

Due Thursday, 8 Nov 2018, by 11:59pm to Gradescope.

Covers material up to Lecture 10.

100 points total.

1. (18 points) **Fourier Series**

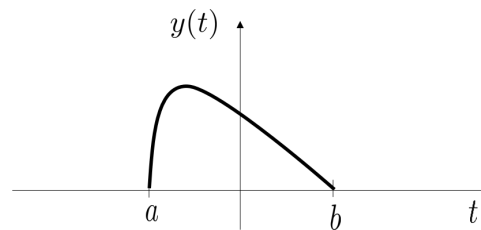
(a) (7 points) When the periodic signal $f(t)$ is real, you have seen in class some properties of symmetry for the Fourier series coefficients of $f(t)$ (handout 8, slide 41). How do these properties of symmetry change when $f(t)$ is pure imaginary?

(b) (7 points) A *real* and *even* signal $x(t)$ has the following properties:

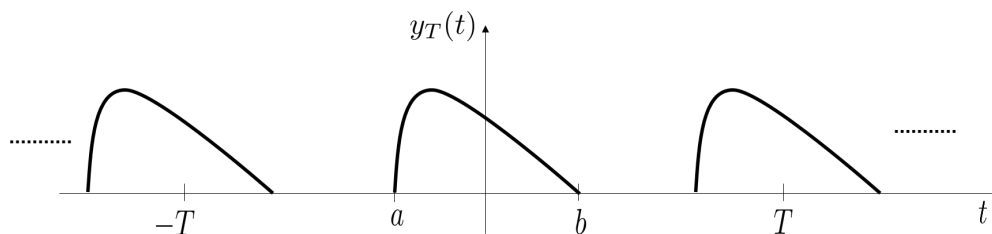
- it is a periodic signal with period 1 s;
- it has a DC component of 1 and one positive frequency component;
- it has a power of 9.

What is $x(t)$?

(c) (4 points) Consider the signal $y(t)$ shown below and let $Y(j\omega)$ denote its Fourier transform.



Let $Y_T(t)$ denote its periodic extension:



How the Fourier series coefficients of $y_T(t)$ can be obtained from the Fourier transform $Y(j\omega)$ of $y(t)$? (Note that the figures given in this problem are for illustrative purposes, the question is for any arbitrary $y(t)$).

2. (32 points) **Symmetry properties of Fourier transform**

(a) (16 points) Determine which of the signals, whose Fourier transforms are depicted in Fig. 1, satisfy each of the following:

- i. $x(t)$ is even
- ii. $x(t)$ is odd
- iii. $x(t)$ is real
- iv. $x(t)$ is complex (neither real, nor pure imaginary)
- v. $x(t)$ is real and even
- vi. $x(t)$ is imaginary and odd
- vii. $x(t)$ is imaginary and even
- viii. There exists a non-zero ω_0 such that $e^{j\omega_0 t}x(t)$ is real and even

