ECE141 - Principles of Feedback Control

Homework 7 ,

Note: You do not need to submit the solutions for this homework, but you are expected to know how to solve similar problems in the final exam.

Problem 1. The z-transform of a discrete-time filter $h(k)$ at a 1-Hz sample rate is given by:

$$H(z) = \frac{1 + (1/2)z^{-1}}{[1 - (1/2)z^{-1}][1 + (1/3)z^{-1}}$$

(a) Let $u(k)$ and $y(k)$ be the discrete input and output of this filter. Find the difference equation relating $u(k)$ and $y(k)$.

(b) Find the natural frequency and the damping coefficient for both of the filter's poles.

(c) Is this filter stable or unstable? Why?

Problem 2. Use the z-transform to solve the following difference equation:

$$y(k) - 3y(k-1) + 2y(k-2) = 2u(k-1) - 2u(k-2)$$

where:

$$\begin{aligned} u(k) &= \begin{cases} k, & k \geq 0 \\ 0, & k < 0 \end{cases} \\ y(k) &= 0 \end{aligned}$$

Problem 3. Consider a lead compensator with the following transfer function, which has been designed to add about 60° of phase at $\omega_1 = 3$ rad/sec:

$$H(s) = \frac{s+1}{0.1s+1}$$

(a) Assume a sampling period of $T = 0.25$ sec, and compute and plot in the z-plane the pole and zero locations of the digital implementation of $H(s)$ obtained using (1) Tustin's method, and (2) matched pole-zero method. For each case, compute the amount of phase lead provided by the compensator at $z_1 = e^{j\omega_1 T}$.

(b) For the frequency range $\omega = 0.1$ to $\omega = 100$ rad/sec, plot the magnitude Bode diagrams for $H(s)$ as well as of each of the equivalent digital implementations you obtained in Part (a), and compare the three magnitude plots. (*Hint:* Magnitude Bode plots are given by $\left|H(z)\right| = \left|H(e^{j\omega T})\right|$.

These homework problems are compiled using the different textbooks listed on the course syllabus