Parallel MCMC Recombination for Big Data Analysis

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Introduction

- Bayesian inference is usually intractable
- Popular inference algorithm–Markov chain Monte Carlo
- “Big data” motivated need for parallel algorithms

Bayesian Inference

- In Bayesian modeling we have a prior: $P(\theta)$
- And we have a likelihood:

$$P(X|\theta) = \prod_{i=1}^{N} P(X_i|\theta)$$

- Our posterior is:

$$P(\theta|X) = \frac{P(X|\theta)P(\theta)}{\int P(X|\theta)P(\theta)\,\mathrm{d}\theta}$$

The Metropolis-Hastings Algorithm

- Classic inference algorithm is Metropolis-Hastings (MH)
- MH algorithm is:
  1. Draw $\theta' \sim Q(\theta)$
  2. Let
$$r = \min\left\{1, \frac{P(X|\theta')P(\theta')Q(\theta^{(t)})}{P(X|\theta^{(t)})P(\theta^{(t)})Q(\theta')}\right\}$$
  3. Set $\theta^{(t+1)} := \theta'$ with probability r or $\theta^{(t+1)} := \theta^{(t)}$ with probability $1 - r$
- Exhibits detail balance

Problem Statement

- We cannot trivially parallelize MCMC
- The likelihood function, $\prod_{i=1}^{N} P(X_i|\theta)$, could be difficult to compute
- Our idea: parallelizable MCMC without excessive likelihood evaluation.

Divide-and-Conquer Inference

- Common parallel technique: Divide and conquer
  1. Divide the data randomly across P processors
  2. Perform MCMC independently
  3. Combine subposteriors
- This is *embarrassingly parallel*.

Divide-and-Conquer Inference

- Popular combination methods: Consensus Monte Carlo, KDE, geometric median, etc.
- Different variants of combining subposteriors
- Ours avoids excessive likelihood calculation

Local Polynomial Regression

- Suppose we want to approximate the log likelihood

$$\sum_{i=1}^{N} \log P(X_i|\theta) \approx \hat{\ell}(\theta)$$

- We fit local polynomial regression model to subposterior samples.
- Complexity is $O(MD^3)$

Parallel Sampler

- Our sampler works as follows:
  1. Sample subposterior $P(\theta|X^{(p)})$ for $p = 1, \ldots, P$ processors
  2. Pool subposteriors, calculate log likelihood
  3. Fit local regression model
  4. For $t = 1, \ldots, T$ iterations on master processor, accept new state with probability

$$r = \min \left\{ 1, \frac{\hat{\ell}(\theta')P(\theta')Q(\theta^{(t)})}{\hat{\ell}(\theta^{(t)})P(\theta^{(t)})Q(\theta')} \right\}$$

Proposal Distribution

- Choice of proposal distribution $Q(\theta)$ is important
- Our recommendation: Dirichlet process mixture of subposterior samples $G \sim \mathsf{DP}(\alpha, H) = \sum_{k=1}^{\infty} \pi_k \delta_{\phi_k}$:

$$\theta_m | z_m, \phi_{z_m} \sim P(\theta_m | \phi_{z_m}),\ z_m | \pi \sim \mathsf{Multinomial}(\pi),$$
$$\pi | \alpha \sim \mathsf{GEM}(\alpha), \phi_k \sim H.$$

  for $m = 1, \ldots, M$ subposterior samples
- Idea: propose samples with good estimates of log likelihood.

