Electrical Engineering 113, Homework 3

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Problem 1

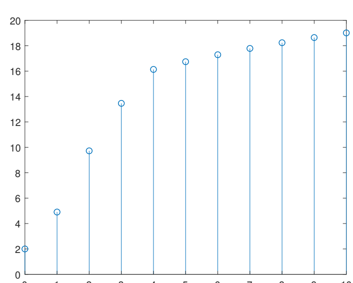
a)

$$h_{eq}[n] = h_1[n] + h_2[n] * (h_3[n] + h_4[n])$$

b)

$$\begin{aligned}
h_{eq}[n] &= e^{-0.1n}u[n] + (u[n] - u[n-3]) * (u[n] - u[n-3] + \delta[n-2]) \\
&= e^{-0.1n}u[n] + u[n] * u[n] - 2u[n] * u[n-3] \\
&\quad + u[n-3] * u[n-3] + u[n-2] - u[n-5] \\
&= e^{-0.1n}u[n] + (n+1)u[n] - 2(n-2)u[n-3] \\
&\quad + (n-5)u[n-6] + u[n-2] - u[n-5] \\
&= (e^{-0.1n} + n + 1)u[n] + u[n-2] - 2(n-2)u[n-3] \\
&\quad - u[n-5] + (n-5)u[n-6]
\end{aligned}$$

c) The output of the system when $x[n] = u[n]$ is shown below..



Problem 2

a) Let $h[n]$ be the impulse response of the system. Then we have the following relationship.

$$y[n] = u[n] * h[n] = 0.5^n u[n]$$

Assume that $h[n]$ is causal. Then if we evaluate both sides for given values of $n = 0, 1, 2, \ldots$ we obtain the following equations.

$$\begin{aligned} 1 &= h[0] \\ \frac{1}{2} &= h[0] + h[1] \\ \frac{1}{2^2} &= h[0] + h[1] + h[2] \\ \frac{1}{2^3} &= h[0] + h[1] + h[2] + h[3] \\ &\vdots \end{aligned}$$

Solving these yields the following function.

$$h[n] = -\frac{1}{2^n}u[n] + 2\delta[n]$$

b) This is true. Consider the system $y[n] = x[n] * h[n]$ where $x[n]$ and $h[n]$ are both even. Using the definition of convolution this system becomes the following.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

We can show at $-n$ this evaluates to the same value as at n.

$$\begin{aligned}
y[-n] &= \sum_{k=-\infty}^{\infty} x[k]h[-n-k] \\
&= \sum_{k=-\infty}^{\infty} x[-k]h[-n-k] \\
&= \sum_{k'=-\infty}^{\infty} x[k']h[-n+k'] \\
&= \sum_{k'=-\infty}^{\infty} x[k']h[n-k'] \\
&= y[n]
\end{aligned}$$

We were able to flip the signs of the indices for $x[n]$ and $h[n]$ since they are even, and we used the substitution $k' = -k$. Thus since $y[-n] = y[n]$ the output is even.

Problem 3

a) By plugging in some values we can see that the output has the following behaviour.

$$\begin{aligned}
y[0] &= 0 \\
y[1] &= \frac{1}{2} \\
y[2] &= \frac{3}{4} \\
y[3] &= \frac{7}{8} \\
&\vdots
\end{aligned}$$

The following output equation fits this behaviour.

$$y[n] = \left(1 - \frac{1}{2^n}\right) u[n-1]$$

b) By plugging in some values we can see that the output has the following behaviour.

$$
\begin{aligned}
y[0] &= 1 \\
y[1] &= 1 \\
y[2] &= \frac{1}{2} \\
y[3] &= \frac{1}{4} \\
&\vdots
\end{aligned}
$$

The following output equation fits this behavior.

$$y[n] = u[-n] + \frac{1}{2^{n-1}} u[n-1]$$

Problem 4

This function has a period of $N = 25$. Thus our fourier transform coefficients have the following form.

$$c_k = \frac{1}{25} \sum_{n=0}^{24} x[n] e^{-j\frac{2\pi k}{25} n}$$

We can rewrite our function as follows.

$$
\begin{aligned}
x[n] &= 1 + \cos(0.24\pi n) + 3\sin(0.56\pi n) \\
&= 1 + \frac{1}{2} e^{j\frac{6\pi}{25} n} + \frac{1}{2} e^{-j\frac{6\pi}{25} n} + \frac{3}{2j} e^{j\frac{14\pi}{25} n} - \frac{3}{2j} e^{-j\frac{14\pi}{25} n} \\
&= 1 + \frac{1}{2} e^{j\frac{6\pi}{25} n} + \frac{3}{2j} e^{j\frac{14\pi}{25} n} - \frac{3}{2j} e^{j\frac{36\pi}{25} n} + \frac{1}{2} e^{j\frac{44\pi}{25} n}
\end{aligned}
$$

