EE113 Digital Signal Processing

Spring 2019

Homework 2

Due: Monday, April 15 at 12pm

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Total: 100 points

Problem 1. ***(20 points)*** *Is the system linear? causal? time-invariant? stable?*

(a) ***(10 points)*** *Consider the moving average system with exponential weighting*

$$y[n] = \frac{1}{M+1} \sum_{k=0}^{M} \lambda^k x[n-k]$$

where $|\lambda| < 1$.

(b) ***(10 points)*** *Consider the system*

$$y[n] = \frac{1}{|n|+1} x[-n^2]$$

Problem 2. ***(10 points)*** *The response of a linear system to* $0.5^n u[n]$ *is* $u[n]$ *and to* $0.5^{n-1}u[n-1]$ *is* $u[n-1]$. *Is the system time-invariant? Why or why not?*

Problem 3. ***(10 points)*** *Consider a first-order differential equation in the form*

$$\frac{dy_a}{dt} + A\ y_a(t) = A\ x_a(t)$$

The derivative can be approximated using the backward difference

$$\frac{dy_a}{dt} \approx \frac{y_a(t) - y_a(t-T)}{T}$$

where T *is the step size.*

(a) ***(5 points)*** *Using the approximation for the derivative in the differential equation, express* $y_a(t)$ *in terms of* $y_a(t-T)$ *and* $x_a(t)$.

(b) ***(5 points)*** *Convert the differential equation to a difference equation by defining discrete-time signals*

$$\begin{aligned} x[n] &= x_a(nT) \\ y[n] &= y_a(nT) \\ y[n-1] &= y_a(nT - T) \end{aligned}$$

Show that the resulting difference equation corresponds to an exponential smoother. That is, the difference equation is of the form

$$y[n] = (1-\alpha)y[n-1] + \alpha x[n].$$

Determine the parameter α *in terms of* A *and* T.

Problem 4. ***(15 points)***

(a) ***(5 points)*** *Compute the convolution of* $x[n]$ *and* $h[n]$, $y[n] = x[n] * h[n]$:

$x[n] = \left\{0, 1, 2, 3, \underset{\substack{\uparrow \\ n=0}}{4}, \quad 3, 2, 1, 0\right\}$ *and* $h[n] = \left\{\underset{\substack{\uparrow \\ n=0}}{1}, \quad 1, -1, -1\right\}$.

(b) ***(10 points)*** *Let $y[n]$ be the convolution of two discrete-time signals $x[n]$ and $h[n]$, that is*

$$y[n] = x[n] * h[n]$$

Show that time shifting either $x[n]$ or $h[n]$ by m samples causes $y[n]$ to be time shifted by m as well. Mathematically prove that

$$x[n-m] * h[n] = y[n-m]$$

and

$$x[n] * h[n-m] = y[n-m]$$

Problem 5. ***(25 points) (Use MATLAB for part (c) of this problem)*** *Consider the DTLTI system shown in Fig. 1:*

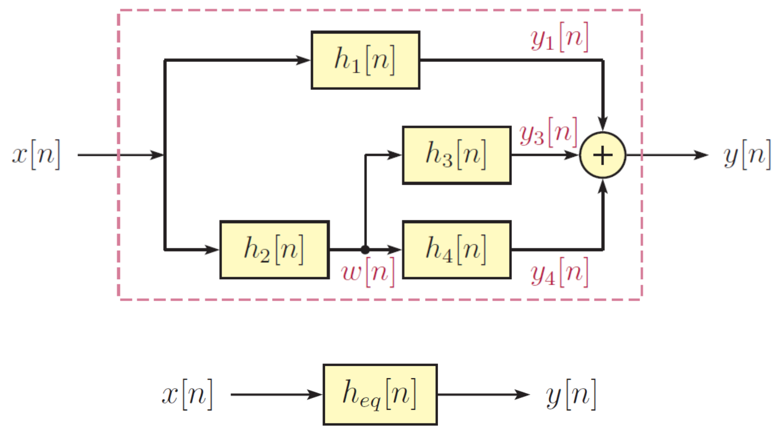

Figure 1: System Block diagram for Problem 5

(a) ***(5 Points)*** *Express the impulse response of the system as a function of the impulse responses of the subsystems.*

(b) ***(10 points)*** *Let*

$$h_1[n] = e^{-0.1n}u[n]$$
$$h_2[n] = h_3[n] = u[n] - u[n-3]$$

and

$$h_4[n] = \delta[n-2]$$

Determine the impulse response $h_{eq}[n]$ of the equivalent system. Note that you need to do the convolution on paper, but are free to use MATLAB ("conv" command) to verify if your calculations are correct.

(c) ***(10 points)*** *Let the input signal be a unit-step, that is, $x[n] = u[n]$. Determine and plot $y[n]$ in MATLAB.*
(Include the MATLAB plots for $n = 0, 1, \ldots, 10$ samples only, and use "stem" for the plot command)

Problem 6. ***(20 points) (Use MATLAB for this problem)***
Consider the problem 4 of Exercise 1, let's say that the goal of that problem is to play the music slower and still make it sound natural. In this problem, we will implement a MATLAB program to achieve the same goal using a system of upsampler and smoother as shown in the figure below.

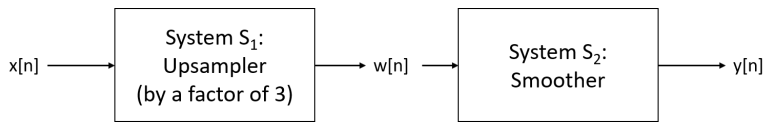

Figure 2: System block diagram for Problem 5

Write a MATLAB program and report your observations for a system that has the cascade of two systems, S_1 and S_2, as shown in the Fig. 2. System S_1 is an upsampler, such that the response of the system to an input of $x[n]$ is $w[n]$:

$$w[n] = \begin{cases} x[n/3], & \textit{if } n/3 \textit{ is an integer} \\ 0, & \textit{otherwise} \end{cases}$$

The system S_2 is is a smoother that can be implemented using:

(a) ***(10 points)*** *Moving average filter. Choose window lengths to be 1, 5, 10, 50, and 100, and play the output audio signal $y[n]$ from the system corresponding to each window length. Report the window length that makes the output signal $y[n]$ sound better, and explain the differences, based on what you hear, as you change the window lengths.*

(b) ***(10 points)*** *Exponential smoother. Report the best $\alpha = 0, 0.3, 0.5, 0.8, 1$ that makes the output signal $y[n]$ sounds better, and explain the differences with the other values of α.*

Consider the output of the overall system is $y[n]$.
The snippet of the code and the audio file are available in a compressed file "HW2_Prob6.rar" in week 2 folder.