# A Bank Run Model: What drives failure?

Elisa Gonzalez, Ty Geri, Leah Margulies, Michael Chen

March 2023


## 1 Abstract

We examine a two-bank system using a Markov Chain to understand what drives liquidity problems- individual bank instability, bank size, macroeconomic conditions, or a combination of the three. We ultimately find that increasing the initial failure rate of bank 1 beyond a threshold of about 0.1 had a more dramatic effect on the system's failure rate than did increasing the contagion factor as well as bank 1's proportion of reserve holdings. We also investigate whether federal policies can potentially reverse a banking system doomed for failure. We find that bank instability increases probability of system collapse, size does not affect failure substantially, and federal intervention can help delay, but not prevent system collapse. Moreover, increasing volatility leads to accelerated bank failure, but is dependent on bank size.


## 2 Introduction

In the wake of modern bank failures, like the fall of SVB bank and the resulting failures of other regional and sector specific banks like Silvergate and Signature Bank, civilians and economists alike are eager to discover what elements drive bank liquidity problems. Moreover, little is known about what can help revive the banking system after a bank run. Nonetheless there are current systems in place, for example, the federal government has multiple policies in place to prevent bank runs including FDIC insurance, established after a series of bank runs during the Great Depression, bank reserve requirements, and, at certain points, a willingness to act as a lender of last resort. However, each of these policies have been criticized for increasing the amount of moral hazard in the banking sector which, especially with the advent of mobile banking, makes bank runs far easier than in the past. In our paper, we will explore the core forces that drive bank runs, experimenting with a two bank Markov Chain model to determine, at a simple level, what factors impact liquidity issues. In particular, this paper will focus on four key elements – independent bank instability, bank size relative to ecosystem, macroeconomic pressure, and federal intervention.

## 3 Description of the Model

### 3.1 Description

Our model is inspired by Professor Dror Parnes, who published a generic Markov model for tracking and forecasting the contagion of bank runs over economic cycles [1]. The model includes N banks, each with an initial instability rate, and predicts the probability of the failure of the banking system after customers start to withdraw their deposits. Under our model, upon the failure of a bank, depositors increase withdrawals from other banks which relies on an assumption of bank run contagion. The failure of a bank depends both on its initial instability rate and the impact of other bank failures via increased cash withdrawals.

We explore both the homogeneous case, where banks have uniform instability and withdrawal rates, as well as a heterogeneous case, where bank instability and withdrawal rates differ. The model runs on a time-horizon of five Federal Reserve planning cycles, with each cycle being nine quarters or 27 months long. Therefore, all time units in the model are months.

Our model relies on a number of important assumptions: First, we assume bank behavior is sequential. Second, banks cannot become operational again once they fail, unless they receive a Federal bailout. Third, no Federal bailout will occur if the entire system fails.

## 3.2 Parameters and Variables

$$\alpha_i \geq 0 \tag{1}$$

Represents the initial instability rate of a bank $i$. This rate represents the probability of bank failure within the time-horizon, reflecting the bank's balance sheet and exposure to macroeconomic conditions. A higher $\alpha_i$ value results in a more unstable bank.

$$\beta \geq 0 \tag{2}$$

Represents the contagion factor in the banking system. This parameter dictates the strength of the 'domino-effect,' how one bank failure will affect the rate of withdrawals across the system. When set to 0, there is no contagion in the system and therefore the failure of one bank will not impact the rate at which other banks in the system fail.

$$\omega_i \geq 0 \tag{3}$$

Represents the proportion of deposit withdrawals of the entire banking system that bank $i$ is experiencing. A higher $w_i$ means the bank is larger.

$$\alpha_{i|j^F} = (\frac{\omega_i + \omega_j}{\omega_i})^\beta \alpha_i \tag{4}$$

Represents the probability bank $i$ fails given that bank $j$ has failed. This is a function of the increased withdrawal pressure on bank $i$ resulting from the failure of bank $j$. This factor is intensified by $\beta$ which represents the contagion sensitivity of the banking system. When one bank fails, the other banks absorb its withdrawals aligning with our assumption that the failure of one bank increases withdrawal pressure on other banks.

