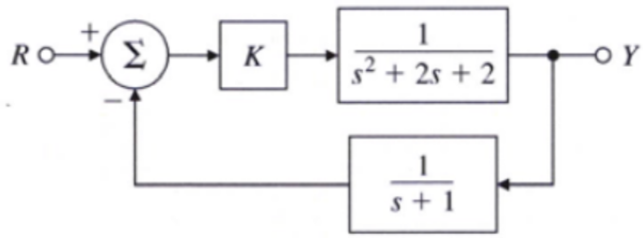
ECE141 - Principles of Feedback Control

Homework 6 , Due: 03/01/19, 9:00am

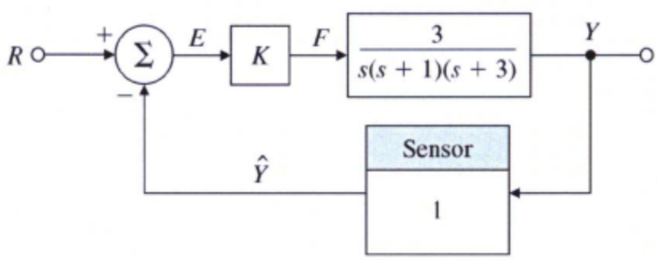
<u>Note:</u> While for these homework problems, you may feel free to use MATLAB to plot all your Bode and Nyquist diagrams, please remember that, for simple systems, you are expected to know how to sketch these diagrams by hand in the exam.

Problem 1. Consider the closed-loop system below:



Using Nyquist stability criterion, find the acceptable range for gain K (positive or negative) for the closed-loop system to remain stable.

Problem 2. Consider the closed-loop system below:



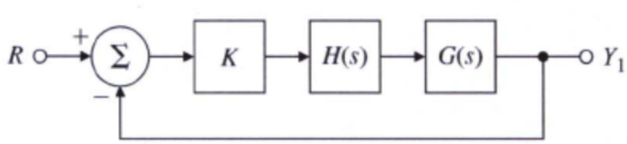
(a) Plot the Nyquist diagram of the loop transfer function with $K = 1$.

(b) Use the Nyquist stability criterion, to find out the range of gain K (positive or negative) such that the closed-loop system will remain stable.

(c) For those values of K that make the closed-loop system unstable, use the Nyquist stability criterion to determine the number of closed-loop poles in RHP.

(d) Verify your results by roughly sketching the root loci for both $K > 0$ and $K < 0$.

Problem 3. Consider the feedback system shown below:



These homework problems are compiled using the different textbooks listed on the course syllabus

Assume:

$$H(s) = 1, \quad \text{and,} \quad G(s) = \frac{5000(s+2)(s+3)}{s^3(s+20)(s+30)}$$

(a) Plot the Nyquist diagram of the loop transfer function $L(s) = KH(s)G(s)$ where $K = 1$ and $H(s) = 1$.

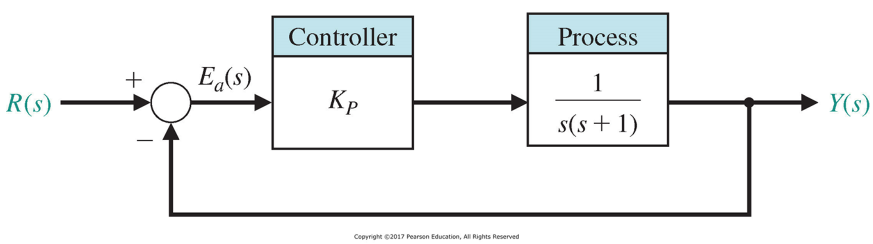
(b) How many times does the Nyquist diagram encircle the critical point -1? Is the closed-loop system stable? (Remember again that `nyquist` function in MATLAB does not properly handle open-loop poles on the $j\omega$ axis. You may instead want to use `lnyquist1(num,den)` function posted on CCLE.)

(c) Using the Nyquist stability criterion, find the upward gain margin and the downward gain margin.

(d) Confirm your result in Part (c) by plotting root-locus and finding the gain values associated with the $j\omega$ axis crossings. (You can use `rlocus` and `rlocfind` functions).

(e) Plot the Bode diagram of the loop transfer function, and again confirm the gain margin values you obtained in Part (c). Also find out the phase margin. (You can use `bode` function, and then right-click on your Bode diagram, and select the `Characteristics -> All Stability Margins` submenu to show all the crossover frequencies and the associated stability margins.)

Problem 4. Consider the feedback system shown below:



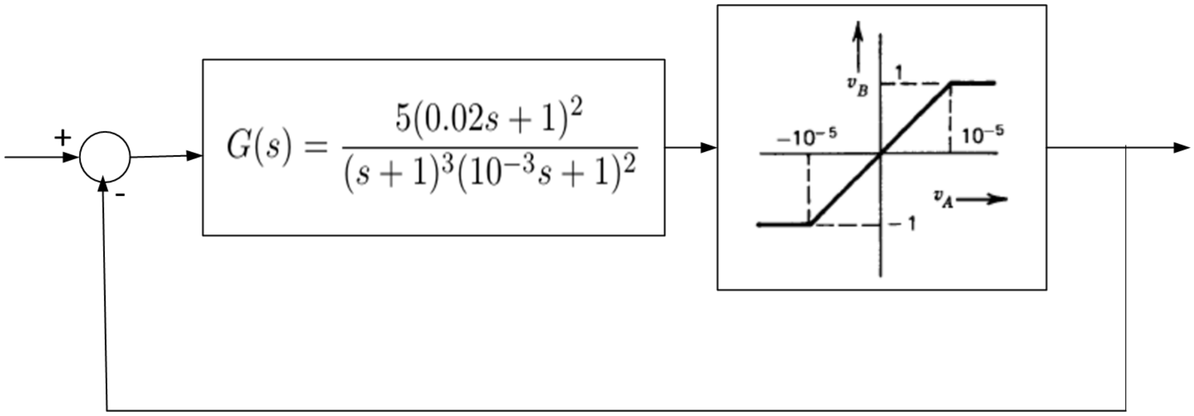
(a) Determine the value of the gain K_p in order to achieve 45° phase margin.

