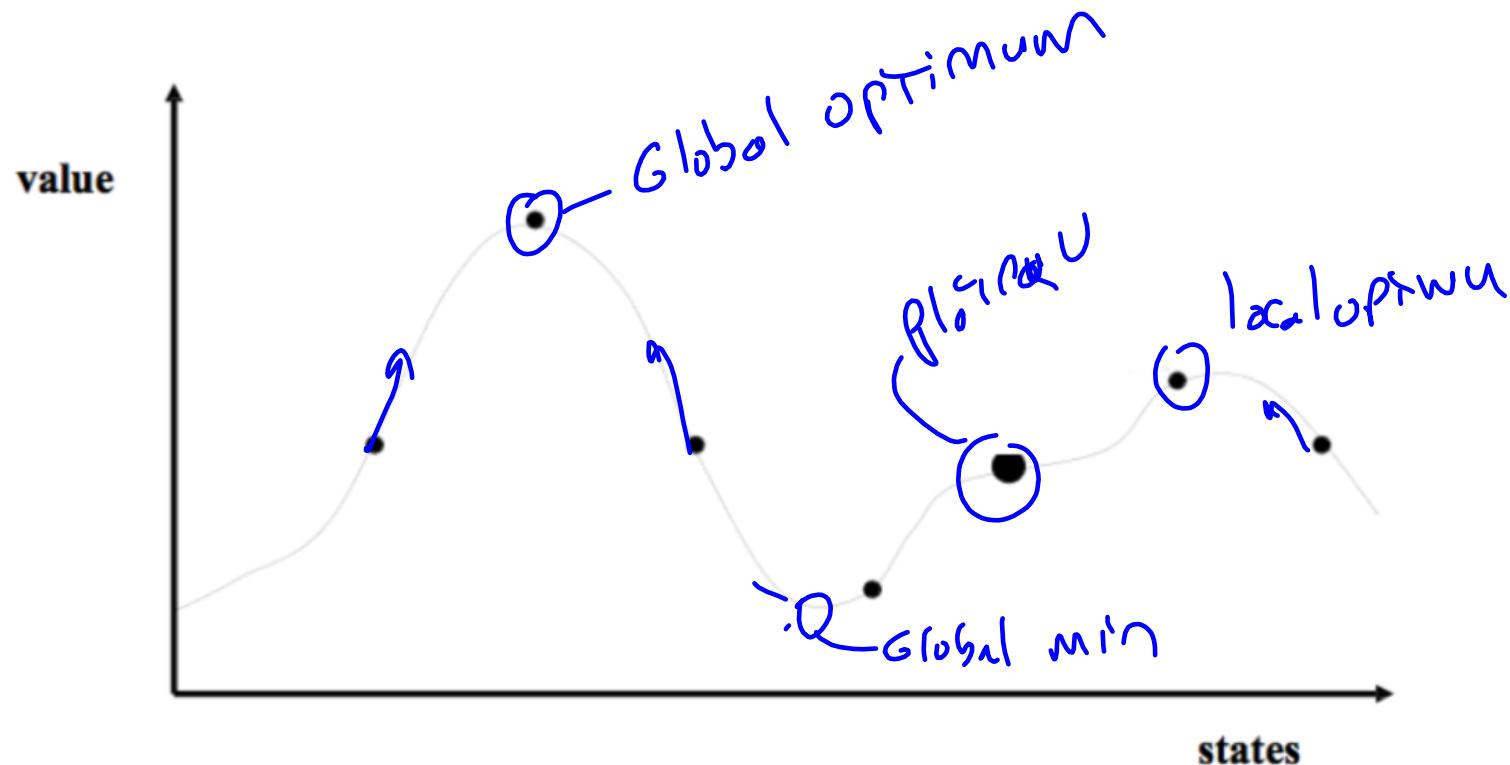
$$(y - \hat{y})^2$$

## Fitness Landscape, Movement



<http://tex.stackexchange.com/questions/97502/2d-colors-above-3d-surface-plot>

## Landscape Features



## Key points

- ▶ Your definitions + algorithm, *define* the search space.
- ▶ Your evaluation function *defines* the fitness landscape.
- ▶ You want them to match.