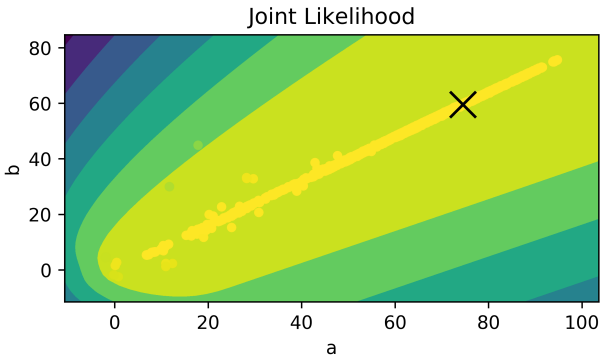
Numerical Tests: Logistic Regression

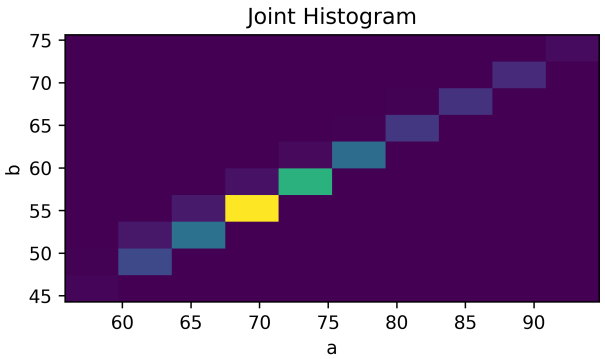
- Assume data generated from model

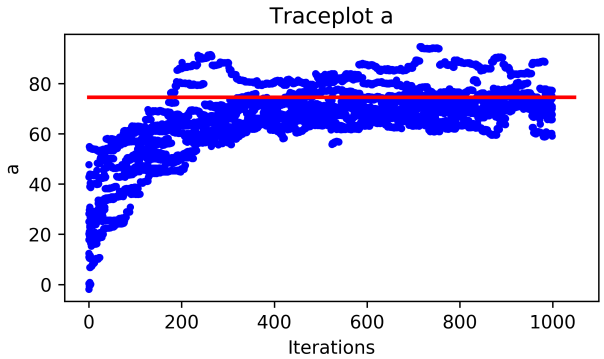
$$y_i \sim \text{Binomial}\left(z_i, \frac{1}{1+\exp\{a+bx_i\}}\right), \quad (a,b) \sim N(0, \tau^2 I)$$

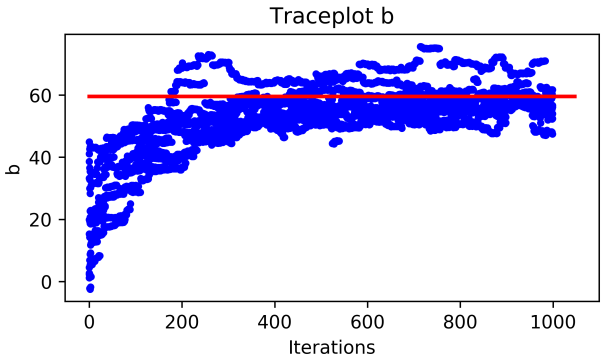
- N = 10,000 and P = 5

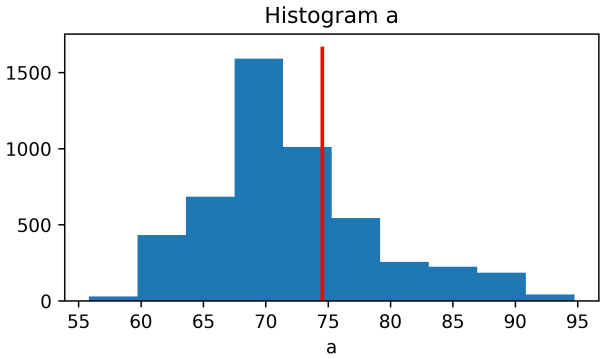
Numerical Tests: Logistic Regression, Subset Posterior

