# Differentiating the **tangent** function

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## 1 The first derivative of the tangent function

Let $f$ be $f = tan(x)$. Then differentiate $f(x)$ with the respect to $x$, such that $f'(x)$:

$$f'(x) = \left[\frac{sin(x)}{cos(x)}\right]' = \frac{[sin(x)]'cos(x) - [cos(x)]'sin(x)}{cos^2(x)} = \frac{cos^2(x) + sin^2(x)}{cos^2(x)} = \frac{1}{cos^2(x)} = sec^2(x), \implies f'(x) = sec^2(x), \text{ where:}$$

$$\forall x \in \mathbb{R} : sin^2(x) + cos^2(x) = 1 \wedge \frac{1}{cos(x)} = sec(x) \wedge \frac{sin(x)}{cos(x)} = tan(x).$$

The proposed steps are based on the principle of the **Quotient rule** (and shall be used in the consecutive sections) which states:

$$f'(x) = \frac{P(x)}{Q(x)} = \frac{P'(x)Q(x) - Q'(x)P(x)}{Q^2(x)}.$$

## 2 The second derivative of the tangent function

Differentiate $f(x)$ with the respect to $x$, such that $f''(x)$:

$$f''(x) = [sec^2(x)]' = \left[\frac{1}{cos^2(x)}\right]' = \frac{0 - (-2sin(x)cos(x))}{cos^4(x)} = \frac{2sin(x)cos(x)}{cos^4(x)} = 2\left(\frac{sin(x)}{cos(x)}\frac{1}{cos^2(x)}\right) = 2tan(x)sec^2(x), \implies f''(x) = 2tan(x)sec^2(x)$$

The proposed steps are based on the principle of the **Product rule** (and shall be used in the consecutive section) which states:

$$f'(x) = P(x)Q(x) = P'(x)Q(x) + Q'(x)P(x).$$

## 3 The third derivative of the tangent function

Differentiate $f(x)$ with the respect to $x$, such that $f'''(x)$:

$$f'''(x) = \big[2tan(x)sec^2(x)\big]' = 2\big[tan(x)sec^2(x)\big]' = 2\big[[tan(x)]'sec^2(x) + [sec^2(x)]'tan(x)\big] = 2\big[sec^4(x) + 2tan^2(x)sec^2(x)\big] = 2\Big[sec^2(x)\big[sec^2(x) + 2tan^2(x)\big]\Big] = 2sec^2(x)\big[sec^2(x) + 2tan^2(x)\big], \implies f'''(x) = 2sec^2(x)\big[sec^2(x) + 2tan^2(x)\big]$$