Electrical Engineering 113, Homework 1

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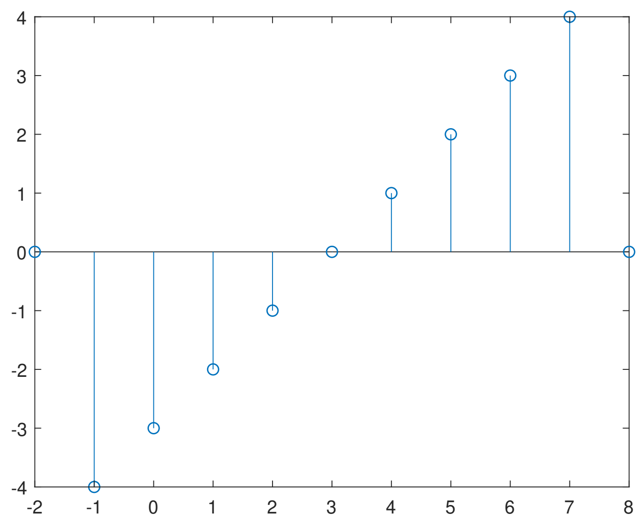
Problem 1

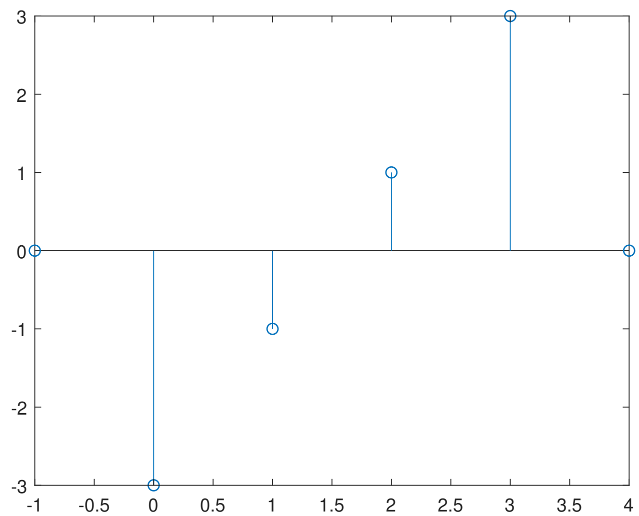
a)

$$g_1[n] = (n-3)(u[n+1] - u[n-8])$$



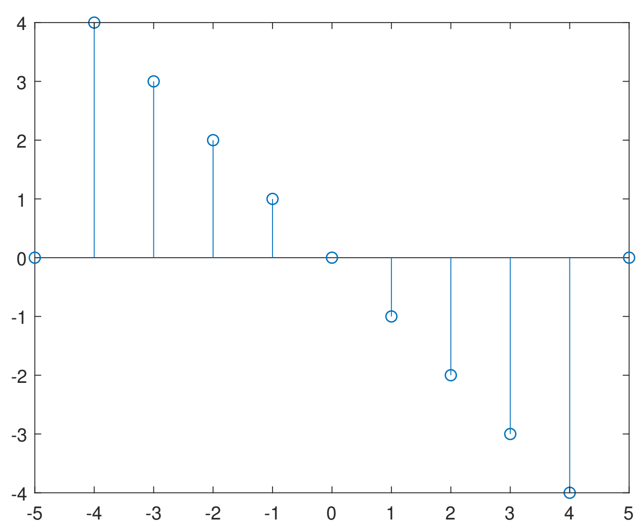
b)

$$g_2[n] = (2n-3)(u[2n+1] - u[2n-8])$$



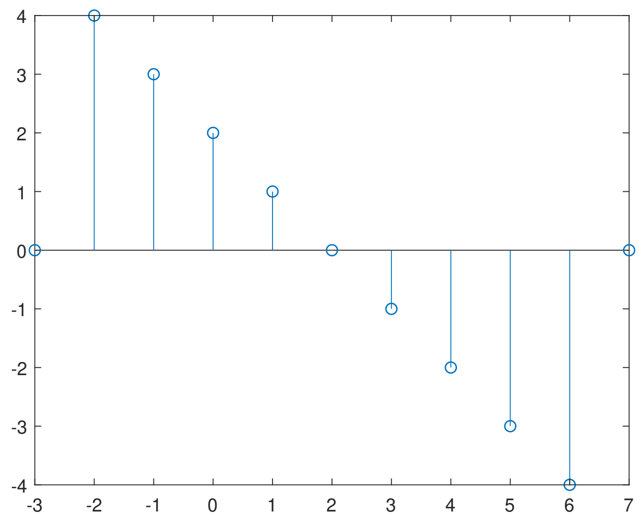
c)

