

CS 181 Spring 2018 Homework Week 8

Due Monday 28 May 4:00pm online

0. [4 points] Give a PDA for the following language over $\Sigma = \{ (,), [,], a, \# \}$ with end-mark “#”:

$L_0 = \{ w\# \mid w \text{ is a string over } \{ (,), [,], a \} \text{ consisting of lists of zero or more } a\text{'s between balanced parentheses of two types} \}$

E.g., L_0 contains: $\#, ()\#, (a)\#, [a a]\#, [](a)\#, ([a])(a)\#, [[[a a a]]]\#,$ etc.

E.g., L_0 does not contain: $\varepsilon, a\#, \#(), ()\#, ()a\#, ((a)\#, [\#]\#, [](() a)\#, (a)()(a)\#, [[a a a]]\#,$ etc.

Scoring: 3 points for correct PDA + 1 point if your PDA is deterministic.

1. Consider the following CFG $G = (V, \Sigma, R, P)$, where:

$V = \{ P, \text{Type}, \text{PI}, \text{PI1}, \text{Body}, \text{St}, \text{Else} \}$

$\Sigma = \{ \text{main}, (,), \text{int}, \text{char}, \text{void}, \{, \}, ;, ,, =, \text{if}, \text{exp}, \text{else}, \text{id} \}$

$R = \{ P \rightarrow \text{Type main} (\text{PI}) \{ \text{Body} \}$

$\text{Type} \rightarrow \text{int} \mid \text{char} \mid \text{void}$

$\text{PI} \rightarrow \varepsilon \mid \text{id PI1}$

$\text{PI1} \rightarrow , \text{id PI1} \mid \varepsilon$

$\text{Body} \rightarrow \text{St} ; \text{Body} \mid \text{St} ;$

$\text{St} \rightarrow \text{id} = \text{exp}$

$\text{St} \rightarrow \text{if} (\text{exp}) \text{St Else} \mid \text{if} (\text{exp}) \{ \text{Body} \} \text{Else}$

$\text{Else} \rightarrow \text{else St} \mid \text{else} \{ \text{Body} \} \mid \varepsilon$

$\}$

This grammar describes a very small portion of the syntax of a programming language similar to the C programming language.

- 1.a. [1 points] Show a derivation tree in G for the string:

`void main () { if (exp) id = exp ; if (exp) id = exp else id = exp ; }`

- 1.b. [1 points] Show a left-most reduction in G for the same string, and underline the handle at each step as shown in class.

Some groups of production rules describe particular language features. E.g., the rule:

`“ P \rightarrow Type main (PI) { Body } ”`

describes the overall structure of programs. The rules for variables PI and PI1 describe parameter lists. The rules for variables St and Else describe the assignment statement and `if` statements. Some of these language features could be represented using only a DFA or equivalent model; while some features can only be described correctly using the Context Free Grammar (CFG) model.

- 1.c. [2 points] Identify the language feature(s) which require the CFG model and very briefly explain why they require a CFG to define them.
2. [4 points] Consider the following language over $\Sigma = \{ 0, 1 \}$:

$L_2 = \{ 0^r 1 0^s 1 0^t \mid r, s, t \geq 0 \text{ and } r < s > t \}$

Prove that L_2 is not context free using the Pumping Lemma for context free languages.

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