CS 181 Homework Week 3

Assigned 4/16; Due Mon 4/23 @ 3:00pm at Box A1 CS181 Room BH 2432

Postponed from Week 2:

5. Consider the following two languages over $\Sigma = \{ a, b, c \}$.

   $L_{five} = \{ xby \mid x, y \in \{ a, c \}^*$ and the number of a's in x = the number of a's in y $\}$

   $L_{NFS} = \{ a^n b a^n \mid n \geq 0 \}$

   As discussed in class, languages like L_{NFS} are *not* finite state. Prove that L_{five} is not finite state using closure properties of finite state languages and the fact that L_{NFS} is not finite state.

zero. Show a DFA over the alphabet { a, b } for the following language via the construction in the proof of Theorem 1.25 (that Finite State Languages are closed under union) *and* the footnote 3 on p. 46 (that Finite State Languages are closed under intersection). First construct two DFAs such that the language below is the intersection of the languages of the two DFAs, as in Sipser exercise 1.4:

$$\{ w \in \Sigma^* \mid w \text{ starts with symbol b and contains at most one symbol a} \}$$

one. Consider the language over alphabet $\Sigma = \{ a, b, c \}$:

$$L_{one} = \{ w \in \Sigma^+ \mid w \text{ begins and ends with c} \}$$

a. Show an NFA for $(L_{one})^*$ by defining a DFA for L_{one} and then using the Kleene star construction in the proof of Theorem 1.49.

b. Is $(L_{one})^*$ the same as L_{one}? Very briefly explain your answer.

two. Consider the following language over $\Sigma = \{ 0, 1 \}$:

$$L_{two} = \{ w \in \Sigma^+ \mid \text{if w begins with a 1, } |w| \text{ is odd; and if w begins with a 0, } |w| \text{ is even} \}$$

a: Show an NFA for the language $L_{two} \cdot L_{two}$, by defining a DFA for L_{two} and then using the concatenation construction in the proof of Theorem 1.47.

b: Does $L_{two} \cdot L_{two} = L_{two}$? Very briefly explain your answer.

three: Let $\Sigma = \{ 0, 1 \}$. Give a regular expression for the language:

$$\{ w \in \Sigma^* \mid w \text{ contains at least two 0s and at most one 1} \}$$

four. Refer to Figure 1.61, Sipser page 70, with the internal states named as shown here:

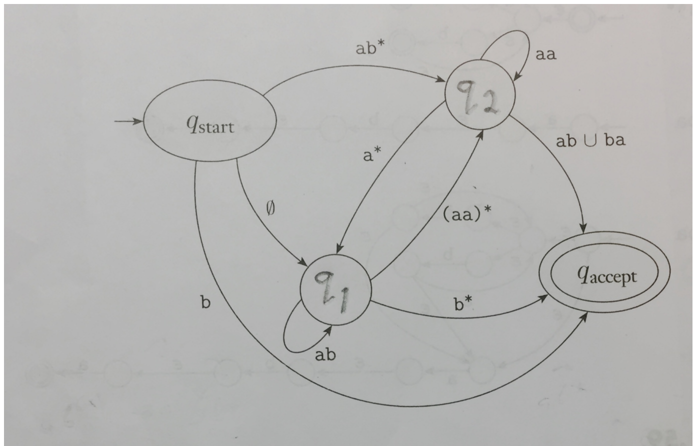

FIGURE 1.61
A generalized nondeterministic finite automaton

a: Verify that ababaaba is accepted by the GNFA.
Show a sequence of states q_{start}, q_1, ... q_{k-1}, q_{accept} , and the corresponding sequence of words $w_1 \ldots w_k$ in the accepting computation.

b: Verify or refute the following claim and *briefly* justify your answer:
Every string accepted by the GNFA that contains more then one b has an odd number of a's.

five. Prove that the following language over alphabet $\Sigma = \{ 0, 1 \}$ is *not* finite state using the pumping lemma:

$\{ 0^i 1^j 0^k \mid i, j, k \geq 0 \text{ and } k = | i - j | \}$

"| i - j |" denotes the absolute value of (i - j)

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