# Simple Technical Trading Rules and the Stochastic Properties of Stock Returns

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## ABSTRACT

This paper tests two of the simplest and most popular trading rules - moving average and trading range break, by utilitizing a very long data series, the Dow Jones index from 1897 to 1986. Standard statistical analysis is extended through the use of bootstrap techniques. Overall our results provide strong support for the technical strategies that are explored. The returns obtained from buy (sell) signals are not consistent with the three popular null models: the random walk, the AR(1) and the GARCH-M. Consistently, buy signals generate higher returns than sell signals. Moreover, returns following sell signals are negative which is not easily explained by any of the currently existing equilibrium models. Furthermore the returns following buy signals are less volatile than returns following sell signals.

The term, "technical analysis," is a general heading for a myriad of trading techniques. Technical analysts attempt to forecast prices by the study of past prices and a few other related summary statistics about security trading. They believe that shifts in supply and demand can be detected in charts of market action. Technical analysis is considered by many to be the original form of investment analysis, dating back to the 1800's. It came into widespread use before the period of extensive and fully disclosed financial information, which in turn enabled the practice of fundamental analysis to develop. In the U.S., the use of trading rules to detect patterns in stock prices is probably as old as the stock market itself. The oldest technique is attributed to Charles Dow and is traced to the late 1800's. Many of the techniques used today have been utilized for over 60 years. These techniques for discovering hidden relations in stock returns can range from extremely simple to quite elaborate.

The attitude of academics towards technical analysis, until recently, is well described by Malkiel(1981):

> "Obviously, I am biased against the chartist. This is not only a personal predilection, but a professional one as well. Technical analysis is anathema to the academic world. We love to pick on it. Our bullying tactics are prompted by two considerations: (1) the method is patently false; and (2) it's easy to pick on. And while it may seem a bit unfair to pick on such a sorry target, just remember: it is your money we are trying to save."

Nonetheless, technical analysis has been enjoying a renaissance on Wall Street. All major brokerage firms publish technical commentary on the market and individual securities, and many of the newsletters published by various "experts" are based on technical analysis.

In recent years the efficient market hypothesis has come under serious siege. Various papers suggested that stock returns are not fully explained by common risk measures. A significant relationship between expected return and fundamental variables such as price-earnings ratio, market-to-book ratio and size was documented. Another group of papers has uncovered systematic patterns in stock returns related to various calendar periods such as the weekend effect, the turn-of-the-month effect, the holiday effect and the January effect. A line of research directly related to this work provides evidence of predictability of equity returns from past returns. De Bondt and Thaler(1985), Fama and French(1986), and Poterba and Summers(1988) find negative serial correlation in returns of individual stocks and various portfolios over three to ten year intervals. Rosenberg, Reid, and Lanstein(1985) provide evidence for the presence of predictable return reversals on a monthly basis at the level of individual securities. Jegadeesh(1990) finds negative serial correlation for lags up to two months and positive serial correlation for longer lags. Lo and MacKinlay(1990a) report positive serial correlation in weekly returns for indexes and portfolios and a somewhat negative serial correlation for individual stocks. Lehmann(1990) and French and Roll(1986) report negative serial correlation at the level of individual securities for weekly and daily returns. Cutler, Poterba and Summers(1990) present results from many different asset markets generally supporting the hypothesis that returns are positively correlated at the horizon of several months and negatively correlated at the 3-5 year horizon.

Two competing explanations for the presence of predictable variation in stock returns have been suggested: (1) market inefficiency in which prices take swings from their fundamental values; and (2) markets are efficient and the predictable variation can be explained by time-varying equilibrium returns. In general, the results of these studies are in sharp contrast with most earlier studies that supported the random walk hypothesis and concluded that the predictable variation in equity returns was economically and statistically very small.

Although many earlier studies concluded that technical analysis is useless, the recent studies on predictability of equity returns from past returns suggest that this conclusion might have been premature. In this paper we explore two of the simplest and most popular technical rules: moving average-oscillator and trading range break (resistance and support levels). In the first method, buy and sell signals are generated by two moving averages, a long period, and a short period. In the second method signals are generated as stock prices hit new highs and lows. These rules will be evaluated by their ability to forecast future price changes. For statistical inferences, standard tests will be augmented with the bootstrap methodology inspired by Efron(1979), Freedman and Peters(1984a,b), and Efron and Tibshirani(1986). Following this methodology returns from an artificial Dow series are generated, and the trading rules are applied to the series. Comparisons are then made between returns from these simulated series and the actual Dow Jones series.

Another contribution of this paper is the introduction of technical analysis as a way of evaluating the adequacy of time series models fitted to stock return data. This use of technical analysis as a model evaluation tool is a new and useful idea. When the null class of models is rejected by such a statistical test, information is provided on how to modify the model to achieve a better description of the series. In addition, the trading rules used in this paper may have power against certain alternatives which are difficult to detect using standard statistical tests.

Few, if any, empirical tests in financial economics are free of the data instigated pre-test biases discussed in Leamer(1978).[1] The more scrutiny a collection of data receives, the more likely "interesting" spurious patterns will be observed. Stock prices are probably the most studied financial series and therefore, most susceptible to data-snooping. In addition, Merton(1987) suggests that there is a tendency to produce "exciting" results (anomalies):

> "All this fits well with what the cognitive psychologists tell us is our natural individual predilection to focus, often disproportionately so, on the unusual... This focus, both individually and institutionally, together with little control over the number of tests performed, creates a fertile environment for both unintended selection bias and for attaching greater significance to otherwise unbiased estimates than is justified."

Therefore, the possibility that various spurious patterns were uncovered by technical analysts cannot be dismissed. Although a complete remedy for data-snooping biases does not exist, we mitigate this problem: (1) by reporting results from all our trading strategies, (2) by utilizing a very long data series, the Dow Jones index from 1897 to 1986, and (3) attaching importance to the robustness of results across various non-overlapping subperiods for statistical inference.

Our study reveals that technical analysis helps to predict stock price changes. The patterns uncovered by technical rules cannot be explained by first order autocorrelation and by changing expected returns caused by risk changes. To put it differently, the trading profits are not consistent with a random walk, an AR(1), or a GARCH-M model. The results generally show that returns during buy periods are larger and less volatile than returns during sell periods. For example, the variable length moving average produced on average a daily return for buy periods of 0.042 percent which is about 12 percent at an annual rate. The corresponding daily return for the sell periods is -0.025 percent which is about -7 percent at an annual rate.

The remainder of the paper is organized as follows: section I reviews some previous research on technical trading rules; section II describes our trading rules and bootstrap methodologies; section III presents the empirical results of the tests utilizing traditional techniques; section IV presents further results using bootstrap methods, and section V concludes and summarizes our results.

## I. Literature Review of Technical Trading Rules

There is very little recent work on technical trading rules published in the mainstream academic journals. Most of the work in this area predates the 1970's. Standard statistical methods were used, and the results were generally consistent with the random walk model for stock prices. Cootner(1962) and Fama(1965) computed serial correlation coefficients for successive daily, weekly, and monthly price changes and found that the correlations are very close to zero. Fama(1965) analyzed runs of successive price changes of the same sign and also found support for the random walk hypothesis. Finally, Granger and Morgenstern(1963) introduced a more rigorous method, spectral analysis, and confirmed the results that returns are unpredictable.

These tests were criticized as not being powerful enough to reject the random walk hypothesis because dependencies might be more complicated, i.e. non-linear, and hence, to a large extent, undetected by the previous tests. Alexander(1961,1964) was one of the first to test a mechanical trading rule - the filter technique. He used filters ranging from 5 percent to 50 percent on the Dow Jones and Standard and Poors indices and concluded that before commissions such a trading rule would outperform a buy-and-hold strategy. Fama and Blume(1966) challenged Alexander's results by examining the profitability of the filter technique at the level of individual stocks. They found that, in general, across all the filters, even before commissions, the trading strategies did not outperform a buy-and-hold policy. However, dependencies were observed for some of the filters. In a recent study, Sweeney(1988) shows that a subset of these firms produced statistically significant profits, after transactions costs, for floor traders during the period, 1970-1982.

The moving average method is one of the most popular tools and the 200-day moving average is depicted in numerous publications for various financial series. Cootner(1962) was the first to test such a rule for individual stocks. Although he admitted that his results suffer from lack of good statistical tests, the moving average strategy substantially outperformed the buy-and-hold strategy. He concluded:

> "Even more interesting, perhaps, is that my model is perfectly compatible with much of what I interpret Wall Street Chart reading is all about. Like the Indian folk doctors who discovered tranquilizers, the Wall Street witch doctors, without the benefit of scientific method, have produced something with their magic, even if they can't tell you what it is or how it works."

Van Horne and Parker(1967) and James(1968) examined various moving average rules for individual stocks and concluded that these trading rules do not outperform a buy-and-hold strategy. Goldberg and Schulmeister(1988) combine moving averages with filters and other rules in a comprehensive study. They conclude that while transaction costs eliminate profits in the cash market, this is not the case for the futures markets.

Relative Strength, another popular technical tool, deals exclusively with individual stocks. According to this strategy, stocks are ranked based on their relative performance, typically during a period of 3 to 6 months. The idea is that there is "momentum", or positive auto-correlation, and the most attractive stocks are stocks that had the best performance in the prior period. This trading strategy was tested by Levy(1967a,b), who concluded, "stock prices follow discernable patterns which have predictive significance, and the theory of random walk has been refuted." Jensen(1967) pointed out that Levy's procedures overstated the performance of his trading strategy. In a subsequent paper, Jensen and Bennington(1970), re-examined the "Relative Strength" rule on a more comprehensive data base, using an improved methodology and concluded that the trading rule outperformed buy-and-hold only before accounting for transaction costs. In a more recent paper, Bohan(1981) implements the "Relative Strength" analysis at the industry level and finds strong support for the method.

In summary, most of the studies in this area suffer from a number of problems: (1) relatively short time periods are examined, for example Van Horne and Parker(1967) study the period 1960 to 1966, (2) the studies, in general, do not compute significance levels (the ones that do utilize questionable methods) and (3) the studies suffer from data-snooping biases. As pointed out by Jensen and Bennington(1970), the rules in many studies were both designed and tested on the same data. Moreover, numerous variations of the trading rules were tested, few of which were actually presented.

