ECE102, Fall 2018 **Homework #1**

Signals & Systems Prof. J.C. Kao

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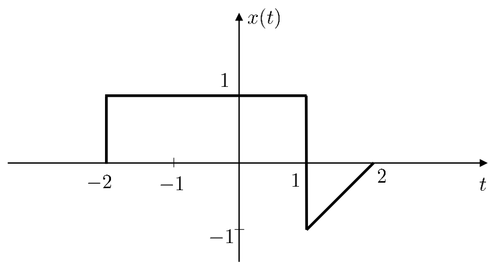
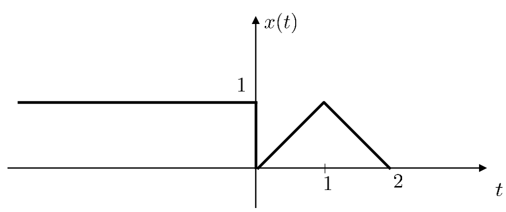
Due Wednesday, 10 Oct 2018, by 11:59pm to Gradescope.
Covers material up to Lecture 2.
100 points total.

1. (10 points) **Even and odd parts.**
   Sketch the even and odd components of the following signal:

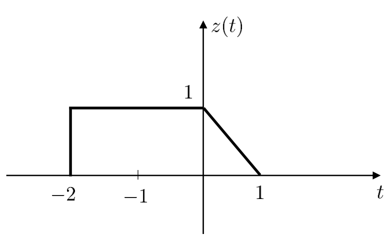


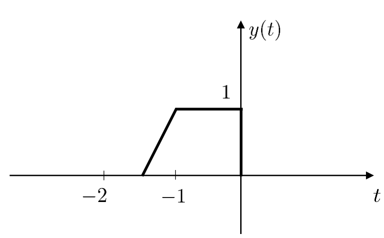
2. (15 points) **Time scaling and shifting.**
   (a) (10 points) Consider the following signal.



   Sketch the following:
   i. $x(1 - 3t)$
   ii. $x(\frac{t}{2} - 2)$

   (b) (5 points) The figure below shows two signals: $z(t)$ and $y(t)$. Can you express $y(t)$ in terms of $z(t)$?





3. (22 points) **Periodic signals.**

   (a) (14 points) For each of the following signals, determine whether it is periodic or not. If the signal is periodic, determine the fundamental period and frequency.

      i. $x_1(t) = \sin(2t + \pi/3)$
      ii. $x_2(t) = \cos(\sqrt{2}\pi t)$
      iii. $x_3(t) = \sin^2(3\pi t + 3)$
      iv. $x_4(t) = x_1(t) + x_2(t)$
      v. $x_5(t) = x_1(\pi t) + x_3(t)$
      vi. $x_6(t) = e^{-t} x_1(t)$
      vii. $x_7(t) = e^{j(\pi t + 1)} x_2(t)$

   (b) (4 points) Assume that the signal $x(t)$ is periodic with period T_0, and that $x(t)$ is odd (i.e. $x(t) = -x(-t)$). What is the value of $x(T_0)$?

   (c) (4 points) If $x(t)$ is periodic, are the even and odd components of $x(t)$ also periodic?

4. (21 points) **Energy and power signals.**

   (a) (15 points) Determine whether the following signals are energy or power signals. If the signal is an energy signal, determine its energy. If the signal is a power signal, determine its power.

      i. $x(t) = e^{-|t|}$
      ii. $x(t) = \begin{cases} \frac{1}{\sqrt{t}}, & \text{if } t \geq 1 \\ 0, & \text{otherwise} \end{cases}$
      iii. $x(t) = \begin{cases} 1 + e^{-t}, & \text{if } t \geq 0 \\ 0, & \text{otherwise} \end{cases}$

   (b) (6 points) Show the following two properties:

      - If $x(t)$ is an even signal and $y(t)$ is an odd signal, then $x(t)y(t)$ is an odd signal;
      - If $z(t)$ is an odd signal, then for any $\tau > 0$ we have:

$$\int_{-\tau}^{\tau} z(t)dt = 0$$

   Use these two properties to show that the energy of $x(t)$ is the sum of the energy of its even component $x_e(t)$ and the energy of its odd component $x_o(t)$, i.e.,

$$E_x = E_{x_e} + E_{x_o}$$

   Assume $x(t)$ is a real signal.

5. (17 points) **Euler's identity and complex numbers.**

   (a) (9 points) Use Euler's formula to prove the following identities:

      i. $\frac{d}{d\theta} \sin(\theta) = \cos(\theta)$

ii. $\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$

iii. $e^{j\alpha} + e^{j\beta} = 2\cos\left(\frac{\alpha-\beta}{2}\right) e^{j\frac{\alpha+\beta}{2}}$

(b) (8 points) Let $x(t) = -(1+j)e^{j(1+2t)}$.

i. Compute the real and imaginary parts of $x(t)$.

ii. Compute the magnitude and phase of $x(t)$.

6. (15 points) **MATLAB tasks**

(a) (2 points) **Task 1**

Plot the waveform

$$x(t) = e^{-t^2}\cos(2\pi t)$$

for $-5 \leq t \leq 5$, with a step size of 0.1. Label the time axis.

(b) (3 points) **Task 2**

Create a vector x corresponding to the function given in Problem 1. Use a sample spacing of 0.01 over the range -2 to 2. Plot this vector. Properly label the time axis.

(c) (4 points) **Task 3**

Create two vectors that represent the even and odd components of the vector x created in Task 2. A vector in MATLAB is reversed by

```
>> xrev = x(length(x):-1:1);
```

Plot the even and odd components of x.

(d) (6 points) **Task 4**

Consider the following three signals:

$$x_1(t) = \cos(2\pi t)$$
$$x_2(t) = \cos(60\pi t)$$
$$x_3(t) = x_1(t)x_2(t)$$

Plot the signals separately (you can use the function subplot) for $-3 \leq t \leq 3$, with a step size of 0.001.