ECE141 - Principles of Feedback Control

Homework 1 , Due: 1/18/19, 9:00am

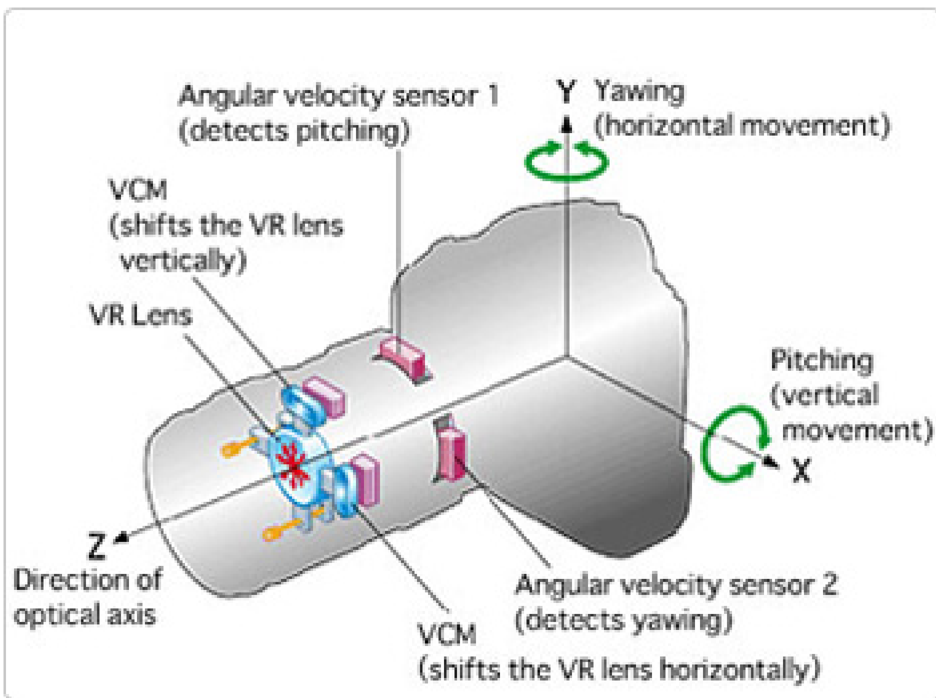
Problem 1. Think about the different systems around you ...

1. Identify three feedback systems.
2. For each feedback system, draw a simple high-level block diagram, and identify the plant/process, the inputs/outputs, the sensing mechanism, the actuating mechanism, the control mechanism, as well as any possible disturbances.
3. Also for each system, determine whether it uses manual or automatic control, whether it is single-input-single-output (SISO) or multi-input-multi-output (MIMO), and whether it is a tracking or a regulator system.

Problem 2. A control systems that has always fascinated me is the system used to stabilize camera/lens motion which enables photographers to shoot at lower shutter speeds (well, as long as the subject itself is steady), and thus be able to shoot at lower light while still keeping the ISO sensitivity level low and thus maintain low levels of noise in the image.

Here is a New York Times article about this technology: NY Times Article on Image Stabilization

Different camera manufacturers call it with different names. For example, Nikon calls it "Vibration Reduction (VR)" whereas Canon calls it "Image Stabilization (IS)". The basic control system is shown in the following picture (taken from the Nikon website linked below):



And here is a description of how the control system works on the Nikon website: Nikon Vibration Reduction Technology

Draw a block diagram for the feedback system, and identify inputs/outputs (including any disturbance and noise inputs), as well as sensing and actuating mechanisms.

These homework problems are compiled using the different textbooks listed on the course syllabus

Problem 3.1. Recall the Bilateral (two-sided) Laplace Transform:

$$\mathcal{L}(x(t)) = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt, \quad s \in \mathbb{C}, \quad X(s) \in \mathbb{C}$$

Find the bilateral Laplace transform for $x_1(t) = e^{-at}u(t)$ and $x_2(t) = -e^{-at}u(-t)$, where $u(t)$ is the unit step function. For each case, determine the associated Region of Convergence (ROC) for the Laplace transform. That is the region in the complex s-plane where the Laplace integral converges and thus the Laplace transform exists.

Problem 3.2. Now, let:

$$H(s) = \frac{5}{s^2 + s - 6}$$

Using Partial Fraction Expansion, find the different possible Inverse Laplace Transforms (i.e., $h(t)$ functions) and, for each $h(t)$, find the associated ROC, and discuss both causality and stability of the system whose impulse response is given by $h(t)$. Once you are done, be happy that we mostly have to deal with causal systems, and can use single-sided Laplace transforms.

Problem 4. Consider a causal signal $x(t)$ with the following Laplace transform:

$$X(s) = \frac{2s^2 + \omega^2}{s(s^2 + \omega^2)}$$

1. Using the Final Value Theorem, find $\lim_{t\to\infty} x(t)$
2. Find the inverse Laplace transform $x(t) = \mathcal{L}^{-1}(X(s))$ and see what its value will be as $t \to \infty$. Compare with the previous result and discuss.