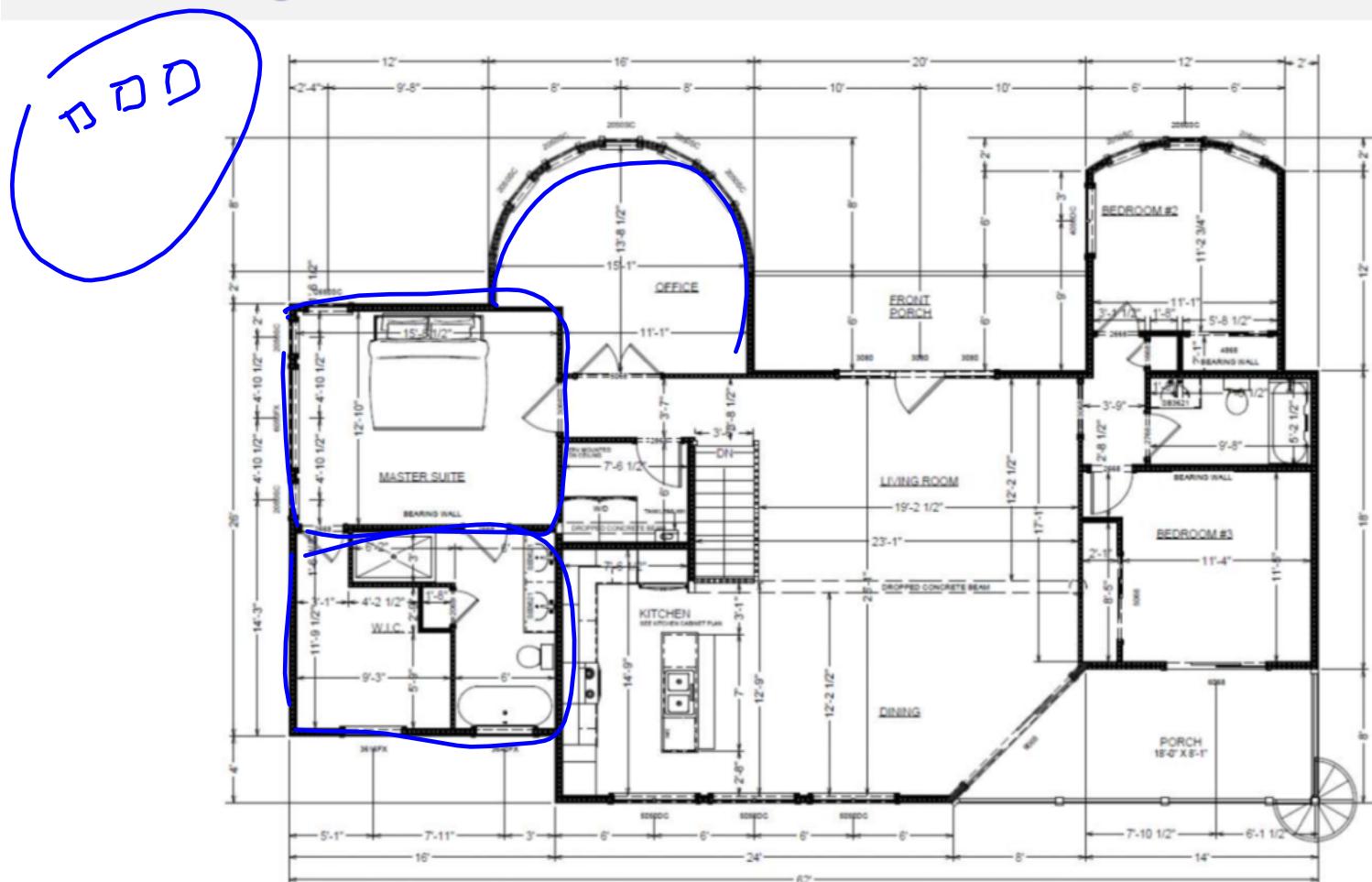
Each one  
use (client)

