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Advanced Programming

State Monad

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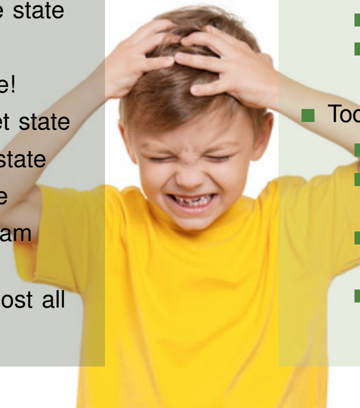
SOFTWARE QUALITY RESEARCH

Does FP Eliminate State?

Picture: you after three weeks of ADPRO

- **Variables** and fields are state
- Program **stack&heap** are state
- **Databases** are state
- **Program counter** is state!
- Variable **assignments** set state
- Variable **accesses** read state
- **Loops** must change state
- **Exceptions** modify program counter and stack
- We have **disallowed** almost all of these?



- In FP we often make **state explicit**:
  - Converted loops to **recursive functions**
  - A function is a **state transform**
  - **Arguments** are the state explicitly
  - **No other implicit**, hidden state that can be **changed** by others
- Today, a pure **pattern** for state transforms:
  - A more **recipe-like** way to encode state
  - Allows to hide the state, make it **implicit** like in imperative programming
  - Still the state is **encapsulated** in a well defined value
  - Still **no other** encapsulated state that can be **changed** by others

- `RNG`: a random **integer generator**; we maintain the state
- `Rand[A]`: a random **A generator**; we hide the RNG in the state
- `State[S,A]`: a general pattern for **stateful computations** producing A, where the state is of type S

AGENDA

A Typical Stateful Imperative API

```scala
var rng = new scala.Util.Random

// returns a random number form 0 to 5
def rollDie: Int = rng.nextInt(6)
```

- We call `rollDie` and observe a value `5`
- Mentimeter: What is the result of `rollDie + rollDie`?
- What does it tell us about referential transparency of `rollDie`?
- `rng` is an external, implicit state, that can be changed by others
- To make `rollDie` referentially transparent, make the state explicit

Converting RNG to explicit state

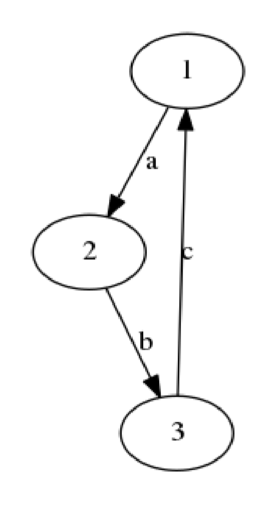
- We had: `RNG.nextInt: () =>Int`
- Lets **return new state explicitly**, instead of modifying old (RT)

```
trait RNG { def nextInt: (Int, RNG) }
object RNG{
  def nextInt (rng: RNG) : (Int, RNG) = rng.nextInt
}
```

- **In general** a function: `State =>(Output,State)`
- **Wrap** this as `case class State[S,+A] (run: S =>(A,S) )`
- So `RNG` becomes `State[RNG,Int] { run =RNG.nextInt }`
- **Intuition 1**: `Automaton` or `Transition` would be better names than `State`
- **Intuition 2**: `step` would be a better name than `run`

Consider a Simple Automaton

Stateful **by definition**




```
var state = 1
while (true)
  state match {
    case 1 => { print "a"; state = 2 }
    case 2 => { print "b"; state = 3 }
    case 3 => { print "c"; state = 1 }
  }
```

```
def step (State: Int): (String,Int) = state match {
 case 1 => ( "a", 2 )
 case 2 => ( "b", 3 )
 case 3 => ( "c", 1 )
}
```

- We need a simple **recursive loop** to run the step like above
- **One general general loop** for the `State` type
- This automaton as an **instance of** `State: State[Int,String] (step)`

Exercise

Use 5 minutes to write down an instance of `State` implementing this imperative code

```
x=0
while (true) {
  println (x)
  x+=1
}
```

States vs Streams

- We can **unroll** (unfold) a state machine from an initial state, producing a stream of actions.
- **Discuss:** What is the stream from our first automaton?
- **Discuss:** What is the stream from our second automaton?
- **Mentimeter:** another stream
- An **exercise** implementing this mapping as a function
- **Laziness** of streams is useful here, why?

Anything stateful maps to the state pattern

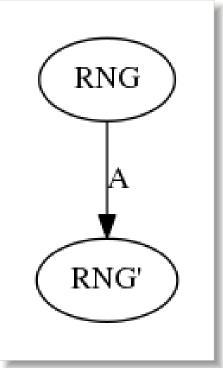
Recap

- Random number generators (state: RNG seed)
- Websites with modality (session state)
- Database backed applications (DB state)
- Communication protocols (protocol state)
- etc.

Random Number Generator as an Instance of State

```
type Rand[A] =State[RNG,A]
```

- RNG is the state of the random generator (usually some large number encapsulated)
- The textbook gives a simple implementation of RNG based on multiplication with large primes module 64 bits
- `Rand[A]` is a **computation** that we can run, then it will produce a random `A` and a new state `RNG`
- Another useful intuition: `Rand[A]` is **a generator of random `A`'s**
- Or even just a **"random `A`"**
- **Question:** What stream can we unfold from `State[RNG,Int] (_.nextInt)`?



How do I use this generator of random numbers?

```
type Rand[A] =State[RNG,A]
val r : Rand[Int] =SimpleRNG (42)
val (r1,i) =r.run
```

- `SimpleRNG` is the book's concrete implementation of the `RNG` trait
- `42` is the initial seed (state)
- `(r1,i)` is a new state and a random number
- **Question:** How do I get the next random number?
- **Question:** What happens if I call `r.run` again?

What can we do with Automata/State?

State is a monad, similar key operations as for `List`, `Option`, and `Stream`

```
def map[S,A,B] (s: State[S,A]) (f: A =>B): State[S,B]
```

Can use this to generate even numbers:

```
val even: Rand[Int] =map[Int] (r) (n =>n * 2)
```

Automata can be composed [1/2]

`flatMap` can be used to compose generators:

```
def flatMap[S,A,B] (s: State[S,A]) (f : A =>State[S,B]): State[S,B]
```

In our context of generators:

```
def flatMap[A,B] (r :Rand[A]) (f =>Rand[B]) : Rand[B] =
  flatMap[A,B] (r: State[RNG,A]) (f :State[RNG,B]) :State[RNG,B]
```

`flatMap` can compose generators (compute a random size list of random even integers):

```
val int :Rand[Int] =... (assume you have it)
val ints: Int =>Rand[List[Int]] =... (assume creates a random list of given length)
val ns :Rand[List[Int]] =int.flatMap( x =>ints(x).map (xs =>xs.map (_*2))
```

the state RNG passed **implicitly**; size generated with different state than each number

Automata can be composed [2/2]

The `map2` function can compute a zipping of two automata over the same state space for us:

```
map2 [S,A,B,C] (sa: State[S,A]) (sb: State[S,B]) (f: (A,B)=>C) :State[S,C]
```

Could be used to create a product automaton
– interleaving computations, then `C` is `Either[A,B]`
– synchronizing two computations, then `C` is `(A,B)`

More fun in exercises :)

Next week

- Next week we will design a parallel computation library, in purely functional style
- This shows (a bit) how Akka is implemented
- In two weeks, we will use the generators of random numbers to implement a modern testing framework
- So: keep reading the chapters and solve the exercises!