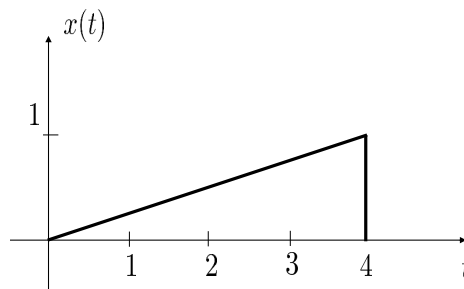Due Wednesday, 17 Oct 2018, by 11:59pm to Gradescope.
Covers material up to Lecture 4.
100 points total.

1. (23 points) **Elementary signals.**

   (a) (12 points) Consider the signal $x(t)$ shown below. Sketch the following:

   i. $x(t)\,(u(t-1) - u(2t-3))$
   ii. $x(t)\delta(t-1) - \int_{-\infty}^{6} x(\tau)\delta(\tau-5)d\tau + \int_{-\infty}^{\infty} x(t)\,(u(\tau-4) - u(\tau-5))\,d\tau$
   iii. $\frac{d}{dt}x(t)$
   iv. $x(t) - \frac{1}{2}r(t) + \frac{1}{2}r(t-4) + 2u(t-4)$ (*Note: $r(t)$ is the ramp function*)

   (b) (8 points) Evaluate these integrals:

   i. $\int_{-\infty}^{\infty} f(t+1)\delta(t+1)dt$
   ii. $\int_{t}^{\infty} e^{-2\tau}u(\tau-1)d\tau$
   iii. $\int_{0}^{\infty} f(t)(\delta(t-1) + \delta(t+1))dt$
   iv. $\int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)\delta(t-2)d\tau$

   (c) (3 points) Let b be a positive constant. Show the following property for the delta function:
 $$\delta(bt) = \frac{1}{b}\delta(t)$$

   Hint: Use the rect approximation of the Dirac delta

2. (23 points) **Expression for signals.**

(a) (15 points) Write the following signals as a combination (sums or products) of unit rectangles $\text{rect}(t)$ ($\text{rect}(t) = 1$ for $|t| \leq 1/2$, and 0 otherwise) and unit triangles $\Delta(t)$ ($\Delta(t) = 1 - |t|$ for $|t| \leq 1$, and 0 otherwise). *Note: You might use a combination of only unit triangles for some signals.*
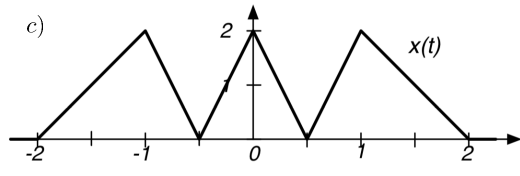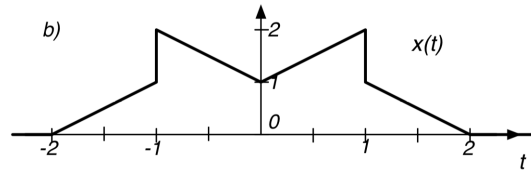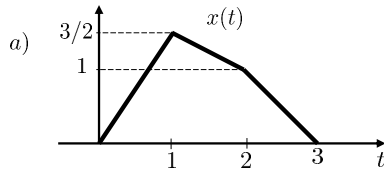
(b) (8 points) Express each of the signals shown below using scaled and time shifted unit-step functions.
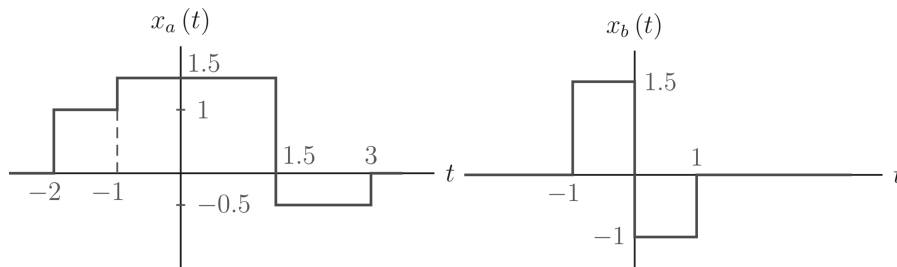
3. (34 points) **System properties.**

(a) (21 points) A system with input $x(t)$ and output $y(t)$ can be time-invariant, causal or stable. Determine which of these properties hold for each of the following systems. Explain your answer.

   i. $y(t) = |x(t)| + x(2t)$

   ii. $y(t) = \int_{t-T}^{t+T} x(\lambda)d\lambda$, where T is positive and constant.

