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Due Thursday, 25 Oct 2018, by 11:59pm to Gradescope.
Covers material up to Lecture 6.
100 points total.

1. (20 points) **Linear systems** Determine whether each of the following systems is linear or not. Explain your answer.

   (a) $y(t) = |x(t)| + x(t)$

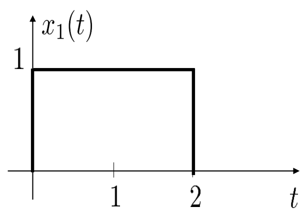
   (b) $y(t) = 1 + x(t)\cos(\omega t)$

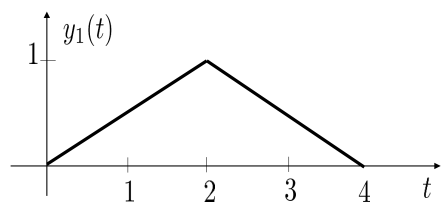
   (c) $y(t) = \cos(\omega t + x(t))$

   (d) $y(t) = (x(t) + x(-t))\,u(t)$

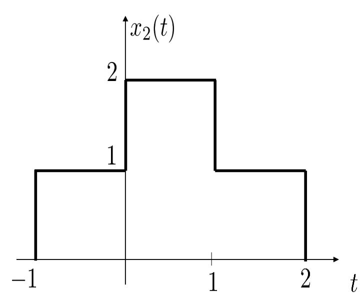
2. (13 points) **LTI systems**

   (a) (7 points) Consider an LTI (linear time-invariant) system whose response to $x_1(t)$ is $y_1(t)$, where $x_1(t)$ and $y_1(t)$ are illustrated as follows:





   Sketch the response of the system to the input $x_2(t)$.



   (b) (6 points) Assume we have a linear system with the following input-output pairs:

   - the output is $y_1(t) = e^{-t}u(t)$ when the input is $x_1(t) = u(t)$;
   - the output is $y_2(t) = e^{-t}(u(t) - u(t-1))$ when the input is $x_2(t) = \mathrm{rect}(t - \frac{1}{2})$.

Is the system time-invariant?

3. (42 points) **Convolution**

(a) (10 points) For each pair of the signals given below, compute their convolution using the flip-and-drag technique. Please provide a piecewise formula for $y(t)$.

i. $f(t) = 2\,\mathrm{rect}(t - \frac{3}{2})$, $\quad g(t) = 2\,r(t-1)\mathrm{rect}(t - \frac{3}{2})$

ii. $f(t) = u(-t-1)$, $\quad g(t) = e^{-t}u(t)$

(b) (12 points) For each of the following, find a function $h(t)$ such that $y(t) = x(t) * h(t)$.

i. $y(t) = \int_{t-T}^{t} x(\tau)d\tau$

ii. $y(t) = x(t-1)$

iii. $y(t) = \sum_{n=-\infty}^{\infty} x(t - nT_s)$
Note: this last operation creates a periodic extension of $x(t)$ where the period is T_s.

(c) (12 points) Use the properties of convolution to simplify the following expressions:

i. $\left[\int_{-\infty}^{t} u(-\tau+3)\delta(\tau-1)d\tau\right] * (\delta(t-2) + \delta(2t-8))$

ii. $[\delta(t-3) + \delta(t+2)] * \left[e^{3t}u(-t) + \delta(t+2) + 2\right]$

iii. $\frac{d}{dt}\left[(u(t) - u(t-1)) * u(t-2)\right]$, *Hint: Show first that $u(t) * u(t) = r(t)$ where $r(t)$ is the ramp function.*

(d) (8 points) Explain whether each of the following statements is true or false.

i. If $x(t)$ and $h(t)$ are both odd functions, and $y(t) = x(t) * h(t)$, then $y(t)$ is an even function.

ii. If $y(t) = x(t) * h(t)$, then $y(2t) = h(2t) * x(2t)$.

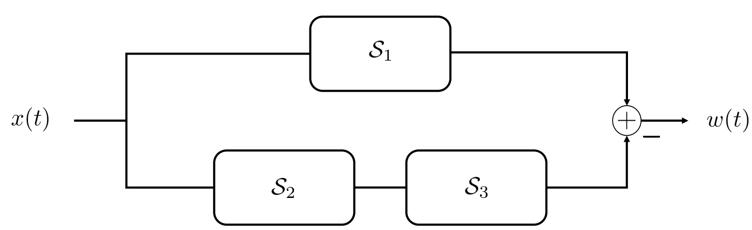
4. (9 points) **Impulse response and LTI systems**

Consider the following three LTI systems:

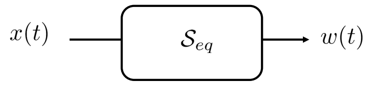
- The first system $\mathcal{S}_1$ is given by its input-output relationship: $y(t) = \int_{-\infty}^{t+t_0} x(\tau)d\tau$;
- The second system $\mathcal{S}_2$ is given by its impulse response: $h_2(t) = u(t-2)$;
- The third system $\mathcal{S}_3$ is given by its impulse response: $h_3(t) = \delta(t-3)$.

(a) (3 points) Compute the impulse responses $h_1(t)$ of system $\mathcal{S}_1$.

(b) (3 points) The three systems are interconnected as shown below.



Determine the impulse response $h_{eq}(t)$ of the equivalent system.



(c) (3 points) Determine the response of the overall system to the input $x(t) = \delta(t) + 2\delta(t-3)$.

5. (16 points) **MATLAB**

To complete the following MATLAB tasks, we will provide you with a MATLAB function (`nconv()`), which numerically evaluates the convolution of two continuous-time functions. Make sure to download it from CCLE and save it in your working directory in order to use it.

The function syntax is as follows:

```
[y, ty] = nconv(x,tx,h,th)
```

where the inputs are:

`x` : input signal vector
`tx`: times over which `x` is defined
`h` : impulse response vector
`th`: times over which `h` is defined

and the outputs are:

`y` : output signal vector
`ty`: times over which `y` is defined.

The function is implemented with the MATLAB's `conv()` function. You are encouraged to look at the implementation of the function provided (the explanations are included as comments in the code).

(a) (7 points) **Task 1**

Use the `nconv()` function to check your result for problem 3(a.i). Plot the output for each problem (you can consider either function to be the input). Properly label the axes of the plots. Make sure to use the same step size for `tx` and `th`.

(b) (5 points) **Task 2**

Using the `nconv()` function, perform the convolution of two unit rect functions: $\text{rect(t)} * \text{rect(t)}$. Plot and label the result.

(c) (4 points) **Task 3**

Using the result of task 2 and the same MATLAB function, calculate $y(t) = \text{rect(t)} * \text{rect(t)} * \text{rect(t)}$. Plot and label the result.

(d) (Optional) **Task 4**

Now, what happens if we consider $\text{rect(t)} * \text{rect(t)} * \cdots * \text{rect(t)} = \text{rect}^{(N)}(t)$? Using for loop, calculate the result of convolving N rect(t) functions together. Plot and label the result (use $N = 100$).

Side note in case you have taken any probability course before: Convolution is an operator that is also useful in statistics. We use it to compute the pdf (probability density function) of the sum of N independent random variables. So if we have $Y = X_1 + X_2 + X_3$, the pdf of Y is the convolution of the pdfs of X_1, X_2 and X_3. In task 4, we are computing the pdf of the sum of N uniform random variables (the pdf of a uniform random variable is a rect function), by convolving N times the rect function. The resulting curve will have a bell-shape. This is related to a theorem in statistics called 'The Central Limit Theorem'.