$$\langle A(\theta,\phi)\rangle = \int_0^{2\pi}\int_0^{\pi} A(\theta,\phi)\, f(\theta,\phi)\, \sin\theta\, d\theta\, d\phi$$

$$\langle A(\theta)\rangle = \int_0^{2\pi} d\phi \int_0^{\pi} A(\theta)\, f(\theta)\, \sin\theta\, d\theta$$

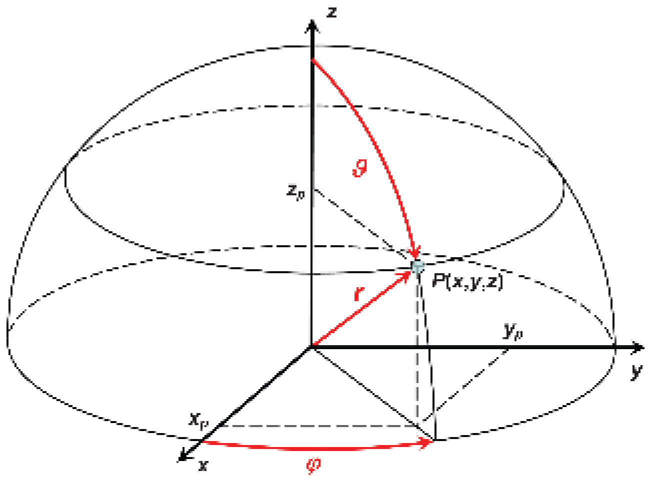
$$= \left(\int_0^{2\pi} d\phi\right)\cdot\left(\int_0^{\pi} A(\theta)\, f(\theta)\, \sin\theta\, d\theta\right) = 2\pi \cdot \int_0^{\pi} A(\theta)\, f(\theta)\, \sin\theta\, d\theta$$

probability distribution

$$\int_0^{\pi} f(\theta)\, \sin\theta\, d\theta = 1$$

to compute averages

$$\langle A(\theta)\rangle = \int_0^{\pi} A(\theta)\, f(\theta)\, \sin\theta\, d\theta$$



a) all molecules point perfectly along director, so:

$$\theta = 0$$

no distribution needed

$$S = \frac{3\cos^2(0) - 1}{2} = \frac{3\cdot 1 - 1}{2} = 1$$

b) perfect alignment perpendicular to the director

$$S = \frac{3\cos^2(\frac{\pi}{2}) - 1}{2} = \frac{3\cdot 0 - 1}{2} = -\frac{1}{2}$$

c) in an isotropic liquid all orientations are equally likely

$$f(\theta) = \text{constant}$$

$$\int_0^{\pi} C\cdot \sin\theta\, d\theta = 1 \Rightarrow C = \frac{1}{2}$$

$$\langle A(\theta)\rangle = \int_0^{\pi} A(\theta)\cdot\frac{1}{2}\cdot\sin\theta\, d\theta$$

$$S = \left\langle \frac{3\cos^2\theta - 1}{2}\right\rangle = \int_0^{\pi} \frac{3\cos^2\theta - 1}{2}\cdot\frac{1}{2}\cdot\sin\theta\, d\theta$$

$$S = \int_0^{\pi}\left(\frac{3\cos^2\theta - 1}{2}\right)\cdot\frac{1}{2}\cdot\sin\theta\, d\theta$$

$$S = \frac{1}{4}\int_0^{\pi}(3\cos^2\theta - 1)\sin\theta\, d\theta$$

$$u = \cos\theta \quad \Rightarrow \quad du = -\sin\theta\, d\theta$$

$$S = \frac{1}{4}\int_1^{-1}(3u^2 - 1)(-du)$$

$$S = \frac{1}{4}\int_{-1}^{1}(3u^2 - 1)\, du$$

$$\int_{-1}^{1}(3u^2 - 1)\, du = 3\int_{-1}^{1} u^2\, du - \int_{-1}^{1} 1\, du$$

$$= 3\cdot\frac{2}{3} - 2 = 2 - 2 = 0$$

$$S = \frac{1}{4}\cdot 0 = 0$$