## AI Housing



Credit: SanAntonio Express News

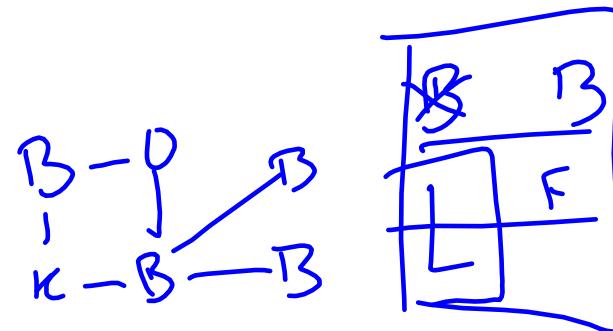
## AI Housing



Credit: SanAntonio Express News

## Problem Definition

1. *Solution representation.*



2. *Modification operations.*

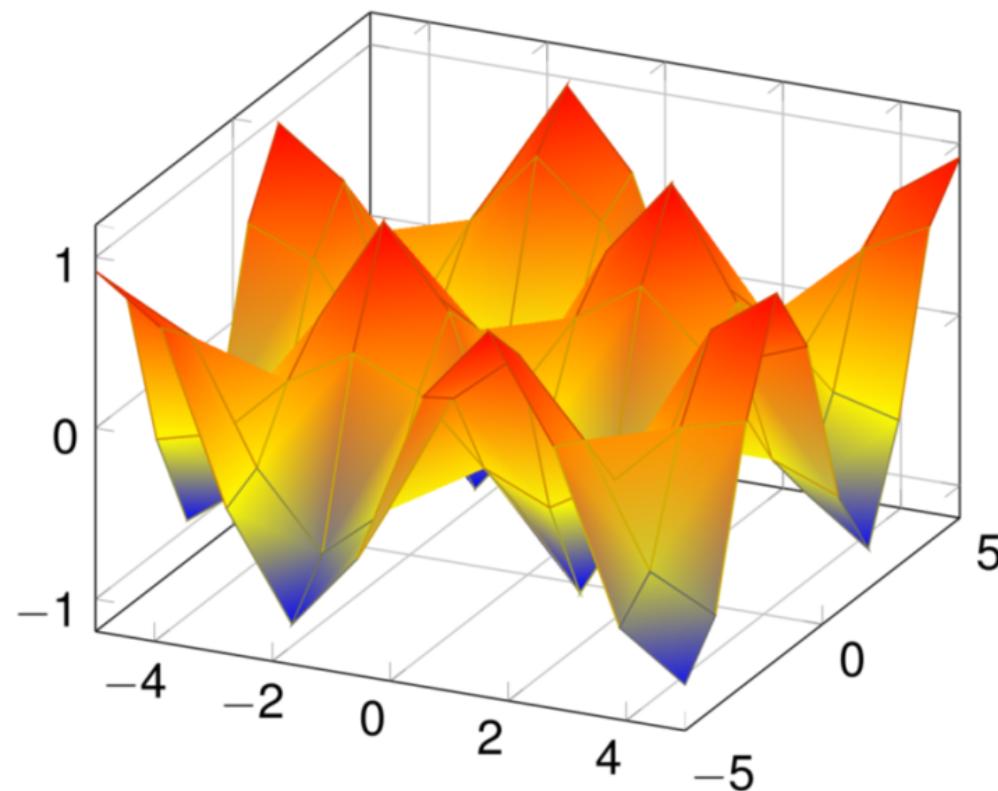
3. *Fitness function.*

Computable

Surroundence

Estimate Pow

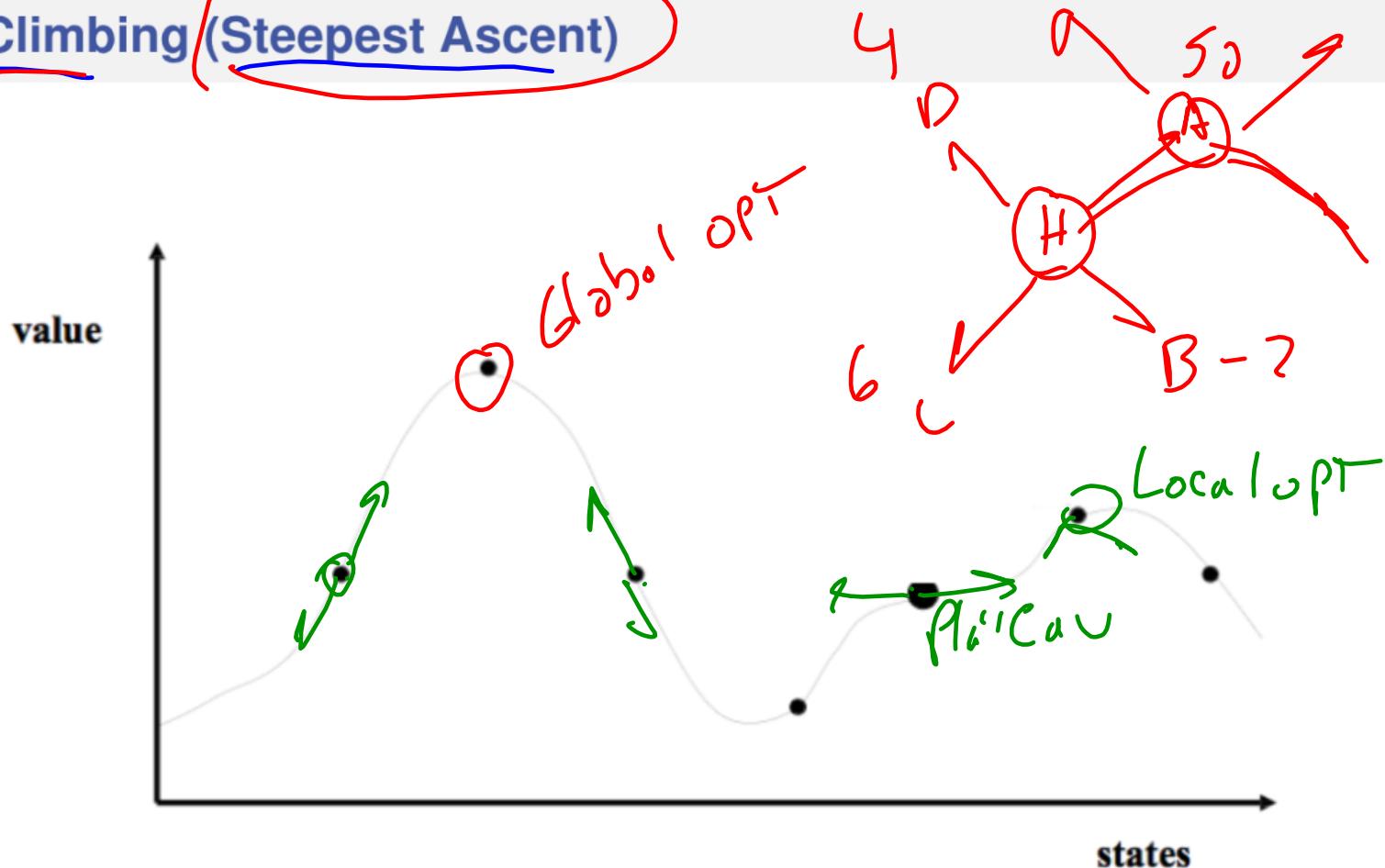
## State Search Space & Fitness Landscape



