

## Task to complete

Given is a square  $ABCD$ . We label the sides of the square as  $a$  and the diagonal as  $d$ . We know the measure of  $d - a$ . Create the steps of such a construction.

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### Steps of construction.

Suppose a plane  $\alpha$ . We have a line segment  $\overline{AX}$ , such that  $\overline{AX} \subset \alpha$ . Furthermore,  $\overline{AX} = d - a$ . Then, we create a ray  $\overrightarrow{AX}$ .

Suppose we create line segments from  $X$  to any of the 4 vertices. We then observe such 4 triangles:

1.  $\Delta_{ABX}$ , where  $\overline{AB} = a$ ,  $\overline{BX} = b$ ,  $\overline{AX} = d - a$
2.  $\Delta_{BCX}$ , where  $\overline{BC} = a$ ,  $\overline{BX} = b$ ,  $\overline{CX} = a$
3.  $\Delta_{CDX}$ , where  $\overline{CD} = a$ ,  $\overline{CX} = a$ ,  $\overline{DX} = b$
4.  $\Delta_{ADX}$ , where  $\overline{AD} = a$ ,  $\overline{AX} = d - a$ ,  $\overline{DX} = b$

... we observe  $\Delta_{ABX} \cong \Delta_{ADX} \wedge \Delta_{BCX} \cong \Delta_{CDX}$ .

\*We don't need to specify the size  $b$  as it's not for this particular example. Moreover, the triangle  $\Delta_{ABX}$  (likewise  $\Delta_{ADX}$ ) are of no significance for this particular exercise. Since,  $\Delta_{BCX} \cong \Delta_{CDX}$ , we will use  $\Delta_{BCX}$  for brevity.

We see that  $\Delta_{BCX}$  is an isosceles triangle with an angle  $\angle BCX = 45^\circ$ . Therefore,  $\angle CBX = \angle CXB = (180 - 45) \div 2 = 67.5^\circ$ . We continue with the next step, namely, we create  $\overleftrightarrow{p}$ , such that  $X \in \overleftrightarrow{p} \wedge \angle \overrightarrow{AX}, \overleftrightarrow{p} = 67.5^\circ$  (in the upper half-plane). Additionally, we have  $\overleftrightarrow{q}$ , such that  $A \in \overleftrightarrow{q} \wedge \angle \overrightarrow{AX}, \overleftrightarrow{q} = 45^\circ$ . Consequently, we get the vertex  $B$ , where  $\overleftrightarrow{p} \cap \overleftrightarrow{q} = \{B\}$ . In order to obtain the vertex  $C$ , we create a line  $\overleftrightarrow{r}$ , such that  $B \in \overleftrightarrow{r} \wedge \overleftrightarrow{q} \perp \overleftrightarrow{r}$ . So, we have  $C$ , where  $\overrightarrow{AX} \cap \overleftrightarrow{r} = \{C\}$ . Lastly, we create a line segment  $\overline{CD}$ , where  $\overline{CD} \parallel \overleftrightarrow{q} \wedge \overline{CD} = \overline{AB}$ . Finally, we observe the square  $ABCD$  in the plane  $\alpha$ .