Problem 5. Consider a causal LTI system with a rational Transfer Function $H(s) = \frac{b(s)}{a(s)}$. Find a necessary and sufficient condition for the associated Impulse Response of this system ($h(t)$) to be continuous at $t = 0.0$. (*hint:* Initial Value Theorem)

Problem 6. Dynamical Signficance of Poles and Zeros: In the second lecture, we discussed how the poles of the transfer function determine the modes of the system and the form of its natural response. We also saw how the zeros impact the amplitudes of both the forced response and the natural response of the system. We also discussed the significance of the exponential inputs e^{st} as the eigenfunctions of LTI systems. The objective of this problem is to summarize the key points on the dynamical significance of poles and zeros. Consider the following proper rational transfer function:

$$H(s) = \frac{b(s)}{a(s)}$$

where $b(s)$ and $a(s)$ are polynomials in s and the order of $b(s)$ is equal or smaller than the order of $a(s)$ (hence a *proper* transfer function) and $b(s)$ and $a(s)$ have no common factors (i.e., no shared roots). Recall that the output of the system to an input $x(t) \overset{\mathcal{L}}{\leftrightarrow} X(s)$ can be written as:

$$y(t) \overset{\mathcal{L}}{\leftrightarrow} Y(s) = \frac{b(s)}{a(s)}X(s) + \frac{I(s)}{a(s)}$$

where $I(s)$ is another polynmial in s of order $n-1$ where n is the order of $a(s)$, and whose coefficients are all determined by the initial conditions of input and output, i.e.,

$$y(0^-), \dot{y}(0^-), \ddot{y}(0^-), \ldots, x(0^-), \dot{x}(0^-), \ddot{x}(0^-), \ldots$$

(a) Assume an exponential input is applied to the sytem at time $t = 0$, i.e., $x(t) = e^{s_0 t}u(t)$, where $u(t)$ is the unit step function, and s_0 is not a pole of the tranfer function. Show that the Laplace transform of the output can be written as follows:

$$Y(s) = \frac{K_0}{s - s_0} + \frac{\beta(s)}{a(s)} + \frac{I(s)}{a(s)}$$

where $\beta(s)$ is a polynomial in s of order $n-1$. What is the value for K_0? Discuss how you can conclude that the output initial conditions $y(0^-), \dot{y}(0^-), \ldots$ can indeed be set such that system response is obtained as:

$$y(t) = H(s_0)e^{s_0 t}u(t)$$

(b) What will now happen if s_0 happens to be a zero of the transfer function, i.e., $b(s_0) = 0$? This is the main reason why transfer function zeros are sometimes called *transmission zeros*.

(c) Now, assume the input to the system is identically 0, i.e., $x(t) = 0, \forall t$. Therefore, the Laplace transform of the zero-input system response can be written as:

$$Y(s) = \frac{I(s)}{a(s)}.$$

Assume $s = p$ is a pole of the transfer function, i.e, $a(s)|_{s=p} = 0$ (for simplicity, lets assume it is not a repeated pole). Show that the output initial conditions can be set such that:

$$y(t) = Ke^{pt}u(t)$$

where K is a constant, and, as before, $u(t)$ is the unit step function.

Problem 7. Use the (single-sided) Laplace transform to solve the following Ordinary Differential Equation and find $y(t)$ given the initial conditions:

$$\ddot{y}(t) + 6\dot{y}(t) + 5y(t) = 6u(t), \quad \dot{y}(0^-) = y(0^-) = 1.0, \quad u(t): \quad \text{unit step function}$$

Problem 8. Find the Laplace Transforms for the following functions ($u(t)$ is the unit step function):

(a) $x(t) = (e^{-2t} + t\cos(2t))u(t)$

(b) $x(t) = 4te^{-4t}u(t)$

Problem 9. Find the (causal) Inverse Laplace Transforms for each of the following:

(a) $X(s) = \frac{10}{(s+1)^2(s+3)}$

(b) $X(s) = \frac{100(s+2)}{(s^2+0.4s+0.4)(s+1)(s+5)}e^{-4s}$

Problem 10. Time Delay: Consider the transfer function in Part (b) of the previous problem again. As you know by now, we end up with an exponential term in the transfer function due to presence of time delay somewhere in our loop. This is one main exception where we would have to deal with non-rational transfer functions.

(a) Ignore the time delay for a moment, and use `zp2tf` command in MATLAB followed by `step` command to plot the step response of the system with no time delay.

(b) Now, use the `InputDelay` option in the `tf` command to form the transfer function with the 4 sec delay. Use `step` command again to plot the delayed step response on the same figure. (use `hold` command to keep the current figure handle active)

(c) To deal with the delay-induced exponential term in transfer functions, one option could just be to approximate it using the power series expansion of the exponential term:

$$e^{-sT} \approx 1 - sT + \frac{1}{2}(sT)^2 - \frac{1}{3!}(sT)^3 + \ldots$$

But it turns out that a better approximation would be obtained using the so-called Padé approximation which approximates the exponential term (or almost any function for that matter) with a rational transfer function. The 1st-order and 2nd-order Padé approximations for the exponential term are given below:

$$\begin{aligned} e^{-sT} &\approx \frac{1-\frac{sT}{2}}{1+\frac{sT}{2}} \\ e^{-sT} &\approx \frac{1-\frac{sT}{2}+\frac{(sT)^2}{12}}{1+\frac{sT}{2}+\frac{(sT)^2}{12}} \end{aligned} \tag{1}$$

So by substituting the Padé approximation for the exponential term, one can turn the transfer function into a rational transfer function again. Thankfully MATLAB makes our life a bit easier here too. Use `pade(sys,n)` command where `sys` is the same system (transfer function) you just created in Part (b) above, and `n` is the desired order for Padé approximation. Try it with both 1st-order ($n = 1$) and 3rd-order ($n = 3$), and then use `step` command again to plot step responses of the approximated systems for both cases on the same figure as Part (b). As one would expect, note how increasing the order of Padé approximation improves the accuracy of the approximated step response. Also note how using Padé approximation can lead to a zero on the right-half-plane (RHP) and thus create a *non-minimum phase* system with a negative slope for the step response at $t = 0.0$.