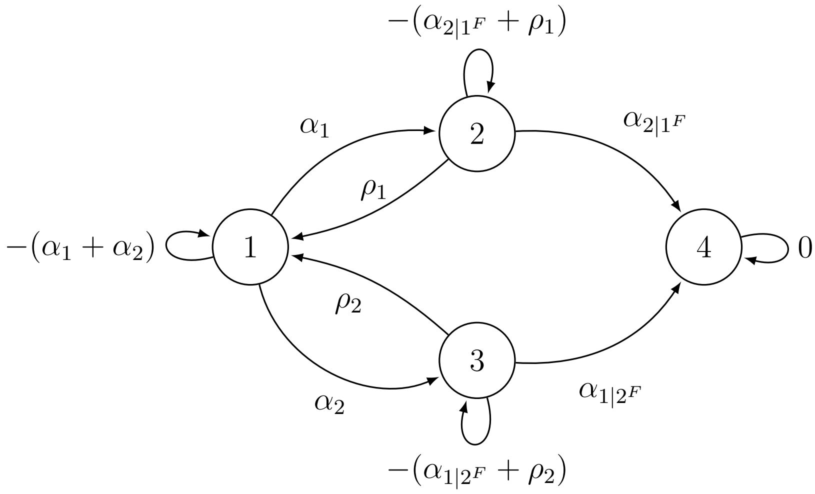
$$\rho_i \geq 0 \tag{5}$$

Represents the probability bank $i$ will be bailed out by the government returning it from bust to operational. When set to 0, there will be no government bailout.

## 3.3 Markov Model

Our matrix takes the form of an infinitesimal generator for a continuous time Markov chain. Each row sums to zero, and non-diagonal entries are related to the holding periods for each state. Q is defined as P'(0) for the transition matrix P, of the embedded discrete Markov chain of this continuous process.

$$Q = \begin{pmatrix} -(\alpha_1+\alpha_2) & \alpha_1 & \alpha_2 & 0 \\ \rho_1 & -(\alpha_{2|1^F}+\rho_1) & 0 & \alpha_{2|1^F} \\ \rho_2 & 0 & -(\alpha_{1|2^F}+\rho_2) & \alpha_{1|2^F} \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{6}$$




- State 1: both banks are operational.
- State 2: bank 1 failed, bank 2 operational.
- State 3: bank 2 failed, bank 1 operational.
- State 4: both banks failed.

# 4 Scenario Results

## 4.1 Homogeneous Banking

To develop a control for the testing of our bank model, we will first explore a situation in which the two banks are homogeneous under stable economic conditions. Firstly, $\alpha_1, \alpha_2$ pertain to the independent failure risk of the two banks. In this situation, with low volatility, we will set $\alpha_1, \alpha_2 = 0.05$ slightly higher than the work done by Dr. Parnes during less tense economic conditions [1]. Secondly, we will assume the banks represent equal proportions of the 2-bank ecosystem, thus $w_1, w_2 = 0.5$. Thirdly, we will assume stable macroeconomic conditions and, for simplicity in our control, we will set $\beta = 1$. Lastly, we will assume there is no federal intervention, so $\rho_1, \rho_2 = 0$. Under these conditions we observe the following behavior in the model as shown in Figure 1.

By construction, a non-zero independent failure rate will inevitably result in the banking system's collapse. However, given the homogeneity between the two banks, both banks will experience analogous failure rates over time, as observed by the overlap between states 2 and 3. Importantly, when we examine time to absorption, that is, how long it takes for both banks to fail, we find it takes roughly 78.4 months or almost 3 Fed cycles.