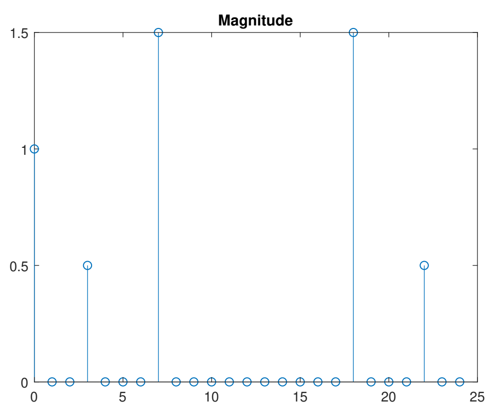
We were able to rewrite the terms with negative normalized frequencies by multiplying with $1^n = e^{j2\pi n}$. Thus the only coefficients that are not zero would be c_0, c_3, c_7, c_{18}, and c_{22}, since these values of k have a corresponding frequency component in $x[n]$. We can select the correct frequency components and calculate the coefficients as follows.

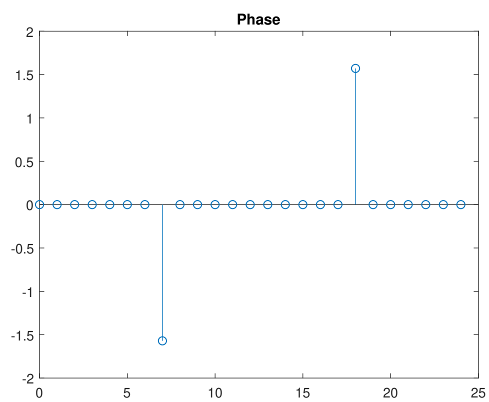
$$
\begin{aligned}
c_0 &= \frac{1}{25} \sum_{n=0}^{24} 1 e^{-j\frac{2\pi \times 0}{25}n} \\
&= \frac{1}{25} \sum_{n=0}^{24} 1 \\
&= 1
\end{aligned}
$$

Similar calculations yield the following coefficients.

$$
c_0 = 1 \qquad c_3 = \frac{1}{2} \qquad c_7 = \frac{3}{2j} \qquad c_{18} = -\frac{3}{2j} \qquad c_{22} = \frac{1}{2}
$$

The magnitude and phase plots of the DTFS coefficients is shown below.





Problem 5

a) We will use the following definition for the DTFS coefficients.

$$
\tilde{d}_k = \frac{1}{5} \sum_{n=0}^{4} \tilde{g}[n] e^{-j\frac{2\pi k}{5}n}
$$

Then we have the following results.

$$\begin{aligned}
\tilde{d}_0 &= \frac{1}{5}(4+1+2+3) = 2 \\
\tilde{d}_1 &= \frac{1}{5}\left(4+e^{-j\frac{4\pi}{5}}+2e^{-j\frac{6\pi}{5}}+3e^{-j\frac{8\pi}{5}}\right) = 0.5+0.68819j \\
\tilde{d}_2 &= \frac{1}{5}\left(4+e^{-j\frac{8\pi}{5}}+2e^{-j\frac{12\pi}{5}}+3e^{-j\frac{16\pi}{5}}\right) = 0.5+0.16246j \\
\tilde{d}_3 &= \frac{1}{5}\left(4+e^{-j\frac{12\pi}{5}}+2e^{-j\frac{18\pi}{5}}+3e^{-j\frac{24\pi}{5}}\right) = 0.5-0.16246j \\
\tilde{d}_4 &= \frac{1}{5}\left(4+e^{-j\frac{16\pi}{5}}+2e^{-j\frac{24\pi}{5}}+3e^{-j\frac{32\pi}{5}}\right) = 0.5-0.68819j
\end{aligned}$$

b) We have that $\tilde{g}[n] = \tilde{x}[n-1]$. Then we can apply the time shifting property as shown below.

$$\begin{aligned}
\tilde{d}_0 &= e^{-j\frac{2\pi}{5}0}\tilde{c}_0 = 2 \\
\tilde{d}_1 &= e^{-j\frac{2\pi}{5}1}\tilde{c}_0 = 0.5+0.68819j \\
\tilde{d}_2 &= e^{-j\frac{2\pi}{5}2}\tilde{c}_0 = 0.5+0.16243j \\
\tilde{d}_3 &= e^{-j\frac{2\pi}{5}3}\tilde{c}_0 = 0.5-0.16243j \\
\tilde{d}_4 &= e^{-j\frac{2\pi}{5}4}\tilde{c}_0 = 0.5-0.68819j
\end{aligned}$$

This agrees with my previous results, except for rounding errors.