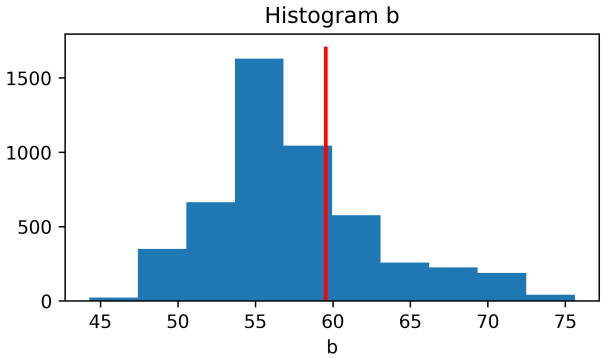




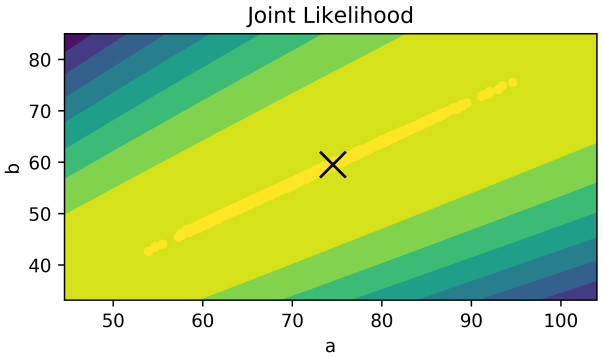


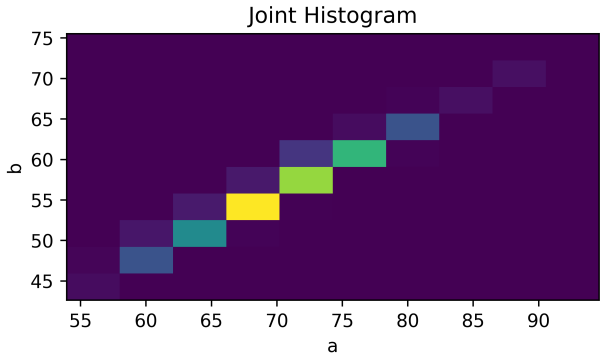


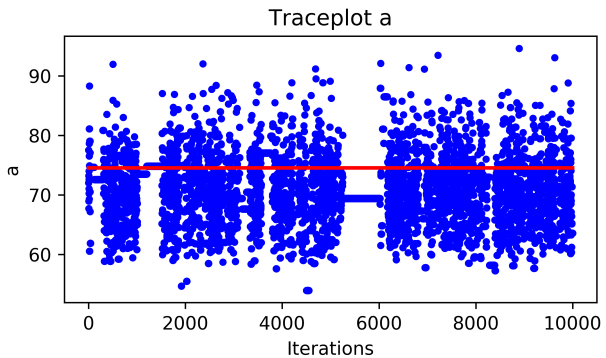


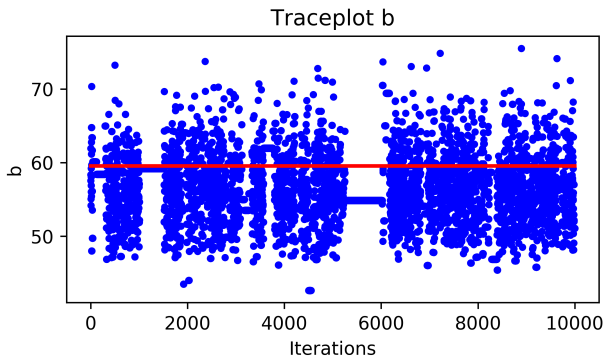


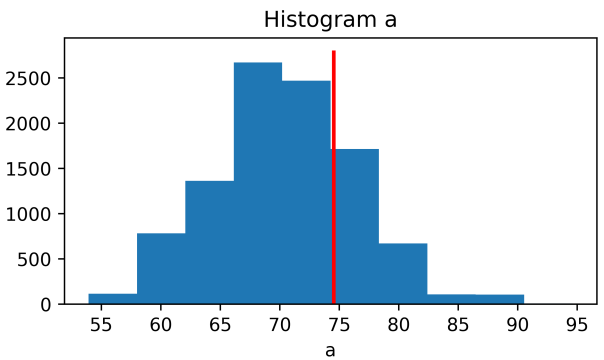
Numerical Tests: Logistic Regression, Final Posterior

