

# Week 4-Seminar 1

## Q2-EM & MLE

### Q2: EM & MLE Exercise

In the world's largest international science competition, young scientists from more than 80 countries, regions and territories will be selected for different levels of awards: Gold, Silver, and Bronze. Each candidate can win *no more than* one award. The probability that a candidate would receive each level of award is:  $P(\text{Gold}) = \frac{2}{3} - 5\mu$ ,  $P(\text{Silver}) = \mu$ ,  $P(\text{Bronze}) = \frac{1}{3}$ , and finally  $P(\text{None}) = 4\mu$  for no prize at all. Ultimately, that 1) a total of  $C$  candidates got either "Gold" or "Silver" prize, that is  $g + s = C$ , 2)  $b$  candidates got a Bronze medal, and 3)  $n$  candidates got no award. Given the information above, use expectation maximization to obtain a maximum likelihood estimate of  $\mu$ .

**Expectation step (E-step):** What are the expected values of  $g$  and  $s$  for given  $\mu$ ?

*Hint: Your answers should be expressed in terms of  $\mu$  and  $C$  only.*

**Solution:**

**Maximization step (M-step):** Use  $g$  and  $s$  to compute the maximum likelihood estimate of  $\mu$ . Show your work.

**Solution:**