Proposal for a Dagstuhl seminar on

Sparsity

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Abstract

The theory of *sparsity* studies abstract notions of uniform sparsity for classes of graphs, as well as more general logical structures. The main goal of the theory is to understand why, and to what extent, sparsity of a given structure can be used to describe its properties, and to develop tools helpful for designing sparsity-based methods. Since the work of Nešetřil and Ossona de Mendez, who laid solid foundations of the theory in the late 2000s, a huge body of work has shown that the two main notions — *bounded expansion* and *nowhere denseness* — have deep connections with classic concepts from combinatorics, model theory, and algorithm design, and can be used to obtain new, powerful results in these areas. It is the synergy of these three fields that makes sparsity a mathematically rich and exciting theory, which is currently under rapid development. The notions of bounded expansion and nowhere denseness also constitute borders of computational tractability for natural classes of problems, most notably for the model-checking problem for first-order logic; this witnesses the fundamental nature of the studied concepts.

The aim of the proposed seminar is to bring together researchers working on various aspects of sparsity, in order to facilitate the exchange of ideas, methods, and questions between different communities. An important part of the seminar will be the discussion of the (still fledgling) area of real-life applications of sparsity-based methods, where theory and practice could meet.

A Metadata

1. **Organizers**:

   - Daniel Kráľ, Masaryk University in Brno, Czech Republic, `kral@ucw.cz`;
   - Michał Pilipczuk, University of Warsaw, Poland, `michal.pilipczuk@mimuw.edu.pl`;
   - Sebastian Siebertz, Humboldt University in Berlin, Germany, `siebertz@mimuw.edu.pl`;
   - Blair D. Sullivan, North Carolina State University, Raleigh, USA, `blair_sullivan@ncsu.edu`.

2. **Title**: Sparsity
3. **Type**: Dagstuhl Seminar
4. **Size and duration**: Large (45 participants) and Long (5 days)
5. **Classification**: TODO
6. **Keywords**: TODO

B Proposal text

Blah blah blah [1]

Introduction.

Combinatorics.

Model theory. The notion of nowhere denseness is intimately linked to model theory and finite model theory. Model theory is the study of logics on classes of mathematical structures, and finite model theory restricts this study to finite structures. Nevertheless, finite model theory is a separate subject, and not just a small chapter in classical model theory. One of the reasons is that many standard methods of classical model theory fail when only finite structures are considered. These include the compactness theorem, the completeness theorem and various interpolation and preservation theorems. Another reason is that finite model theory is an area that grew out of computer science applications. This led to the development of strong tools to study logics over finite structures, which helped to answer many questions about complexity theory, databases, formal languages, algorithmics, and many more.

The failure of the classical tools from model theory on the class of all finite structures led researchers to investigate whether there are subclasses of finite structures that may be better behaved. In finite model theory such classes are called *tame*. One success story in this line of research is the study of homomorphism preservation theorems in the finite. A result of classical model theory states that a first-order formula is preserved under homomorphisms on all structures if, and only if, it is logically equivalent to an existential positive formula. A surprising result by Rossmann (LICS 2005) shows that this result also holds when restricted to finite structures. Furthermore, Dawar (JCSS 2010) introduced the notion of *uniform quasi-wideness* and proved that the homomorphism preservation theorem holds on all uniformly quasi-wide finite structures. It was later observed by Nešetřil and Ossona de Mendez (JSL 2010) that this notion of model-theoretic tameness corresponds exactly to the graph-theoretic notion of nowhere denseness. A result that sparked much interest of finite model theorists for nowhere dense graph classes.

The model-theoretic notion of uniform quasi-wideness was also soon picked up in algorithmic research. Using a constructive, algorithmic version of uniform quasi-wideness, it was shown by Dawar and Kreutzer (FSTTCS 2009) that various algorithmic domination and and independence problems can be solved efficiently on nowhere dense graph classes.

Besides providing tools for solving individual problems efficiently on restricted graph classes, model theory offers a very elegant approach to explain algorithmic techniques that work not only for individual problems but for whole classes of problems. This approach is to classify problems by their descriptive complexity, that is, by the resources required to describe the problem in a suitable logic. Because tractability results for logically defined classes of problems establish tractability for a large number of problems, such results are often called *algorithmic meta theorems*. The prototypical example of an algorithmic meta theorem is Courcelles Theorem, stating that every property of graphs that is definable in monadic second-order logic is decidable in linear time on every class of bounded tree-width. There is a long line of meta theorems for first-order logic on sparse structures, culminating in the result of Grohe et al. (STOC 2014), which states that every property of graphs that is definable in first-order logic is decidable in nearly linear time on every nowhere dense class of graphs. This result (under standard complexity theoretic assumptions) cannot be extended to classes which are not nowhere dense and closed under taking subgraphs, as shown by Kreutzer (ECCC 2009) and Dvořák et al. (JACM 2013)

Hence, under standard complexity theoretic assumptions, the classification of subgraph closed graph classes that admit efficient first-order model-checking is complete. Consequently, recent research has shifted to investigating the complexity of the model-checking problem for first-order logic on dense classes which are not closed under taking subgraphs. And again, it is model theory which may guide the way for this line of research. Adler and Adler (EJC 2014) observed that the notion of nowhere denseness is strongly connected to the notion of *stability*, which is a key dividing line in classical model theory between well-behaved and complicated first-order theories. More precisely, they showed that every nowhere dense class of finite graphs is stable, and conversely, any stable class of finite graphs which is closed under taking subgraphs is nowhere dense. This result again manifests the notion of nowhere denseness as a model-theoretic property. It also suggests stability theory as the framework for extending the border of tractability for first-order model-checking to dense graphs. This connection to stability theory has already been exploited for the design of efficient algorithms on nowhere dense graphs. As a first step, it was used by Kreutzer et al. (SODA 2016) to strongly improve the parameter dependence between the equivalent notions of uniform quasi-wideness and nowhere denseness. Based on these tools, Eickmeyer et al. (ICALP 2017) presented a near optimal kernelization algorithm for the distance-r dominating set problem on nowhere dense graph classes. We believe that this synergy between classical model theory and algorithms will lead to strong and very general algorithmic results. Our goal is to bring together researchers from model theory and algorithms to explore these new connections between classical model theory and algorithms.

Algorithms.

Applications.

Expected results.

Structure of the seminar.

Other seminars, workshops, and projects.

C Invitee list

TODO

D Organizers' CVs

E Proposed dates

TODO

References

[1] J. Nešetřil and P. Ossona de Mendez. *Sparsity — Graphs, Structures, and Algorithms*, volume 28 of *Algorithms and combinatorics*. Springer, 2012.