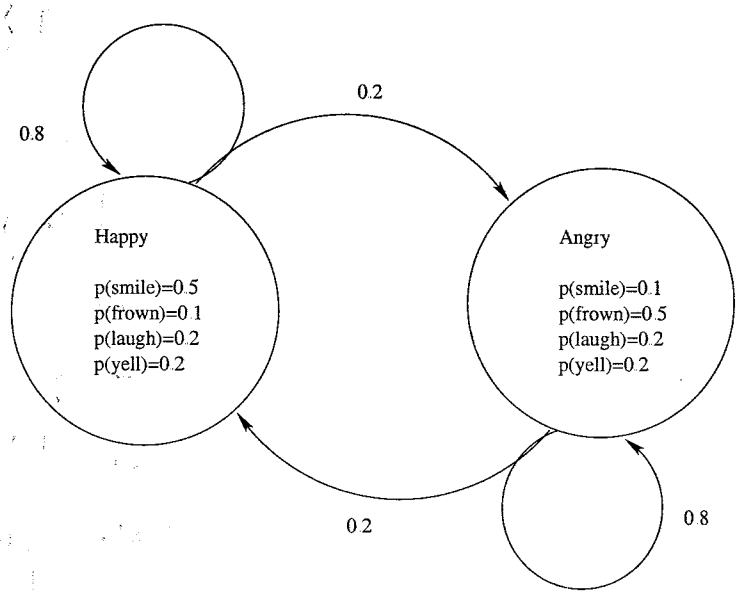
3 HMMs

Andrew lives a simple life. Some days he's Angry and some days he's Happy. But he hides his emotional state, and so all you can observe is whether he smiles, frowns, laughs, or yells. We start on day 1 in the Happy state, and there's one transition per day.



Definitions:
q_t = state on day t.
O_t = observation on day t.

(a) What is $P(q_2 = Happy)$? 0.8

(b) What is $P(O_2 = frown)$? $\frac{8}{10} \times \frac{1}{10} + \frac{2}{10} \times \frac{1}{2} = \frac{8}{100} + \frac{10}{100} = \frac{18}{100}$

(c) What is $P(q_2 = Happy | O_2 = frown)$? $\frac{P(O_2 = frown \mid q_2 = H)\, P(q_2 = H)}{P(O_2 = frown)}$

$= \frac{\frac{1}{10} \times \frac{8}{10}}{\left(\frac{18}{100}\right)} = \frac{4}{9}$

(d) What is $P(O_{100} = yell)$?

$P(O_{100} = yell) = P(O_{100} = yell \mid q_{100} = H) P(q_{100} = H) + P(O_{100} = yell \mid q_{100} = A) P(q_{100} = A)$

$= \frac{2}{10} \times \left(P(q_{100} = H) + P(q_{100} = A)\right) = \frac{2}{10} \times 1 = \frac{2}{10}$

(e) Assume that $O_1 = frown$, $O_2 = frown$, $O_3 = frown$, $O_4 = frown$, and $O_5 = frown$. What is the most likely sequence of states?

HAAAA