There are two recent theoretical papers that deal with the usefulness of technical analysis whose focus is not on the design of profitable trading strategies. Treynor and Ferguson(1985) assume that an investor receives what he believes to be non-public information about a security and must decide how to act. Using past prices, the trader will be able to assess whether he received the information before the market, or whether the information has already been incorporated in the stock price. In this setting, non-price information creates the opportunities, and past prices (technical analysis) permit efficient exploitation of such information. Brown and Jennings(1989) show that individuals employ technical analysis (use past prices), although, the current price is set competitively by rational investors incorporating all public information. Past prices are useful because current equilibrium spot prices do not fully reveal the private information. More accurate inferences can be made when historical and spot prices are utilized. Hence, technical analysis is useful to investors in forming their demands.

## II. Methodology

### A. Data

The data series used in this study is the Dow Jones Industrial Average (DJIA) from the first trading day in 1897 to the last trading day in 1986 - - 90 years of daily data. The DJIA is available on a daily basis back to September 8, 1886. Before this date, Charles H. Dow, editor of the Wall Street Journal, had occasionally published stock averages of various kinds, but not on a regular basis. No other index of U.S. stocks has been available for so long a period of time.

The stocks included in the index have changed from time to time. Changes were more frequent in the earlier days. From the beginning, the list included large, well known and actively traded stocks. In recent years the 30 stocks in the index represent about 25 percent of the market value of all NYSE stocks. All the stocks are very actively traded and problems associated with non-synchronous trading should be of little concern with the DJIA.

In addition to the full sample, results are presented for four subsamples: 1/1/97 - 7/30/14, 1/1/15 - 12/31/38, 1/1/39 - 6/30/62, and 7/1/62 - 12/31/86. These subsamples are chosen for several reasons. The first subsample ends with the closure of the stock exchange during World War I. The second subsample includes both the rise of the twenties and the turbulent times of the depression. The third subsample includes the period of World War II and ends in June 1962, which is the date at which the Center for Research in Securities Prices (CRSP) begins its daily price series. The last subsample covers the period that was extensively researched because of data availability.

### B. Technical Trading Rules

Two of the simplest and most widely used technical rules are investigated: moving-average-oscillator and trading range break-out (resistance and support levels). According to the moving average rule, buy and sell signals are generated by two moving averages – a long period average and a short period average. In its simplest form this strategy is expressed as buying (selling) when the short period moving average rises above (falls below) the long period moving average. The idea behind computing moving averages is to smooth out an otherwise volatile series. When the short period moving average penetrates the long period moving average, a trend is considered to be initiated. The most popular moving average rule is 1-200, where the short period is one day and the long period is 200 days. While there are numerous variations of this rule used in practice, we attempted to select several of the most popular ones. We examined the following rules: 1-50, 1-150, 5-150, 1-200, and 2-200. The moving average decision rule is often modified by introducing a band around the moving average. The introduction of a band reduces the number of buy (sell) signals by eliminating "whiplash" signals when the short and long period moving averages are close. We test the moving average rule both with and without a 1 percent band.

Our first rule, called the variable length moving average (VMA), initiates buy (sell) signals when the short moving average is above (below) the long moving average by an amount larger than the band. If the short moving average is inside the band no signal is generated. This method attempts to simulate a strategy where traders go long as the short moving average moves above the long and short when it is below. With a band of zero this method classifies all days into either buys or sells. Other variations of this rule put emphasis on the crossing of the moving averages. They stress that returns should be different for a few days following a crossover. To capture this we test a strategy where a buy (sell) signal is generated when the short moving average cuts the long moving average from below (above). Returns during the next ten days are then recorded. Other signals occurring during this ten day period are ignored. We call this rule a fixed length moving average (FMA).

There are numerous variations of the moving average rule that we do not examine. We focus on the simplest and most popular versions. Other variants of the moving average rule also consider the slope of the long period moving average in addition to whether the short period moving average penetrated from above or below. In other versions changes in trading volume are examined before buy (sell) decisions are reached. Thus, numerous moving average rules can be designed. Some, without a doubt, will work. The dangers of data-snooping are immense. We present results for all the rules that we examined and special emphasis is placed on robustness of the results over time.

Our second technical rule is trading range break-out (TRB). A buy signal is generated when the price penetrates the resistance level. The resistance level is defined as the local maximum. Technical analysts believe that many investors are willing to sell at the peak. This selling pressure will cause resistance to a price rise above the previous peak. However, if the price rises above the previous peak, it has broken through the resistance area. Such a breakout is considered to be a buy signal. Under this rule, a sell signal is generated when the price penetrates the support level which is the local minimum price. The underlying rationale is that the price has difficulties penetrating the support level because many investors are willing to buy at the minimum price. However, if the price goes below the support level, the price is expected to drift downward. In essence, technical analysts recommend buying when the price rises above its last peak and selling when the price sinks below its last trough.

To implement the trading range strategy, we defined rules in accordance with the moving average strategy. Maximum (minimum) prices were determined based on the past 50, 150, and 200 days. In addition, the rule is implemented with and without a one percent band. As with the moving average rule, there exist numerous variations of the trading range strategy implemented in this study.

### C. Empirical Methods

Buy and sell signals are generated by the various technical rules explored in the study. To test whether the returns following a buy or sell signal are "abnormal" we first apply traditional tests for significance which rely on the assumption of independent, identically distributed returns. Secondly we use the bootstrap methodology under which distributions for the statistics of interest are generated by simulating various null models. The bootstrap methodology is described in detail in the Appendix. The techniques used here are inspired by Efron(1982), Freedman(1984), Freedman and Peters(1984a,b), and Efron and Tibshirani(1986). We provide a very informal description here.

The returns conditional on buy (sell) signals using the raw Dow Jones data are compared to conditional returns from simulated comparison series. These simulated comparison series are generated from a null model estimated on the Dow Jones data. Comparison series are generated using computer simulations and then processed by the same technical rules used for the raw Dow Jones data. Each of the simulations is based on 250 repetitions of the null model. This should provide a good approximation of the return's distribution under the null model for each of our trading rules. The null hypothesis is rejected at the $\alpha$ percent level if returns obtained from the actual Dow Jones data are greater than the $\alpha$ percent cutoff of the simulated returns under the null model.

This use of technical analysis as a model evaluation tool is a new and useful idea. When a null class of models is rejected by model specification tests based upon technical trading strategies, we are likely to learn more about the structure and formation of stock market prices than we are by applying statistical methods that were not designed with the financial application in mind. For example, in our study, not only do we reject the GARCH-M model for describing the evolution of stock market returns over time, but our technique also points out why the GARCH-M model fails. Why was this method not invented before? First, the bootstrap methodology is relatively new and it needed cheap computer time before it became practical. Second, as discussed earlier, technical analysis was somewhat discredited in the academic literature. Using technical analysis as a model specification test is a new idea that can be evaluated on its own right independently of the hoary issue of whether technical analysis produces superior profits. The combination of the bootstrap methodology with technical analysis generates a rich arsenal of model evaluation techniques that are especially useful for financial applications.

In this study representative price series are simulated from the following widely used processes for stock prices: a random walk with a drift, autoregressive process of order one (AR(1)) and generalized autoregressive conditional heteroscedasticity in-mean model (GARCH-M). The random walk with a drift series was simulated by taking the returns from the original Dow Jones series and "scrambling" them. The term "scrambling" refers to the formal bootstrap sampling process described in the Appendix. The "scrambling" procedure forms a new time series of returns by randomly drawing from the original series with replacement. This scrambled series will have the same drift in prices, the same volatility, and the same unconditional distribution. However, by construction the returns are independent, indentically distributed.

The second model for the simulation is an AR(1),

$$R_t = b + \rho R_{t-1} + e_t, \quad |\rho| < 1 \tag{1}$$

where $R_t$ is the return on day t and $e_t$ is independent, identically distributed. The parameters $(b, \rho)$ are estimated from the Dow Jones series. To the extent that returns over short periods of time are positively correlated, the technical strategies might produce "abnormal" returns. Conrad and Kaul(1990) report first-order autocorrelation of 0.20 for a value weighted portfolio of the largest companies during the period 1962-1985. They find that higher-order-autocorrelation, beyond a lag of one day, is essentially zero. Our data also reveals the presence of significant autocorrelation in some of the subsamples. Hence, in the presence of positive autocorrelation, the "abnormal" returns from our trading strategy might be a result of an autoregressive process that generates stock returns.

The third model for simulation is a GARCH-M model:

$$R_t = a + \gamma h_t + b e_{t-1} + e_t \tag{2}$$

$$h_t = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta h_{t-1}$$

$$e_t = h_t^{1/2} z_t \qquad z_t \sim N(0,1).$$

In this model, the error, $e_t$, is conditionally normally distributed and serially uncorrelated, and the conditional variance, $h_t$, is a linear function of the square of the last period's errors and of the last period's conditional variance. This specification of the conditional variance implies positive serial correlation in the conditional second moment of the return process – periods of high (low) volatility are likely to be followed by periods of high (low) volatility. The conditional returns in this model are a linear function of the conditional variance (see Engle, Lilien, and Robins(1987)) and the past disturbance, $e_{t-1}$. Under this return generating process, volatility can change over time and the expected returns are a function of volatility as well as past returns. This is the richest specification and is popular in financial economics literature.[2] As before, the parameters for the simulations based on the GARCH-M model are estimated from the Dow Jones series. The

GARCH-M might be consistent with the efficient market hypothesis; higher ex ante returns are expected when conditional volatility is high under the GARCH-M specification.

As we have stressed before, the bootstrapping methodology implemented in this study is innovative and is superior to the traditional approach utilized in numerous studies to measure the significance of trading profits. Under the traditional approach, which is also utilized in this study, the returns from the trading strategy are compared to unconditional benchmark returns. The traditional approach assumes that returns and volatility are constant across time, and that returns are serially uncorrelated. Utilizing the bootstrapping methodology, more meaningful confidence intervals can be generated for various specifications of return generating processes.

## III. Empirical Results: Traditional Tests

### A. Sample Statistics and Model Estimation

Table I contains summary statistics for the entire series and four subsamples for 1 and 10 day returns on the Dow Jones series. Returns are calculated as log differences of the Dow level. In Panel A the results for the daily returns are presented. These returns are strongly leptokurtic for the entire series and all the subsamples. All of the subperiods except one show some signs of skewness. Volatility is largest for the subperiod containing the Great Depression, and it appears to have declined in the most recent subperiods. Serial correlations are generally small with the exception of a few large values at the first lag in the most recent two subperiods. Panel B reports the values for 10 day nonoverlapping returns. Again, returns are calculated as log differences of the Dow levels. Here, as we move to a longer horizon, we see a reduction in kurtosis for all subperiods. Volatility shows a similar pattern to the daily returns with the largest value in the period containing the great depression, and the lowest values in the two most recent subperiods. For the 10 day returns the autocorrelations are generally quite small with no indication of an increase in the latter subperiods. The largest autocorrelation is observed for the 1 day lag during the period of the Great Depression.

