Computer Science 181, Homework 9

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Problem 1

The leftmost reduction show below has an unforced handle.

$$\underline{a}$$
$$\underline{R}$$
$$S$$

The first line is an unforced handle, since the leftmost reduction for the string shown below begins with the same prefix as the previous string up until the first character, but has a different first handle.

$$\underline{a;}a$$
$$L\underline{a}$$
$$L\underline{R}$$
$$\underline{LS}$$
$$S$$

Problem 2

L_2 is a recursive language. This is because we are given a finite input string, which means that we have a finite graph. Starting from the start node, we can travel along unexplored edges until we can no longer reach any new nodes. At this point, the Turing machine can halt and accept or reject depending on if the end node has been explored. So for any input string, a Turing machine for L_2 will halt and not loop. Thus L_2 is recursive.

Problem 3

For a string w not in L, the machine M may not halt and reject since if L is recursively enumerable and not recursive. It may loop on w. Then the machine $\bar{M}$ must accept w, but will never do so in this construction because $\bar{M}$ would not halt and accept on w. Thus this construction of $\bar{M}$ is not recursively enumerable, and cannot be used to show that recursively enumerable languages are closed under complementation.

Problem 4

We want to simulate the Turing machine and the DFA on every possible input string. Since the set of input strings is countably infinite, map each possible input string string to an index to generate a sequence of strings $s_1, s_2, s_3, \ldots$ and then do the following.

1. Begin at iteration $i = 1$ with string index $j = 1$.
2. Run the DFA on the string s_j and the Turing machine for i steps on s_j.
3. If both accept, then accept.
4. If $j = i$, increase i by one, reset the string index j to 1, and go to step two.
5. Otherwise increment the string index j by one and go back to step two.

This will have the effect of inputting every possible input string into the DFA and the Turing machine and checking to see if they both accept. Note that this is a recursively enumerable language, as this procedure will never halt on a string not in the language.