$$\langle A( heta,\phi)
angle = \int_0^{2\pi} \int_0^\pi A( heta,\phi) \, f( heta,\phi) \, \sin heta \, d heta \, d\phi$$

$$\langle A( heta) 
angle = \int_0^{2\pi} d\phi \int_0^\pi A( heta) \, f( heta) \, \sin heta \, d heta$$

$$=\left(\int_0^{2\pi}d\phi
ight)\cdot\left(\int_0^\pi A( heta)\,f( heta)\,\sin heta\,d heta
ight)=2\pi\cdot\int_0^\pi A( heta)\,f( heta)\,\sin heta\,d heta$$

probability distribution

$$\int_0^\pi f( heta) \, \sin heta \, d heta = 1$$

to compute averages

$$\langle A( heta)
angle = \int_0^\pi A( heta)\,f( heta)\,\sin heta\,d heta$$

c) in an isotropic liquid all orientations are equally likely

$$f(\theta) = \text{constant}$$

$$\int_0^{\pi} C \cdot \sin \theta \, d\theta = 1 \Rightarrow C = \frac{1}{2}$$

$$\langle A(\theta) \rangle = \int_0^{\pi} A(\theta) \cdot \frac{1}{2} \cdot \sin \theta \, d\theta$$

$$S = \left\langle \frac{3\cos^2\theta - 1}{2} \right\rangle = \int_0^\pi \frac{3\cos^2\theta - 1}{2} \cdot \frac{1}{2} \cdot \sin\theta \, d\theta$$

$$S = \int_0^\pi \left( rac{3\cos^2 heta - 1}{2} 
ight) \cdot rac{1}{2} \cdot \sin heta \, d heta$$

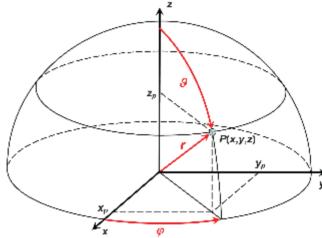
$$S = rac{1}{4} \int_0^\pi (3\cos^2 \theta - 1) \sin \theta \, d\theta$$

$$u = \cos \theta \quad \Rightarrow \quad du = -\sin \theta \, d\theta$$

$$S=rac{1}{4}\int_{1}^{-1}(3u^{2}-1)(-du)$$

$$S = rac{1}{4} \int_{-1}^{1} (3u^2 - 1) \, du$$

$$\int_{-1}^{1} (3u^2 - 1) du = 3 \int_{-1}^{1} u^2 du - \int_{-1}^{1} 1 du$$
$$= 3 \cdot \frac{2}{3} - 2 = 2 - 2 = 0$$
$$S = \frac{1}{4} \cdot 0 = 0$$



a) all molecules point perfectly along director, so:

$$\theta = 0$$

no distribution needed

$$S = \frac{3\cos^2(0) - 1}{2} = \frac{3 \cdot 1 - 1}{2} = 1$$

b) perfect alignment perpendicular to the director

$$S = \frac{3\cos^2(\frac{\pi}{2}) - 1}{2} = \frac{3 \cdot 0 - 1}{2} = -\frac{1}{2}$$