$$g_3[n] = -n(u[-n+4] - u[-n-5])$$



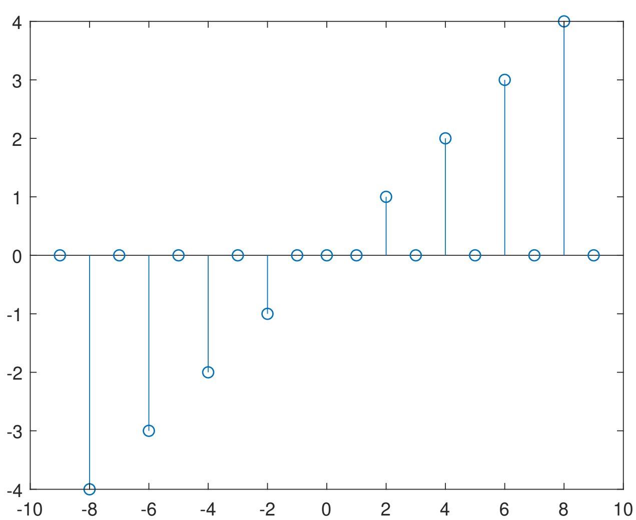
d)

$$g_4[n] = (2-n)(u[-n+6] - u[-n-3])$$



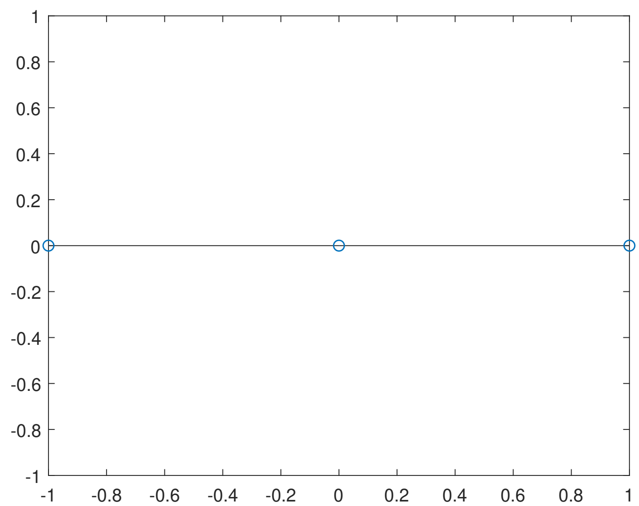
e)

$$g_5[n] = \begin{cases} \frac{n}{2}\left(u\left[\frac{n}{2}+4\right] - u\left[\frac{n}{2}-5\right]\right) & \frac{n}{2} \in \mathbb{Z} \\ 0 & \text{otherwise} \end{cases}$$



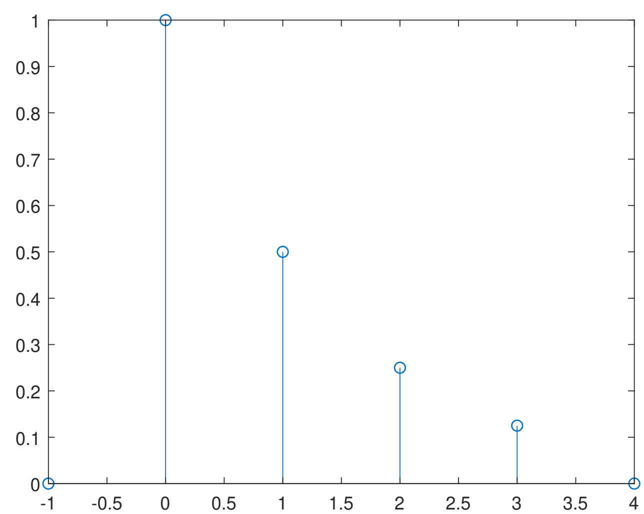
f)