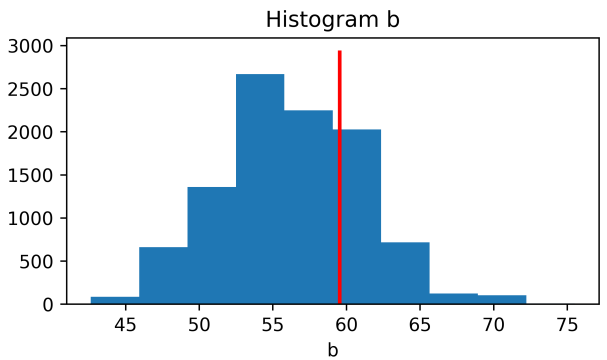




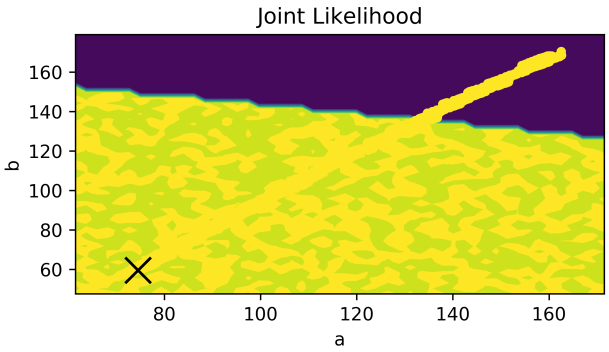


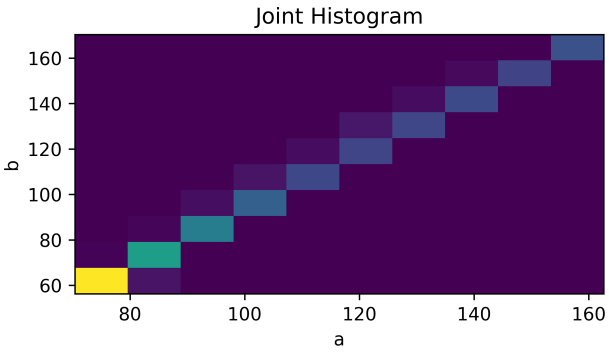


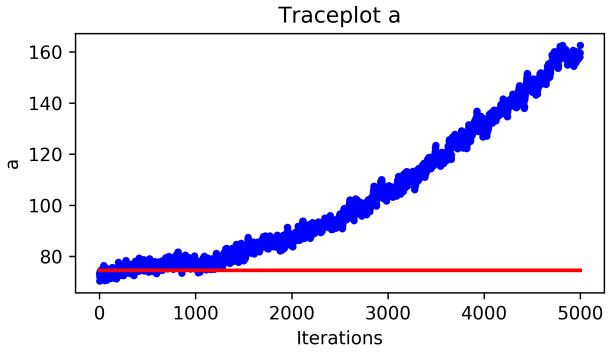


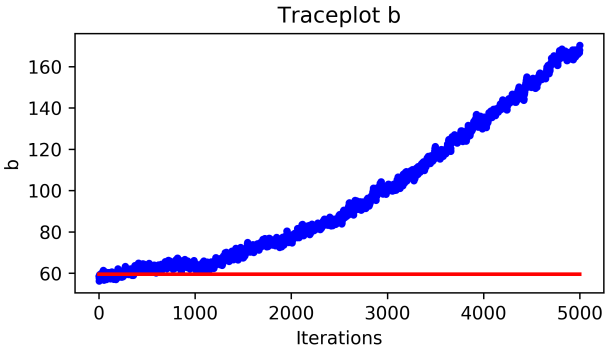


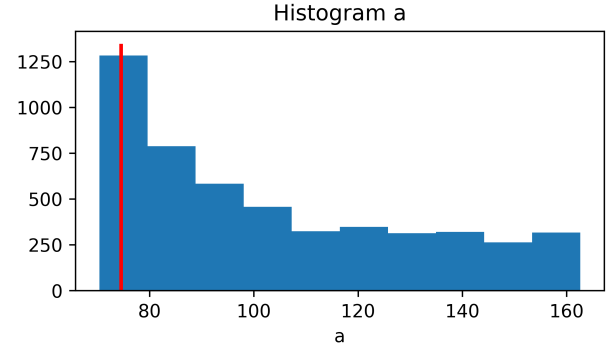
Numerical Tests: Logistic Regression, Proposal Distribution