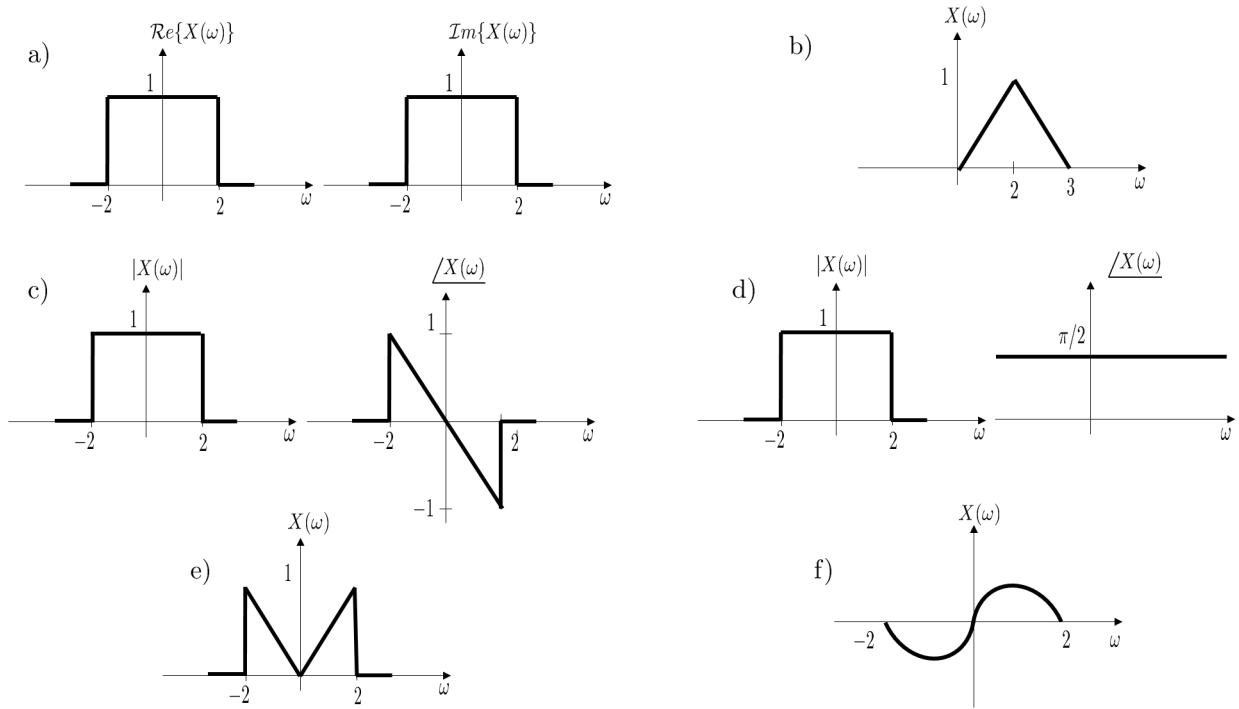


Figure 1: P2.a

(b) (8 points) Using the properties of Fourier transform, determine whether the assertions are true or false.

- i. The convolution of a real and even signal and a real and odd signal is odd.
- ii. The convolution of a signal and the same signal reversed is an even signal.

(c) (8 points) Show the following statements:

- i. If $x(t) = x^*(-t)$, then $X(j\omega)$ is real.

- ii. If $x(t)$ is a real signal with $X(j\omega)$ its Fourier transform, then the Fourier transforms $X_e(j\omega)$ and $X_o(j\omega)$ of the even and odd components of $x(t)$ satisfy the following:

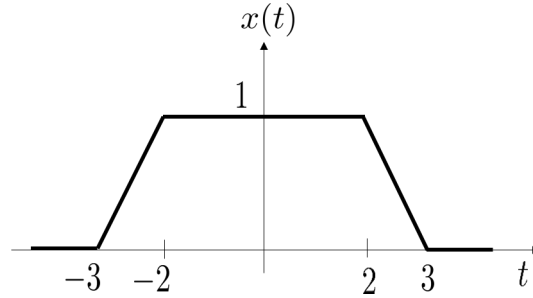
$$X_e(j\omega) = \mathcal{Re}\{X(j\omega)\}$$

and

$$X_o(j\omega) = j\mathcal{Im}\{X(j\omega)\}$$

3. (15 points) **Fourier transform properties**

Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ sketched below:



Evaluate the following quantities without explicitly finding $X(j\omega)$:

- (a) $\int_0^\infty X(j\omega) d\omega$
- (b) $X(j\omega)|_{\omega=0}$
- (c) $\angle X(j\omega)$
- (d) $\int_{-\infty}^\infty e^{-j\omega} X(j\omega) d\omega$
- (e) Plot the inverse Fourier transform of $\mathcal{Re}\{e^{-3j\omega} X(j\omega)\}$

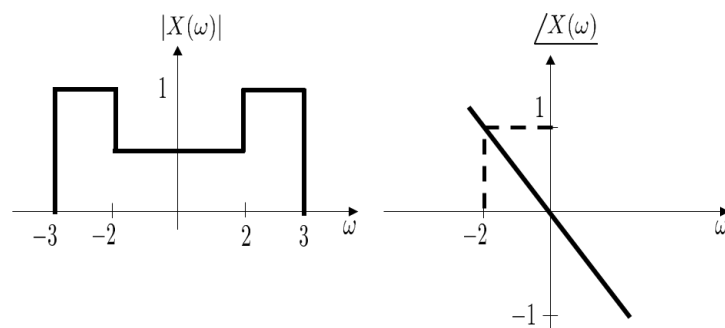
4. (35 points) **Fourier transform and its inverse**

- (a) (20 points) Find the Fourier transform of each of the signals given below:

Hint: you may use Fourier Transforms derived in class.

- i. $x_1(t) = 2\text{rect}\left(\frac{-t-3}{2}\right) \cos(10\pi t)$
- ii. $x_2(t) = e^{(2+3j)t} u(-t+1)$
- iii. $x_3(t) = \begin{cases} 1 + \cos(\pi t), & |t| < 1 \\ 0, & \text{otherwise} \end{cases}$
- iv. $x_4(t) = te^{-t} u(t)$

(b) (7 points) Find the inverse Fourier transform of the signal shown below:



(c) (8 points) Two signals $f_1(t)$ and $f_2(t)$ are defined as

$$f_1(t) = \text{sinc}(2t)$$

$$f_2(t) = \text{sinc}(t) \cos(3\pi t)$$

Let the convolution of the two signals be

$$f(t) = (f_1 * f_2)(t)$$

- i. Find $F(j\omega)$, the Fourier transform of $f(t)$.
- ii. Find $f(t)$.