Table II contains estimation results for the two models which will be used for comparison with the actual Dow series, the AR(1) and the GARCH-M. Panel A presents the results from estimation of an AR(1) using OLS. The results reveal a significant first order autocorrelation for the Dow series. Panel B shows the results from estimation of a GARCH-M using maximum likelihood. The GARCH-M captures both changing volatility and conditional means. The model estimated also contains an MA(1) to account for the short horizon autocorrelations. The results are consistent with previous studies, i.e. French, Schwert, and Stambaugh(1987) (FSS) that utilized the Standard and Poors index over the time period 1928-84. The estimates of the GARCH-M model indicate that the conditional variance of stock returns is time varying and is highly autocorrelated. The conditional variance in t-1, $h_{t-1}$, and the shock in t-1, $\epsilon_{t-1}^2$, are highly significant in estimating the conditional volatility at time t, $h_t$. (Since $\alpha_1 + \beta < 1$ the process is stationary.) The estimated parameters also show a significantly positive relation between conditional variance and conditional mean, $\gamma$ of 2.95. This is in close agreement with (FSS) who estimated a value a $\gamma$ of 2.41 for their entire sample. The $b$ parameter, capturing the first order autocorrelation in the series, is also significantly positive.

### B. The Moving Average Strategy

Results from trading strategies based on moving average rules for the full sample are presented in Table III. The rules differ by the length of the short and long period and by the size of the band. For example (1,200,0) indicates that the short period is one day, the long period is 200 days and the band is zero percent. We present results for the 10 rules that we examined. The moving average rule is used to divide the entire sample into either buy or sell periods depending on the relative position of the moving averages. If the short moving average is above (below) the long, the day is classified as a buy (sell). This rule is designed to replicate returns from a trading rule where the trader buys when the short moving average penetrates the long from below and then stays in the market until the short moving average penetrates the long moving average from above. After this signal the trader moves out of the market or sells short. We will refer to this rule as the "variable length moving average" (VMA). In Table III we report daily returns during buy and sell periods and corresponding t-statistics.[3]

The results in Table III are striking. The last column lists the differences between the mean daily buy and sell returns. All the buy-sell differences are positive and, the t-tests for these differences are highly significant rejecting the null hypothesis of equality with zero. In every case the introduction of the one percent bands increased the spread between the buy and sell returns. The first two columns in Table III are the number of buy and sell signals generated. For each of the trading rules about 50 percent more buy signals are generated than sells which is consistent with an upward trending market.

The mean buy and sell returns are reported separately in columns 3 and 4. The buy returns are all positive with an average one day return of 0.042 percent, which is about 12 percent at an annual rate. This compares with the unconditional one day return of 0.017 percent from Table I. Six of the ten tests reject the null hypothesis that the returns equal the unconditional return at the 5 percent significance level using a two tailed test. The other four tests are marginally significant. For the sells, the results are even stronger. All the sell returns are negative with an average daily return for the 10 tests of -0.025 percent which is -7 percent at an annual rate. The t-tests for equality with the unconditional mean return are all highly significant with all t-values being less than -2.5.

The fifth and sixth columns in Table III present the fraction of buys and sells greater than zero. For buys this fraction ranges from 53 to 54 percent and and for sells it is about 49 percent. Under the null hypothesis that technical rules do not produce useful signals the fraction of positive returns should be the same for both buys and sells. Performing a binomial test all these differences are highly significant and the null hypothesis of equality can be rejected.

The negative returns in Table III for sell signals are especially noteworthy. These returns cannot be explained by various seasonalities since they are based on about 40 percent of all trading days. [4] Many previous studies found as we did that returns are predictable. This predictability can reflect either (1) changes in expected returns that result from an equilibrium model, or (2) market inefficiency. In general, it is difficult to distinguish between these two alternative explanations. Although rational changes in expected returns are possible it is hard to imagine an equilibrium model that predicts negative returns as we found.

In Table IV we repeat the procedures in Table III for several subsamples. To save space results are presented for only 4 of the 10 tests. The results in this table are consistent with the results in the previous table. The buy-sell differences are significantly different from zero for all four tests for 3 of the 4 subperiods. For the most recent subperiod two of the tests are significant while the other two are marginally significant. For all tests, in all subperiods, the addition of a 1 percent band increases the buy-sell spread. The results for the buys and sells are in line with the previous findings although less significant.

The last subsample, starting in July, 1962, is chosen to line up with the time period investigated in many studies using the CRSP (Center for Research in Securities Prices) data. We present results for all the trading rules during this subperiod in Table V. Once again all the buy-sell differences are positive with 5 of the 10 rules significant at the 5 percent level. Three of the remaining 5 rules are significant at the 10 percent level. These results do not provide any indication that the usefulness of these technical trading rules has diminished over time.

The second moving average test examines fixed 10 day holding periods after a crossing of the two moving averages. This is the fixed length moving average (FMA) rule previously described. Table VI presents results on the full sample using this test. For all the tests the buy-sell differences are positive. The average difference without a band is 0.77 percent while the average with a 1 percent band is 1.09 percent. These are quite substantial returns given that the unconditional ten day return from Table I is only 0.17 percent. For 7 of these 10 tests the null hypothesis that the difference is equal to zero can be rejected at the 5 percent level. The remaining 3 tests are marginally significant. As before, in all cases the addition of a 1 percent band to the trading rule increases the buy-sell difference. [5]

The table also reports the buy and sell returns separately. For all of the buys the returns are greater than the unconditional mean 10 day return with an average of 0.53 percent. The sells all fall below the unconditional mean 10 day return with an average of -0.40 percent. Checking the t-values reveals that none of the tests reject the null hypothesis that the mean buy is different from the unconditional mean. For the sells 5 out of the 10 tests reject the null hypothesis. The fifth and sixth columns in Table VI present the fraction of buys and sells greater than zero. For all the tests the fraction of buys greater than zero exceeds the fraction of sells greater than zero.

The profits that can be derived from these trading rules depend on the number of signals generated. The lowest number of signals is for the (2,200,0.01) rule which generates an average of 2.8 signals per year over the 90 years of data. The largest number of signals is generated by the (1,50,0) rule with 7.6 signals per year. The profits that investors can expect to obtain from the implementation of moving average rules depend on the specifics of the rules and transactions costs. We explore the following strategy; upon a buy signal, we borrow and double the investment in the Dow index, upon a sell signal, we sell shares and invest in a risk free asset. Given that the number of buy and sell signals is similar we make the following assumptions: (1) the borrowing and lending rates are the same, (2) the risk during buy periods is the same as the risk during sell periods. Under these assumptions such a strategy, ignoring transaction costs, should produce the same return as a buy and hold strategy. Using the (1,50,0.01) rule as an example, there are on average about 3.5 buy and sell signals per year. On the buy side, because of the leverage, we will gain on average, 1.8 percent (3.5 x 0.0052). On the sell side, by not being in the market, we gain 1.6 percent (3.5 x 0.0046). This results in an extra expected return of 3.4 percent which is substantial when compared to the annual return on the Dow Index of around 5 percent per year (excluding dividends).

### C. Trading Range Break

Results for the trading range break rule are presented in Table VII. With this rule buy and sell signals are generated when the price level moves above or below local maximums and minimums. Local maximums and minimums are computed over the previous 50, 150, and 200 days. We also use a band technique where the price level must exceed the local maximum by 1 percent, or fall below the minimum by 1 percent. For the trading range break rule we compute 10 day holding period returns following buy and sell signals.

The results are presented in the same format as Table VI. The average buy-sell return is 0.86 percent. Of the six tests all reject the null hypothesis of the buy-sell difference being equal to zero. The buy return is positive across all the rules with an average of 0.55 percent. For 3 out of the 6 rules the buy returns are significantly different from the unconditional ten day return at the 5 percent level and the remaining 3 rules are marginally significant. One possible reason for this relatively strong rejection when compared to the moving average rules is the fact that this rule generates more buy and sell signals. The sell returns are negative across all the rules with an average of -0.24 percent. For the individual rules 1 out of the 6 are significantly different from the unconditional ten day return. Results for the subperiods are similar and are not presented to save space.

## IV. Empirical Results: Bootstrap Tests

### A. Random Walk Process

The results of the previous two sections are intriguing, but there are still some missing pieces. First, we do not compute a comprehensive test across all rules. Such a test would have to take into account the dependencies between results for different rules. We develop a joint test of significance for our set of trading rules. This is accomplished by utilizing bootstrap distributions for these tests. This is a major advantage of the bootstrap methodology, since constructing such tests using traditional statistical methods would require properly accounting for the complex dependencies across the different rules, which is an extremely difficult task. Second, the t-ratios reported assume normal, stationary, time independent distributions. For stock returns there are several well known deviations from this assumed distribution. Among these are: leptokurtosis, autocorrelation, conditional heteroskedasticity, and changing conditional means. These important aspects of the data will be addressed using distributions generated from simulated null models for stock prices. Using this strategy we can address all the issues brought up earlier. A third benefit of this methodology is that we can examine the standard deviations of returns during the buy and sell periods. This gives us an indication of the riskiness of the various strategies during buy and sell periods.

Three simulated null models for stock returns are used to generate a bootstrapped price series. The first is a random walk model for prices. This simulation takes the log first differences of the original Dow series and scrambles these with replacement, generating a scrambled returns series. This is then exponentiated to form a price series. This series contains all the same unconditional distributional properties of the original series. However, all intertemporal dependencies are lost. All of the trading rules are analyzed on 250 replications of random walks generated to be of the same length as the original Dow series.