<http://tex.stackexchange.com/questions/97502/2d-colors-above-3d-surface-plot>

# Approaches

## Hill Climbing (Steepest Ascent)



## Criteria

- ▶ Completeness

NO

- ▶ Optimality

NO

- ▶ Time Complexity

how much do you have?

- ▶ Space Complexity

how much do you have?  
solution representation

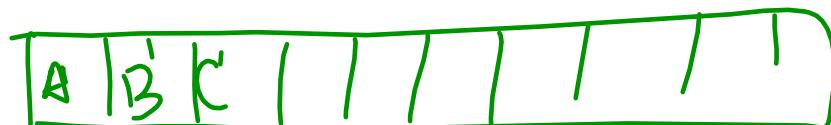
Berm

## Other Hill Climbing

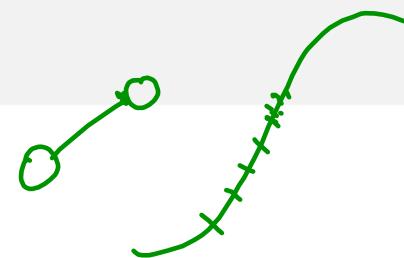
- ▶ Stochastic Hill-Climbing.
- ▶ Random Restart.
- ▶ Simulated Annealing:  $\delta E \propto t$
- ▶ Beam Search.

Y<sub>o</sub> → worse choice  
Outer : Restart  
Inner : Hillclimbing

Why have just one?



## Continuous Gradients



- ▶ Continuous applications allow for a true fitness landscape.
- ▶ Represent gradient as:  $\nabla f = \left( \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \dots \right)$
- ▶ Calculate (estimate) it empirically.
- ▶ Select optimal update.

## Evolutionary Computation

Submodulw.

