Integrals exercise 1

Given is the function $f(x) = \frac{1}{4}\sin(2x) + 3$.

a) Evaluate the **indefinite** integral
b) Evaluate the **definite** integral over a period

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a) The **indefinite** integral

$\int f(x)dx = \int[\frac{1}{4}\sin(2x) + 3]dx = \int[\frac{1}{4}\sin(2x)]dx + \int[3]dx = \frac{1}{4}\int[\sin(2x)]dx + \int[3]dx$

1. Evaluate $\frac{1}{4}\int[\sin(2x)]dx$

Let $u = 2x$, thus $\frac{1}{4}\int[\sin(u)]dx$.

$\frac{du}{dx} = 2x$
$du = [2x]dx$
$du = 2dx$
$\frac{1}{2}du = dx$

$\Rightarrow \frac{1}{8}\int[\sin(u)]du$
$\frac{1}{8}\int[\sin(u)]du = \frac{1}{8}\int -\cos(u) = \frac{1}{8}\int -\cos(2x) = -\frac{1}{8}\int\cos(2x)$

2. Evaluate $\int[3]dx = \int 3x$

Lastly, evaluate the complete integral, $\int f(x) = -\frac{1}{8}\int\cos(2x) + \int 3x = 3x - \frac{1}{8}\cos(2x) + c$

Solution: $\int f(x) = 3x - \frac{1}{8}\cos(2x) + c$

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b) The **definite** integral over a period

The period of $f(x)$ is π, since $\sin(2x) \iff period = \frac{2\pi}{2} = \pi$

$\int_0^\pi f(x) = \int_0^\pi[\frac{1}{4}\sin(2x) + 3]dx = -\frac{1}{8}\int_0^\pi \cos(2x) + \int_0^\pi 3x = -\frac{1}{8}\left[\cos(2x)\right]_0^\pi + \left[3x\right]_0^\pi = -\frac{1}{8}[0\pi] + [3\pi - 0\pi]$

$= 3\pi$

Solution: 3π