# Computer Science 181, Homework 4

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## Postponed Problem 5

Assume for contradiction that this language

$$L = \{0^i 1^j 0^k \mid i, j, k \geq 0 \wedge k = |i - j|\}$$

is finite state. Then by the pumping lemma if a language is finite state then there exists some $p$ such that for any string $s$ in $L$ with $|s| > p$, $s$ can be split into three strings $x$, $y$, and $z$ where $s = xyz$, $|y| \geq 1$, $|xy| \leq p$, and $xy^*z \subseteq L$. Take the string $0^p 1^p \in L$, and note that $xy$ must be a string made entirely of 0's, since $|xy| \leq p$. Additionally, $y$ must be in $0^+$, since $|y| \geq 1$. Let $x = 0^a$ and $y = 0^b$ for some constants $a \geq 0$ and $b > 0$. Then $z$ must be $0^{p-a-b}1^p$. By the pumping lemma $xy^*z \subseteq L$, so

$$\{0^a (0^b)^* 0^{p-a-b} 1^p \mid a \geq 0 \wedge b > 0\} \subseteq L$$

This can be stated equivalently as

$$\{0^{p-b} 0^{xb} 1^p \mid \forall x \in \{0, 1, 2, \ldots\} \wedge b > 0\} \subseteq L$$

When $x = 2$, this means that the string $0^{p+b}1^p \in L$ for some constant $b > 0$. But in the definition of $L$, this string has the parameters $i = p + b$, $j = p$, and $k = 0$. Thus $k \neq |i - j|$ and $0^{p+b}1^p \notin L$. This is a contradiction, so $L$ cannot be finite state.

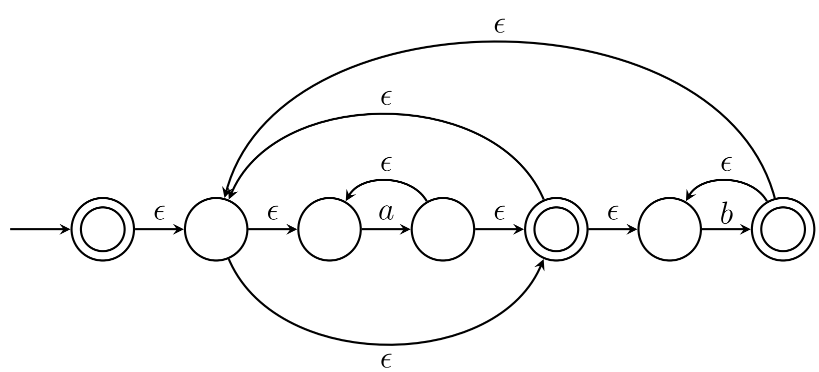
## 1 Problem 1

**a)** Always. The intersection of a finite language and any other language must be a finite language, and thus $L_\text{f} \cap L_\text{nfs}$ is finite state.

**b)** Sometimes. $\{\} \cup L_=$ is not finite state but $\Sigma^* \cup L_=$ is finite state.

**c)** Never. Non finite state languages are infinite languages, so there are an infinite number of strings in $L_{\text{nfs}}$ that cannot be expressed through a finite state machine. Because $L_{\text{f}}$ is finite, there aren't enough strings in $L_{\text{f}}$ to simplify $L_{\text{nfs}}$ into a finite state language. For example, consider the finite language $L_x$ consisting of all the strings with length smaller than $x$. $L_x \cup L_{\text{nfs}}$ would still be a non finite state language, as there would be no finite state machine to express the strings in $L_x \cup L_{\text{nfs}}$ that have a length greater than $x$, since these are strings that are exclusively from $L_{\text{nfs}}$. For any finite language $L_{\text{f}}$ a similar argument could apply, as there must be a maximum length string in $L_{\text{f}}$. After this maximum length, all the strings in $L_x \cup L_{\text{nfs}}$ would come from $L_{\text{nfs}}$ and could not be expressed through a finite state machine.

