# Electrical Engineering 141, Homework 7

Michael Wu
UID: 404751542

March 18th, 2019

## Problem 1

**a)**

$$\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{3}z^{-1}\right)Y(z) = \left(1 + \frac{1}{2}z^{-1}\right)U(z)$$
$$\left(1 + \left(\frac{1}{3} - \frac{1}{2}\right)z^{-1} - \frac{1}{6}z^{-2}\right)Y(z) = \left(1 + \frac{1}{2}z^{-1}\right)U(z)$$
$$y(k) - \frac{1}{6}y(k-1) - \frac{1}{6}y(k-2) = u(k) + \frac{1}{2}u(k-1)$$

**b)** For the pole at $z = \frac{1}{2}$ we can find the associated continuous pole with the following equation.

$$z = e^{sT}$$
$$sT = \ln\left(\frac{1}{2}\right)$$
$$s = -\frac{\ln(2)}{T}$$

Since the poles are also given by $s = \omega_n(-\zeta \pm \sqrt{\zeta^2 - 1})$, we can solve this for $\omega_n$ and $\zeta$ to obtain $\omega_n = \frac{\ln(2)}{T}$ and $\zeta = 1$. We have $T = 1$ so $\omega_n = \ln(2)$.

For the pole at $z = -\frac{1}{3}$ we can find the associated continuous pole with the following equation.

$$\begin{aligned}
z &= e^{sT} \\
sT &= \ln\left(-\frac{1}{3}\right) \\
s &= -\frac{\ln(-3)}{T} \\
s &= -\frac{\ln(3) + j\pi}{T}
\end{aligned}$$

We can then solve for $\omega_n$ and $\zeta$ as follows.

$$\begin{aligned}
\omega_n(-\zeta \pm \sqrt{\zeta^2 - 1}) &= -\ln(3) - j\pi \\
\omega_n &= \sqrt{\ln(3)^2 + \pi^2} \\
\zeta &= \frac{\ln(3)}{\sqrt{\ln(3)^2 + \pi^2}}
\end{aligned}$$

**c)** This is stable because both poles are within the unit circle.

# Problem 2

We have the following transfer function.

$$\begin{aligned}
(1 - 3z^{-1} + 2z^{-2})Y(z) &= (2z^{-1} - 2z^{-2})U(z) \\
\frac{Y(z)}{U(z)} &= \frac{2z^{-1} - 2z^{-2}}{1 - 3z^{-1} + 2z^{-2}}
\end{aligned}$$

The Z-transform of $u(k)$ is $\frac{z}{(z-1)^2}$. Then we have the following result.

$$\begin{aligned}
Y(z) &= \frac{2z^{-1} - 2z^{-2}}{1 - 3z^{-1} + 2z^{-2}} \frac{z}{(z-1)^2} \\
&= \frac{2z - 2}{z^2 - 3z + 2} \frac{z}{(z-1)^2} \\
&= \frac{z(2z-2)}{(z-1)^3(z-2)} \\
&= \frac{2z}{(z-1)^2(z-2)} \\
&= \frac{2z}{z-2} - \frac{2z}{z-1} - \frac{2z}{(z-1)^2} \\
y(k) &= 2^{k+1} - 2 - 2k
\end{aligned}$$