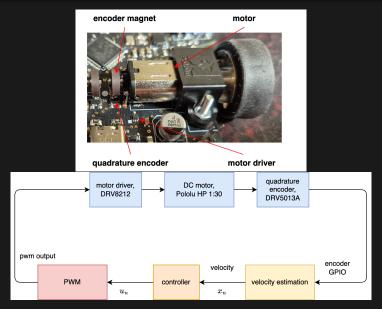


motor velocity controller



motor parameters

- motor maximum input range $u_{max} = 1.0$
- motor constant k = 139.084
- motor time constant $\tau = 8.276[ms]$

$$\alpha = -\frac{1}{\tau}$$

$$\beta = k \frac{1}{\tau}$$

$$\frac{d\omega(t)}{dt} = \alpha\omega(t) + \beta u(t)$$

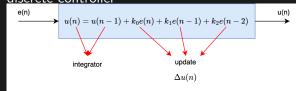
$$\frac{d\omega(t)}{dt} = -120.83\omega(t) + 16805.7u(t)$$

PID control

textbook continuous controller

$$\underbrace{u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt}} \longrightarrow \underbrace{u(t)}$$

discrete controller



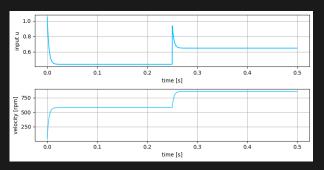
$$k_0 = K_p + K_i \Delta t + rac{K_d}{\Delta t}$$
 $k_1 = -K_p - 2rac{K_d}{\Delta t}$
 $k_2 = rac{K_d}{\Delta t}$

PID control - P only

- target value : 1000rpm
- P-only control causes steady state error

$$u(n+1) = u(n-1) + k_p e(n) - k_p e(n-1)$$

 $k_p = 0.01$

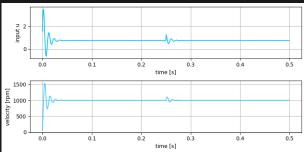


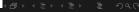
PID control - PI

- target value : 1000rpm
- PI control removes steady state error
- too high I term causes oscilations and overshot

$$u(n+1) = u(n-1) + (k_p + k_i \Delta t)e(n) - k_p e(n-1)$$

 $k_p = 0.005$



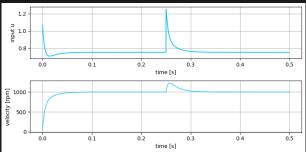


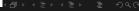
PID control - PI

- target value : 1000rpm
- correct tunned PI controller for 1st order system
- no overshot, no steady state error

$$u(n+1) = u(n-1) + (k_p + k_i \Delta t)e(n) - k_p e(n-1)$$

 $k_p = 0.01$
 $k_i = 0.0002$





complete generic discrete PID

calculate u-change candidate :

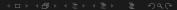
$$\Delta \hat{u}(n) = k_0 e(n) + k_1 e(n) + k_2 e(n)$$

2 clip maximum allowed u-change, to avoid u-kick :

$$\Delta u(n) = clip(\Delta \hat{u}(n), -du_{min}, du_{max})$$

clip maximum allowed u value, to avoid saturation / windup :

$$u(n) = clip(u(n-1) + \Delta u(n), -u_{min}, u_{max})$$



complete generic discrete PID

```
def init (self, kp, ki, kd, antiwindup = 10**10, du max=10**10):
    self_k0 = kp + ki + kd
   self.k1 = -kp -2.0*kd
   self.k2 = kd
   self.e0 = 0.0
   self.e1 = 0.0
   self.e2 = 0.0
    self.antiwindup = antiwindup
    self.du max = du max
def forward(self, xr, x, u_prev):
   # error compute
   self.e2 = self.e1
   self.e1 = self.e0
   self.e0 = xr - x
   du = self.k0*self.e0 + self.k1*self.e1 + self.k2*self.e2
   #kick clipping, maximum output value change limitation
   du = numpv.clip(du, -self.du max . self.du max)
   #entiwindup, maximum output value limitation
   u = numpy.clip(u prev + du, -self.antiwindup, self.antiwindup)
    return u
```

PID control - issues

- mostly hand tuned parameters
- only single input, single output systems
- no implicit noise filtering