CS 181 Spring 2017 Homework Week 4

Due Mon 5/30, by 3:00, online submission preferred

From last week:
five. Prove that the following language over alphabet $\Sigma = \{ 0, 1 \}$ is *not* finite state using the pumping lemma:

$\{ 0^i 1^j 0^k \mid i, j, k \geq 0 \text{ and } k = | i - j | \}$

"| i - j |" denotes the absolute value of (i - j)

1: Let alphabet $\Sigma = \{ a, b, c \}$. Consider the following languages over Σ such that:

L_f is a finite language, i.e. a set consisting of a finite *number* of strings

L_{fs} is a finite state language

$L_= = \{ xcy \mid x, y \in \{ a, b \}^* \text{ and } |x| = |y| \}$ (as discussed in class, this is *not* a finite state language)

L_{nfs} is not finite state language

Answer each question below about the given combinations of languages.
The possible answers are "always", "sometimes", or "never".
Explain your answer by providing a *brief* justification, example, or counterexample, as appropriate. Note that if your answer is "sometimes", then to justify your answer you need to provide two different examples: one where the combination is a finite state language and one where the combination is not a finite state language. Briefly explain why the language is or isn't finite state, but you do not need to prove it.

a. Is $L_f \cap L_{nfs}$ a finite state language?

b. Is $L_{fs} \cup L_=$ a finite state language?

c. Is $L_f \cup L_{nfs}$ a finite state language?

2: Let $\Sigma = \{ 0, 1 \}$. Convert the following Regular Expression to an NFA via the compositional construction in the proof of Sipser Lemma 1.55:

$(0^* + 1^*)^*$

3: Let $\Sigma = \{ a, b \}$. Prove that the following is finite state:

$L_3 = \{ w \mid$ for some string, $x \in \Sigma^*$ of length 3, w contains a substring x, and w also contains a non-overlapping substring $x^R \}$

Here, "non-overlapping" means that in string w, the occurrence of x and the occurrence of x^R do not overlap each other.

4: Let $\Sigma = \{ a, b \}$, and let $G = (V, \Sigma, R, S)$ be the context free grammar where:

$V = \{ S, A, B \}$, and

$$
\begin{aligned}
R = \; S &\to A \mid B \mid \varepsilon \\
A &\to aC \mid Ca \\
C &\to bS \mid Sb \\
B &\to bD \mid Db \\
D &\to aS \mid Sa
\end{aligned}
$$

(a) Show a derivation tree in G of the string "aabb".

(b) Show the left-most derivation of the string "aabb" corresponding to your tree in part (a).

(c) Show a *different* derivation tree in G of the string "aabb".

(d) Show the left-most derivation of the string "aabb" corresponding to your tree in part (c).

5: Give a CFG for the following language over $\Sigma = \{ a, b, c \}$:

$L_5 = \{ a^i b^j c^k \mid i, j, k \geq 0, \text{ and } j = i + k \}$

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