(b) For the same gain you found in Part (a) (i.e., for PM=45°), what would be the approximate damping ratio of the closed-loop system? What would be the associated approximate percentage overshoot (P.O.)?

(c) For the same gain you found in Part (a), plot the actual step response of the closed-loop system, and find out the actual P.O. and the damping ratio. How do the actual values compare with what you had predicted approximately based on the 45° phase margin?

(d) For the gain you found in Part (a), plot the Nyquist diagram of the loop transfer function. On the same figure, plot a unit circle centered at the origin. And by showing on the figure, confirm that the system does indeed have 45° phase margin. (You can use `lnyquist1` and `plot_unit_circle` functions which are posted on CCLE).

Problem 5. Conditional Stability: The closed-loop system below shows the model of a feedback amplifier where the output saturation is also modeled within the loop:



(a) Assume the output of $G(s)$ remains smaller than 10 μVolts. Therefore, we can ignore the saturation nonlinearity and replace it with a constant gain $K = 10^5$. Hand sketch the asymptotes for the Bode magnitude and phase plots of the loop transfer function $L(s) = KG(s)$. Then use `bode` function in MATLAB to verify your plots.

(b) How many times does the phase curve cross -180 degrees? This is a common characteristics of conditionally stable systems where the large slope (i.e., high roll-off) of the magnitude curve results in a phase cruve that drops below $-180°$ over a range of frequencies before it recovers and goes back up such that we will still get positive phase margin at the gain crossover frequency.

(c) We know that larger loop gain at low frequencies is desirable to achieve lower sensitivity for the closed-loop system. At the same time, we know that the gain curve should not have very large slope at the gain crossover frequency, or else we will end up with poor phase margin. This is one reason why conditionally stable systems may sometimes be desirable, in that they can give us larger loop gain and thus lower sensitivity while still maintaining a small slope at the gain crossover frequency and thus a decent phase margin. To see that, consider the following alternative loop transfer function:

$$L_2(s) = \frac{5 \times 10^5}{(2.5 \times 10^3 s + 1)(10^{-3}s + 1)^2} \tag{1}$$

Use `bode` function in MATLAB and plot the Bode diagrams of both $L(s)$ and $L_2(s)$ on the same figure. How do the two Bode diagrams compare? Which one will lead to lower sensitivity at low frequencies for the closed-loop system? Please discuss.

(d) Use `margin` function in MATLAB to find and compare the gain margins and the phase margins of $L(s)$ and $L_2(s)$. Remember that the gain margin returned by `margin` function is in linear scale. So the gain margin in dB will be $20\log(g_m)$ where g_m is returned by the `margin` function.

(e) Notice that in case of the conditionally stable $L(s)$, the gain margin returned by `margin` function is smaller than 1.0. That is because the `margin` function has only returned the *downward gain margin*. Find out the *upward gain margin* for $L(s)$. To do that, right-click on the Bode diagram you plotted in Part (c), and select the `Characteristics -> All Stability Margins` submenu to show all the crossover frequencies and the associated stability margins.

(f) To better appreciate why $L(s)$ is conditionally stable, and thus, unlike $L_2(s)$, has both a nonzero downward gain margin and a finite upward gain margin, use `nyquist` function in MATLAB to plot the Nyquist diagrams for both $L(s)$ and $L_2(s)$ on the same figure. Then to see the behavior around the critical point -1, open a new figure (using `figure` command) and plot the two Nyquist diagrams again, but this time set your axis limits using the command `axis([-10 1 -2 2])`. How many times does each Nyquist diagram encircle the -1 point? Please discuss the differences between the Nyquist diagrams of $L(s)$ and $L_2(s)$.

(g) [**Optional, 5 pt. Extra Credit**] Now let's look at why the saturation block, which is quite common in feedback amplifiers, can be problematic for the conditionally stable $L(s)$. Recall that nonlinear systems do not have transfer functions. But the *transfer characteristics* of some nonlinear elements may sometimes be represented by the so-called *Describing Functions*, which essentially look at the *fundamental frequency component* at the output in the presence of a sinusoidal input, while ignoring all other harmonics that could be generated due to the nonlinearity. Now, for the saturation element, the Describing Function can be obtained as:

$$G_D(E) = \frac{2 \times 10^5}{\pi} \left(\sin^{-1} \frac{10^{-5}}{E} + \frac{10^{-5}}{E} \sqrt{1 - \frac{10^{-10}}{E^2}} \right), \ \measuredangle 0^\circ \tag{2}$$

where E is the amplitude of the input sinusoid. Obtain the value of G_D assuming that the input voltage of the saturation block increases to $E = 92.5\mu$Volts. The effective loop gain will thus be reduced by the amount equal to $\frac{G_D}{10^5}$. Compare this value with the downward gain margin you had obtained for $L(s)$ in Part (d). Then replace the nonlinear saturation block with the constant gain equal to G_D, and use the `feedback` and `pole` functions in MATLAB to obtain the closed-loop pole locations. As you shall notice, the closed-loop system will move into instability. And that is the challenge with the conditionally stable systems where the closed-loop system can become unstable due to a nonzero downward gain margin, and the effective loop gain reduction that may happen due to the nonlinearities in the loop.