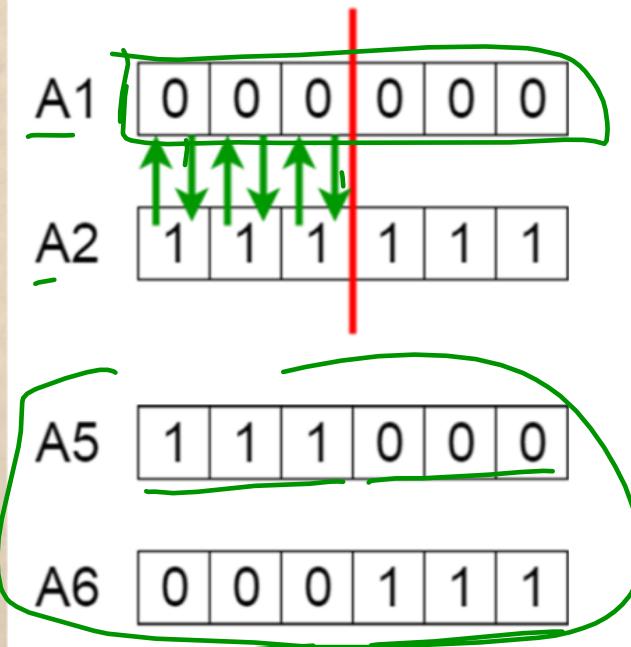
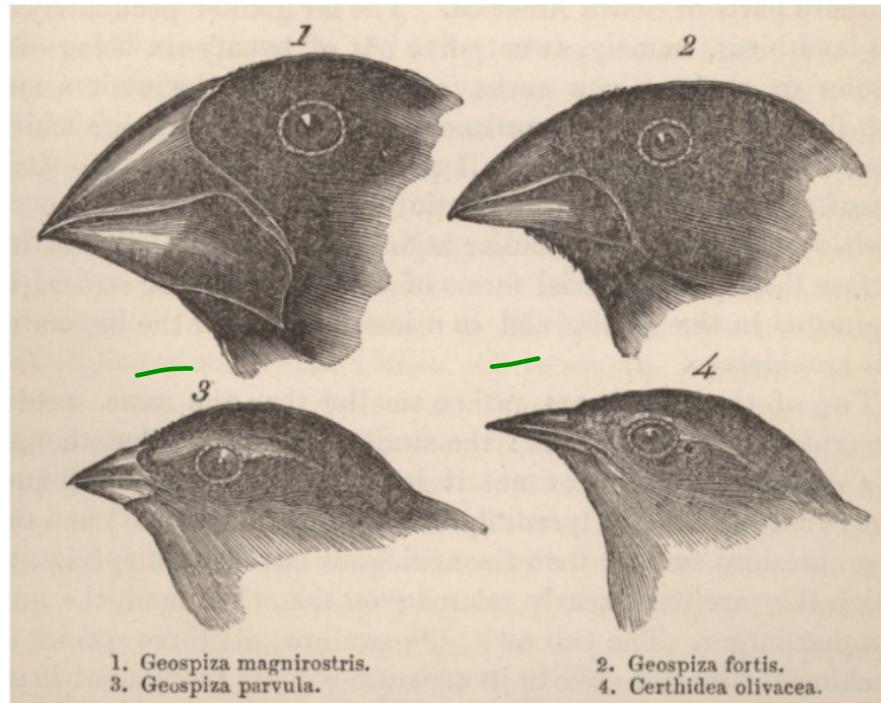
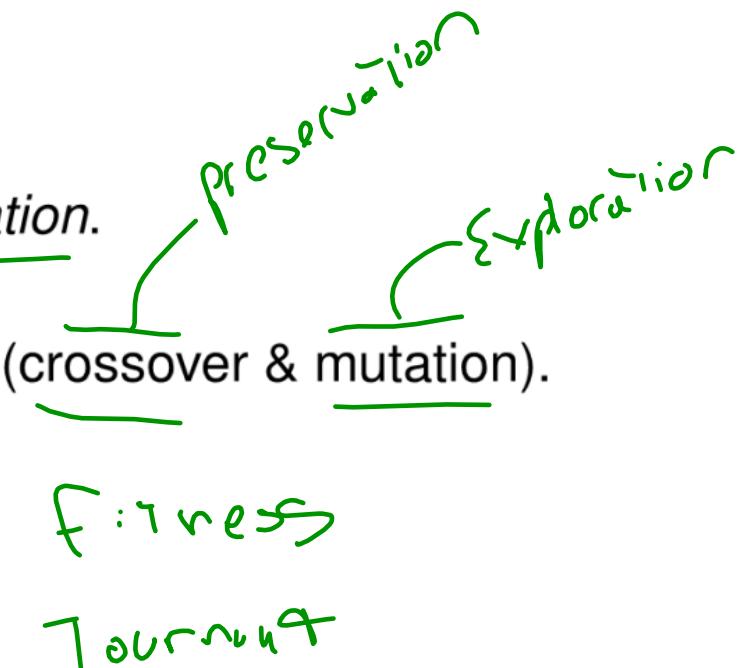


Image credit: Yale Scientific Publishers, & Vijini Mallawaarachchi

## Problem Definition

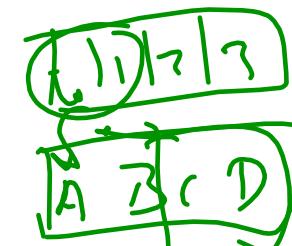
A\*  
AST

1. *Solution representation.*
2. *Genetic operations (crossover & mutation).*
3. *Selection function.*



## Evolving Test Cases

- ▶ Evolve test suites for a designated block of code.
- ▶ Solution Representation: A full suite of black-box tests.
- ▶ Crossover: Exchange of tests between suites.
- ▶ Mutation: Splitting, altering IO parameters.
- ▶ Fitness Function: Code coverage and error cases.
- ▶ EvoSuite: Fraser 2018



# *Machine Learning is Search*

Over Representations.

# Basics of Probability

## Foundation of Probability



Antoine Gombaud, Chevalier de Méré.

骰子: one double six during 24 throws.

Credit: <http://www.mon-poeme.fr>

## Variables Worlds and Samples

- ▶ Logical space defined by variables (e.g. Dice<sub>1</sub>).
- ▶ Sample Space ( $\Omega$ ): The space of all possible worlds.
- ▶ Probability Model: Assignment of probabilities to states:
  - $0 \leq p(\omega) \leq 1 \forall \omega$
  - $\sum_{\omega \in \Omega} p(\omega) = 1$