In Table VIII we display the results of random walk simulations for all three types of trading rules. To save space we present only a subset of the rules used. In order to describe the format of Table VIII we start with the first row of Panel A which presents the results for the (1,50,0) VMA rule. All the numbers presented in this row are the fractions of the simulated results which are larger than the results for the original Dow series. Results for returns are presented in the columns labled, Buy, Sell, and Buy-Sell, while the results for standard deviations are presented in the columns labled $\sigma_b$ and $\sigma_s$. The number in the column labled Buy, which is 0.00, shows that none of the simulated random walks generated a mean buy return as large as that from the original Dow series. This number can be thought of as a simulated "p-value". Turning to the Sell column the fraction is 1.00, showing that all of the simulated random walks generated mean sell returns larger than the mean sell return from the Dow series. The fraction in the Buy-Sell column, 0.00, reports that none of the simulated random walks generated mean buy-sell differences larger than the mean differences for the Dow series. In the column $\sigma_b$ ($\sigma_s$) the reported numbers are 1.00 (0.00) showing that all (none) of the standard deviations for the simulated random walks were greater (smaller) than those from the Dow series. The results for the returns are consistent with the previous tables. However, the results for the standard deviations are new. Not only do the buy signals select out periods with higher conditional means, they also pick periods with lower volatilities. This is in contrast to sell periods where the conditional return is lower, but the volatility is higher. An often used explanation for predictability in returns is changing risk levels. These results are not in accord with this explanation. Aside from the negative returns during sell periods being inconsistent with existing equilibrium models, the higher returns for buys do not seem to arise during riskier periods.

Results for some of the other moving averages, (1,50,0.01), (1,150,0), (1,150,0.01), (1,200,0), (1,200,0.01), using the VMA strategy are presented in other rows of Table VIII, Panel A. These results are essentially the same as for the rule (1,50,0) and provide further support for our earlier findings.

To conclude the discussion of the VMA method we turn to Panel B of Table VIII where the results are summarized across all rules. As mentioned earlier we would like to construct a statistic to jointly test our set of trading rules. This is almost impossible using the traditional statistical approach, but is easily accomplished using bootstrap techniques. The bootstrap technique allows us to choose any function to aggregate our results across the ten rules. We decided to use a simple average over the 10 rules. Thus, for each of our 250 simulations we compute an average over all rules for both returns and standard deviations. The first row of Panel B, labeled Fraction > Dow, follows the same format as the results presented in Panel A. Not surprisingly, these results strongly agree with those for the individual rules. The second row, labeled Bootstrap Mean, presents the returns for buys, sells, and buy-sells, and the standard deviations for buys and sells averaged over the 250 simulated random walks. The values reported are as expected. The returns and standard deviations for buys and sells are essentially the same and close to their unconditional values reported in Table I. The third row, labeled Dow Series, presents the same statistics for the original Dow series. For both returns and standard deviations large differences not seen in the bootstrap results are now observed. The average difference between the buys and sells is 0.067 percent as presented previously in Table III. The standard deviations for buys and sells are 0.89 and 1.34 percent, respectively. This shows that the standard deviation for the buys is substantially below the unconditional standard deviation, whereas the standard deviation for the sells is substantially above.

For the FMA test the results are similar to those for the VMA test although somewhat weaker for the individual trading rules. Turning to the rule averages in Panel B of Table VIII the results are almost exactly the same as those reported for the VMA rule. For the buys only 1.2 percent of the simulated random walks produce larger returns than the Dow series. In contrast, for the sells, 99.6 percent of the simulated random walks have larger returns than the Dow series. For the buy-sell difference none of the simulated random walks generated a value as large as the difference for the actual series which was 0.93 percent. Results for the standard deviations follow a similar pattern to the VMA rule. For the buys all of the standard deviations from the simulated random walks are larger than those from the Dow series, and for the sells none of the standard deviations are as large as those from the Dow series. For the buys the 10 day standard deviation is 3.1 percent whereas for the sells we observe a substantially higher standard deviation of 4.2 percent. The last test presented in Table VII is the trading range break (TRB). The results agree with the FMA and VMA tests and therefore will not be discussed.

Although most of the results from this table agree with earlier findings using traditional methods (Tables III-V) some differences can be observed. For example, focusing on sell returns for the TRB (1,200,0) rule, Table VIII indicates that only 2.4 percent of the random walk simulations generated a value as low as that in the original series. The entry in Table VII, based on traditional tests, is a t-statistic of -1.49. The probability for a standard normal being less than this value is about 7.7 percent. This suggests that the distributional assumptions made for the standard tests may have some impact on statistical inferences.

In summary Table VIII strongly rejects the random walk model for stock prices. The simulations generate lower (higher) returns during buy (sell) periods than the actual Dow series. In addition, the buy (sell) returns on the Dow are less (more) volatile than under the simulated random walks. This makes it less plausible that the extra buy returns are a compensation for additional risk. Moreover, returns following sell signals are negative which is inconsistent with any of the currently existing equilibrium models.

### B. AR(1) Process

Table IX repeats the previous results for a simulated AR(1) process by utilizing the estimated residuals from the original series. The residuals are scrambled with replacement and reconstructed into a new return series using the estimated AR(1) model. This experiment is designed to detect whether the results from the trading rules could be caused by daily serial correlations in the series. For all our trading rules the return on a day in which a buy (sell) signal is received is expected to be large (small). If the returns are positively autocorrelated we should also expect higher (lower) returns on the following days. Indeed, the results reported in Table II document some degree of positive autocorrelation.

Table IX, Panel B, confirms that some differences between buys and sells can occur with an AR(1) process. For the VMA rules the average buy return from the simulated AR(1) is 0.020 percent, and the average sell return is 0.013 percent. This compares with an unconditional return of 0.017 percent for the entire sample. The AR(1) clearly creates a buy-sell spread as predicted, but the magnitude of this spread is not large when compared with the Dow series which produces a spread of 0.067 percent. The "p-value" of zero confirms that this difference cannot be explained by the AR(1). For the FMA and TRB rule the spreads produced by the AR(1) process are 0.11 percent and 0.14 percent, respectively. These spreads should be compared to the much larger spreads from the original Dow series which are 0.93 percent and 0.87 percent, respectively.

The buy-sell column in Panel A of Table IX reveals that the AR(1) process has some impact on the rules tested. For example, the (1,200,0) filter for the fixed length moving average produces a "p-value" of 0.096 compared with 0.028 for the random walk. This change is one of the largest changes between Table IX and Table VIII. For the VMA test, for example, the results for the 6 rules on the buy-sell returns show no difference in "p-values" from Table VIII to Table IX. They all continue to be zero. Also, for buys and sells separately, the results are similar. Furthermore, the changes for the standard deviations are small too. This should be expected since we do not have conditional variances changing over time in this simulated model. In summary, the AR(1) provides a partial explanation for the differences between buys and sells generated by our trading rules, but clearly does not account for the large difference in returns observed in the Dow series.

### C. GARCH-M Process

The next simulations use a GARCH-M process. In this model both conditional means and variances are changing over time. A changing conditional mean can account for some of the difference between buy and sell returns. The GARCH-M is simulated using parameter values estimated by maximum likelihood. The model is simulated using computer generated normal random numbers.

Checking the buy-sell column in Table X, Panel B, the VMA rule shows that the GARCH-M generates an average spread of 0.022 percent, compared with 0.067 percent for the Dow series. Of the simulations only 1.6 percent generated buy-sell returns as large as those from the Dow series. The GARCH-M generates a positive buy-sell spread which is substantially larger than the spread under the AR(1), but this spread is still small when compared with that from the original Dow series. [6] These findings are repeated for the FMA and TRB rules, although the results are somewhat weaker.

The discrepancies for sell returns is in particular large. For example, for the VMA rule, the GARCH-M generates a sell average return of 0.054 percent which should be compared with an actual return of -0.025 percent for the Dow series. This difference is highly significant as indicated by the "p-value" of 1.00. Results for the FMA and TRB rules strongly support the VMA results, and overall present strong evidence that the GARCH-M is incapable of generating returns consistent with the negative returns for the sell periods. For the buys the GARCH-M generates very large returns. For all three tests the bootstrap mean return for the GARCH-M is substantially larger than the return from the Dow series. This is in sharp contrast to previous tests, and seems to indicate some possible further problems with the GARCH-M.[7] In summary, the GARCH-M model fails in replicating returns during both buy and sell periods. In both cases the actual return is substantially lower than the GARCH-M return.

The GARCH-M model not only fails in replicating returns, but also is unable to match the results for volatility. For the Dow series standard deviations are lower for the buy periods than for the sells. Examining Table X, Panel B, for the VMA rule the GARCH-M average standard deviation for buys is 1.23 percent which should be compared with 0.89 percent for the Dow series. The "p-value" of 1.00 supports the significance of this difference. Hence, the GARCH-M is substantially overestimating the volatility for buy periods. This provides a partial explanation for the high returns that the GARCH-M generates during buy periods. The GARCH-M model also fails in predicting volatility during sell periods. The model predicts a standard deviation of 1.18 while the actual standard deviation is 1.34, the "p-value" for this difference is 4.8 percent. The performance of the GARCH-M in predicting volatility during sell periods is somewhat better for the other two tests. In summary, the GARCH-M model fails to replicate the conditional returns for the Dow Index. Moreover, the focal point of the GARCH models is to predict volatility where, as with returns, the GARCH-M model also fails.

## V. Conclusions

The recent studies on predictability of equity returns from past returns suggest that the conclusion reached by many earlier studies that found that technical analysis is useless, might have been premature. In this paper we investigate two of the simplest and most popular trading rules – moving averages and trading range breaks. Overall, 20 moving average rules and 6 trading range break rules are evaluated. The possibility that various spurious patterns were uncovered by technical analysts cannot be dismissed. To mitigate this problem we present results from all the trading strategies that we examined. Moreover, we utilize a very long data series, the Dow Jones Industrial Average index from 1897 to 1986.

For statistical inferences, in addition to the standard tests, we apply the bootstrap methodology. The returns conditional on buy (sell) signals from the actual Dow Jones data are compared to returns from simulated comparison series generated by a fitted model from the null hypothesis class being tested. The null models tested are: random walk with a drift, AR(1), and GARCH-M. Each of the simulations is based on 250 repetitions of the null model which should provide a good approximation of the returns distribution utilizing the technical rules under the null model.

Our statistical design can be viewed as the use of technical analysis as a model evaluation tool where popular models such as GARCH-M can be evaluated. This is a new and potentially important application that enables us not only to test the null model, but gain important insight about the structure and formation of stock market prices.

Overall our results provide strong support for the technical strategies that we explored. The returns obtained from buy (sell) signals are not consistent with the three popular null models, the random walk, the AR(1), or the GARCH-M model. Consistently, buy (sell) signals generate returns which are higher (lower) than "normal" returns. A typical difference in returns over a ten day period between a buy and a sell signal is about 0.8 percent, which is sizable when compared to a "normal" 10-day upward drift of about 0.17 percent. The small positive autocorrelation in returns cannot explain the observed patterns and accounts for less than 10 percent of the differences in returns between buys and sells. Furthermore, the difference in returns between buys and sells is not easily explained by risk. The results reveal that following a buy signal stock returns are substantially less volatile than following a sell signal. The most intriguing result is for the moving average rule with a variable length holding period. Under this rule, an investor is always in the market assuming a long or a short position. On average, this strategy called for a long (short) position about 60 (40) percent of all the trading days. During buy signals, the market went up by 2.5 times the "normal" day return, or at an annual rate of 12 percent. During 40 percent of all the trading days, a sell signal was received and the market declined at an annual rate of around seven percent. This negative conditional return over a large fraction of trading days is an intriguing result as predictably negative returns are inconsistent with currently existing equilibrium models. Moreover, it is unlikely that the seasonal anomalies documented in the literature could cause a negative return averaged over 40 percent of all the trading days.

