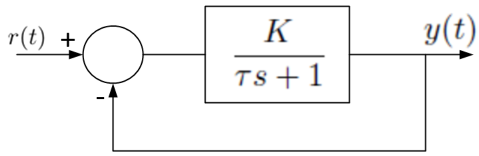
# ECE141 - Principles of Feedback Control

## Homework 3 , Due: 02/01/19, 9:00am

**Problem 1.** Consider the following first-order closed-loop feedback system:




(a) Find the closed-loop Transfer Function $H_{cl}(s) = \frac{Y(s)}{R(s)}$

(b) Find and plot the unit step response for the closed-loop system.

(c) What is the DC gain of the closed-loop system? What is the Time Constant of the closed-loop step response?

(d) What is the steady-state error with the unit step input?

(e) Find the minimum value for the open-loop time constant $\tau$ to ensure the maximum 2% settling time of 2 sec and the maximum steady-state error of 10% for the step response of the closed-loop system.

**Problem 2.** Consider a thermometer at room temperature 20.0°C. We now place it inside a water tank whose temperature is at 30.0°C. After one minute, it shows the temperature at 29.8°C.

(a) Assuming the thermometer follows a 1st-order dynamics, what is its time constant?

(b) If we place the same thermometer in another tank whose temperature increases linearly with a uniform rate of 2°C/min, and we wait long enough, what would be the measurement error (in °C) in the thermometer reading?

**Problem 3.** Consider the following 2nd-order system:

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For each of the following sets of specifications on the step response, show the desired region(s) in the s-plane where the complex poles should be located:

(a) $\zeta \leq 0.7, \quad \text{and} \quad 5 \leq \omega_n \leq 10$

(b) $0.5 \leq \zeta \leq 0.8, \quad \text{and} \quad \omega_d \leq 2$

(c) $t_s(2\%) \leq 2\,\text{sec}, \quad \text{and} \quad P.O. \leq 20\%$

**Problem 4.** Consider a 2nd-order system with the following transfer function:

$$G(s) = \frac{2(1-s)}{s^2 + 3s + 2}$$

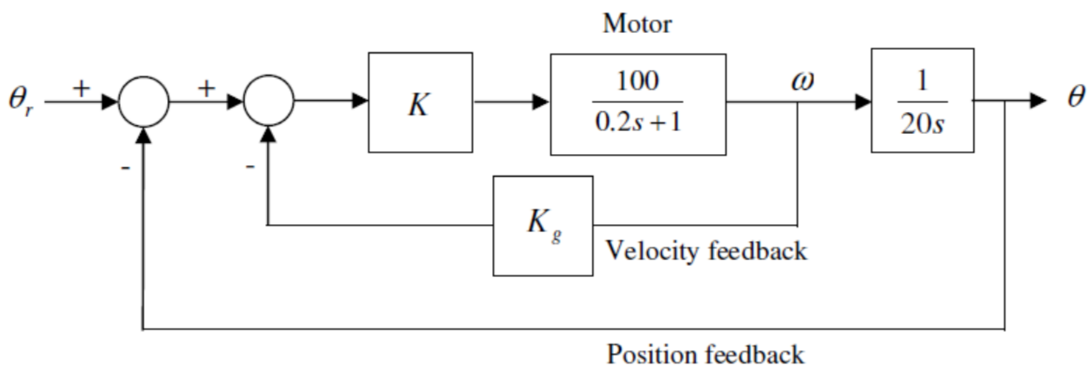
These homework problems are compiled using the different textbooks listed on the course syllabus

(a) Obtain and plot the step response of the system.

(b) Find the slope of the step response at $t = 0.0$.

(c) Do you notice anything unusual with the step response? Please discuss.

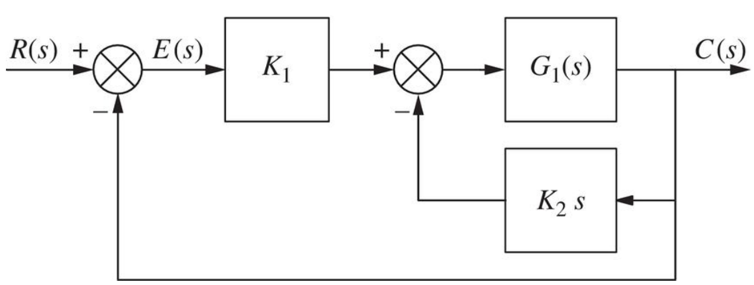
**Problem 5.** Consider the following closed-loop servo-motor control system:




(a) Find acceptable values for the forward gain $K$ and the velocity feedback gain $K_g$ such that the output angle response to a step reference angle input has a maximum settling time (2%) of 0.05 sec and the maximum percentage overshoot of 10%.

(b) Based on your result in Part (a), pick some values for gains $K$ and $K_g$ that meet your spec in Part (a). Obtain the tarnsfer function $\frac{\theta(s)}{\theta_r(s)}$. Then using `step` command in MATLAB, plot the step response of position angle $\theta(t)$ to a step input $\theta_r(t) = u(t)$. Then use `stepinfo` command and obtain the settling time as well as the percentage overshoot. Confirm that the values your picked for the gains do indeed meet your specs.