   iii. $y(t) = (t+1)\int_{-\infty}^{t} x(\lambda)d\lambda$

   iv. $y(t) = 1 + x(t)\cos(\omega t)$

    v. $y(t) = \cos(\omega t + x(t))$

   vi. $y(t) = \int_{-\infty}^{t/2} x(\lambda)d\lambda$

  vii. $y(t) = \frac{1}{1+x^2(t)}$

(b) (4 points) Consider a system H that takes a signal $x(t)$ as input and returns the even part of $x(t)$ as output, i.e.,

$$x_e(t) = H(x(t))$$

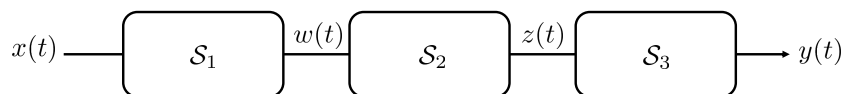where $x_e(t)$ is the even part of $x(t)$. Is it time invariant? Is it stable?

(c) (5 points) Consider the following three systems:

$$\mathcal{S}_1 : w(t) = x(t/2)$$

$$\mathcal{S}_2 : z(t) = \int_{-\infty}^{t} w(\tau)d\tau$$

$$\mathcal{S}_3 : y(t) = \mathcal{S}_3(z(t))$$

The three systems are connected in series as illustrated here:


Choose the third system $\mathcal{S}_3$, such that overall system is equivalent to the following system:

$$y(t) = \int_{-\infty}^{t-1} x(\tau)d\tau$$

(d) (4 points) In part (c), you saw an example of three systems connected in series. In general, systems can be interconnected in series or in parallel to form what we call cascaded systems. The figure below shows the difference between a series cascade and a parallel cascade. *Note that parts (c) and (d) are unrelated.*

   i. (2 points) Show that the series cascade of any two time-invariant systems is also time-invariant.

  ii. (2 points) Show that the parallel cascade of any two time-invariant systems is also time-invariant.

 iii. (*Optional*) Can you think of two **time-variant** systems, whose series cascade is **time-invariant**? Can you think of two **time-variant** systems, whose parallel cascade is **time-invariant**?
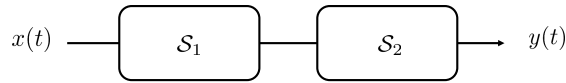
(a) Series Cascade
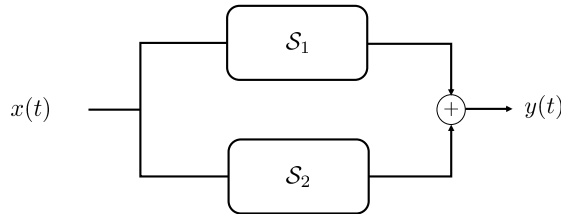
(b) Parallel Cascade

4. (10 points) **Power and energy of complex signals**

   (a) (5 points) Is $x(t) = Ae^{j\omega t} + Be^{-j\omega t}$ a power or energy signal? A and B are both real numbers, not necessarily equal. If it is an energy signal, compute its energy. If it is a power signal, compute its power. (*Hint: Use the fact that the square magnitude of a complex number v is: $|v|^2 = v^*v$, where v^* is the complex conjugate of the complex number v.*)

   (b) (5 points) Is $x(t) = e^{-(1+j\omega)t}u(t-1)$ an energy signal or power signal? Again, if it is an energy signal, compute its energy. If it is a power signal, compute its power.

5. (10 points) **MATLAB**

   (a) (4 points) **Task 1**

   A complex sinusoid is denoted:
   $$y(t) = e^{(\sigma + j\omega)t}$$

   First compute a vector representing time from 0 to 10 seconds in about 500 steps (You can use `linspace`). Use this vector to compute a complex sinusoid with a period of 2 seconds, and a decay rate that reduces the signal level at 10 seconds to half its original value. What σ and ω did you choose? If your complex exponential is y, plot:

   `>> plot(y);`

   What is MATLAB doing here?

   (b) (3 points) **Task 2**

   Use the `real()` and `imag()` MATLAB functions to extract the real and imaginary parts of the complex exponential, and plot them as a function of time (plot them separately, you can use `subplot` for this task). This should look more reasonable. Label your axes, and check that your signal has the required period and decay rate.

   (c) (3 points) **Task 3**

   Use the `abs()` and `angle()` functions to plot the magnitude and phase angle of the complex exponential (plot them in the same figure). Scale the `angle()` plot by dividing

4

it by `2*pi` so that it fits well on the same plot as the `abs()` plot (i.e. plot the angle in cycles, instead of radians, the function `angle(x)` returns the angle in radians).