Computer Science 161, Homework 6

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Problem 1

a) Let O = Oil, G = Gas, and T = Test. Then we have the following.

$$\Pr(O = \text{True}) = \frac{1}{2}$$
$$\Pr(G = \text{True} \mid O = \text{True}) = 0$$
$$\Pr(G = \text{True} \mid O = \text{False}) = \frac{2}{5}$$
$$\Pr(T = \text{True} \mid O = \text{False}, G = \text{False}) = \frac{1}{10}$$
$$\Pr(T = \text{True} \mid O = \text{False}, G = \text{True}) = \frac{3}{10}$$
$$\Pr(T = \text{True} \mid O = \text{True}, G = \text{False}) = \frac{9}{10}$$
$$\Pr(T = \text{True} \mid O = \text{True}, G = \text{True}) = \text{Undefined}$$

b) We wish to find $\Pr(O = \text{True} \mid T = \text{True})$. This can be done as follows.

$$\begin{aligned}\Pr(O = \text{True} \mid T = \text{True}) &= \frac{\Pr(O = \text{True}, T = \text{True})}{\Pr(T = \text{True})} \\ &= \frac{\frac{1}{2} \times \frac{3}{10}}{\frac{1}{2} \times \frac{3}{10} + \frac{1}{2} \times \frac{2}{5} \times \frac{9}{10} + \frac{1}{2} \times \frac{3}{5} \times \frac{1}{10}} \\ &= \frac{5}{12}\end{aligned}$$

Problem 2

a)

$$\begin{aligned}\Pr(A,B,C,D,E,F,G,H) = {} & \Pr(H \mid E,F)\Pr(G \mid F)\Pr(F \mid C,D)\Pr(E \mid B) \\ & \Pr(D \mid A,B)\Pr(C \mid A)\Pr(B)\Pr(A)\end{aligned}$$

b)

$$\begin{aligned}\Pr(E,F,G,H) = {} & \Pr(H \mid E,F)\Pr(G \mid F)\sum_A \Pr(A)\sum_B \Pr(E \mid B)\Pr(B) \\ & \sum_C \Pr(C \mid A)\sum_D \Pr(D \mid A,B)\Pr(F \mid C,D)\end{aligned}$$

$$\begin{aligned}\Pr(E,F,G,H) = {} & f_8(E,F,H)f_7(F,G)\sum_A f_6(A)\sum_B f_5(B,E)f_4(B) \\ & \sum_C f_3(A,C)\sum_D f_2(A,B,D)f_1(C,D,F)\end{aligned}$$

$$\begin{aligned}f_9(A,B,C,F) &= \sum_D f_2(A,B,D)f_1(C,D,F) \\ f_{10}(A,B,F) &= \sum_C f_3(A,C)f_9(A,B,C,F) \\ f_{11}(A,E,F) &= \sum_B f_5(B,E)f_4(B)f_{10}(A,B,F) \\ f_{12}(E,F) &= \sum_A f_6(A)f_{11}(A,E,F) \\ f_{13}(E,F,G,H) &= f_8(E,F,H)f_7(F,G)f_{12}(E,F) \\ \Pr(E,F,G,H) &= f_{13}(E,F,G,H)\end{aligned}$$

c)

$$\begin{aligned}\Pr(a,\neg b,c,d,\neg e,f,\neg g,h) = {} & \Pr(h \mid \neg e,f)\Pr(\neg g \mid f)\Pr(f \mid c,d)\Pr(\neg e \mid \neg b) \\ & \Pr(d \mid a,\neg b)\Pr(c \mid a)\Pr(\neg b)\Pr(a)\end{aligned}$$

$$\begin{aligned}\Pr(a, \neg b, c, d, \neg e, f, \neg g, h) = {} & \Pr(h \mid \neg e, f) \Pr(\neg g \mid f) \Pr(f \mid c, d) \\ & \times 0.1 \times 0.6 \times \Pr(c \mid a) \times 0.3 \times 0.2\end{aligned}$$

$$\Pr(a, \neg b, c, d, \neg e, f, \neg g, h) = 0.0036 \Pr(h \mid \neg e, f) \Pr(\neg g \mid f) \Pr(f \mid c, d) \Pr(c \mid a)$$

d) Because A and B are independent of each other, we have the following.

$$\Pr(\neg a, b) = \Pr(\neg a) \Pr(b) = 0.8 \times 0.7 = 0.56$$

Since E only depends on B, we have the following.

$$\begin{aligned}\Pr(\neg e \mid a) &= \Pr(\neg e) \\ &= \Pr(\neg e \mid b) \Pr(b) + \Pr(\neg e \mid \neg b) \Pr(\neg b) \\ &= 0.9 \times 0.7 + 0.1 \times 0.3 \\ &= 0.66\end{aligned}$$

e)

1. A is independent of B and E.
2. B is independent of A and C.
3. Given A, C is conditionally independent of B, D, and E.
4. Given A and B, D is conditionally independent of C and E.
5. Given B, E is conditionally independent of A, C, D, F, and G.
6. Given C and D, F is conditionally independent of A, B, and E.
7. Given F, G is conditionally independent of A, B, C, D, E, and H.
8. Given E and F, H is conditionally independent of A, B, C, D, and G.

f)

$$A, B, C, F$$

g)

$$f(A,B,D,E) = \Pr(D \mid A,B)\Pr(E \mid B)$$

A	B	D	E	$f(A,B,D,E)$
0	0	0	0	0.02
0	0	0	1	0.18
0	0	1	0	0.08
0	0	1	1	0.72
0	1	0	0	0.81
0	1	0	1	0.09
0	1	1	0	0.09
0	1	1	1	0.01
1	0	0	0	0.04
1	0	0	1	0.36
1	0	1	0	0.06
1	0	1	1	0.54
1	1	0	0	0.45
1	1	0	1	0.05
1	1	1	0	0.45
1	1	1	1	0.05

h)

$$f(A,B,E) = \sum_D \Pr(D \mid A,B)\Pr(E \mid B)$$

A	B	E	$f(A,B,E)$
0	0	0	0.1
0	0	1	0.9
0	1	0	0.9
0	1	1	0.1
1	0	0	0.1
1	0	1	0.9
1	1	0	0.9
1	1	1	0.1