**Problem 6.** Consider the feedback system shown below:




Assume:

$$G_1(s) = \frac{1}{s(s+2)(s+4)}$$

(a) Find the value of gain $K_2$ for which the inner loop will have two equal negative real poles. Then use Routh-Hurwitz technique to find an acceptable range for the forward gain $K_1$ to ensure system stability.

(b) Find the value of $K_1$ for which the system will oscillate and find the associated frequency of oscillation.

(c) Find the value of $K_1$ at which there will be a real closed-loop pole at $s = -5$. Can the step response, $c(t)$ be approximated by a second-order, underdamped response in this case? Why, or why not? Please discuss.

(d) Assuming that the answer to Part (c) is yes, find the percentage overshoot and the 2% settling time $t_s$ of the appoximated 2nd-order system.

(e) Use the `impulse` command in MATLAB, and, on the same figure, plot the impulse responses of both the original 3rd-order system in Part (c) as well as the impulse response of the approximated 2nd-order system.

**Problem 7.** We have seen how the Routh-Hurwitz method can be used to find the number of unstable poles (i.e., roots of the characteristic equation that have positive real parts). Now, consider a system with the following characteristic equation:

$$s^3 + (6 + K)s^2 + (5 + 6K)s + 5K + 1 = 0$$

We want to find the acceptable range of values for the parameter $K$ such that all of the poles fall to the left of $\sigma = -1$ line on the complex plane, i.e., all the poles have real parts smaller than -1. Obtain the desired range for $K$ by showing how you can come up with a modified equation and still using the Routh-Hurwitz method.

**Problem 8.** Consider a 4th-order system with the following transfer function:

$$G(s) = \frac{(s + 1.05)}{(s + 1)(s + 20)(s^2 + 4s + 8)}$$

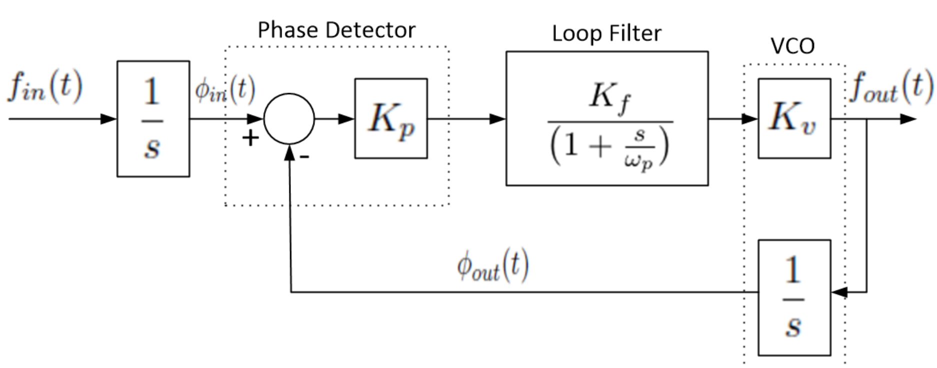
We want to identify the dominant poles and approximate this system by the following second-order system with the same DC gain:

$$G_a(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

(a) Find $G_a(s)$.

(b) Using `step` command in MATLAB, draw the step responses for the original system as well as the approximated 2nd-order system on the same plot.

**Problem 9. Phase-Locked-Loop (PLL)**: A Phase-Locked-Loop (PLL) is a circuit block that synchronizes the output of a Voltage-Controlled-Oscillator (VCO) to a reference input such as to maintain a constant frequency relationship (and sometimes phase relationship too) between the VCO output and the reference. PLL's are widely used in many applications. Some examples include frequency synthesizers to generate accurate and stable high-frequency clocks from a reference crystal oscillator, frequency and phase modulation and demodulation in communication transcievers, and phase tracking for carrier-phase positioning in high-accuracy GPS receivers. As the name suggests, a PLL uses a feedback loop with the basic structure shown below:



As shown, the key blocks within a PLL include a Phase Detector which compares the phases between the reference input and the VCO output, and forms an error signal, a VCO which generates a signal whose frequency depends on the VCO input voltage, and a Loop Filter which is designed to achieve the desired characteristics for the PLL (e.g., in terms of the performance specifications on the transient and steady-state frequency/phase tracking). The integrators shown on the feedforward and feedback paths are inherently present due to the fact that, by definition, integrating frequency leads to phase, or, putting it another way, frequency is the derivative of phase.

(a) With the given loop filter, find the closed-loop transfer function $H(s) = \frac{F_{out}(s)}{F_{in}(s)}$.

(b) Show that, by establishing proper relationship between the gains $K_p$, $K_f$, $K_v$, and the loop filter pole location at $-\omega_p$, you can write the closed-loop transfer function in the familiar second-order form:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

(c) Let $\zeta = 0.5$, and $\omega_n = 100$ rad/sec. Assume the input reference frequency follows a ramp with a uniform rate of 1 KHz/sec. What would be the PLL frequency tracking error (in Hz) after 20 msec? *Hint*: While you can certainly obtian the time-response of the 2nd-order system to the ramp input analytically and find the error at $t = 20$ msec, you may feel free to use MATLAB for this problem (e.g., see `lsim` command).