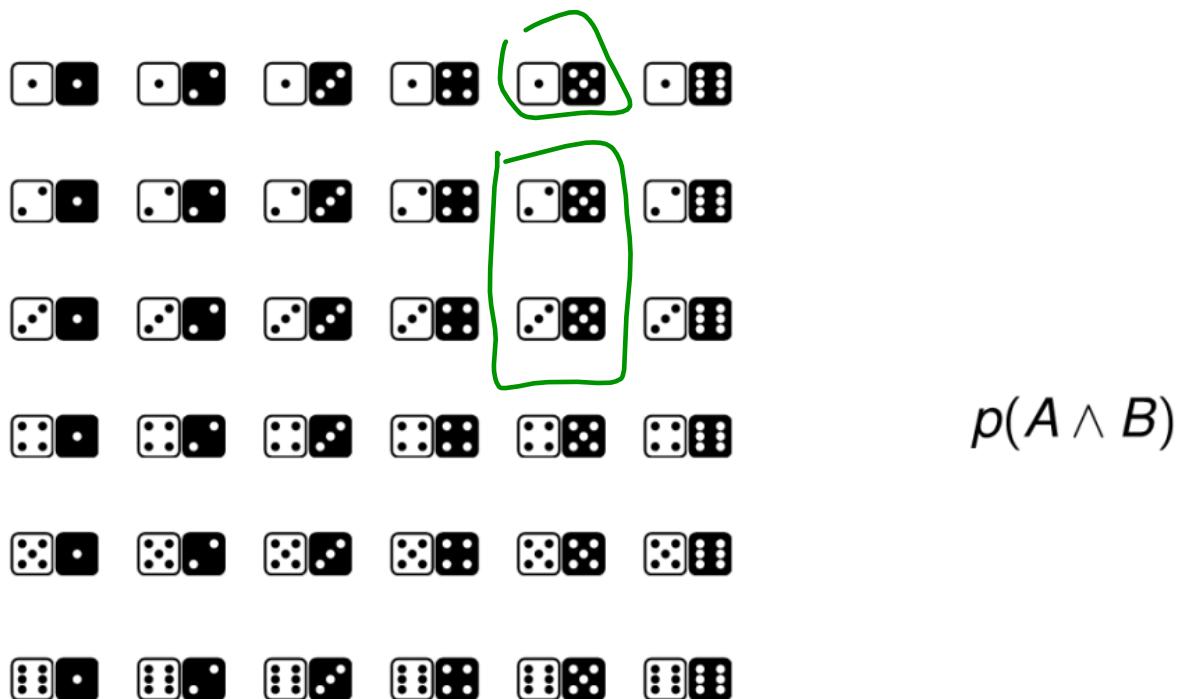
## Single Dice

D<sub>1</sub>

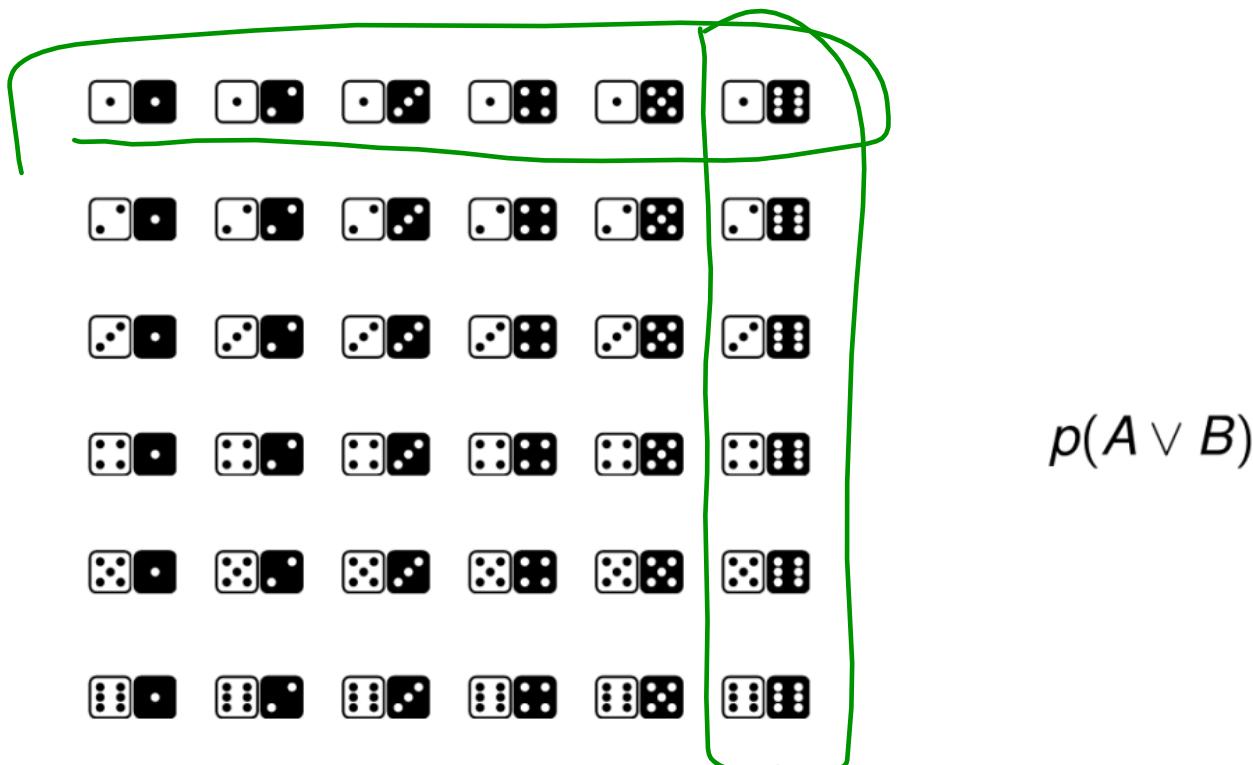


1/6

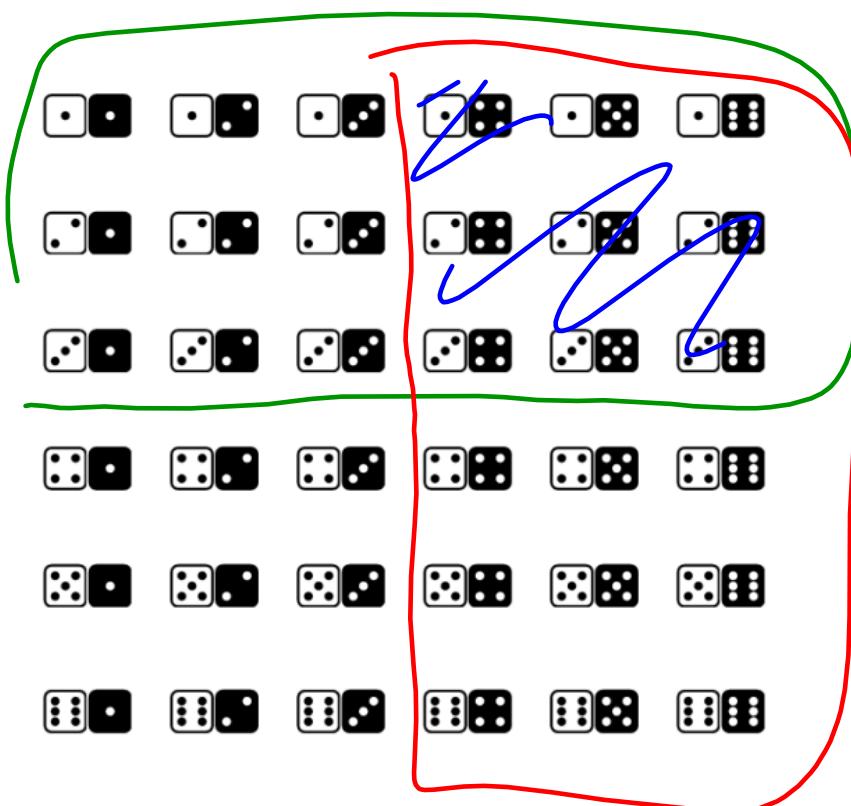
## Joint Probability $\wedge$



## Joint Probability $\vee$

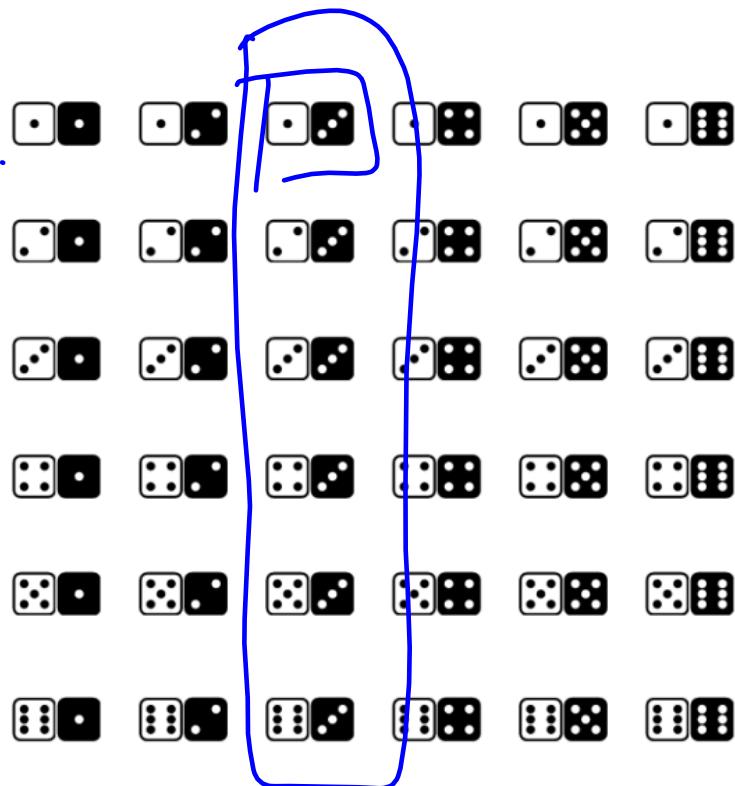


## Joint Probability Distribution



$$p(D_1 < 4), p(D_2 > 2)$$

## Conditional Probability |



$$p(A|B) = \frac{p(A \wedge B)}{p(B)}$$

## The Axioms of Probability

Name	Unconditional
Axiom	$P(A) + P(\neg A) = 1$
Fundamental Law	$P(A, B) = P(A B)P(B)$
Bayes Theorem	$P(B A)P(A) = P(A B)P(B)$
Independence	$A, B$ ind. iff $P(A B) = P(A)$
Independence V2	$A, B$ ind. iff $P(A, B) = P(A)P(B)$
Marginalization	$\sum_B P(A, B) = P(A)$
Expectation	$E[f] = \sum_X f(X)P(X)$

