EE113 Digital Signal Processing

Spring 2019

Homework 7

Due: Wednesday, June 5

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100 points

Problem 1. ***(24 points) z-transform:*** *Determine the z-transforms of the signals given below. Indicate the ROC for each.*

a) ***(8 points)***

$$x[n] = \begin{cases} n, & n = 0, \ldots, 9 \\ 0 & \textit{otherwise.} \end{cases}$$

b) ***(8 points)***

$$x[n] = \begin{cases} n, & n = 0, \ldots, 9 \\ 10, & n \geq 10 \\ 0, & \textit{otherwise.} \end{cases}$$

c) ***(8 points)***

$$x[n] = \begin{cases} n, & n = 0, \ldots, 9 \\ -n + 20, & n = 10, \ldots, 19 \\ 0, & \textit{otherwise.} \end{cases}$$

Problem 2. ***(28 points) Inverse z-transform:*** *Determine the inverse z-transform of*

$$X(z) = \frac{2 - 3z^{-1}}{1 - 3z^{-1} + 2z^{-2}},$$

for the following two cases
a) ***(14 points)*** *The ROC is* $|z| > 2$.
b) ***(14 points)*** *The ROC is* $1 < |z| < 2$.
Hint: use partial fraction expansion.

Problem 3. ***(24 points)*** *An input-output response pair of a relaxed causal and stable LTI system is given by*

$$x[n] = \left(\frac{1}{2}\right)^n u[n], \qquad y[n] = n\left(\frac{1}{2}\right)^{n-1} u(n-1).$$

a) ***(8 points)*** *Determine the transfer function of the system and indicate its ROC.*
b) ***(8 points)*** *Determine the poles and zeros of the system.*
c) ***(8 points)*** *Determine a difference equation relating any input sequence* $x[n]$ *to the corresponding output sequence* $y[n]$.

Problem 4. ***(24 points)*** *Find the impulse response sequences of the LTI systems with the following transfer functions:*
a) ***(8 points)*** $H(z) = \frac{z^2}{(z-\frac{1}{2})(z+\frac{1}{3})}$, $|z| > \frac{1}{2}$.
b) ***(8 points)*** $H(z) = \frac{1}{z^2+\frac{1}{4}}$, $|z| < \frac{1}{2}$.
c) ***(8 points)*** $H(z) = \frac{z+\frac{1}{3}}{(z-\frac{1}{2})(z+\frac{1}{4})}$, $|z| < \frac{1}{4}$.