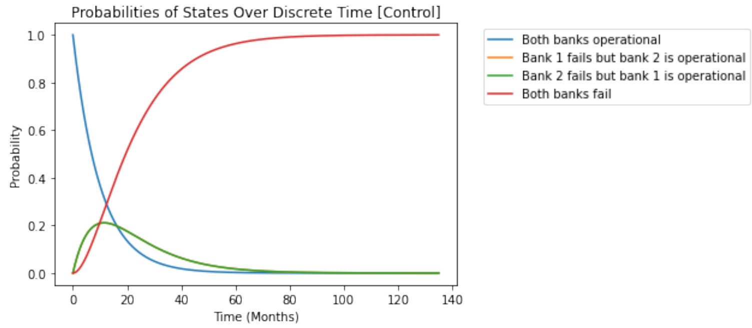



Figure 1: Control Group: Banking System States Over Time

## 4.2 Fed Intervention

A key assumption of our model was that it assumed a lack of intervention. That is, as soon as a bank failed there was no hope for the bank to become operational again. However, in the real world, the existence of federal intervention via bailouts allow banks to regain functionality. In this section, we will explore a change in our probabilities such that a bank that once failed can become operational again.

Here we return to our control case, which as a reminder has $\alpha_1, \alpha_2 = 0.05$, $w_1, w_2 = 0.5$, and $\beta = 1$. To introduce $\rho_i$ we make two key assumptions. The first is that the federal bank cannot bring the whole banking system back, so once both banks have failed (the absorbing state) they are failed forever. The second is that federal policies neither apply to all banks nor guarantee success if applied. Thus, we believe that $\rho_i$ should not be greater than one, allowing for potential failure even with intervention. In order to create more nuance, for the below simulation, we will assume that federal policies will only offset potential failure in the independent market. We make this assumption as reserves are likely only to be used for individual banks rather than to resolve consequences created by the entire banking system. Thus, we will set $\rho$ equal to $\alpha$ so that $\rho_1, \rho_2 = 0.05$. The outcome is displayed in Figure 3.

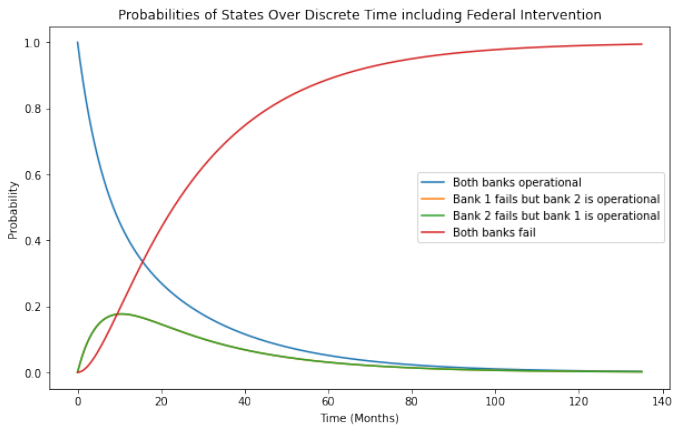



Figure 2: Federal Intervention: Banking System States Over Time

As we can see, the behavior observed in the control is mimicked in the case of federal intervention, with Figure 2 being a slightly elongated version of Figure 1. Such a system implies that, federal intervention, as defined in this model, will not prevent the inevitability of collapse but rather slightly delay its onset. As seen by the time to absorption, the federal intervention model will take roughly 119.3 months, far offsetting future failure by about 3 years. Moreover, as we increase the $\rho_i$, our bailout factor, the time to absorption increases but inevitably occurs. This indicates that federal buyouts are short-term solutions that do not fundamentally reverse a skewed banking system. Note that his model does not include deposit insurance which may be able to prolong the impact of a federal intervention. This and other factors indicate that deeper exploration into the mechanics and aftermath of federal intervention is an interesting topic that future papers could consider.

### 4.3 Heterogeneous Banking

To extend on our homogeneous bank model, we introduce heterogeneity to examine the effects of differences in individual volatility (banks 1 and 2 no longer have the same $\alpha$) and size of the bank as a function of total withdrawals (banks 1 and 2 no longer have the same proportion of withdrawals), to better understand the dynamics of our model. Introducing heterogeneity in our model is important because it allows for a more realistic representation of the system being modeled. In real-world bank systems like that in the US, individual bank entities often have different characteristics, behaviors, and interactions with each other, which can significantly impact the system's overall behavior.

We begin with heterogeneity in individual volatility, while holding all other parameters constant. We model this by setting $\alpha_1 = 0.05$ to model a bank with low individual probability of failure, while setting $\alpha_2 = 0.10$ to model a bank with higher individual probability of failure.

