Computer Science 181, Homework 7

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Problem 0

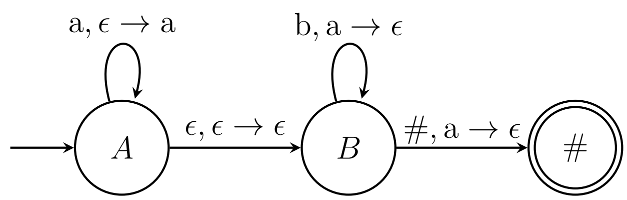
The language L_0 is context free and not finite state. The context free grammar G that describes the language is shown below.

$$
\begin{aligned}
G &= (V, \Sigma, R, S) \\
V &= \{A, B, C, D, T\} \\
R &= \{T \to AB \mid CD, \\
&\quad\; A \to \mathrm{a}A\mathrm{b} \mid \epsilon, \\
&\quad\; B \to \mathrm{c}B \mid \epsilon, \\
&\quad\; C \to \mathrm{a}C \mid \epsilon, \\
&\quad\; D \to \mathrm{b}D\mathrm{c} \mid \epsilon\} \\
S &= T
\end{aligned}
$$

To prove that L_0 is not finite state, first assume for contradiction that L_0 is finite state. Then by the pumping lemma for finite state languages, there exists some p such that $\forall w \in L_0$ with $|w| \geq p$, w can be split into three substrings x, y, and z such that $w = xyz$, $|y| \geq 1$, $|xy| \leq p$, and $\forall n \geq 0$ we have that $xy^nz \in L_0$. But consider the string $w = \mathrm{b}^p\mathrm{c}^p$, which is in L_0 and has a length greater than p. When splitting w into x, y, and z, because $|xy| \leq p$, we have that x and y must consist entirely of b's. We know that $|y| \geq 1$, so for any $n \geq 2$ we have that xy^nz must take the form $\mathrm{b}^i\mathrm{c}^j$ with $i > j$. This is not in the language L_0, which is a contradiction. Thus L_0 is not a finite state language.

Problem 1

The language L_1 is context free and not finite state. The PDA with the set of stack symbols $\Gamma = \{a\}$ is shown below.



To prove that L_1 is not finite state, first assume for contradiction that L_1 is finite state. Then by the pumping lemma for finite state languages, there exists some p such that $\forall w \in L_1$ with $|w| \geq p$, w can be split into three substrings x, y, and z such that $w = xyz$, $|y| \geq 1$, $|xy| \leq p$, and $\forall n \geq 0$ we have that $xy^n z \in L_1$. But consider the string $w = \text{a}^i\text{b}^j\#$ in L_1 such that $i + j \geq p$ and $i = j + 1$. Because $|xy| \leq p$ when splitting w into the three substrings, y can only be chosen such that it consists entirely of a's, entirely of b's, or contains a sequence of a's followed by a sequence of b's. If y consists entirely of a's then the string $xy^n z$ with $n = 0$ will not be in L_1, as this will result in an equal or lesser amount of a's than b's in the string. So y cannot consist entirely of a's. If y consists entirely of b's then the string $xy^n z$ with $n \geq 2$ will not be in L_1, as this will result in an equal or greater amount of b's than a's in the string. So y cannot consist entirely of b's. If y consists of a sequence of a's followed by a sequence of b's, then the string $xy^n z$ with $n \geq 2$ will not be in L_1, as this string will have interleaved a's and b's, which does not fit the format of $\text{a}^i\text{b}^j\#$. So y cannot consist of a sequence of a's followed by a sequence of b's. Thus there exists some $w \in L_1$ with $|w| \geq p$ that cannot be split into three substrings x, y, and z such that $w = xyz$, $|y| \geq 1$, $|xy| \leq p$, and $\forall n \geq 0$ we have that $xy^n z \in L_1$. This is a contradiction, and so L_1 cannot be finite state.

Problem 2

Assume for contradiction that L_2 is a context free language. Then by the pumping lemma for context free languages, there exists some p such that $\forall s \in L_2$ with $|s| \geq p$, s can be split into five substrings u, v, w, x, and y such that $|vwx| \leq p$, $|vx| \geq 1$, and $\forall n \geq 0$ we have that $uv^n wx^n y \in L_2$. Consider the string $s = t\#t^R\#t$ in L_2, with $|t| \geq p$. Then when attempting to split s, vwx can only contain one or zero #'s because $|vwx| \leq p$, which means vx can contain only one or zero #'s. If vx contains one #, then uv^nwx^ny would contain three #'s for $n = 2$, and thus uv^nwx^ny would not be in L_2 since it does not fit the format of $t\#t^R\#t$. If vx contains zero #'s, then w contains either one or zero #'s. If w contains zero #'s and vx contains zero #'s, then v and x must lie entirely within one t or within the t^R. Thus uv^nwx^ny will be in the form $r\#t^R\#t$, $t\#r^R\#t$, or $t\#t^R\#r$ with $r \neq t$ for $n \geq 2$. Then uv^nwx^ny would not be in L_2. If w contains one #, then uv^nwx^ny will be in the form $q\#r^R\#t$ or $t\#q^R\#r$ with $q \neq t$ and $r \neq t$ for $n \geq 2$. Then uv^nwx^ny would not be in L_2. So there exists some string s such that s cannot be split into five substrings that make the conditions of the pumping lemma hold. This is a contradiction, and thus L_2 is not a context free language.