EE113 Digital Signal Processing

Spring 2019

Homework 1

Due: Monday, April 8 at 12pm

Instructor: Prof. Christina Fragouli

TA: Dhaivat Joshi and Joris Kenanian

March 31, 2019

Problem 1. ***(30 points)*** *For the signal* $x[n] = n(u[n+4] - u[n-5])$, *compute and sketch the following:*

(a) ***(5 points)*** $g_1[n] = x[n-3]$

(b) ***(5 points)*** $g_2[n] = x[2n-3]$

(c) ***(5 points)*** $g_3[n] = x[-n]$

(d) ***(5 points)*** $g_4[n] = x[2-n]$

(e) ***(5 points)*** $g_5[n] = \begin{cases} x[n/2], & \textit{if } n/2 \textit{ is integer} \\ 0, & \textit{else} \end{cases}$

(f) ***(5 points)*** $g_6[n] = x[n]\delta[n]$

Problem 2. ***(15 points)*** *Consider the sequences*

$$x[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & 0 \le n \le 3 \\ 0, & \textit{otherwise} \end{cases}$$

and

$$y[n] = \begin{cases} \left(\frac{1}{4}\right)^{n-1}, & 0 \le n \le 5 \\ 0, & \textit{otherwise} \end{cases}$$

(a) ***(5 points)*** *Plot the samples of* $x[n]$ *and* $y[n]$.

(b) ***(5 points)*** *Plot the samples of the sequence* $z[n] = x[n]y[n]$, *which are obtained from the point-wise product of the samples of* $x[n]$ *and* $y[n]$.

(c) ***(5 points)*** *What is the energy of the sequences* $x[n]$ *and* $z[n]$*?*

Problem 3. ***(10 points)*** *For each of the discrete-time signals listed below, determine if the signal is periodic or not. If the signal is periodic, determine the fundamental period.*

(a) ***(5 points)*** $x[n] = 3\cos(\frac{5}{17}n + \frac{\pi}{4})$

(b) ***(5 points)*** $x[n] = 2\ \cos(0.48\pi n) + 1.5\ \sin(0.8\pi n + \pi/3)$

Problem 4. ***(15 points)*** *Consider an arbitrary sequence* $x[n]$ *with even and odd parts denoted by* $x_e[n]$ *and* $x_o[n]$, *respectively. Show that the DC level of* $x[n]$ *coincides with the DC level of its even part.*
The DC level of a sequence $x[n]$ *is defined as the average value of the sequence:*

$$\bar{x} \triangleq \lim_{N\to\infty}\left(\frac{1}{2N+1} \cdot \sum_{n=-N}^{N} x[n]\right)$$

Problem 5. ***(15 points)*** *Let* $x[n] = x_1[n]x_2[n]$. *Show that:*

(a) ***(5 points)*** *If both* $x_1[n]$ *and* $x_2[n]$ *are even, then* $x[n]$ *is even.*

(b) ***(5 points)*** *If both* $x_1[n]$ *and* $x_2[n]$ *are odd, then* $x[n]$ *is even.*

(c) ***(5 points)*** *If* $x_1[n]$ *is even and* $x_2[n]$ *is odd, then* $x[n]$ *is odd.*

Problem 6. ***(15 points)*** *Consider a discrete-time signal, $x[n]$, defined over the interval $n \geq 0$. At each time n, let $\bar{x}[n]$ denote the average value of the samples $x[n]$ from time $k = 0$ up to time $k = n$, i.e.,*

$$\bar{x}[n] = \frac{1}{n+1} \sum_{k=0}^{n} x[k]$$

This is an example of one processing algorithm; it acts on the data and generates $\bar{x}[n]$. We would like to motivate an alternative processing algorithm that operates on the data in a recursive manner to generate the same $\bar{x}[n]$. Show that $\bar{x}[n]$ satisfies the recursion:

$$\bar{x}[n] = \frac{n}{n+1}\bar{x}[n-1] + \frac{1}{n+1}x[n]$$

with initial condition $\bar{x}[0] = x[0]$. Note that the above algorithm is in terms of the previous value $\bar{x}[n-1]$ and the most recent term in the sequence, $x[n]$. In this way, the second procedure for evaluating the mean of the sequence does not need to save all prior data; the history of the prior data is incorporated into $\bar{x}(n-1)$ and only the most recent sample, $x[n]$, is needed along with $\bar{x}[n-1]$ to evaluate $\bar{x}[n]$.