We next model heterogeneity in bank size. We model this by setting $w_1 = 0.33$ and $w_2 = 0.67$ to assume that one bank is larger than the other.

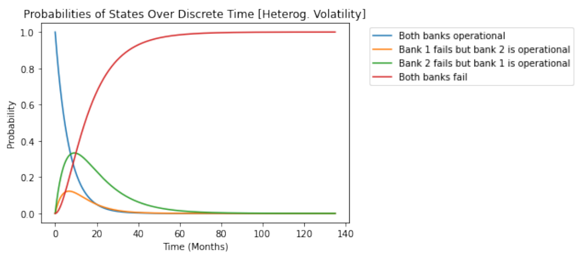



Figure 3: Heterogeneous Group: Banking System States Over Time Given Differences in Initial Volatility

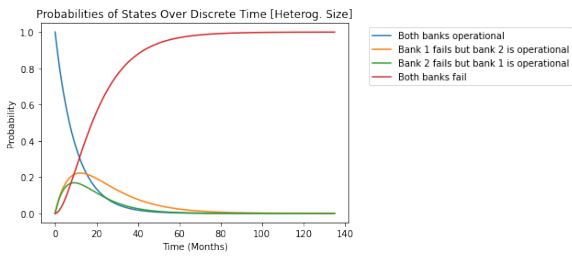



Figure 4: Heterogeneous Group: Banking System States Over Time Given Differences in Bank Size

We see that time to absorption where there's heterogeneity in individual bank volatility is 65.5 months (one bank is 2x the other), almost a whole year earlier than our base case, while the time to absorption where there's heterogeneity in bank size (one bank is 2x the size) is 76.5 months, only 2 months earlier than our base case. This implies that individual volatility has a larger impact than size on the health of the overall bank ecosystem. Note that it is difficult to compare effects across size and initial volatility, given their different units.

# 5 Analysis

### 5.1 $\alpha_1$ Sensitivities

The likelihood of both banks failing increases dramatically as we increase alpha1 from 0.1 to 0.35. This makes sense given that, in the original model, alpha 1 was set to 0.05.

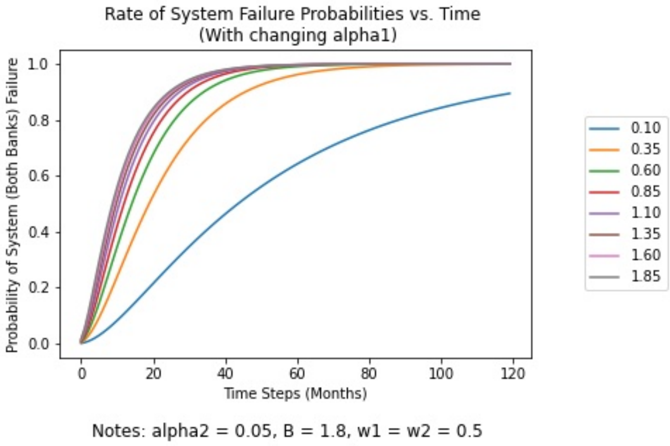



Figure 5: Rate of Both Banks Failing vs. Prior Probability of Bank 1 Failing

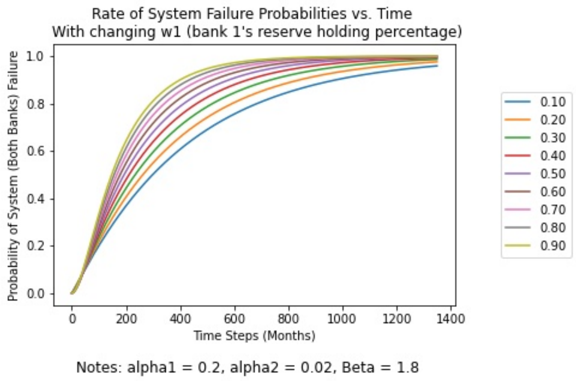



Figure 6: Rate of Both Banks Failing vs. Proportion of Reserve Holdings for Bank 1