## The Axioms of (Conditional) Probability

Name	Conditional
Axiom	$P(A C) + P(\neg A C) = 1$
Fundamental Law	$P(A, B C) = P(A B, C)P(B C)$
Bayes Theorem	$P(B A, C)P(A C) = P(A B, C)P(B C)$
Independence	$A, B \text{ CI given } C \text{ iff } P(A B, C) = P(A C)$
Independence V2	$A, B \text{ CI given } C \text{ iff } P(A, B C) = P(A C)P(B C)$
Marginalization	$\sum_B P(A, B C) = P(A C)$
Expectation	$E[X C] = \sum_X f(X)P(X C)$

## Probability vs. Likelihood

- ▶ We often use probability and likelihood interchangeably.
- ▶ The likelihood of B given A is  $L(B|A)$
- ▶ This is the reversed conditional:  $p(A|B)$

# Bandits

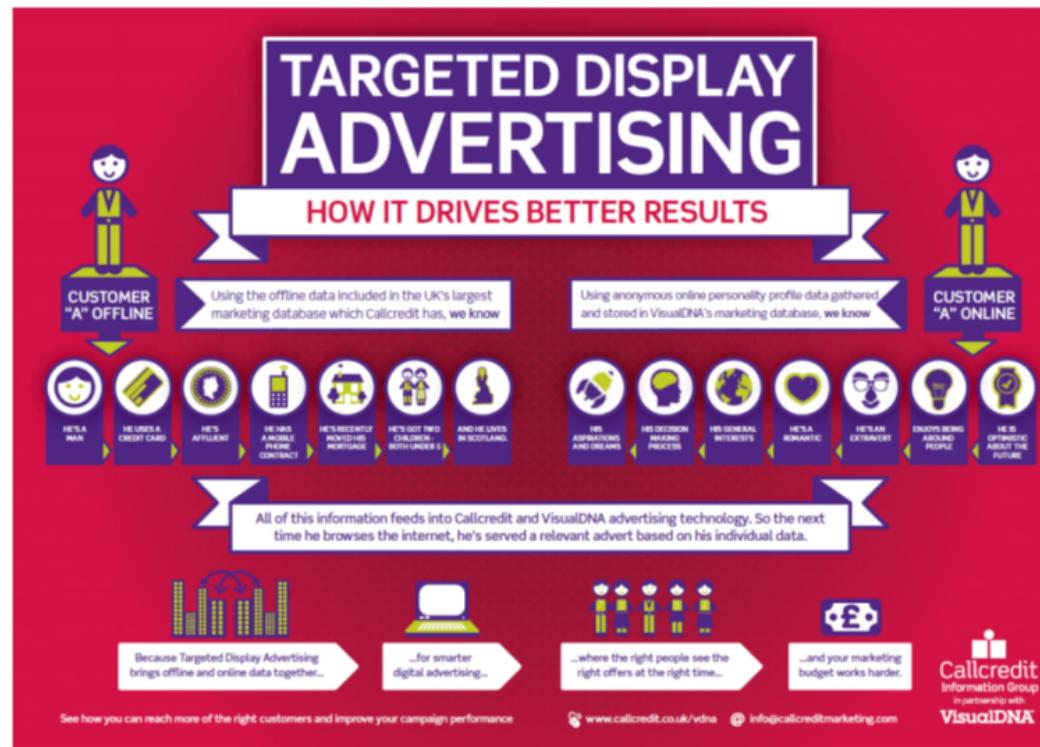
## Bandits

Exploration/Exploitation



Credit: [wwwlibertygames.co.uk](http://wwwlibertygames.co.uk)

## Applications: Recommendation Systems



## Applications: Social Networks



## Core Bandits

Stationary



$$\underline{p_i} = \tilde{w}_a^i(1 - \gamma) + \gamma \xi_a \quad (2)$$

- ▶  $\tilde{w}_a^i$  normalized weight for action  $a$  at time  $i$ .
- ▶  $\gamma$  exploration rate.
- ▶  $\xi_a$  uniform distribution parameter.

$$\underline{w}_a^i \leftarrow \beta w_a^{i-1} + \eta r \quad (3)$$

- ▶  $\beta$  decay parameter for the weight.
- ▶  $\eta$  reward weight.
- ▶  $r$  reward from current selection.

- 📄 Fraser, Gordon (2018). “A Tutorial on Using and Extending the EvoSuite Search-Based Test Generator”. In: *Search-Based Software Engineering*. Springer, pp. 106–130.