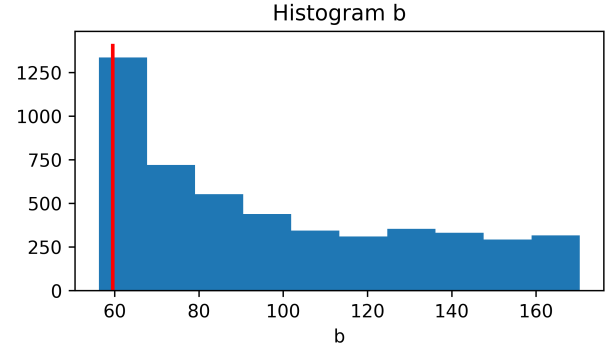




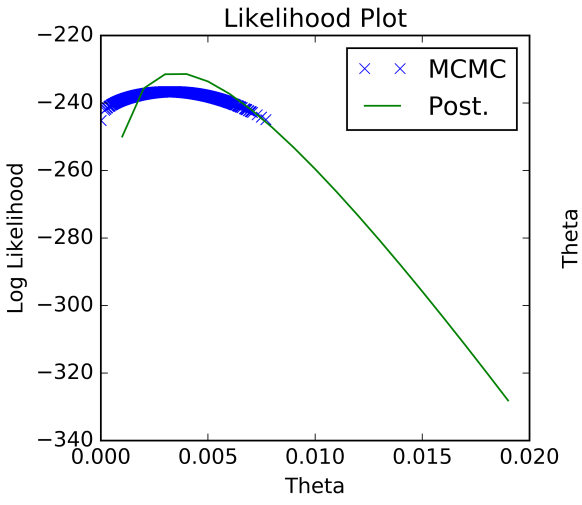


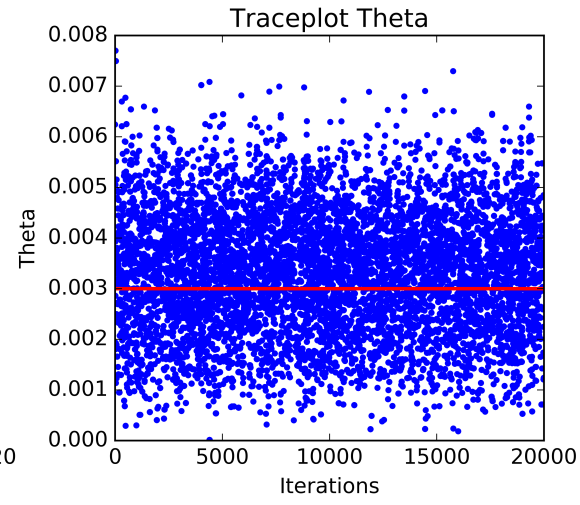


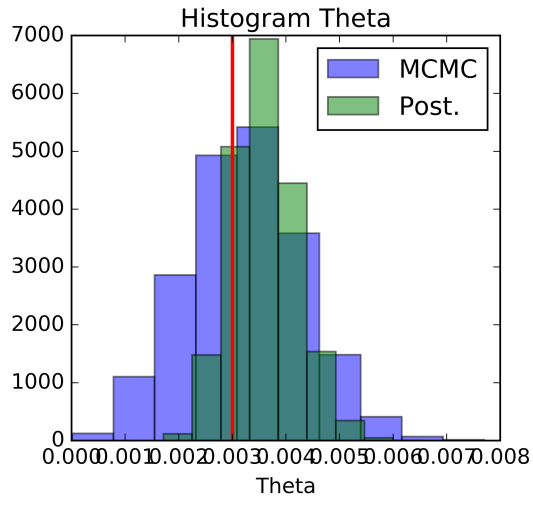




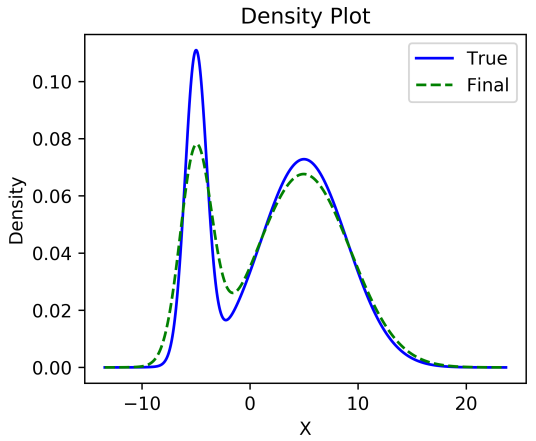
Numerical Tests: Rare Bernoulli

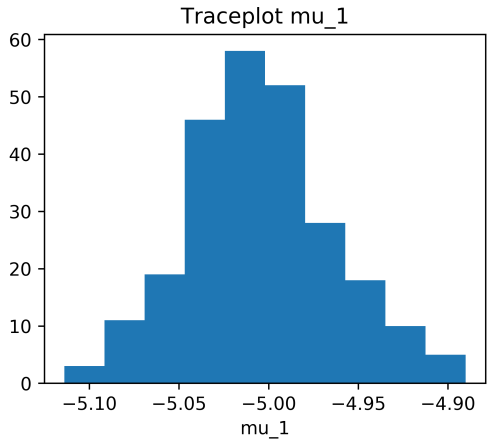


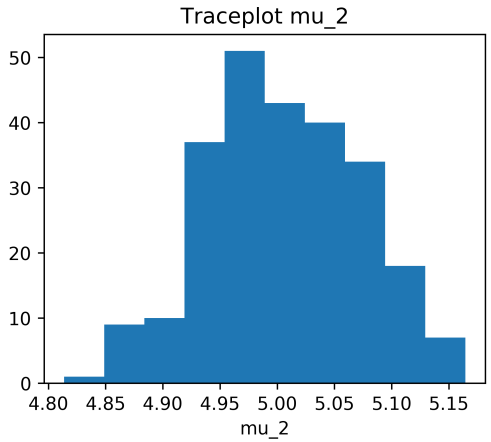


