Week 4-Seminar 1
Q2-EM & MLE

Q2: **EM** & MLE **Exercise**

In the world's largest international science competition, young scientists from more than 80 countries, regions and territories will be selected for different levels of awards: Gold, Silver, and Bronze. Each candidate can win *no more than* one award. The probability that a candidate would receive each level of award is: $P(Gold) = \frac{2}{3} - 5\mu$, $P(Silver) = \mu$, $P(Bronze) = \frac{1}{3}$, and finally $P(None) = 4\mu$ for no prize at all. Ultimately, that 1) a total of C candidates got either "Gold" or "Silver" prize, that is $g + s = C$, 2) b candidates got a Bronze medal, and 3) n candidates got no award. Given the information above, use expectation maximization to obtain a maximum likelihood estimate of μ.

Expectation step (E-step): What are the expected values of g and s for given μ?
Hint: Your answers should be expressed in terms of μ and C only.

Solution:

$\frac{2/3-5\mu}{2/3-4\mu} = \frac{g}{C}$

$\frac{\mu}{2/3-4\mu} = \frac{s}{C}$

$g = \frac{2-15\mu}{2-12\mu} \times C$

$s = \frac{3\mu}{2-12\mu} \times C$

Maximization step (M-step): Use g and s to compute the maximum likelihood estimate of μ. Show your work.

Solution:

$log(P) = g * log(\frac{2}{3} - 5\mu) + s * log(\mu) + b * log(\frac{1}{3}) + n * log(4\mu)$

After derivation:

$g * \frac{-5}{2/3-5\mu} + \frac{s}{\mu} + \frac{n}{\mu} = 0$

$\frac{-15g}{2-15\mu} + \frac{s+n}{\mu} = 0$

$-15g\mu + 2(s+n) - 15(s+n)\mu = 0$

$2(s+n) = 15(g+s+n)\mu$

$\mu = \frac{2(s+n)}{15(C+n)}$