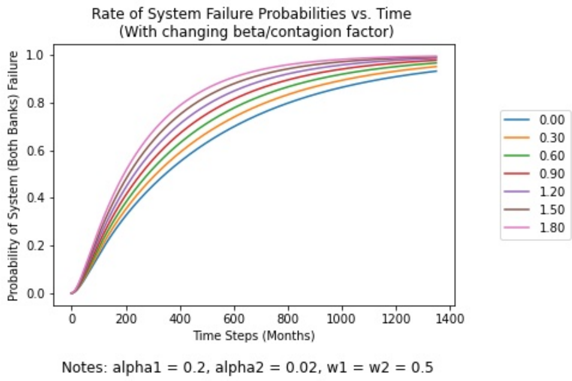



Figure 7: Rate of Both Banks Failing vs. Bank Contagion Factor

### 5.2 $\beta$ Sensitivities

As expected, with a higher contagion factor the bank system fails faster, however, the difference is perhaps not as dramatic as the previous slide. The effect of beta would be stronger if we put more reserves in the bank that was more likely to fail first (creating an unbalanced reserve environment), and making one bank far less likely to fail than another.

### 5.3 $w_1$ $(w_2 = 1 - w_1)$ Sensitivities

Bank 1 is more likely to fail than bank 2 is in this case with alpha 1 and 2 being 0.2 and 0.02 respectively. As expected, we can see that when bank 1 holds a smaller proportion of the reserves, this puts less strain on bank 2 (the probability of bank 2 failing given bank 1 has failed increases by (w1+w2)/w2 so we would prefer a large w2 and small w1).

## 6 Summary and Limitations

One limitation of our model is the absence of observable data, particularly for $\alpha_1, \alpha_2$. Since banks are inherently intertwined, it is impossible to exactly quantify an individual bank's probability of failure regardless of external conditions. A future application of this model would be finding ways to approximate real values of these parameters.

Another limitation of our model is that we only model a 2 bank ecosystem. Although we could generalize this model and include parameters such that $\alpha_1 ... \alpha_n$ and so on to make our model more realistic, we decide that a 2 bank state allows us to dive deeper into the parameter dynamics and align with the scope of this paper.

Finally, one last limitation of our model is that we decide to not model the moral hazard that arises given federal intervention. In the long run, there are dangerous consequences if the federal government continues to backstop all banks. Concretely, consumers take on no risk in putting their money into a bank and theoretically the interest rate offered should be 0. Additionally, because banks are no longer liable, they will invest deposits in riskier and riskier assets because even if those assets fail, the federal government will bail them out. As such, future work should model bank failure probability increasing after each bailout.

# 7 Attribution of Work

## 7.1 Elisa Gonzalez

Paper Writing and Editing, Research on fed intervention, Initial Code for Markov chain, and Homogeneous and Federal Sections

## 7.2 Michael Chen

Paper Writing and Editing, Research and formulation of extensions to Professor Parnes' Markov Model, Heterogeneous Section and Code, Limitations and Further Work

## 7.3 Leah Margulies

Paper Writing and Editing, Research and Explanation of the infinitesimal generator Q, sensitivity analysis to parameters alpha1, beta, and w1

## 7.4 Ty Geri

Paper Writing and Editing, background research and formulation of extension to Prof. Parnes model, model section

# References

[1] Parnes, Dror. "Modeling the Contagion of Bank Runs with a Markov Model." The Quarterly Review of Economics and Finance, Elsevier BV, Aug. 2021, pp. 174–87. Crossref, doi:10.1016/j.qref.2021.05.009.

[2] "Capital Planning and Stress Capital Buffer Requirement." Code of Federal Regulations, United States Federal Reserve, 29 2023, https://www.ecfr.gov/current/title-12/chapter-II/subchapter-A/part-238/subpart-S/section-238.170.

# 8 Appendix

## 8.1 Assumptions and Justification

Note that we chose to use a discrete time transition matrix. Rather than directly incorporate the time parameter into our matrix, the time parameter is incorporated into the power that the transition matrix is raised to, and reflects our simulation over time of multiple bank cycles. By choosing a small time step, we attempt to mimic a continuous process.

We define absorption to mean the time to reach state 4 where both banks have failed and the banking system has collapsed.