Although our results are consistent with technical rules having predictive power, transactions costs should be carefully considered before such strategies can be implemented. Of course, there are cases where the marginal transaction costs are zero, such as for pension funds that must reinvest dividends and funds contributed by sponsors. Opportunities also might exist in the futures markets where transactions costs are very small.

The popular GARCH-M model cannot explain the returns that our various rules generate. Moreover, our statistical design enables us to better understand why the GARCH-M model performs so poorly. This insight cannot be obtained when traditional statistical procedures are applied. For example, under the GARCH-M model, the results reveal that when a buy signal is generated, the conditional mean is very high because the conditional variance is high. The model predicts a volatile period. The actual results however, reveal the opposite. Following a buy signal the volatility is below "normal" volatility and lower than the volatility following sell signals. Our results therefore show that the GARCH-M model fails not only in predicting returns, but also in predicting volatility.

In sum, this paper shows that the returns generating process of stocks is probably more complicated than suggested by the various studies using linear models. It is quite possible that technical rules pick up some of the hidden patterns. We would like to emphasize that our analysis focused on the simplest trading rules. Other more elaborate rules may generate even larger differences between conditional returns. Why such rules might work is an intriguing issue left for further studies.

## APPENDIX

### Use of the Bootstrap to Estimate Confidence Intervals for Technical Trading Rules Under Null Models

We want to be able to estimate confidence intervals for trading profits under three null models whose parameters will be estimated on our data: (i) A random walk with unknown drift, but driven by IID errors drawn from an unknown distribution, F. (ii) A random walk with unknown drift and augmented by first order autoregressive errors drawn from an unknown distribution F. (iii) A GARCH-M model where the ultimate uncertainty comes from IID errors drawn from an unknown normal distribution, F, with mean zero and unknown variance. To do this we briefly explain the bootstrap method we shall use. A more complete description can be had by writing the authors.

Following (Efron 1982) (chapters 5 and 10) and (Singh 1981), let $(E_1, \ldots, E_n)$ be n IID draws, i.e. a random sample, from distribution F. Let $T(E_1, \ldots, E_n; F)$ be a random variable of interest which may depend directly upon F. Let $F_n$ be the empirical distribution function that puts mass $1/n$ on $E_i, i = 1, 2, \ldots, n$. The bootstrap is a method that approximates the distribution of $T(E_1, \ldots, E_n; F)$ under F by the distribution of $T(E_1, \ldots, E_n; F_n)$ under $F_n$ where $(E_1, \ldots, E_n)$ denotes an IID sample from $F_n$. In other words the bootstrap estimates

$$\text{Prob}\{T(E_1, \ldots, E_n; F) \in A\} \tag{A1}$$

by

$$\text{Prob}\{T(E_1, \ldots, E_n; F_n) \in A\} \tag{A2}$$

Here Prob$\{\cdot\}$ denotes the probability of event $\{\cdot\}$. Bootstrapping is done by taking B bootstrap samples (each of which consists of n IID draws from $F_n$). Put $Z_b = (E_{1,b}, \ldots, E_{n,b})$, $b = 1, 2, \ldots, B$ from $F_n$ and estimate (A2) by

$$\sum_{b=1}^{B} (1/B) I_A(T(Z_b; F_n)) \qquad (A3),$$

where $I_A$ denotes the indicator function of event $A$ which takes the value 1 when the event occurs and zero otherwise. By taking B to infinity with cheap computer time one can get as close to (A2) as one wishes.

We shall show how to apply the bootstrap to estimate quantities based upon technical trading profits. Assume $\{X_t\}$ is IID, drawn from $F$. Start with the random walk,

$$X_t = Y_t - Y_{t-1} = E_t, \qquad E_t \sim F \tag{A4},$$

where $Y = \log(P)$ where P is stock price, X is returns, and $E_t$ is the innovation at date t. We wish to test the adequacy of (A4) using a data set of n+1 stock prices $P_1, \ldots, P_{n+1}$. Build returns by setting

$$X_t = Y_t - Y_{t-1} = \log(P_t) - \log(P_{t-1}), \quad t = 2, 3, \ldots, n+1. \tag{A5}$$

Define date t to be a buy signal if

$$P_t > \mathrm{MA}_{t,L}, \qquad \text{and} \qquad P_{t-1} < \mathrm{MA}_{t-1,L}, \tag{A6}$$

where

$$\mathrm{MA}_{t,L} = \frac{1}{L}\sum_{j=0}^{L-1} P_{t-j}. \tag{A7}$$

For each buy signal purchase one dollar's worth of stock, hold it for H periods, then sell it to obtain

$$\pi_{t;t+H} \equiv \frac{P_{t+H} - P_t}{P_t}. \tag{A8}$$

Let

$$\pi_n = (1/n)\sum_{t \in \mathbf{B}} \pi_{t;t+H}, \tag{A9}$$

where B is the set of all buy signals.

QUESTION: Under the null hypothesis that the stock returns data set of length n is a sample from the random walk model (A4) how can we tell if the actual computed technical trading profit (A9) is compatible with the null hypothesis (A4)?

ANSWER: Reject (A4) at the 5% level if $\pi_n$ is greater than $b_{.95}$ (or less than $a_{.95}$) where ($a_{.95}$,$b_{.95}$) is the 95% confidence interval for $\pi_n$ under the null hypothesis (A4).

A basic issue that must be dealt with is this. The price process $\{P_t\}$ generated by (A4) is not a stationary process. But stationarity of $\{X_t\}$ allows us to write the probability of event (A6) and the quantities (A8) and (A9) as time stationary functions of a finite number of $\{X_t\}$.

Use the homogeneity of degree zero of (A6) in $P_{t-L}$ to write it in the form

$$\begin{aligned}\exp(X_t + \ldots + X_{t-L+2}) &> \{\exp(X_t + \ldots + X_{t-L+2}) + \ldots + \exp(X_{t-L+2})\}/L, \quad \text{and} \\ \exp(X_{t-1} + \ldots + X_{t-L+1}) &< \{\exp(X_{t-1} + \ldots + X_{t-L+1}) + \ldots + \exp(X_{t-L+1})\}/L.\end{aligned} \tag{A6'}$$

Note that the indicator function $I_{\mathrm{B}}$, of event (A6'), which we shall call BUY, can be written as a time stationary function of $(X_t, \ldots, X_{t-L+1})$. Turn now to profits for a given t in B.

Similar algebra show us profits can be written

$$\frac{P_{t+H} - P_t}{P_t} = \exp(\sum_{j=1}^{H} X_{t+j}) - 1, \tag{A10}$$

which is a time stationary function of a finite number of consecutive X's. Hence $\pi_n$ may be written as a time stationary function of a finite number of consecutive X's.

Estimate the cumulative distribution function,

$$\mathrm{Prob}\{\pi_n < x\} \equiv \mathrm{Prob}\{T(Z; F) < x\} = J(x; F). \tag{A11}$$

Do this by drawing B bootstrap samples $Z(b) = (X_{1,b}, \ldots, X_{n,b})$ from $F_n, b = 1, 2, \ldots, B$ and compute

$$\sum_{b=1}^{B} I\{T(Z_b; F_n)) < x\}/B \xrightarrow{d} J(x; F_n), \quad B \to \infty, \tag{A12}$$

This procedure yields another useful byproduct – an estimate of the "p-value" of the null model obtained from the data value of $\pi_n$. Obtain this estimated "p-value" by

$$\hat{p} = J^{-1}(\pi_n; F_n). \tag{A13}$$

The estimated $\hat{p}$ values are reported in the text for the two basic types of technical trading rule statistics for three classes of null models: (i) returns are IID; (ii) returns are autoregressive of order 1 (AR(1)) with IID errors; (iii) returns are autoregressive with GARCH-M errors.

The theory above can be extended to autoregressive processes as in (Freedman 1984). For the stable GARCH-M processes, we have not found any asymptotic justification for the bootstrap method like that of (Freedman 1984) for general stable autoregressions. (Freedman and Peters 1984a) propose a double bootstrap style of test to test the quality of bootstrapped values. We have done a few early experiments here which support the evidence reported in table, but due to the enormous amount of computer time required for these tests our experiments were limited to a small subset of the tests and simulations used.

## Notes

1. Data snooping issues are also discussed in Lakonishok and Smidt(1988) and Lo and MacKinlay(1990).

2. The specification used in this paper, the GARCH(1,1)-M, was determined using the Schwarz(1978) model specification criterion. It is similar to the model specification used in French, Schwert and Stambaugh(1987).

3. The t-statistics for the buys(sells) are,

$$\frac{\mu_r - \mu}{(\sigma^2/N + \sigma^2/N_r)^{1/2}},$$

where $\mu_r$ and $N_r$ are the mean return and number of signals for the buys and sells, and $\mu$ and $N$ are the unconditional mean and number of observations. $\sigma^2$ is the estimated variance for the entire sample. For the buy-sell the t-statistic is,

$$\frac{\mu_b - \mu_s}{(\sigma^2/N_b + \sigma^2/N_s)^{1/2}},$$

where $\mu_b$ and $N_b$ are the mean return and number of signals for the buys and $\mu_s$ and $N_s$ are the mean return and number of signals for the sells.

4. The series used in this study does not contain dividends. Results in Lakonishok and Smidt(1988) suggest that our finding of negative returns during sell periods will not be altered with the inclusion of dividends.

5. There is an unusual effect of the band on the number of buy and sell signals. For several of the rules an addition of the band actually increases the number of buy signals. In our testing method signals which occur within 10 days of a previous signal are blocked out, so the elimination of a few sell signals may actually increase the number of buys.

6. The GARCH-M estimated here also contains an MA(1) component. Part of this returns spread could be coming from this element of the model.

7. Based on our estimated parameters the unconditional mean of the GARCH-M series does not match that of the data on which it was estimated. This is entirely possible since the model considers the weighted mean of the series. The fact that the GARCH-M substantially deviates from the mean of the series might be a serious specification problem which is first noted in French, Schwert, and Stambaugh(1987).

