Electrical Engineering 113, Homework 6

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Problem 1

The period of the cosine term is 10. Then the circular convolution is equivalent to a regular convolution which shifts the original signal to the left by 2. Our final signal is then $x[n] = \cos(-0.2\pi n)$. This has two DTFS coefficients with magnitude $\frac{1}{2}$ and the rest are 0. From Parseval's theorem we know that the power is equal to the sum of the squares of the DTFS coefficients.

$$p = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

Problem 2

a)

$$
\begin{aligned}
X(\Omega) &= \sum_{n=0}^{N_1-1} x[n]e^{-j\Omega n} \\
X\left(\frac{2\pi k}{N_2}\right) &= \sum_{n=0}^{N_1-1} x[n]e^{-j\frac{2\pi k}{N_2}n} \\
&= \sum_{n=0}^{N_2-1} x[n]e^{-j\frac{2\pi k}{N_2}n} + \sum_{n=N_2}^{N_1-1} x[n]e^{-j\frac{2\pi k}{N_2}n} \\
&= \sum_{n=0}^{N_2-1} x[n]e^{-j\frac{2\pi k}{N_2}n} + \sum_{n=0}^{N_1-1-N_2} x[n+N_2]e^{-j\frac{2\pi k}{N_2}n} \\
&= \sum_{n=0}^{N_2-1} (x[n]+x[n+N_2])e^{-j\frac{2\pi k}{N_2}n} + \sum_{n=0}^{N_1-1-2N_2} x[n+2N_2]e^{-j\frac{2\pi k}{N_2}n} \\
&= \sum_{n=0}^{N_2-1} (x[n]+x[n+N_2]+\ldots)e^{-j\frac{2\pi k}{N_2}n} \\
&= \sum_{n=0}^{N_2-1} \sum_{l=-\infty}^{\infty} x[n-lN_2]e^{-j\frac{2\pi k}{N_2}n} \\
&= Y[k]
\end{aligned}
$$

Note that the right sum will go to zero as we continually wrap the sum so that the exponential terms align, because eventually $N_1 - 1 - mN_2 < 0$ for some integer m. We can keep the exponential terms the same because we take out factors of $e^{-j2\pi} = 1$. In the periodic extension we can ignore when $l > 0$ since $x[n] = 0$ for $n < 0$, and they will contribute nothing to the DFT. Thus $X\left(\frac{2\pi k}{N_2}\right)$ is the DFT of $y[n]$.

b)

$$\begin{aligned}
X\left(\frac{2\pi k}{N}\right) &= \sum_{n=0}^{\infty} a^n e^{-j\frac{2\pi k}{N}n} \\
&= \sum_{n=0}^{N-1} a^n e^{-j\frac{2\pi k}{N}n} + \sum_{n=N}^{\infty} a^n e^{-j\frac{2\pi k}{N}n} \\
&= \sum_{n=0}^{N-1} a^n e^{-j\frac{2\pi k}{N}n} + \sum_{n=0}^{\infty} a^{n+N} e^{-j\frac{2\pi k}{N}n} \\
&= \sum_{n=0}^{N-1} (1+a^N) a^n e^{-j\frac{2\pi k}{N}n} + \sum_{n=0}^{\infty} a^{n+2N} e^{-j\frac{2\pi k}{N}n} \\
&= \sum_{n=0}^{N-1} (1+a^N+\ldots) a^n e^{-j\frac{2\pi k}{N}n} \\
y[n] &= (1+a^N+\ldots)a^n = \frac{1}{1-a^N}a^n \qquad \text{for } 0 \le n \le N-1 \\
y_{ps}[n] &= \sum_{l=-\infty}^{\infty} y[n-lN]
\end{aligned}$$

Problem 3

$$\begin{aligned}
X(z) &= \sum_{n=-\infty}^{\infty} x[n]z^{-n} \\
&= \sum_{n=-\infty}^{\infty} \left(x[2n]z^{-2n} + x[2n+1]z^{-(2n+1)}\right) \\
&= \sum_{n=0}^{\infty} (0.2^n z^{-2n} + 0.1^n z^{-2n+1}) \\
&= \sum_{n=0}^{\infty} \left(\left(0.2z^{-2}\right)^n + \frac{\left(0.1z^{-2}\right)^n}{z}\right) \\
&= \frac{1}{1-0.2z^{-2}} + \frac{z^{-1}}{1-0.1z^{-2}}
\end{aligned}$$