Hidden Markov Model (HMM) Workshop

Week 7

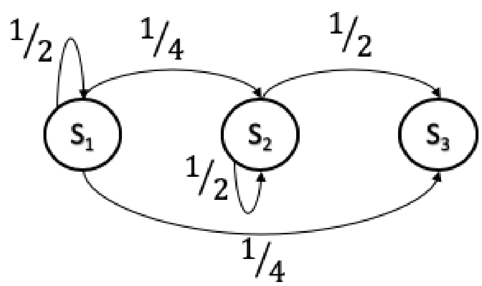
HMM Exercise

Consider an HMM with states $q_t \in \{S_1, S_2, S_3\}$, observations $o_t \in \{A, B, C\}$, and parameters:

$\pi_1 = 1$	$a_{11} = 1/2$	$a_{12} = 1/4$	$a_{13} = 1/4$	$b_1(A) = 1/2$	$b_1(B) = 1/2$	$b_1(C) = 0$
$\pi_2 = 0$	$a_{21} = 0$	$a_{22} = 1/2$	$a_{23} = 1/2$	$b_2(A) = 1/2$	$b_2(B) = 0$	$b_2(C) = 1/2$
$\pi_3 = 0$	$a_{31} = 0$	$a_{32} = 0$	$a_{33} = 1$	$b_3(A) = 0$	$b_3(B) = 1/2$	$b_3(C) = 1/2$

1. Draw the graphical representation of this HMM model.

   Solution:



2. What is $P(q_4 = S_3)$?

   Solution: $1 - P(q_4 = S_1) - P(q_4 = S_2) = 11/16$

   *****For the subquestions in the following, suppose we observe that $O = BABCABC$, starting at time point 1.*****

3. What is $P(q_6 = S_2 | o_{1:7} = BABCABC)$?

   Solution: 0

4. Given the observation $BABCABC$, the $\alpha_t(i)$ is calculated during the forward algorithm as: $\alpha_t(i) = P(o_1, ..., o_t, q_t = i)$. Complete all the cells in the following table. **State your answers as fractions**. No partial credit.

t	$\alpha_t(1)$	$\alpha_t(2)$	$\alpha_t(3)$
1			
2			
3			
4			
5			
6			
7			

Solution:

t	$\alpha_t(1)$	$\alpha_t(2)$	$\alpha_t(3)$
1	2^{-1}	0	0
2	2^{-3}	2^{-4}	0
3	2^{-5}	0	2^{-5}
4	0	2^{-8}	$2^{-6}+2^{-8}$
5	0	2^{-10}	0
6	0	0	2^{-12}
7	0	0	2^{-13}

5. Write down the sequence of $q_{1:7}$ with the maximal posterior probability (most likely path) assuming the observation $BABCABC$. **What is the posterior probability of this path**?

Solution: $S = \{S_1S_1S_1S_2S_2S_3S_3\}$ and $p(S|O) = 1$.