# Nonlinearity Modification of CMOS-MEMS Resonators with Stress Concentration Structures

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### **ABSTRACT**

This paper presents an experimental study on modifying the geometric nonlinearity of micromechanical beam structures using stress concentration designs. Two types of CMOS-MEMS resonators, the cantilever and the fixed-fixed beam, were chosen for this study. We found that integrating small coupling springs at the ends of both structures significantly altered their nonlinear behavior due to the stress concentration effect under high excitation. This approach suggests a method to enhance the power handling capability of micromechanical beam structures.

**Keywords:** CMOS-MEMS, nonlinearity, coupled spring, stress concentration

### 1. INTRODUCTION

CMOS-MEMS resonators play a critical role in designing low-noise, fully integrated oscillators for sensor applications. However, as these devices are miniaturized to enhance responsivity, the limited power handling capabilities of MEMS resonators become a performance bottleneck for the sensor. The significant amplitude-to-phase modulation (AM-PM) effect from nonlinear vibration not only distorts the output signal but also worsens phase noise, thereby reducing sensor performance. To address this issue, innovative methods for controlling resonator nonlinearity are needed.

Typically, electrostatically-actuated micro-mechanical resonators on the CMOS-MEMS platform exhibit two types of nonlinearity: (i) geometric (mechanical) nonlinearity due to significant deformation at resonance and (ii) capacitive nonlinearity, which arises from the inverse relationship between displacement and parallel plate capacitance. In the frequency spectrum, mechanical nonlinearity usually induces a hardening effect, causing the resonance spectrum to shift to higher frequencies. Conversely, capacitive nonlinearity can lead to a softening effect, resulting in a shift to lower frequencies.

In this study, we selected two typical resonators, the cantilever and the fixed-fixed beam, to examine their nonlinear behavior when stress concentration structures are applied. This analysis aimed to explore the interaction between mechanical and electrical nonlinearities.

## 2. DESIGN AND FABRICATION

Generally, cantilever beams are mainly influenced by capacitive nonlinearity, while fixed-fixed beams are predominantly affected by mechanical nonlinearity [1][2]. Fig. 1 presents the schematics of the modified beam

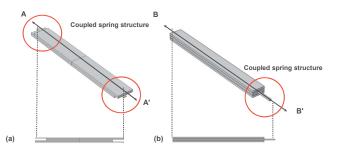


Fig. 1: Schematic of the mechanical structure of (a) fixed-fixed beam integrated with stress concentration through local etching on the fixed end, (b) cantilever beam integrated with geometric nonlinearity through small beam attachment.

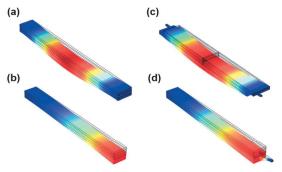


Fig. 2: The 1<sup>st</sup> out of plane (OOP) vibration mode shape of (a) common fixed-fixed beam, (b) common cantilever beam, (c) modified fixed-fixed beam, and (d) modified cantilever beam.

structures, where small beam-type springs are implemented at the device ends. The finite-element method (FEM) simulated mode shapes are shown in Fig. 2. In both devices, these small springs serve as efficient stress concentration structures, enhancing the hardening nonlinearity by storing more tension than the original versions to counter the negative capacitive nonlinearity.

For the device fabrication, the unreleased devices fabricated by the TSMC 0.35 um CMOS process undergo a wet-etching process with sulfuric acid ( $H_2SO_4$ ) to remove the sacrificial metals and suspend the beam structures. Fig. 3 shows the optical image of the beam structure with the proposed design after post-fabrication process. Four identical unit cell are connected in parallel to reduce the motional impedance during measurement These devices were designed with various lengths ( $L_b$ ) and widths ( $W_b$ ) of spring to investigate their effect on the nonlinearity.

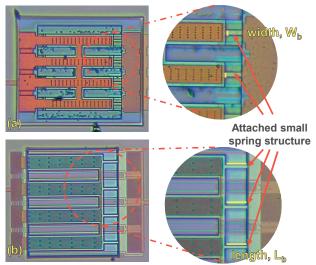


Fig. 3: Optical microscope image of (a) fixed-fixed beam integrated with stress concentration through local etching on fixed end and (b) cantilever beam integrated with geometric nonlinearity through small beam attachment.

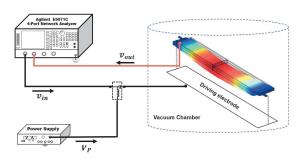


Fig. 4: Measurement scheme of the stress concentrated fixed-fixed beam.

### 3. MEASUREMENT RESULTS

The fabricated devices were measured using the setup shown in Fig. 4. The measurements were taken under vacuum to reduce air damping and squeeze-film damping effects.

Fig. 5 presents the measured S-parameters of standard fixed-fixed beam and cantilever resonators at various applied bias voltages. As expected, the fixed-fixed beam exhibits strong hardening nonlinear effects, while the cantilever beam shows pronounced softening effects.

Next, Fig. 6 illustrates the transmission curves for the modified designs with the integrated coupled spring structure. Fig. 6(a) shows the frequency response for different fixed-fixed beam designs, with the coupled spring structure exhibiting a greater hardening effect compared to the regular design, indicating that the small springs contribute to the hardening nonlinearity. In Fig. 6(b), the addition of small springs increases the mechanical nonlinearity in cantilever beams, counteracting the softening behavior from electrostatic actuation. The shorter spring structure appears to provide greater nonlinear stiffness to mitigate the softening effect.

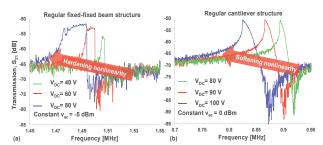


Fig. 5: Measured spectra shows the nonlinear effect for the (a) regular fixed-fixed beam structure and (b) cantilever structure under various dc-bias voltage.

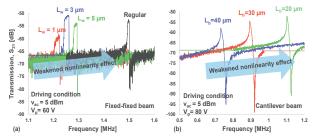


Fig. 6: Measured spectra of (a) modified fixed-fixed beam and (b) modified cantilever beam with different dimension of the small spring structure.

We observed a shift in resonance frequency in both fixed-fixed beam and cantilever designs. This shift is attributed to the addition of small spring structures, which slightly increase the effective length of the beams.

### 4. Conclusions

In this study, we designed and fabricated modified cantilever and fixed-fixed beams. The modified cantilever beam exhibited reduced softening nonlinearity with shorter springs, which tuned the resonator toward a more linear response. In contrast, the modified fixed-fixed beam showed increased mechanical nonlinearity, which became more pronounced with narrower springs. Both effects suggest that the additional small spring structure provides greater hardening nonlinearity, as anticipated.

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