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Table I

Summary statistics for daily and 10 day returns. Returns are measured as log differences of the level of the Dow index. 10 day returns are based on nonoverlapping 10 day periods. Numbers marked with * (**) are significant at the 5% (1%) levels for a two tailed test. Bartlett Std. refers to the Bartlett standard error, $1/\sqrt{N}$.

(A) Daily Returns

| | Full Sample | 97-14 | 15-38 | 39-62 | 62-86 |
|---|---|---|---|---|---|
| N | 25036 | 5255 | 7163 | 6442 | 6155 |
| Mean | 0.00017 | 0.00012 | 0.00014 | 0.00020 | 0.00020 |
| Std. | 0.0108 | 0.0099 | 0.0147 | 0.0075 | 0.0088 |
| Skew | -0.1047** | -0.4804** | 0.0193 | -0.7614** | 0.2707** |
| Kurtosis | 16.00** | 8.86** | 12.75** | 13.60** | 11.57** |
| $\rho(1)$ | 0.033** | 0.013 | 0.009 | 0.117** | 0.079** |
| $\rho(2)$ | -0.026** | -0.020 | -0.029* | -0.068** | -0.001 |
| $\rho(3)$ | 0.012* | 0.041** | -0.006 | 0.036** | 0.009 |
| $\rho(4)$ | 0.046** | 0.085** | 0.055** | 0.028* | -0.012 |
| $\rho(5)$ | 0.022** | 0.042** | 0.027* | 0.014 | -0.011 |
| Bartlett Std. | 0.006 | 0.014 | 0.012 | 0.012 | 0.013 |

(B) 10 Day Returns

| | Full Sample | 97-14 | 15-38 | 39-62 | 62-86 |
|---|---|---|---|---|---|
| Mean | 0.0017 | 0.0012 | 0.0014 | 0.0019 | 0.0019 |
| Std. | 0.0351 | 0.0339 | 0.0486 | 0.0272 | 0.0296 |
| Skew | -0.4583** | -0.1762 | -0.9105** | -1.1551** | -0.0786 |
| Kurtosis | 7.91** | 4.59** | 8.51** | 9.05** | 3.91** |
| $\rho(1)$ | 0.037* | -0.004 | 0.065* | 0.032 | -0.002 |
| $\rho(2)$ | 0.018 | 0.044 | 0.001 | -0.090* | -0.041 |
| $\rho(3)$ | 0.013 | 0.071 | 0.056 | -0.037 | 0.007 |
| $\rho(4)$ | 0.011 | -0.125** | 0.024 | 0.045 | 0.026 |
| $\rho(5)$ | 0.032 | 0.094* | -0.022 | 0.018 | -0.021 |
| Bartlett Std. | 0.019 | 0.043 | 0.037 | 0.039 | 0.040 |

Table II

Parameter estimates for AR(1) and GARCH-M models. Estimated on daily returns series 1897-1986. The AR(1) is estimated by OLS and numbers in parenthesis are t-ratios. The GARCH-M is estimated by maximum likelihood and the numbers in parenthesis are asymptotic t-ratios.

(A) AR(1) Parameter Estimates

$$x_t = a + bx_{t-1} + \epsilon_t$$

| $a$ | $b$ |
|---|---|
| 0.00015 | 0.03330 |
| (2.50) | (5.27) |

(B) GARCH-M Parameter Estimates

$$x_t = a + \gamma h_t + b\epsilon_{t-1} + \epsilon_t \qquad \epsilon_t = h_t^{1/2} z_t$$

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta h_{t-1}$$

$$z_t \sim N(0,1)$$

| $\alpha_0$ | $\alpha_1$ | $\beta$ | $\gamma$ | $b$ | $a$ |
|---|---|---|---|---|---|
| 1.33e-6 | 0.098 | 0.893 | 2.95 | 0.10 | 2.50e-4 |
| (26.6) | (49.0) | (447) | (3.73) | (16.7) | (0.36) |

Table III

Standard test results for the Variable Length Moving (VMA) rules run on daily data from 1897-1986. Rules are identified as (short, long, band), where short and long are the short and long moving averages respectively, and band is the percentage difference that is needed to generate a signal. N(buy) and N(sell) are the number of buy and sell signals reported during the sample. Numbers in parenthesis are standard t-ratios testing the difference of the mean buy and mean sell from the unconditional 1 day mean, and buy-sell from zero. Buy> 0 and Sell> 0 are the fraction of buy and sell returns greater than zero. The last row reports averages across all 10 rules.

| Test | N(buy) | N(sell) | Buy | Sell | Buy>0 | Sell>0 | buy-sell |
|---|---|---|---|---|---|---|---|
| (1,50,0) | 14240 | 10531 | 0.00047 | -0.00027 | 0.5387 | 0.4972 | 0.00075 |
| | | | (2.68473) | (-3.54645) | | | (5.39746) |
| (1,50,0.01) | 11671 | 8114 | 0.00062 | -0.00032 | 0.5428 | 0.4942 | 0.00094 |
| | | | (3.73161) | (-3.56230) | | | (6.04189) |
| (1,150,0) | 14866 | 9806 | 0.00040 | -0.00022 | 0.5373 | 0.4962 | 0.00062 |
| | | | (2.04927) | (-3.01836) | | | (4.39500) |
| (1,150,0.01) | 13556 | 8534 | 0.00042 | -0.00027 | 0.5402 | 0.4943 | 0.00070 |
| | | | (2.20929) | (-3.28154) | | | (4.68162) |
| (5,150,0) | 14858 | 9814 | 0.00037 | -0.00017 | 0.5368 | 0.4970 | 0.00053 |
| | | | (1.74706) | (-2.61793) | | | (3.78784) |
| (5,150,0.01) | 13491 | 8523 | 0.00040 | -0.00021 | 0.5382 | 0.4942 | 0.00061 |
| | | | (1.97876) | (-2.78835) | | | (4.05457) |
| (1,200,0) | 15182 | 9440 | 0.00039 | -0.00024 | 0.5358 | 0.4962 | 0.00062 |
| | | | (1.93865) | (-3.12526) | | | (4.40125) |
| (1,200,0.01) | 14105 | 8450 | 0.00040 | -0.00030 | 0.5384 | 0.4924 | 0.00070 |
| | | | (2.01907) | (-3.48278) | | | (4.73045) |
| (2,200,0) | 15194 | 9428 | 0.00038 | -0.00023 | 0.5351 | 0.4971 | 0.00060 |
| | | | (1.87057) | (-3.03587) | | | (4.26535) |
| (2,200,0.01) | 14090 | 8442 | 0.00038 | -0.00024 | 0.5368 | 0.4949 | 0.00062 |
| | | | (1.81771) | (-3.03843) | | | (4.16935) |
| Average | | | 0.00042 | -0.00025 | | | 0.00067 |

Table IV

Standard test results for the Variable Length Moving (VMA) rules run on four subsamples. Rules are identified as (short, long, band), where short and long are the short and long moving averages respectively, and band is the percentage difference that is needed to generate a signal. N(buy) and N(sell) are the number of buy and sell signals reported during the sample. Numbers in parenthesis are standard t-ratios testing the difference of the mean buy and mean sell from the unconditional 1 day mean, and buy-sell from zero. Buy> 0 and Sell> 0 are the fraction of buy and sell returns greater than zero.

| Period | Test | N(buy) | N(sell) | Buy | Sell | Buy>0 | Sell>0 | buy-sell |
|---|---|---|---|---|---|---|---|---|
| 1897-1914 | (1,50,0) | 2824 | 2370 | 0.00048 | -0.00030 | 0.5322 | 0.4992 | 0.00078 |
| | | | | (1.57453) | (-1.70049) | | | (2.82916) |
| | (1,50,0.01) | 2228 | 1822 | 0.00071 | -0.00028 | 0.5408 | 0.5016 | 0.00099 |
| | | | | (2.35505) | (-1.49087) | | | (3.16818) |
| | (1,150,0) | 2925 | 2170 | 0.00039 | -0.00025 | 0.5323 | 0.4959 | 0.00065 |
| | | | | (1.19348) | (-1.48213) | | | (2.30664) |
| | (1,150,0.01) | 2611 | 1871 | 0.00043 | -0.00025 | 0.5358 | 0.4923 | 0.00067 |
| | | | | (1.28311) | (-1.38532) | | | (2.24555) |
| 1915-1938 | (1,50,0) | 4009 | 2967 | 0.00056 | -0.00053 | 0.5478 | 0.4992 | 0.00109 |
| | | | | (1.44073) | (-2.11557) | | | (3.08064) |
| | (1,50,0.01) | 3473 | 2467 | 0.00071 | -0.00060 | 0.5505 | 0.4978 | 0.00131 |
| | | | | (1.87796) | (-2.17728) | | | (3.40511) |
| | (1,150,0) | 4092 | 2884 | 0.00048 | -0.00045 | 0.5503 | 0.4941 | 0.00092 |
| | | | | (1.16041) | (-1.82639) | | | (2.59189) |
| | (1,150,0.01) | 3877 | 2652 | 0.00052 | -0.00053 | 0.5522 | 0.4932 | 0.00105 |
| | | | | (1.28954) | (-2.03621) | | | (2.85704) |
| 1939-1962 | (1,50,0) | 3894 | 2497 | 0.00048 | -0.00023 | 0.5534 | 0.5010 | 0.00070 |
| | | | | (1.81064) | (-2.43062) | | | (3.66852) |
| | (1,50,0.01) | 3149 | 1787 | 0.00054 | -0.00043 | 0.5526 | 0.4919 | 0.00097 |
| | | | | (2.10878) | (-3.12433) | | | (4.36866) |
| | (1,150,0) | 4170 | 2122 | 0.00036 | -0.00004 | 0.5422 | 0.5151 | 0.00040 |
| | | | | (1.06310) | (-1.26932) | | | (1.98384) |
| | (1,150,0.01) | 3689 | 1667 | 0.00039 | -0.00011 | 0.5468 | 0.5159 | 0.00050 |
| | | | | (1.24510) | (-1.49128) | | | (2.25967) |
| 1962-1986 | (1,50,0) | 3468 | 2636 | 0.00036 | -0.00004 | 0.5167 | 0.4879 | 0.00041 |
| | | | | (0.90076) | (-1.16108) | | | (1.78607) |
| | (1,50,0.01) | 2782 | 1985 | 0.00053 | 0.00003 | 0.5230 | 0.4831 | 0.00049 |
| | | | | (1.64014) | (-0.70959) | | | (1.89872) |
| | (1,150,0) | 3581 | 2424 | 0.00037 | -0.00012 | 0.5205 | 0.4777 | 0.00049 |
| | | | | (0.94029) | (-1.49333) | | | (2.11283) |
| | (1,150,0.01) | 3292 | 2147 | 0.00035 | -0.00018 | 0.5216 | 0.4742 | 0.00052 |
| | | | | (0.80174) | (-1.67583) | | | (2.13824) |

