CS 181 Spring 2018 Homework Week 1 (amended 4/4)

Assigned 4/2; Due Mon 4/9 @ 3:00pm at Box A1 CS 181 Room BH 2432

Zero: Briefly explain the system used in the Sipser textbook to number the sections, subsections, exercises, problems, figures, examples, theorems, etc..

Inspired by Sipser Exercises: pp 25-27:

One: Write formal descriptions *using set notation* for the following sets of strings over the alphabet $\Sigma = \{ 0, 1 \}$. It would be easy to just put the English description in {}'s. Can you write it with fewer/no English words to make it more mathematically precise?

a. L_a = All strings over Σ that change from one symbol to the other at most once as you read the string

b. L_b = All strings over Σ that begin and end with the same symbol

Did you consider all the "corner cases"?

Two: Let $\Sigma = \{ a, b, c, 0, 1 \}$. Let A be the set {a, b, c} and let B be the set {0, 1};

and let L_a and L_b be as in the previous problem.

a. List the elements of the Cartesian Product B x ($L_a \cap$ {strings of length 3})

b. List the elements of the power set $\mathscr{P}$(B)

c. What is the cardinality of the power set $\mathscr{P}$(A) ?

d. Postponed to next week: List the elements of the language concatenation:

$B \bullet (L_b \cap \{\text{strings of length} \leq 2\})$

e. Postponed to next week: What is the language concatenation B • {} ?

f. What is the Cartesian Product {} x L_a ?

g. What is the Cartesian Product { ε } x A ?

Three: Consider the following two definitions of a rooted undirected tree:

Structural:

A rooted tree of height n ($n \geq 0$) edges is a connected undirected graph without cycles where one node is designated as the root; and the longest path from the root to any node is of length n edges. (There could be more than one such path of length n.)

Recursive:

A rooted tree of height 0 edges is a graph with one node (the root) and no edges.

A rooted tree of height $n > 0$ edges is a graph consisting of:

The root node,

One rooted sub-tree of height n-1,

Zero or more additional rooted sub-trees of height $\leq$ n-1, and

For each sub-tree there is one edge between the root of the sub-tree and the root of the tree.

All sub-trees are disjoint from each other.

Prove that these two definitions are equivalent. (You may want to use induction for at least one direction of the proof.)

Inspired by Sipser Exercises: p 84:

Four: Show a fully specified DFA that recognizes the following language over the alphabet, $\Sigma = \{a, b, c\}$. Specify the DFA ~~transition function~~ as a fully specified state diagram, i.e., a labeled digraph as we did in class and similar to the ones below. Be sure to clearly indicate your initial state and Accepting states.

$L_{four} = \{ w \in \Sigma^+ \mid$ w begins and ends with c $\}$

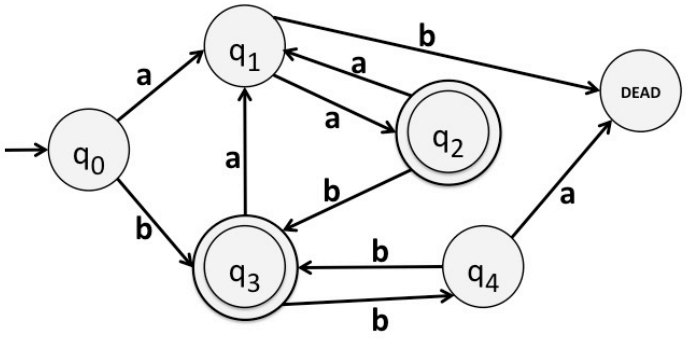
Can you do this with 4 states?

Five: Show a fully specified DFA ~~Give a formal definition using the 5-tuple notation for a DFA~~ that recognizes the following language over the alphabet, $\Sigma = \{0, 1\}$. Specify the DFA ~~transition function δ~~ using a fully specified state diagram ~~function table~~. Be sure to clearly indicate your initial state and Accepting states.

$L_{five} = \{ w \in \Sigma^* \mid$ the number of 1's in w is evenly divisible by three $\}$

Six: Postponed to next week: Consider the two DFAs M_1 & M_2 (Fig. 1.6 on p. 36 & Fig. 1.8 on p. 37, respectively of Sipser) and reproduced below for convenience. What is the language concatenation $\mathcal{L}(M_1) \bullet \mathcal{L}(M_2)$? I.e., give a simple English description of this language.