$$g_6[n] = 0$$

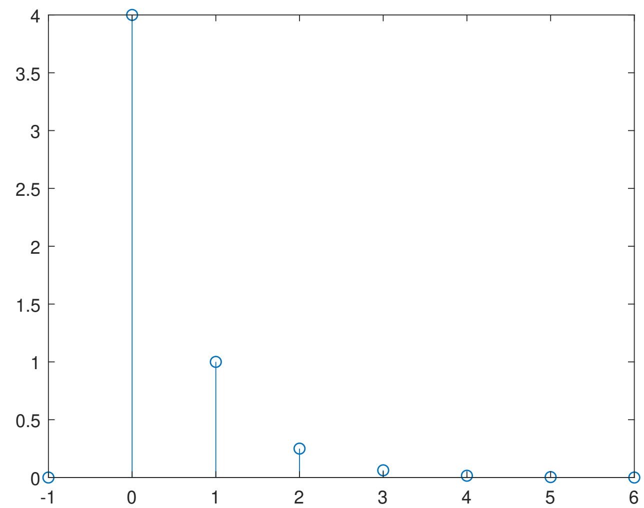


Problem 2

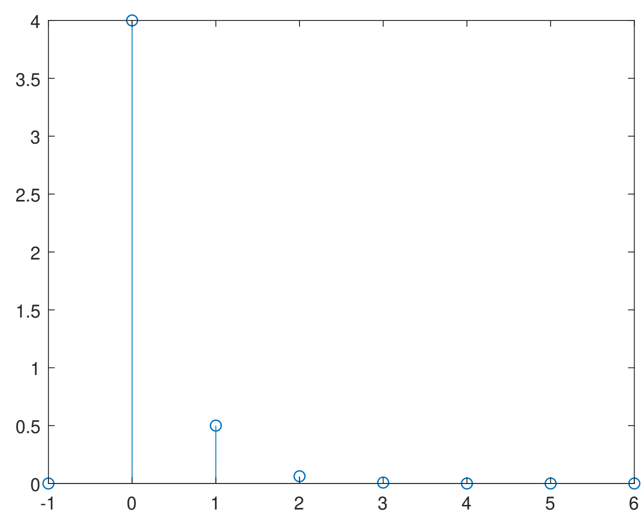
a) The plot of $x[n]$ is shown below.



The plot of $y[n]$ is shown below.



b) The plot of $z[n]$ is shown below



c) The energy of $x[n]$ is equal to the following value.

$$1^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^6 = \frac{85}{64} = 1.328125$$

The energy of $z[n]$ is equal to the following value.

$$4^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^8 + \left(\frac{1}{2}\right)^{14} = \frac{266305}{16384} \approx 16.254$$

Problem 3

a) This is not periodic since the normalized frequency does not contain π and so the value of $x[n]$ will be different for all n.

b) The left sinusoid has a fundamental period of 25 and the right sinusoid has a fundamental period of 5. Thus the overall fundamental period is 25.

Problem 4

We have that $x[n] = x_e[n] + x_o[n]$. By linearity the DC component of $x[n]$ is the sum of the DC component of its even and odd components. The odd component is 0 at $n = 0$, so we can rewrite the DC level as the following.

$$\lim_{N\to\infty} \left(\frac{1}{2N+1} \sum_{n=1}^{N} x[n] + x[-n] \right)$$

Since the definition of an odd function has that $x[n] = -x[-n]$, this expression becomes zero. Thus the DC component of $x[n]$ is equal to only the DC component of $x_e[n]$.

Problem 5

a) We have that $x_1[n] = x_1[-n]$ and $x_2[n] = x_2[-n]$. Then we have the following.

$$x[n] = x_1[n]x_2[n] = x_1[-n]x_2[-n] = x[-n]$$

Therefore $x[n]$ is even.

b) We have that $x_1[n] = -x_1[-n]$ and $x_2[n] = -x_2[-n]$. Then we have the following.

$$x[n] = x_1[n]x_2[n] = x_1[-n]x_2[-n] = x[-n]$$

Therefore $x[n]$ is even.

c) We have that $x_1[n] = x_1[-n]$ and $x_2[n] = -x_2[-n]$. Then we have the following.

$$x[n] = x_1[n]x_2[n] = -x_1[-n]x_2[-n] = -x[-n]$$

Therefore $x[n]$ is odd.

Problem 6

Since our initial conditions at $n = 0$ match, we can solve the recursion for $n > 0$.

$$\bar{x}[n-1] = \frac{1}{n}\sum_{k=0}^{n-1} x[k]$$

$$\bar{x}[n] = \frac{n}{n+1}\frac{1}{n}\sum_{k=0}^{n-1} x[k] + \frac{1}{n+1}x[n]$$

$$\bar{x}[n] = \frac{1}{n+1}\left(\sum_{k=0}^{n-1} x[k] + x[n]\right)$$

$$\bar{x}[n] = \frac{1}{n+1}\sum_{k=0}^{n} x[k]$$

This matches our given definition of $\bar{x}[n]$, so it is a valid solution for the recursion.