EE113 Digital Signal Processing

Spring 2019

Homework 6

Due: Wednesday, May 29

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Total: 100 points

Problem 1. ***(30 points)*** *Find the power of*

$$x[n] = \cos(-0.2\pi n - 0.4\pi) \circledast \sum_{l=-\infty}^{\infty} \delta[n + 2 - 10l]$$

where $\circledast$ *denotes periodic convolution.*

Problem 2. ***(40 points)*** *Assume you are given the DTFT* $X(\Omega)$ *of a signal* $x[n]$ *that starts at* 0 *and has length* N_1*. You sample* $X(\Omega)$ *at* N_2 *points (as you need to store it digitally), and keep these* N_2 *samples*

$$X(\frac{2\pi k}{N_2}), \quad k = 0, \dots, N_2 - 1$$

Unfortunately because you did not know what N_1 *was, you used* $N_2 < N_1$*. You now try to apply IDFT to recover your original signal* $x[n]$*.*

(a) ***(25 points)*** *Show that* $X(\frac{2\pi k}{N_2})$ *is the DFT of a signal* $y[n]$ *(of length* N_2*), whose periodic extension is* $y_{ps}[n] = \sum_{\ell=-\infty}^{+\infty} x[n - \ell N_2]$*. That is,* $y[n]$ *is an "aliased in time" version of* $x[n]$*.*

   (Hint: You can either start from the expression for DTFT $X(\Omega)$*, get DFT via sampling and express this as the DFT of* $y[n]$*. Or you can start from* $y_{ps}[n]$ *and show that its* N_2*-point DFT applied to the interval* $\{0, 1, ..., N_2\}$ *is indeed equal to* $X(\frac{2\pi k}{N_2})$*).*

(b) ***(15 points)*** *Assume that* $x[n] = a^n u[n]$*,* $0 < a < 1$*, and thus* $X(\Omega) = \frac{1}{1 - ae^{-i\Omega}}$*. Assume that we take* N *samples of* $X(\Omega)$*, what will* $y_{ps}[n]$ *be?*

   You may find the infinite geometric summation formula useful:

$$\sum_{i=-\infty}^{a} r^{-i} = \frac{r^{-a}}{1 - r}, \quad \text{for} \quad |r| < 1.$$

Problem 3. ***(30 points) z-transform:*** *Determine the z-transform of*

$$x[n] = \begin{cases} 0.2^n u[n] & n \text{ is even} \\ 0.1^n u[n] & n \text{ is odd} \end{cases}$$