# 2 Problem 2




# 3 Problem 3

We can write a regular expression for this language $L_3$, which shows that it is finite state. Let $x$ be a string in the set

$$\{aaa, aab, aba, abb, baa, bab, bba, bbb\}$$

and $R$ be some constant. Then the regular expression

$$E_x = (\Sigma^* x \Sigma^* x^R \Sigma^*) \cup (\Sigma^* x^R \Sigma^* x \Sigma^*)$$

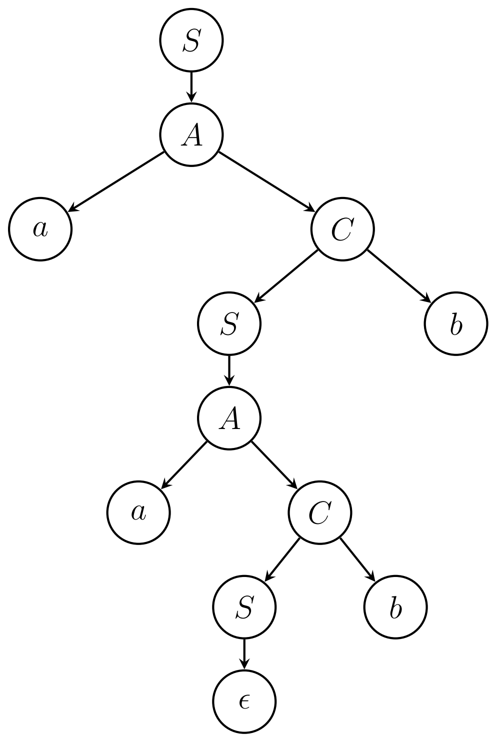
denotes a string in $L_3$ that contains a given $x$ and $x^R$. Then we have that

$$L_3 = E_{aaa} \cup E_{aab} \cup E_{aba} \cup E_{abb} \cup E_{baa} \cup E_{bab} \cup E_{bba} \cup E_{bbb}$$

which is a regular expression that proves that $L_3$ is a finite state language.

# 4 Problem 4

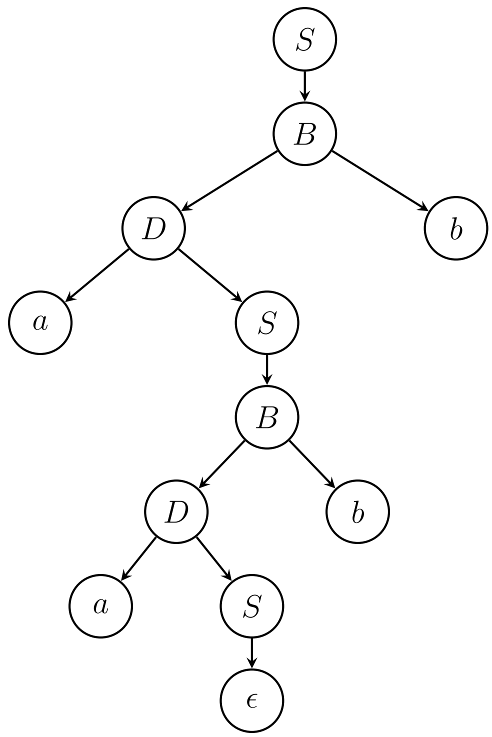
**a)**




**b)** The leftmost derivation yields the following sequence.

$$S, A, aC, aSb, aAb, aaCb, aaSbb, aabb$$

**c)**




**d)** The leftmost derivation yields the following sequence.

$$S, B, Db, aSb, aBb, aDbb, aaSbb, aabb$$

# 5 Problem 5

Given a starting state $S$ the following context free grammar describes the language.

$$\begin{aligned} S &\to LR \\ L &\to aLb \mid \epsilon \\ R &\to bRc \mid \epsilon \end{aligned}$$