

Hidden Markov Model (HMM) Workshop

Week 7

HMM Exercise

Consider an HMM with states $q_t \in \{S_1, S_2, S_3\}$, observations $o_t \in \{A, B, C\}$, and parameters:

$\pi_1 = 1$	$a_{11} = 1/2$	$a_{12} = 1/4$	$a_{13} = 1/4$	$b_1(A) = 1/2$	$b_1(B) = 1/2$	$b_1(C) = 0$
$\pi_2 = 0$	$a_{21} = 0$	$a_{22} = 1/2$	$a_{23} = 1/2$	$b_2(A) = 1/2$	$b_2(B) = 0$	$b_2(C) = 1/2$
$\pi_3 = 0$	$a_{31} = 0$	$a_{32} = 0$	$a_{33} = 1$	$b_3(A) = 0$	$b_3(B) = 1/2$	$b_3(C) = 1/2$

1. Draw the graphical representation of this HMM model.
2. What is $P(q_4 = S_3)$? Starting at time point 1.
*****For the subquestions in the following, suppose we observe that $O = BAB CABC$, starting at time point 1.*****
3. What is $P(q_6 = S_2 | o_{1:7} = BAB CABC)$?
4. Given the observation $BAB CABC$, the $\alpha_t(i)$ is calculated during the forward algorithm as: $\alpha_t(i) = P(o_1, \dots, o_t, q_t = i)$. Complete all the cells in the following table.
State your answers as fractions. No partial credit.

t	$\alpha_t(1)$	$\alpha_t(2)$	$\alpha_t(3)$
1			
2			
3			
4			
5			
6			
7			

t	$\alpha_t(1)$	$\alpha_t(2)$	$\alpha_t(3)$
1		0	0
2			
3			
4		2^{-8}	
5		2^{-10}	
6	0	0	
7	0		

5. Write down the sequence of $q_{1:7}$ with the maximal posterior probability (most likely path) assuming the observation $BAB CABC$. **What is the posterior probability of this path?** (Try it now and it will be covered in week 8)