Simple example

Let events be "grades in a class"

w_1 = Gets an A   P(A) = ½
w_2 = Gets a B   P(B) = μ
w_3 = Gets a C   P(C) = 2μ
w_4 = Gets a D   P(D) = ½-3μ
(Note $0 \leq \mu \leq 1/6$)

Assume we want to estimate μ from data. In a given class there were

a A's
b B's
c C's
d D's

A	B	C	D
14	6	9	10

What's the maximum likelihood estimate of μ given a,b,c,d ?

Maximize likelihood

$P(A) = ½ \quad P(B) = \mu \quad P(C) = 2\mu \quad P(D) = ½-3\mu$

$P(a,b,c,d \mid \mu) = K(½)^a(\mu)^b(2\mu)^c(½-3\mu)^d$

$\log P(a,b,c,d \mid \mu) = \log K + a\log ½ + b\log \mu + c\log 2\mu + d\log (½-3\mu)$

$$\text{FOR MAX LIKE } \mu\text{, SET } \frac{\partial \text{LogP}}{\partial \mu} = 0$$

$$\frac{\partial \text{LogP}}{\partial \mu} = \frac{b}{\mu} + \frac{2c}{2\mu} - \frac{3d}{1/2 - 3\mu} = 0$$

$$\text{Gives max like } \mu = \frac{b+c}{6(b+c+d)}$$

A	B	C	D
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Same Problem with Hidden Information

Someone tells us that

Number of High grades (A's + B's) = h

Number of C's = c

Number of D's = d

What is the max. like estimate of μ now?

REMEMBER
P(A) = ½
P(B) = μ
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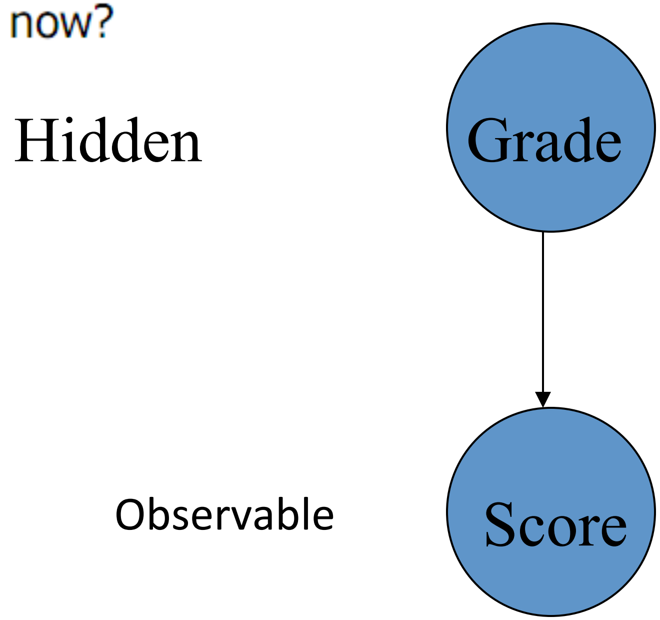
P(D) = ½-3μ



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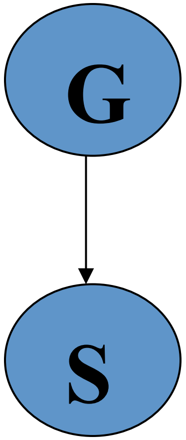

MAXIMIZATION

If we know the expected values of *a* and *b* we could compute the maximum likelihood value of μ

$$\mu = \frac{b+c}{6(b+c+d)}$$

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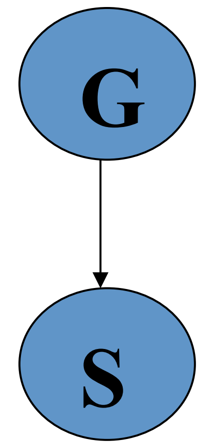

EXPECTATION

If we know the value of μ we could compute the expected value of *a* and *b*

Since the ratio a:b should be the same as the ratio ½ : μ

$$a = \frac{1/2}{1/2 + \mu} h \qquad b = \frac{\mu}{1/2 + \mu} h$$

MAXIMIZATION

If we know the expected values of *a* and *b* we could compute the maximum likelihood value of μ

$$\mu = \frac{b + c}{6(b + c + d)}$$

EM for our example

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We begin with a guess for μ

We iterate between EXPECTATION and MAXIMALIZATION to improve our estimates of μ and *a* and *b*.

Define μ(t) the estimate of μ on the t'th iteration
b(t) the estimate of *b* on t'th iteration

$$\mu(0) = \text{initial guess}$$

$$b(t) = \frac{\mu(\mathrm{t})h}{\frac{1}{2} + \mu(t)} = \mathrm{E}[b \mid \mu(t)]$$

E-step

$$\mu(t+1) = \frac{b(t)+c}{6(b(t)+c+d)}$$

$$= \text{max like est of } \mu \text{ given } b(t)$$

M-step