Instead: Briefly describe in English the language over ∑ = { a, b } accepted by this DFA:



DFA

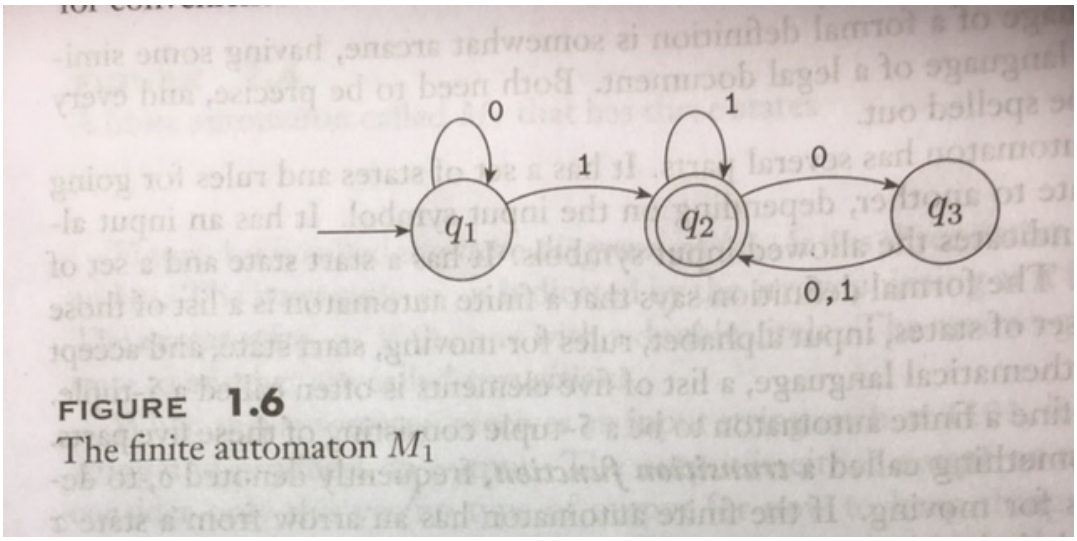


FIGURE 1.6
The finite automaton M_1

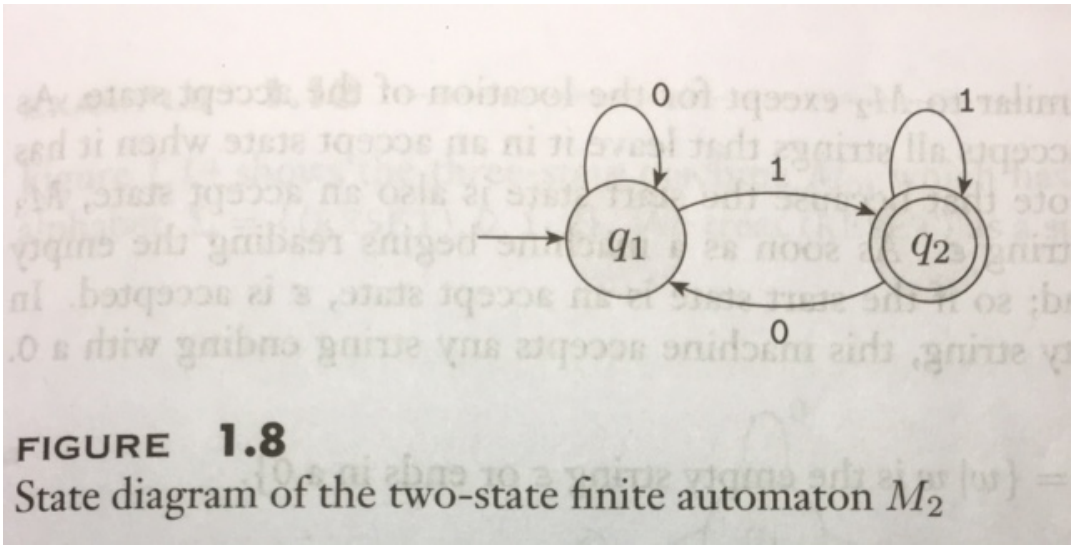


FIGURE 1.8
State diagram of the two-state finite automaton M_2

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