

Simple example

Let events be "grades in a class"

$$w_1 = \text{Gets an A} \quad P(A) = \frac{1}{2}$$

$$w_2 = \text{Gets a B} \quad P(B) = \mu$$

$$w_3 = \text{Gets a C} \quad P(C) = 2\mu$$

$$w_4 = \text{Gets a D} \quad P(D) = \frac{1}{2} - 3\mu$$

(Note $0 \leq \mu \leq 1/6$)

Assume we want to estimate μ from data. In a given class there were

a A's
b B's
c C's
d D's

A	B	C	D
14	6	9	10

What's the maximum likelihood estimate of μ given a, b, c, d ?

Maximize likelihood

$$P(A) = 1/2 \quad P(B) = \mu \quad P(C) = 2\mu \quad P(D) = 1/2 - 3\mu$$

$$P(a, b, c, d \mid \mu) = K(1/2)^a(\mu)^b(2\mu)^c(1/2 - 3\mu)^d$$

$$\log P(a, b, c, d \mid \mu) = \log K + a \log 1/2 + b \log \mu + c \log 2\mu + d \log (1/2 - 3\mu)$$

$$\text{FOR MAX LIKE } \mu, \text{ SET } \frac{\partial \text{LogP}}{\partial \mu} = 0$$

$$\frac{\partial \text{LogP}}{\partial \mu} = \frac{b}{\mu} + \frac{2c}{2\mu} - \frac{3d}{1/2 - 3\mu} = 0$$

$$\text{Gives max like } \mu = \frac{b + c}{6(b + c + d)}$$

A	B	C	D
14	6	9	10