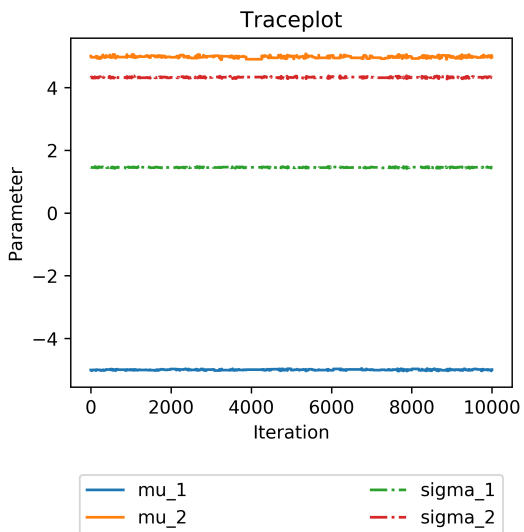


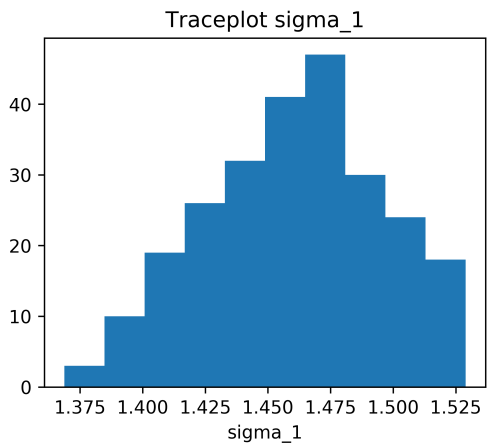
Numerical Tests: Bimodal Distribution

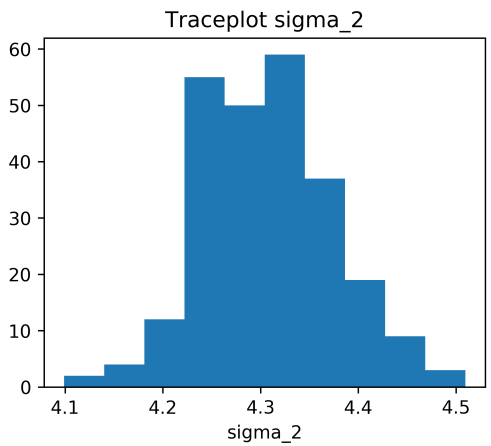






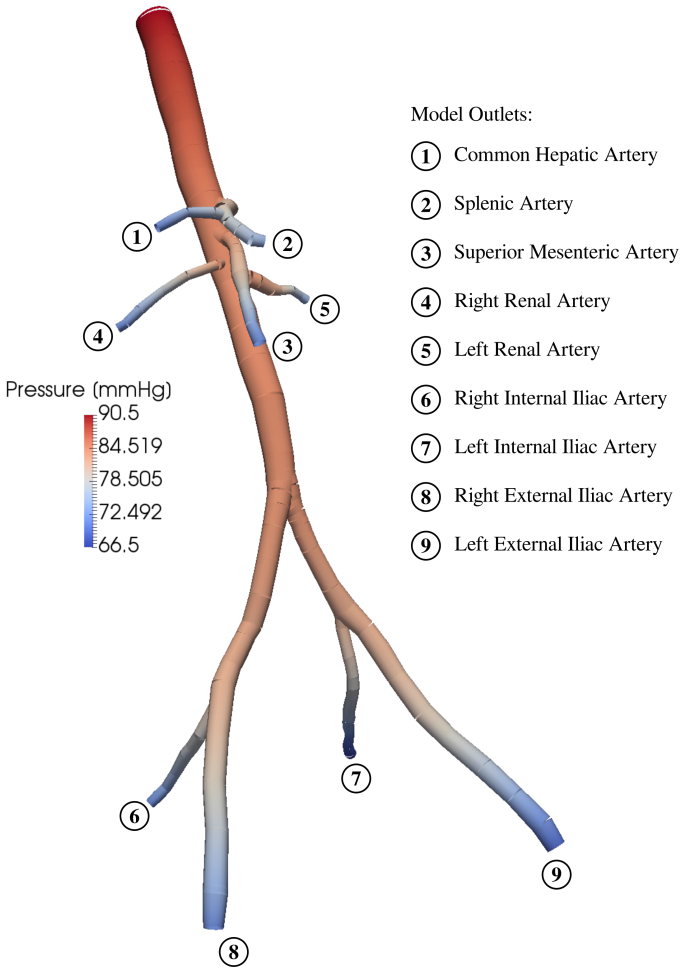






Future Work: Heart Blood Flow Model

- Interest in modeling blood flow dynamics in heart
- Parameter size is small but likelihood is expensive



Conclusion

- Parallel sampler idea: for expensive likelihoods with small parameter sizes
- Generally we need parallel samplers for “big data”
- Also want to quantify uncertainty of divide-and-conquer methods
- To be completed: Theorems and blood flow model