Mathematical proofs

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1 The inner angle of an $n-sided$ convex regular polygon

Suppose an $n-sided$ convex regular polygon. Its 4 consecutive vertices are shown in the figures, $N_1, N_2, N_3, ..., N_i$ respectively. Thus $\Delta_{N_1,N_2,S} \cong \Delta_{N_2,N_3,S}$, i.e. such $n-sided$ polygon consists of n congruent isosceles triangles. That is, $|\measuredangle N_2N_1S| = |\measuredangle N_1N_2S| \implies |\measuredangle N_1N_2S| = |\measuredangle SN_2N_3|$, denoted as β, β' respectively.

An angle $\alpha = |\measuredangle N_1SN_2| = \frac{360}{n}$, since such an angle multiplied by n makes for a perfect circle of $360°$. Likewise $\alpha = 180° - 2\beta$.

Let Φ be an inner angle of the polygon, such that $\phi = 2\beta$ (shown in the figure at the vertex N_2).

Express in terms of β: $\alpha = 180 - 2\beta \iff \beta = \frac{-\alpha + 180}{2}$

Substitute α for $\alpha = \frac{360}{n}$: $\beta = \frac{-\frac{360}{n} + 180}{2} \iff \beta = \frac{180n - 360}{2n}$

Express in terms of Φ: $\Phi = 2\beta = 2\Big(\frac{180n - 360}{2n}\Big) = \frac{180n - 360}{n}$

The expression can be further simplified to the following form:

$$\Phi = \frac{(n-2)\pi}{n}$$

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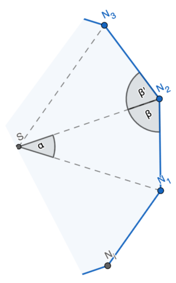

Figure 1: $n-sided$ polygon with its 4 vertices in a plane

2 The number of diagonals of an $n-sided$ convex regular polygon

Suppose a geometrical locus of n points on the plane, i.e. a set of points $A_1, A_2, ..., A_n$ such that $A_1, A_2, ..., A_n$ create an $n-sided$ convex regular polygon. The number of different abscissas in the geometrical locus is $\binom{n}{2}$ and denoted as N_a. It implies that no abscissa in the locus is given by more than two points, i.e. each point is unique. Likewise, $\overbracket{A_1A_2} \equiv \overbracket{A_2A_1}$ holds for any 2 points in the locus, thus such abscissas are counted as one. It can be inferred that the number of diagonals, denoted as N_D, is the same as *the difference of the number of abscissas and the number of sides*:

$$N_D = N_s - n$$