EE113 Digital Signal Processing

Spring 2019

Homework 5

Due: Wednesday, May 15 at 12pm

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Total: 100 points
We thank Prof. Mert Pilanci for problem 5.

Problem 1. ***(15 points)*** *Two signals $x[n]$ and $h[n]$ are given by*

$$x[n] = u[n] - u[n-10] \quad \text{and} \quad h[n] = (0.8)^n u[n]$$

(a). ***(5 points)*** *Determine the DTFT for each signal.*

(b). ***(5 points)*** *Let $y[n]$ be the convolution of these two signals, that is, $y[n] = x[n] * h[n]$. Compute $y[n]$ by direct application of convolution sum.*

(c). ***(5 points)*** *Determine the DTFT of $y[n]$ by direct application of DTFT analysis equation. Verify that it is equal to the product of the individual transforms of $x[n]$ and $h[n]$:*

$$Y(\Omega) = X(\Omega)H(\Omega)$$

Problem 2. ***(10 points)*** *Let $x[n] = \alpha^{2n}u[n-1]$ with $|\alpha| < 1$. Evaluate the following ratio by using the properties of the DTFT:*

$$\frac{\sum_{n=0}^{\infty} n^2 x[n]}{\sum_{n=0}^{\infty} n x[n]}$$

Problem 3. ***Relation between DTFT, DFT and DTFS: (20 points):*** *Let $x[n] = \delta[n+2] - \delta[n] + \delta[n-2]$.*

(a). ***(5 points)*** *Determine its DTFT.*

(b). ***(5 points)*** *Sample the DTFT to obtain the 6-point DFT of $x[n]$.*

(c). ***(10 points)*** *Let the periodic version of $x[n]$ be defined as*

$$\tilde{x}[n] = \sum_{m=-\infty}^{\infty} x[n + m \cdot N].$$

Here $N = 6$, so it is periodic with period 6. Compute the DTFS coefficients of $\tilde{x}[n]$. Show that N times the DTFS coefficients is equal to the corresponding N-point DFT coefficients.

Problem 4. ***(15 points):*** *Let $X[k]$ denote the N-point DFT of sequence $x[n]$. Let $y[n]$ denote the N-point DFT (not inverse DFT) of the sequence $X[k]$. Let $Y[k]$ denote the N-point DFT of the sequence $y[n]$. Let $w[n]$ denote the N-point DFT (not inverse DFT) of the sequence $Y[k]$. Relate the sequences $w[n]$ and $x[n]$. The series of transformations can be illustrated as follows:*

$$x[n] \overset{DFT}{\rightarrow} X[k] \overset{DFT}{\rightarrow} y[n] \overset{DFT}{\rightarrow} Y[k] \overset{DFT}{\rightarrow} w[n]$$

Problem 5. ***(40 points): Visualizing the effect of windowing on DTFT using FFT and MATLAB.***

In this problem we are going to use a N-point FFT to approximate the DTFT of a signal. We will also see the effect of windowing on the DTFT spectrum. Here we are choosing a "long" signal - a signal with a large number of samples. In principle one could choose N as large as possible to plot as many frequency samples of the DTFT as possible, so the resolution in the frequency domain (also called "spectral resolution") could be made arbitrarily fine. However, large N means more storage, longer sampling time, longer computation, etc. In practice, we store L samples of $x[n]$, and then *compute an N-point DFT of $x[n]$ to get $X[k] = X(\frac{2\pi k}{N})$. We can ask: how good is our frequency resolution as a function of N?*

First, we create the signal $x[n]$ on MATLAB, use the following commands. The signal created is thus sampled every 100th of a second and exists for 50 sec. Plot the signal for the first 3 sec as a function of time to see how it looks.

```
fs = 100; % sampling frequency
t = 0:1/fs:50;  % time axis
x = 2 * sin(2*pi*30*t) + 3* sin(2*pi*20*(t-2))
        + 3 * sin(2*pi*10*(t-4)); % the signal as a function of time
N = length(x); % length of the discrete time signal
```

The tasks:

(a). *Plot the DTFT spectrum of the signal using the following commands.*

```
omega = 2*pi*(0:N-1)/N;
omega = fftshift(omega);
omega = unwrap(omega - 2*pi); % creating the frequency axis
X = fft(x,N); % compute N point DFT of x
X = X/max(X); % rescale the DFT
plot(omega,abs(fftshift(X)),'LineWidth',2); % center DFT at 0 and plot the magnitude
title('DTFT of x[n]','fontsize',14)
set(gca,'fontsize',14)
xlabel('Radians','fontsize',14)
```

(b). ***(15 points):*** *Now create a rectangular window $w_r[n]$, with a width of 2 sec. To do so on MATLAB, you must create a N-length vector where only the samples corresponding to $t \leq 2$ are 1 and the other samples are zero. Now multiply $x[n]$ with $w_r[n]$ pointwise to get $x_r[n] = w_r[n] \cdot x[n]$. This gives a rectangular windowed version of $x[n]$. Compute the DTFT of $x_r[n]$ using its N-point FFT and plot it as was done for $x[n]$.*

*(Hint: On MATLAB, elementwise multiplication of two vectors a and b can be accomplished with a.*b.)*

(c). ***(15 points):*** *Now create a Hamming window $w_h[n]$, with a width of 2 sec. To do so on MATLAB, you must create a N-length vector where only the samples corresponding to $t \leq 2$ are non-zero and the other samples are zero. You can use the function `hamming` on MATLAB with appropriate parameters to accomplish this. Now multiply $x[n]$ with $w_h[n]$ pointwise to get $x_h[n] = w_h[n] \cdot x[n]$. This gives a Hamming windowed version of $x[n]$. Compute the DTFT of $x_h[n]$ using its N-point FFT and plot it as was done earlier.*

(d). ***(10 points):*** *Compare the three plots and explain the effects of windowing on the DTFT magnitude spectrum for the rectangular and hamming window.*

Attach all the codes and plots when submitting.