Week 4-Seminar 1
Q3-EM

Q3: EM Theory

Consider a simple dice-throwing game in which we are given a pair of dice A and B with unknown biases, θ_A and θ_B, respectively (that is, on any given throw, die A and B will land on an even number with probability θ_A and θ_B respectively.) We repeat the following procedure four times: randomly select one of the two dice and throw it four times. Thus, the entire procedure involves 4 rounds for a total of 16 throws. At each round, the probability that A is selected is λ.

1. [**Fully Observable Case:**] Assume that you were told which die was selected for each round and that $\lambda = 0.7$. Use maximum likelihood estimation to estimate θ_A, θ_B using the following data. No Partial Credit.

	Selected Dice	Outcomes of Throws
1	B	3 1 4 2
2	A	1 6 4 3
3	A	4 3 1 3
1	A	3 2 5 4

2. [**The selected Die is Hidden.**] Now you were not told which die was tossed on each round. You see the following sequence: $< 3426 >, < 6132 >, < 1351 >, < 2436 >$.

   Apply expectation maximization (EM) to estimate $\lambda, \theta_A, \theta_B$. Initially, we have: $\lambda^0 = 0.4$; $\theta_A^0 = 0.7$; $\theta_B^0 = 0.2$

   (a) [**E-Step 1:**] Given λ^0, θ_A^0, θ_B^0, calculate $P(DiceA| < 3426 >)$.
   [**Answer:**]**:**

   (b) [**M-Step 1:**] Assume from E-step 1 that we have:

   - $P(DiceA| < 3426 >) = a_1$,
   - $P(DiceA| < 6132 >) = a_2$
   - $P(DiceA| < 1351 >) = a_3$,
   - $P(DiceA| < 2436 >) = a_4$.

   Use the results from E-step 1 to determine the new estimates for the three parameters: λ^1, θ_A^1, θ_B^1. Express your answers in terms of a_1, a_2, a_3 and a_4.
   [**Answer:**]**:**