Table V

Standard test results for the Variable Length Moving (VMA) rules run on daily data from 1962-1986. Rules are identified as (short, long, band), where short and long are the short and long moving averages respectively, and band is the percentage difference that is needed to generate a signal. N(buy) and N(sell) are the number of buy and sell signals reported during the sample. Numbers in parenthesis are standard t-ratios testing the difference of the mean buy and mean sell from the unconditional 1 day mean, and buy-sell from zero. Buy> 0 and Sell> 0 are the fraction of buy and sell returns greater than zero. The last row reports averages across all 10 rules.

| Test | N(buy) | N(sell) | Buy | Sell | Buy>0 | Sell>0 | buy-sell |
|---|---|---|---|---|---|---|---|
| (1,50,0) | 3468 | 2636 | 0.00036 | -0.00004 | 0.5167 | 0.4879 | 0.00041 |
| | | | (0.90076) | (-1.16108) | | | (1.78607) |
| (1,50,0.01) | 2782 | 1985 | 0.00053 | 0.00003 | 0.5230 | 0.4831 | 0.00049 |
| | | | (1.64014) | (-0.70959) | | | (1.89872) |
| (1,150,0) | 3581 | 2424 | 0.00037 | -0.00012 | 0.5205 | 0.4777 | 0.00049 |
| | | | (0.94029) | (-1.49333) | | | (2.11283) |
| (1,150,0.01) | 3292 | 2147 | 0.00035 | -0.00018 | 0.5216 | 0.4742 | 0.00052 |
| | | | (0.80174) | (-1.67583) | | | (2.13824) |
| (5,150,0) | 3583 | 2422 | 0.00031 | -0.00003 | 0.5177 | 0.4818 | 0.00034 |
| | | | (0.60258) | (-1.05691) | | | (1.44509) |
| (5,150,0.01) | 3284 | 2137 | 0.00033 | -0.00006 | 0.5195 | 0.4815 | 0.00039 |
| | | | (0.68972) | (-1.15961) | | | (1.58385) |
| (1,200,0) | 3704 | 2251 | 0.00037 | -0.00016 | 0.5173 | 0.4780 | 0.00053 |
| | | | (0.92753) | (-1.64056) | | | (2.23379) |
| (1,200,0.01) | 3469 | 2049 | 0.00038 | -0.00018 | 0.5189 | 0.4763 | 0.00056 |
| | | | (0.96907) | (-1.66579) | | | (2.26328) |
| (2,200,0) | 3706 | 2249 | 0.00037 | -0.00017 | 0.5159 | 0.4802 | 0.00055 |
| | | | (0.96463) | (-1.69355) | | | (2.31148) |
| (2,200,0.01) | 3466 | 2042 | 0.00033 | -0.00014 | 0.5162 | 0.4780 | 0.00047 |
| | | | (0.71888) | (-1.47865) | | | (1.90087) |
| Average | | | 0.00037 | -0.00011 | | | 0.00048 |

Table VI

Standard test results for the Fixed Length Moving (FMA) rules run on daily data from 1897-1986. Cumulative returns are reported for fixed 10 day periods after signals. Rules are identified as (short, long, band), where short and long are the short and long moving averages respectively, and band is the percentage difference that is needed to generate a signal. N(buy) and N(sell) are the number of buy and sell signals reported during the sample. Numbers in parenthesis are standard t-ratios testing the difference of the mean buy and mean sell from the unconditional 1 day mean, and buy-sell from zero. Buy> 0 and Sell> 0 are the fraction of buy and sell returns greater than zero. The last row reports averages across all 10 rules.

| Test | N(buy) | N(sell) | Buy | Sell | Buy>0 | Sell>0 | buy-sell |
|---|---|---|---|---|---|---|---|
| (1,50,0) | 340 | 344 | 0.0029 | -0.0044 | 0.5882 | 0.4622 | 0.0072 |
| | | | (0.5796) | (-3.0021) | | | (2.6955) |
| (1,50,0.01) | 313 | 316 | 0.0052 | -0.0046 | 0.6230 | 0.4589 | 0.0098 |
| | | | (1.6809) | (-3.0096) | | | (3.5168) |
| (1,150,0) | 157 | 188 | 0.0066 | -0.0013 | 0.5987 | 0.5691 | 0.0079 |
| | | | (1.7090) | (-1.1127) | | | (2.0789) |
| (1,150,0.01) | 170 | 161 | 0.0071 | -0.0039 | 0.6529 | 0.5528 | 0.0110 |
| | | | (1.9321) | (-1.9759) | | | (2.8534) |
| (5,150,0) | 133 | 140 | 0.0074 | -0.0006 | 0.6241 | 0.5786 | 0.0080 |
| | | | (1.8397) | (-0.7466) | | | (1.8875) |
| (5,150,0.01) | 127 | 125 | 0.0062 | -0.0033 | 0.6614 | 0.5520 | 0.0095 |
| | | | (1.4151) | (-1.5536) | | | (2.1518) |
| (1,200,0) | 114 | 156 | 0.0050 | -0.0019 | 0.6228 | 0.5513 | 0.0069 |
| | | | (0.9862) | (-1.2316) | | | (1.5913) |
| (1,200,0.01) | 130 | 127 | 0.0058 | -0.0077 | 0.6385 | 0.4724 | 0.0135 |
| | | | (1.2855) | (-2.9452) | | | (3.0740) |
| (2,200,0) | 109 | 140 | 0.0050 | -0.0035 | 0.6330 | 0.5500 | 0.0086 |
| | | | (0.9690) | (-1.7164) | | | (1.9092) |
| (2,200,0.01) | 117 | 116 | 0.0018 | -0.0088 | 0.5556 | 0.4397 | 0.0106 |
| | | | (0.0377) | (-3.1449) | | | (2.3069) |
| Average | | | 0.0053 | -0.0040 | | | 0.0093 |

Table VII

Standard test results for the Trading Range Break (TRB) rules run on daily data from 1897-1986. Cumulative returns are reported for fixed 10 day periods after signals. Rules are identified as (short, long, band), where short and long are the short and long moving averages respectively, and band is the percentage difference that is needed to generate a signal. N(buy) and N(sell) are the number of buy and sell signals reported during the sample. Numbers in parenthesis are standard t-ratios testing the difference of the mean buy and mean sell from the unconditional 1 day mean, and buy-sell from zero. Buy> 0 and Sell> 0 are the fraction of buy and sell returns greater than zero. The last row reports averages across all 6 rules.

| Test | N(buy) | N(sell) | Buy | Sell | Buy>0 | Sell>0 | buy-sell |
|---|---|---|---|---|---|---|---|
| (1,50,0) | 722 | 415 | 0.0050 | 0.0000 | 0.5803 | 0.5422 | 0.0049 |
| | | | (2.1931) | (-0.9020) | | | (2.2801) |
| (1,50,0.01) | 248 | 252 | 0.0082 | -0.0008 | 0.6290 | 0.5397 | 0.0090 |
| | | | (2.7853) | (-1.0937) | | | (2.8812) |
| (1,150,0) | 512 | 214 | 0.0046 | -0.0030 | 0.5762 | 0.4953 | 0.0076 |
| | | | (1.7221) | (-1.8814) | | | (2.6723) |
| (1,150,0.01) | 159 | 142 | 0.0086 | -0.0035 | 0.6478 | 0.4789 | 0.0120 |
| | | | (2.4023) | (-1.7015) | | | (2.9728) |
| (1,200,0) | 466 | 182 | 0.0043 | -0.0023 | 0.5794 | 0.5000 | 0.0067 |
| | | | (1.4959) | (-1.4912) | | | (2.1732) |
| (1,200,0.01) | 146 | 124 | 0.0072 | -0.0047 | 0.6164 | 0.4677 | 0.0119 |
| | | | (1.8551) | (-1.9795) | | | (2.7846) |
| Average | | | 0.0063 | -0.0024 | | | 0.0087 |

Table VIII

Simulation tests from random walk bootstraps for 250 replications. The log difference series is resampled with replacement and exponentiated back to a simulated price series. The rows marked Fraction > Dow refer to the fraction of simulations generating a mean or standard deviation larger than those from the actual Dow series. The trading rules are, variable length moving average (VMA), fixed length moving average (FMA), trading range break (TRB). Panel B presents results for the averages across all the rules for each reported statistic. Dow Series refers to the actual mean return or standard deviation from the Dow, and Bootstrap Mean refers to the mean from the simulated series.

(A) Individual Rules

| Rule | | Result | Buy | $\sigma_b$ | Sell | $\sigma_s$ | Buy-Sell |
|---|---|---|---|---|---|---|---|
| (1,50,0) | VMA | Fraction > Dow | 0.00000 | 1.00000 | 1.00000 | 0.00000 | 0.00000 |
| | FMA | Fraction > Dow | 0.24000 | 0.18000 | 1.00000 | 0.10800 | 0.00400 |
| | TRB | Fraction > Dow | 0.00800 | 1.00000 | 0.89600 | 0.00000 | 0.00000 |
| (1,50,0.01) | VMA | Fraction > Dow | 0.00000 | 1.00000 | 1.00000 | 0.00000 | 0.00000 |
| | FMA | Fraction > Dow | 0.03600 | 0.59600 | 1.00000 | 0.01600 | 0.00000 |
| | TRB | Fraction > Dow | 0.00000 | 0.11600 | 0.88800 | 0.00000 | 0.00000 |
| (1,150,0) | VMA | Fraction > Dow | 0.00800 | 1.00000 | 1.00000 | 0.00000 | 0.00000 |
| | FMA | Fraction > Dow | 0.02400 | 0.73200 | 0.86000 | 0.00000 | 0.00400 |
| | TRB | Fraction > Dow | 0.04000 | 1.00000 | 0.99200 | 0.00000 | 0.00000 |
| (1,150,0.01) | VMA | Fraction > Dow | 0.00400 | 1.00000 | 1.00000 | 0.00000 | 0.00000 |
| | FMA | Fraction > Dow | 0.01600 | 0.96800 | 0.99600 | 0.00000 | 0.00000 |
| | TRB | Fraction > Dow | 0.00000 | 0.95200 | 0.96400 | 0.00000 | 0.00000 |
| (1,200,0) | VMA | Fraction > Dow | 0.00800 | 1.00000 | 1.00000 | 0.00000 | 0.00000 |
| | FMA | Fraction > Dow | 0.10400 | 0.90400 | 0.93200 | 0.05200 | 0.02800 |
| | TRB | Fraction > Dow | 0.07600 | 1.00000 | 0.97600 | 0.00000 | 0.01200 |
| (1,200,0.01) | VMA | Fraction > Dow | 0.00400 | 1.00000 | 1.00000 | 0.00000 | 0.00000 |
| | FMA | Fraction > Dow | 0.06400 | 1.00000 | 0.99600 | 0.00000 | 0.00000 |
| | TRB | Fraction > Dow | 0.00800 | 0.99600 | 0.97600 | 0.00000 | 0.00400 |

