

1. How to find equilibrium points for nonlinear systems and how to linearize them around a given equilibrium point.  
 $F(x, u) = 0$  and linearize with the Jacobian.
2. How to determine stability from the state-space representation and how to go from state-space to transfer function and vice versa.

Given the following transfer function.

$$\frac{b_n s^n + \dots + b_0}{a_n s^n + \dots + a_0}$$

We have the following controller canonical form.

$$\mathbf{x}' = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ -\frac{a_0}{a_n} & -\frac{a_1}{a_n} & \dots & -\frac{a_{n-1}}{a_n} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \frac{1}{a_n} \end{bmatrix} \mathbf{u}$$

$$\mathbf{y} = \left[ b_0 - \frac{b_n a_0}{a_n} \quad \dots \quad b_{n-1} - \frac{b_n a_{n-1}}{a_n} \right] \mathbf{x} + \frac{b_n}{a_n} \mathbf{u}$$

$$\begin{aligned} sX(s) &= AX(s) + BU(s) \\ (sI - A)X(s) &= BU(s) \\ X(s) &= (sI - A)^{-1}BU(s) \\ Y(s) &= C(sI - A)^{-1}BU(s) + DU(s) \\ \frac{Y(s)}{U(s)} &= C(sI - A)^{-1}B + D \end{aligned}$$

The eigenvalues of  $A$  are the poles of the transfer function.

3. How to go from ordinary differential equations to transfer functions and vice versa.  
 Use the derivative rule of the Laplace transform.
4. How to find various input/output relationships in feedback systems.
5. How to find the steady-state response of LTI systems to sinusoidal inputs using the fundamental properties of transfer functions.

Final value theorem.

6. How the Internal Model Principle can help you design proper compensators in order to track a given reference input with zero steady-state error and/or reject the given disturbances at the output in the steady-state.

The open loop transfer function  $C(s)P(s)$  must have the input's closed right hand plane poles and the compensator  $C(s)$  must have the disturbance's closed right hand plane poles.

7. How to plot Root Locus for a given loop transfer function. That includes all the rules we have learned, e.g., the angles condition and the magnitude condition, finding the centroid and the angles of the asymptotes, finding the angles of arrivals to zeros and departures from poles, finding the crossing points with the  $j\omega$  axis using Routh-Hurwitz method, etc.

We have the conditions on magnitude and angle.

$$|K| \frac{|b(s_0)|}{|a(s_0)|} = 1 \quad \sum_{i=1}^m \angle(s_0 - z_i) - \sum_{j=1}^n \angle(s_0 - p_j) = \begin{cases} (2l+1)\pi & K > 0 \\ 2l\pi & K < 0 \end{cases}$$

If the number of poles and zeros on real axis to the right is odd, it is on the root locus, otherwise it is on the complementary root locus. With  $n$  poles and  $m$  zeros,  $n - m$  roots go to infinity. The asymptote centroid and angles are given by

$$\sigma = \frac{\sum_{j=1}^n p_j - \sum_{i=1}^m z_i}{n - m} \quad \theta_r = \begin{cases} \frac{(2r+1)\pi}{n-m} & K > 0 \\ \frac{2r\pi}{n-m} & K < 0 \end{cases}$$

for  $r = \{0, \dots, n - m - 1\}$ . Breakaway points of the characteristic equation  $1 + KG(s)H(s) = 0$  correspond to when there are multiple real roots at the same value. This occurs when

$$\frac{d(G(s)H(s))}{ds} = 0$$

has a roots that also satisfy the characteristic equation for a real  $K$ . Use condition on angles to find angle of approach and departure from zeros and poles. Use Routh-Hurwitz method for intersection with imaginary axis.

8. How to find out the desired closed-loop locations given the specifications on the transient response (e.g., settling time, overshoot percentage, etc.)

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}$$

$$\omega_n(-\zeta \pm \sqrt{\zeta^2 - 1})$$

$$M_p = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}}$$

$$t_s = -\frac{1}{\zeta\omega_n} \ln(\beta\sqrt{1-\zeta^2})$$

9. What Lead, Lag, and Lead-Lag compensators are.

The take the form of  $\frac{s-z_0}{s-p_0}$ , and a Lead-Lag compensator is two chained together. Lead compensator has  $p_0 < z_0 < 0$  while lag has  $z_0 < p_0 < 0$ .

10. How to use Root Locus and design Lead compensators in order to place the closed-loop poles in desired locations.
11. How to design Lag compensators in order to improve the steady-state tracking performance without disturbing the desired locations of the closed-loop poles.

$f(t)$	$\mathcal{L}\{f(t)\}$
1	$\frac{1}{s}, \quad s > 0$
$e^{at}$	$\frac{1}{s-a}, \quad s > a$
$t^n, \quad n = \text{positive integer}$	$\frac{n!}{s^{n+1}}, \quad s > 0$
$t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
$\sin at$	$\frac{a}{s^2 + a^2}, \quad s > 0$
$\cos at$	$\frac{s}{s^2 + a^2}, \quad s > 0$
$\sinh at$	$\frac{a}{s^2 - a^2}, \quad s >  a $
$\cosh at$	$\frac{s}{s^2 - a^2}, \quad s >  a $
$e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}, \quad s > a$
$e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$
$t^n e^{at}, \quad n = \text{positive integer}$	$\frac{n!}{(s-a)^{n+1}}$
$u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$
$e^{ct}f(t)$	$F(s-c)$
$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
$\int_0^t f(t-\tau)g(\tau) dt$	$F(s)G(s)$
$\delta(t-c)$	$e^{-cs}$
$f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$
$(-t)^n f(t)$	$F^{(n)}(s)$