Rules of Logarithms

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$$\forall x, y \in \mathbf{R}^+, \forall a > 0 \wedge a \neq 1, \forall s \in \mathbf{R} :$$

1. **Rule 1**
$$\log_a 1 = 0$$
$a^0 = 1 \ldots \log_a 1 = 0$

2. **Rule 2**
$$\log_a a = 1$$
$a^1 = a \ldots \log_a a = 1$

3. **Rule 3**
$$a^{log_a x} = x$$
$\log_a x = \ell \ldots a^\ell = x \Rightarrow a^{\log_a x} = x$

4. **Rule 4**
$$\log_a(x.y) = \log_a x - \log_a y$$
$x = a^{\log_a x}, y = a^{\log_a y} \ldots (x.y) = a^{\log_a x}.a^{\log_a y} \ldots (x.y) = a^{\log_a x + \log_a y}$;
Rule 3 $\Rightarrow \log_a(x.y) = \log_a x - \log_a y$

5. **Rule 5**
$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$
$x = a^{\log_a x}, y = a^{\log_a y} \ldots \left(\frac{x}{y}\right) = \frac{a^{\log_a x}}{a^{\log_a y}} \ldots \left(\frac{x}{y}\right) = a^{\log_a x - \log_a y}$;
Rule 3 $\Rightarrow \log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

6. **Rule 6**
$$\log_a x^s = s(\log_a x)$$
$x = a^{\log_a x} \ldots x^s = \left(a^{log_a x}\right)^s \ldots x^s = a^{s(log_a x)}$; **Rule 3** $\Rightarrow \log_a x^s = s(\log_a x)$

7. **Rule 7**
$$\log_a x = \frac{\log_b x}{\log_b a}$$
$x = a^{\log_a x} \ldots \log_b x = \log_b a^{\log_a x}$; **Rule 6** $\Rightarrow \log_b a^{\log_a x} = \log_a x\big(\log_b a\big)$
$\ldots \log_b x = \log_a x\big(\log_b a\big) \; / \div \big(\log_b a\big) \Rightarrow \log_a x = \frac{\log_b x}{\log_b a}$