(B) Rule Averages

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Rule Average | VMA | Fraction > Dow | 0.00400 | 1.00000 | 1.00000 | 0.00000 | 0.00000 |
| | | Bootstrap Mean | 0.00016 | 0.01080 | 0.00017 | 0.01078 | -0.00000 |
| | | Dow Series | 0.00042 | 0.00890 | -0.00025 | 0.01342 | 0.00067 |
| Rule Average | FMA | Fraction > Dow | 0.01200 | 1.00000 | 0.99600 | 0.00000 | 0.00000 |
| | | Bootstrap Mean | 0.00168 | 0.03398 | 0.00162 | 0.03389 | 0.00007 |
| | | Dow Series | 0.00531 | 0.03064 | -0.00399 | 0.04156 | 0.00930 |
| Rule Average | TRB | Fraction > Dow | 0.00000 | 0.99600 | 0.98800 | 0.00000 | 0.00000 |
| | | Bootstrap Mean | 0.00147 | 0.03394 | 0.00159 | 0.03390 | -0.00012 |
| | | Dow Series | 0.00633 | 0.03000 | -0.00238 | 0.05398 | 0.00871 |

Table IX

Simulation tests from AR(1) bootstrap for 250 replications. The AR(1) residual series is resampled with replacement and simulated using the AR(1) estimated parameters. The rows marked Fraction > Dow refer to the fraction of simulations generating a mean or standard deviation larger than those from the actual Dow series. The trading rules are, variable length moving average (VMA), fixed length moving average (FMA), trading range break (TRB). Panel B presents results for the averages across all the rules for each reported statistic. Dow Series refers to the actual mean return or standard deviation from the Dow, and Bootstrap Mean refers to the mean from the simulated series.

(A) Individual Rules

| Rule | | Result | Buy | $\sigma_b$ | Sell | $\sigma_s$ | Buy-Sell |
|---|---|---|---|---|---|---|---|
| (1,50,0) | VMA | Fraction > Dow | 0.00400 | 1.00000 | 1.00000 | 0.00000 | 0.00000 |
| | FMA | Fraction > Dow | 0.34800 | 0.38000 | 1.00000 | 0.26800 | 0.02000 |
| | TRB | Fraction > Dow | 0.02000 | 1.00000 | 0.70800 | 0.00000 | 0.05200 |
| (1,50,0.01) | VMA | Fraction > Dow | 0.00000 | 1.00000 | 1.00000 | 0.00000 | 0.00000 |
| | FMA | Fraction > Dow | 0.03600 | 0.82000 | 1.00000 | 0.01600 | 0.00000 |
| | TRB | Fraction > Dow | 0.00400 | 0.32000 | 0.74000 | 0.00000 | 0.01200 |
| (1,150,0) | VMA | Fraction > Dow | 0.01600 | 1.00000 | 1.00000 | 0.00000 | 0.00000 |
| | FMA | Fraction > Dow | 0.04000 | 0.87200 | 0.78800 | 0.00000 | 0.03200 |
| | TRB | Fraction > Dow | 0.07200 | 1.00000 | 0.98400 | 0.00000 | 0.01600 |
| (1,150,0.01) | VMA | Fraction > Dow | 0.00800 | 1.00000 | 1.00000 | 0.00000 | 0.00000 |
| | FMA | Fraction > Dow | 0.02000 | 0.99200 | 0.97200 | 0.00000 | 0.00000 |
| | TRB | Fraction > Dow | 0.01600 | 0.99600 | 0.90800 | 0.00000 | 0.00400 |
| (1,200,0) | VMA | Fraction > Dow | 0.02000 | 1.00000 | 1.00000 | 0.00000 | 0.00000 |
| | FMA | Fraction > Dow | 0.13600 | 0.91600 | 0.79600 | 0.11200 | 0.09600 |
| | TRB | Fraction > Dow | 0.11200 | 1.00000 | 0.93600 | 0.00000 | 0.04400 |
| (1,200,0.01) | VMA | Fraction > Dow | 0.01200 | 1.00000 | 1.00000 | 0.00000 | 0.00000 |
| | FMA | Fraction > Dow | 0.09600 | 1.00000 | 0.99200 | 0.00000 | 0.00000 |
| | TRB | Fraction > Dow | 0.03600 | 1.00000 | 0.94800 | 0.00000 | 0.01200 |

(B) Rule Averages

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Rule Average | VMA | Fraction > Dow | 0.00400 | 1.00000 | 1.00000 | 0.00000 | 0.00000 |
| | | Bootstrap Mean | 0.00020 | 0.01076 | 0.00013 | 0.01078 | 0.00007 |
| | | Dow Series | 0.00042 | 0.00890 | -0.00025 | 0.01342 | 0.00067 |
| Rule Average | FMA | Fraction > Dow | 0.04400 | 1.00000 | 1.00000 | 0.00000 | 0.00000 |
| | | Bootstrap Mean | 0.00196 | 0.03497 | 0.00081 | 0.03487 | 0.00115 |
| | | Dow Series | 0.00531 | 0.03064 | -0.00399 | 0.04156 | 0.00930 |
| Rule Average | TRB | Fraction > Dow | 0.01600 | 1.00000 | 0.94800 | 0.00000 | 0.00400 |
| | | Bootstrap Mean | 0.00224 | 0.03502 | 0.00082 | 0.03490 | 0.00142 |
| | | Dow Series | 0.00633 | 0.03000 | -0.00238 | 0.05398 | 0.00871 |

Table X

Simulation tests GARCH-M simulations for 250 replications. GARCH-M returns series are simulated using the estimated parameters. The rows marked Fraction > Dow refer to the fraction of simulations generating a mean or standard deviation larger than those from the actual Dow series. The trading rules are, variable length moving average (VMA), fixed length moving average (FMA), trading range break (TRB). Panel B presents results for the averages across all the rules for each reported statistic. Dow Series refers to the actual mean return or standard deviation from the Dow, and Bootstrap Mean refers to the mean from the simulated series.

(A) Individual Rules

| Rule | | Result | Buy | $\sigma_b$ | Sell | $\sigma_s$ | Buy-Sell |
|---|---|---|---|---|---|---|---|
| (1,50,0) | VMA | Fraction > Dow | 1.00000 | 1.00000 | 1.00000 | 0.08000 | 0.04000 |
| | FMA | Fraction > Dow | 0.99200 | 0.94000 | 1.00000 | 0.92400 | 0.08800 |
| | TRB | Fraction > Dow | 0.97200 | 1.00000 | 0.99200 | 0.14800 | 0.19200 |
| (1,50,0.01) | VMA | Fraction > Dow | 0.98000 | 1.00000 | 1.00000 | 0.05200 | 0.02000 |
| | FMA | Fraction > Dow | 0.89600 | 1.00000 | 1.00000 | 0.80800 | 0.00800 |
| | TRB | Fraction > Dow | 0.80400 | 1.00000 | 0.98000 | 0.24400 | 0.10400 |
| (1,150,0) | VMA | Fraction > Dow | 1.00000 | 1.00000 | 1.00000 | 0.04800 | 0.02800 |
| | FMA | Fraction > Dow | 0.68000 | 0.99600 | 0.98000 | 0.27200 | 0.15200 |
| | TRB | Fraction > Dow | 0.98000 | 1.00000 | 0.99600 | 0.08000 | 0.09200 |
| (1,150,0.01) | VMA | Fraction > Dow | 1.00000 | 1.00000 | 1.00000 | 0.04400 | 0.02400 |
| | FMA | Fraction > Dow | 0.66000 | 1.00000 | 1.00000 | 0.25600 | 0.04400 |
| | TRB | Fraction > Dow | 0.76800 | 1.00000 | 0.95200 | 0.14400 | 0.10400 |
| (1,200,0) | VMA | Fraction > Dow | 1.00000 | 1.00000 | 1.00000 | 0.04400 | 0.02800 |
| | FMA | Fraction > Dow | 0.83600 | 1.00000 | 0.98000 | 0.89600 | 0.20800 |
| | TRB | Fraction > Dow | 0.98800 | 1.00000 | 0.98800 | 0.09600 | 0.20800 |
| (1,200,0.01) | VMA | Fraction > Dow | 1.00000 | 1.00000 | 1.00000 | 0.04400 | 0.02000 |
| | FMA | Fraction > Dow | 0.78000 | 1.00000 | 1.00000 | 0.45600 | 0.02400 |
| | TRB | Fraction > Dow | 0.86000 | 1.00000 | 0.96800 | 0.18000 | 0.14000 |

(B) Rule Averages

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Rule Average | VMA | Fraction > Dow | 1.00000 | 1.00000 | 1.00000 | 0.04800 | 0.01600 |
| | | Bootstrap Mean | 0.00076 | 0.01225 | 0.00054 | 0.01183 | 0.00022 |
| | | Dow Series | 0.00042 | 0.00890 | -0.00025 | 0.01342 | 0.00067 |
| Rule Average | FMA | Fraction > Dow | 0.88800 | 1.00000 | 1.00000 | 0.54800 | 0.03200 |
| | | Bootstrap Mean | 0.00804 | 0.04229 | 0.00591 | 0.04299 | 0.00213 |
| | | Dow Series | 0.00531 | 0.03064 | -0.00399 | 0.04156 | 0.00930 |
| Rule Average | TRB | Fraction > Dow | 0.89600 | 1.00000 | 0.99600 | 0.10000 | 0.07600 |
| | | Bootstrap Mean | 0.01007 | 0.04979 | 0.00687 | 0.04681 | 0.00320 |
| | | Dow Series | 0.00633 | 0.03000 | -0.00238 | 0.05398 | 0.00871 |

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