Electrical Engineering 113, Homework 7

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Problem 1

a)

$$X(z) = \sum_{n=1}^{9} n z^{-n}$$

This is a polynomial with 9 terms and converges for $|z| > 0$.

b)

$$X(z) = \sum_{n=1}^{9} n z^{-n} + \frac{10z^{-10}}{1 - z^{-1}}$$

This has a region of convergence for $|z| > 1$.

c)

$$X(z) = \sum_{n=1}^{9} n z^{-n} + \sum_{n=10}^{19} (20 - n) z^{-n}$$

This is a polynomial with 19 terms and converges for $|z| > 0$.

Problem 2

a) We can use partial fraction expansion to rewrite the equation in the following form.

$$X(z) = \frac{1}{1 - z^{-1}} + \frac{1}{1 - 2z^{-1}}$$

If $|z| > 2$ then each of these terms take the form $a^n u[n]$ and we obtain the following inverse.

$$x[n] = (1 + 2^n)u[n]$$

b) If $1 < |z| < 2$ then the right term takes the form $-a^n u[-n-1]$ and we obtain the following inverse.

$$x[n] = u[n] - 2^n u[-n-1]$$

Problem 3

a) We have the following z transforms.

$$X(z) = \frac{1}{1 - \frac{z^{-1}}{2}}$$

This has a ROC of $|z| > \frac{1}{2}$.

$$\begin{aligned}
Y(z) &= -z\frac{d}{dz}\left(z^{-1}\frac{1}{1 - \frac{z^{-1}}{2}}\right) \\
&= -z\frac{d}{dz}\left(\frac{1}{z - \frac{1}{2}}\right) \\
&= \frac{z}{\left(z - \frac{1}{2}\right)^2} \\
&= \frac{z}{z^2\left(1 - \frac{z^{-1}}{2}\right)^2} \\
&= \frac{z^{-1}}{\left(1 - \frac{z^{-1}}{2}\right)^2}
\end{aligned}$$

This also has a ROC of $|z| > \frac{1}{2}$. Thus the transfer function is the following.

$$\frac{Y(z)}{X(z)} = \frac{z^{-1}}{1 - \frac{z^{-1}}{2}}$$

This has a ROC of $|z| > \frac{1}{2}$.

b) There is a zero at infinity and a pole at $z = \frac{1}{2}$.

c)

$$
\begin{aligned}
\left(1 - \frac{z^{-1}}{2}\right) Y(z) &= z^{-1} X(z) \\
y[n] - \frac{1}{2} y[n-1] &= x[n-1]
\end{aligned}
$$

Problem 4

a)

$$
\begin{aligned}
H(z) &= \frac{1}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{3}z^{-1}\right)} \\
&= \frac{3}{5\left(1 - \frac{1}{2}z^{-1}\right)} + \frac{2}{5\left(1 + \frac{1}{3}z^{-1}\right)} \\
h[n] &= \left(\frac{3}{5}\left(\frac{1}{2}\right)^n + \frac{2}{5}\left(-\frac{1}{3}\right)^n\right) u[n]
\end{aligned}
$$

b)

$$
\begin{aligned}
H(z) &= \frac{z^{-2}}{\left(1 - \frac{i}{2}z^{-1}\right)\left(1 + \frac{i}{2}z^{-1}\right)} \\
&= z^{-2} \frac{1}{\left(1 - \frac{i}{2}z^{-1}\right)\left(1 + \frac{i}{2}z^{-1}\right)} \\
&= z^{-2} \frac{1}{\left(1 - \frac{i}{2}z^{-1}\right)\left(1 + \frac{i}{2}z^{-1}\right)} \\
&= z^{-2} \left(\frac{1}{2\left(1 - \frac{i}{2}z^{-1}\right)} + \frac{1}{2\left(1 + \frac{i}{2}z^{-1}\right)}\right) \\
h[n] &= -\frac{1}{2}\left(\left(\frac{i}{2}\right)^{n-2} + \left(-\frac{i}{2}\right)^{n-2}\right) u[-n+1]
\end{aligned}
$$

c)

$$
\begin{aligned}
H(z) &= \frac{10}{9\left(1 - \frac{1}{2}z^{-1}\right)} - \frac{1}{9\left(1 + \frac{1}{4}z^{-1}\right)} \\
h[n] &= -\frac{1}{9}\left(10\left(\frac{1}{2}\right)^n - \left(-\frac{1}{4}\right)^n\right) u[-n-1]
\end{aligned}
$$