Mathematical proofs

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1 The inner angle of an $n - sided$ convex regular polygon

Suppose an $n - sided$ convex regular polygon. Its 4 consecutive vertices are shown in the figures, $N_1, N_2, N_3, ..., N_i$ respectively. Thus $\Delta_{N_1,N_2,S} \cong \Delta_{N_2,N_3,S}$, i.e. such $n - sided$ polygon consists of n congruent isosceles triangles. That is, $|\measuredangle N_2 N_1 S| = |\measuredangle N_1 N_2 S| \implies |\measuredangle N_1 N_2 S| = |\measuredangle S N_2 N_3|$, denoted as β, β' respectively.

An angle $\alpha = |\measuredangle N_1 S N_2| = \dfrac{360}{n}$, since such an angle multiplied by n makes for a perfect circle of $360°$. Likewise $\alpha = 180° - 2\beta$.

Let Φ be an inner angle of the polygon, such that $\phi = 2\beta$ (shown in the figure at the vertex N_2).

Express in terms of β: $\alpha = 180 - 2\beta \iff \beta = \dfrac{-\alpha + 180}{2}$

Substitute α for $\alpha = \dfrac{360}{n}$: $\beta = \dfrac{-\dfrac{360}{n} + 180}{2} \iff \beta = \dfrac{180n - 360}{2n}$

Express in terms of Φ: $\Phi = 2\beta = 2\Big(\dfrac{180n - 360}{2n}\Big) = \dfrac{180n - 360}{n}$

The expression can be further simplified to the following form:

$$\Phi = \frac{(n-2)\pi}{n}$$

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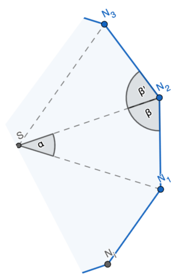

Figure 1: $n - sided$ polygon with its 4 vertices in a plane